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A MATHEMATICAL MODEL FOR NEW PRODUCT DIFFUSION: THE INFLUENCE OF INNOVATORS AND IMITATORS

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Abstract—A simple mathematical model describing the diffusion of a new, infrequently purchased product is proposed. Many previous papers have reported that the differing behaviour of "innovators" and "imitators" is fundamental to the diffusion process. However, previous models have failed to represent this differing behaviour effectively. The model presented considers a new and, importantly, simple way of viewing the diffusion process for consumer durables based on a simplified representation of "innovative" and "imitative" response of adopters to certain types of product information. The resulting model can accommodate bimodal first purchase sales curves, which are found to occur quite frequently, in addition to the more traditional unimodal curves. The resulting shape of the diffusion curve, which is the early product life cycle (PLC), can be explained in terms of key dynamic parameters of the model. Data for six consumer durables are analysed to test the model's performance.

1. INTRODUCTION

Diffusion modelling of new product adoption has been an active area of marketing research since the pioneering work of Bass [1969]. These models are concerned with representing the dynamic nature of the adoption (first purchases) of a new product. Typically, durable products are considered, since first purchases represent a large majority of total sales early in the product's life. There have been significant extensions and generalizations to the fundamental model proposed by Bass (see [Mahajan and Wind, 1986]).

Several reasons have motivated the development of mathematical diffusion models in marketing [Kalish and Sen, 1986]. First, these models are used for forecasting future first purchase sales. Research in this area has included both modification of the model's structure to enhance its ability to adequately represent first purchase sales (see a review by Mahajan and Peterson [1985]) and also more sophisticated estimation techniques to improve early sales predictions (see, for example, [Srinivasan and Mason, 1986; Lilien *et al.*, 1981]).

A second reason for using these models is normative. Models of this type include managerial decision variables in order to study their effect on the diffusion process. Variables studied include price (for example, [Robinson and Lakhani, 1975; Kalish, 1983]), advertising (for example, [Horsky and Simon, 1983; Dockner and Jorgensen, 1988]), product quality and market entry timing [Kalish and Lilien, 1986]. Managerial decisions have also been studied in a competitive environment (for example, [Thompson and Teng, 1984; Eliashberg and Jeuland, 1986]).

Thirdly, a model may be developed to serve in an explanatory role. These models provide a simplified mathematical representation of diffusion processes which attempts to incorporate the dominant influences or to highlight some aspect of the process. The theoretical basis for these models is the largely qualitative "diffusion of innovation" research (for example, [Rogers, 1983]). Typical of this approach are multistate models (for example, [Midgley, 1976; Mahajan *et al.*, 1984]) which increase the possible number of consumer states by differentiating between such things as awareness/non-awareness and positive/negative attitudes, models which include population heterogeneity (for example, [Jeuland, 1981; Kalish, 1985]) and the model proposed by Easingwood *et al.* [1983] in which the influence of word of mouth varies throughout the diffusion process.

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It is this third consideration which has primarily motivated the work presented here. We present a new and, importantly, simple way of modelling the diffusion process. In particular, we provide a new simplistic representation of transmission of information and innovative/imitative response. The resulting diffusion model is more flexible than previous simple diffusion models in the sense that it can accommodate both unimodal and bimodal penetration curves. Furthermore, the shape of the curve, which represents the early Product Life Cycle (PLC) shape, can be determined by key underlying factors in the system.

The outline of the paper is as follows. In Section 2 we discuss the related literature and the theoretical basis for model development. In Section 3 we formulate the new model and in Section 4 the dynamic behaviour of the model is examined. Data from six consumer durables are analysed in Section 5 and the performance of the new model is compared with some other models. The paper concludes with a discussion in Section 6 of managerial implications and directions for future research.

2. RELATED LITERATURE AND THEORY DEVELOPMENT

Interest in the use of diffusion models in marketing was probably first generated by Bass [1969]. The theoretical foundations for the Bass model and its extensions originate from the "diffusion of innovation" literature (e.g. [Rogers, 1983]). Bass in his original publication, presented as the theoretical foundation for his model this dichotomy of innovators/imitators, each group possessing different response characteristics to the innovation. However his mathematical formulation does not reflect this [Jeuland, 1981]. His model may be written,

$$\frac{dN(t)}{dt} = (K - N(t))(a + bN(t)), \tag{1}$$

where $N(t)$ is the number of people who have adopted the new product at time t, K is the size of the adopting population and a, b are constants. This model instead differentiates between two different communication channels for product information in a homogeneous population. The first channel (represented by the "a" term) includes information sources external to the "social system" (such as advertising) while the second channel (represented by the "b" term) is internal word of mouth (w.o.m.) communication between adopters and non-adopters.

This is true of much of the diffusion modelling in the marketing literature. Despite continued reference to the innovator/imitator interpretation of the Bass model and its extensions (for example, [Easingwood *et al.*, 1983; Mahajan and Peterson, 1985; Mahajan and Wind, p. xiii, 1986] diffusion research has generally failed to distinguish between innovative and imitative behaviour. Instead it has simply concentrated on diffusion of the different types of information which influence innovators or imitators. However, both these elements are central to the theory of innovation diffusion upon which they are based.

Three papers in the new product diffusion literature have differentiated the population on the basis of innovativeness. Schmalen [1982] differentiates between innovators and imitators by the influences which affect their decisions. However, he offers no specific functional form for his model and numerically analyses advertising and pricing decisions in an oligopoly. In their "two compartment model", Tanny and Derzko [1988] divide the population into "innovators" and "imitators". However, their distinction is somewhat ambiguous. They define "innovators" as those who buy in response to learning about the product from external sources, and "imitators" as those who buy in response to either external sources or interaction with prior adopters. Finally, Jeuland [1981] differentiates consumers on the basis of a (continuous) trait he calls "propensity to purchase". However, consumers all respond identically to both internal and external influences.

Other papers have differentiated the type of information being transmitted into categories which influence innovators and imitators differently. However, they have not considered the differing responses of innovative/imitative consumers. In particular, by considering product awareness separately to adoption [Dodson and Muller, 1978; Kalish, 1985], information associated with awareness is differentiated from that associated with product experience.[1] Kalish used Nelson's [1970] distinction between product "search attributes", which can be easily established prior

[1] However, neither paper explicitly models information diffusion about "experience attributes".

to purchase from sources such as advertising and store visits, and "experience attributes", which can only be verified through product usage.

In this paper, we model in a simplified way both the transmission of different types of information, and innovative/imitative response to this information. Like Kalish [1985], we distinguish between information relating to "search attributes" and "experience attributes". The adopting population is then divided into those willing to adopt the product given only "search" type information and those who require "experience" type information before adopting.[2] This distinction is, to a certain extent, an indicator of consumer innovativeness with respect to the product's adoption,[3] so we call the first group "innovators" and the second group "imitators".

Another limiting aspect of present mathematical diffusion models is that they are unimodal in structure. While modifications to the Bass model such as those done by Jeuland [1981] and Easingwood *et al.* [1983] have resulted in models of more flexible structure, they can not produce bimodal diffusion curves. An exception is Norton and Bass's [1987] model of successive generations of a product. However, empirical evidence of product life cycles (for example, [Rink and Swan, 1979; Tellis and Crawford, 1981] indicates bimodal sales curves are common. Though some of this empirical evidence is certainly attributable to repeat purchase recycling phenomenon and possibly successive generations, some of the evidence is clearly associated with first purchases of a non-improved product. In fact, Kluyver [1977] writes "One drawback of such models (diffusion type models) is that only unimodal phenomena can be fitted".

It has also been our observation that early new product sales curves (dominated by first purchases) often rise initially then plateau out or fall, before rising again to adopt the traditional bell shaped diffusion pattern. We propose that this phenomena could be due to early adoption resulting from "innovative" type behaviour, followed by a delay before "imitative" type sales take off as information about product experience diffuses.

We therefore present a very simple model of new product adoption which focuses on innovative and imitative types of behaviour as outlined above. The dynamic behaviour of the system is governed by the spread of two different types of product information. The resultant model is flexible enough to accommodate the bimodal sales curve discussed above, as well as the traditional unimodal curves. In fact, we propose that it is the dynamic nature of the above system which determine the predominant trends of the diffusion curve, or early PLC trends.

3. MODEL FORMULATION

Consider the diffusion process of the adoption of a new durable product within a population, size K. We assume two types of product information is available. Some information about the new product is available prior to purchase, initially from sources such as advertising, store visits and manufacturer's specifications. This type of product information is often referred to as "search attributes" (We will use SAI to refer to this type of information). Further information about the product is gained through its use (e.g., reliability and ease of use). This type of product information is often referred to as "experience attributes" [Nelson, 1970]. We divide the population on the basis of the information required before they are willing to adopt. The first group, which we term "innovators", are willing to adopt based on the product's search attributes, while the second group, which we term "imitators", delay adoption until they are informed of the product's "experience attributes". We let δ_1 and δ_2 be the fraction of innovators and imitators, respectively, within the population, such that:

$$K_1 = \delta_1 K = \text{ total number on innovators,}$$
$$K_2 = \delta_2 K = \text{ total number of imitators,}$$

and $\delta_1 + \delta_2 \leq 1$.

[2] This is related to the concept of information uncertainty used by other modellers. See [Roberts and Urban,1983].

[3] There is little consensus in the literature regarding the definition of innovativeness [Midgley and Dowling, 1978]. Although Rogers [1983] defines innovativeness with respect to the timing of adoption, he does acknowledge that (in general) early and late adopter behave differently. It is this, behavioral distinction between consumers which is more appropriate for modelling purposes. An appropriate definition is given by Midgley [1977, p. 49] "innovativeness is the degree to which an individual makes innovation decisions independently of the communicated experience of others".

To model the adoption of innovators, we must therefore represent the diffusion of search attribute information (SAI) through the population. For parsimony, we assume population groups are homogeneous with respect to SAI and its transmission. Thus,

$$A_1(t) = \delta_1 A(t), \tag{2}$$

where $A(t)$ and $A_1(t)$ are the total number of people and number of innovators, respectively, aware of the new product's SAI at time t.

Next we represent the spread of SAI by a model analogous to the Bass model, which is itself a deterministic version of a classical model for the diffusion of news [Bartholomew, 1967]. That is, we assume SAI may be conveyed either from an external source or via word of mouth. Formalizing this assumption,

$$\frac{dA(t)}{dt} = (K - A(t))\,(a_1 + a_2 A(t)), \tag{3}$$

with $A(0) = 0$. The two parameters a_1 and a_2 are positive.

Let $N_1(t)$ be the number of innovators who have adopted the new product at time t. If we assume innovators buy the product as soon as they receive SAI, then $N_1(t) = A_1(t)$. Therefore, Equations (2) and (3) yield the following equation for adoption by the innovators:

$$\frac{dN_1(t)}{dt} = (K_1 - N_1(t))\,(\alpha + \beta N_1(t)), \tag{4}$$

with $N_1(0) = 0$. The two parameters α and β are positive.

Imitators adopt the product once informed about its experience attributes. We assume this information is communicated directly from a previous adopter.[4] Hence their adoption may be represented as a pure word of mouth term. We assume that adopters take a time T to evaluate the product after its adoption. Further, we neglect possible negative reactions to the new product, assuming that most adopters are satisfied with the product's performance. Let $N_2(t)$ be the number of imitators who have adopted at time t. As a result the equation for adoption by the imitators is given by:

$$\frac{dN_2(t)}{dt} = (K_2 - N_2(t))\,(\gamma_1 N_1(t-T) + \gamma_2 N_2(t-T)), \tag{5a}$$

for $t \geq T$. $N_2(t) = 0$ for $0 \leq t \leq T$ and the parameters γ_1 and γ_2 are positive.

In this paper, we make the following simplifications for the sake of parsimony: (i) the time for adopters to evaluate the product is negligible compared to the time for information transmission, so that $T = 0$, and (ii) innovators and imitators are homogeneous with respect to transmission of this information such that $\gamma_1 = \gamma_2 = \gamma$. This results in a parsimonious model (with two less parameters) which can be solved analytically. As a result, Equation (5a) becomes,

$$\frac{dN_2(t)}{dt} = (K_2 - N_2(t))\,\gamma\,(N_1(t) + N_2(t)), \tag{5b}$$

with $N_2(0) = 0$. We also assume $\gamma \leq a_2$ since it is unrealistic that word of mouth transmission of experience attributes (which also includes search attributes) would be faster than transmission of search attributes.

As a result, Equations (4) and (5b) define the cumulative adoption of the new product over time. The solution is given by, (details given in Appendix A).

$$N_1(t) = K_1 \frac{1 - e^{-zt}}{1 + Ce^{-zt}}, \tag{6}$$

[4] As pointed out by Midgley and Dowling [1978], an interesting point is whether this information can be "hearsay". We assume that it cannot, though the extension for its inclusion is straightforward.

where $z = \alpha + \beta K_1$, $C = \frac{\beta K_1}{\alpha}$, and

$$N_2(t) = K_2 + \frac{e^{-\gamma K_3 t}(1 + Ce^{-zt})^{-\gamma/\beta}}{I(t) - K_2^{-1}(1+C)^{-\gamma/\beta}}, \tag{7}$$

where $K_3 = K_1 + K_2$ and $I(t)$ is the infinite series:

$$I(t) = \gamma \sum_{j=0}^{\infty} \left\{ \frac{1}{j!} \frac{1}{zj + \beta K_3} \left[(-C)^j - (-Ce^{-(zt)})^j e^{-\gamma K_3 t} \right] \prod_{i=0}^{j-1} (\gamma/\beta + i) \right\}.$$

Cumulative sales of the new product are then given by,

$$N(t) = N_1(t) + N_2(t). \tag{8}$$

4. MODEL ANALYSIS

In this section we examine the dynamic behaviour of the above system to determine the underlying factors that influence the shape of the diffusion curve, or early PLC. To do so we introduce these parameter groups with the following interpretations for the groupings:

$$\begin{aligned} \theta_1 = \alpha \quad &= \text{ representative rate of innovator adoption due to external influence,} \\ \theta_2 = \frac{1}{2}\beta K_1 &= \text{ representative rate of innovator adoption due to word of mouth,} \\ \theta_3 = \frac{1}{2}\gamma K \quad &= \text{ representative rate of imitator adoption due to word of mouth.} \end{aligned}$$

These rates together with the number of innovators, K_1, and imitators, K_2, completely define the diffusion curve. However, only three ratios of these are necessary to completely describe the diffusion curve shape (different parameter sets with identical ratios are simply scaled versions of the same shape). We therefore define the following ratios:

$$\begin{aligned} R_1 &= \frac{\theta_1}{\theta_2} &&= \frac{\text{innovator adoption rate due to external sources}}{\text{innovator adoption rate due to word of mouth}}, \\ R_2 &= \frac{\theta_3}{\theta_1 + \theta_2} &&= \frac{\text{imitator adoption rate}}{\text{total innovator adoption rate}}, \\ R_3 &= \frac{K_1}{K} &&= \frac{\text{number of innovators}}{\text{total population}}. \end{aligned}$$

Diffusion curve shapes can now be discussed in terms of these ratios. Large values of R_1 will result in initially declining sales. Since empirical studies have found this situation to be extremely rare, we will exclude this case. Further, extensive simulation showed that provided initial sales increased, varying R_1 has only a small effect on the diffusion curve shape in most cases. This implies that in most cases the shape of the diffusion curve is determined by R_2 and R_3 only.

R_2 is presently defined as the ratio of two representative rates. However, it is more intuitive to think of R_2 in terms of time scales. Hence we define:

$$\begin{aligned} \mathcal{T}_1 &= \text{ time scale for majority } (\sim 95\%) \text{ of innovators to adopt.} \\ \mathcal{T}_2 &= \text{ time scale for majority } (\sim 95\%) \text{ of imitators to adopt.} \end{aligned}$$

Then,

$$R_2 \approx \mathcal{T}_1/\mathcal{T}_2.$$

Once again this is an approximation due to the non-linearity of the equations.

The shape of the diffusion curve is therefore defined in terms of the ratio of innovator to imitator adoption time scales and the proportion of innovators. Below we examine the way in which the shape varies with changes in these two ratios. First note that our assumptions $\gamma \leq a_2$ and $\delta_1 \leq 1$ imply that:

$$R_2 \leq (1 + R_1)^{-1} \leq 1 \quad \text{and} \quad R_3 \leq 1.$$

Hence, we investigate the affect of R_2 and R_3 on the diffusion curve as they vary between 0 and 1.

Figure 1 shows a series of typical diffusion curves for a low value of R_2 (0.2) with R_3 varying (0.05 to 0.5). R_2 small implies that T_1 is small in relation to T_2. When both R_2 and R_3 are low ($\sim$ 0.3 or less), the curve exhibits a bimodal nature. As R_3 increases from very low values ($\sim$ .05) to medium values ($\sim$ 0.3) the shape of the curve varies through a variety of bimodal shapes as shown in Figure 1(a)–(c). For larger values of R_3, a skewed diffusion curve results as shown in Figure 1(d). It can be seen that for low values of R_2 there are two distinct stages—an initial stage in which innovators dominate sales, followed by an imitator dominating stage.

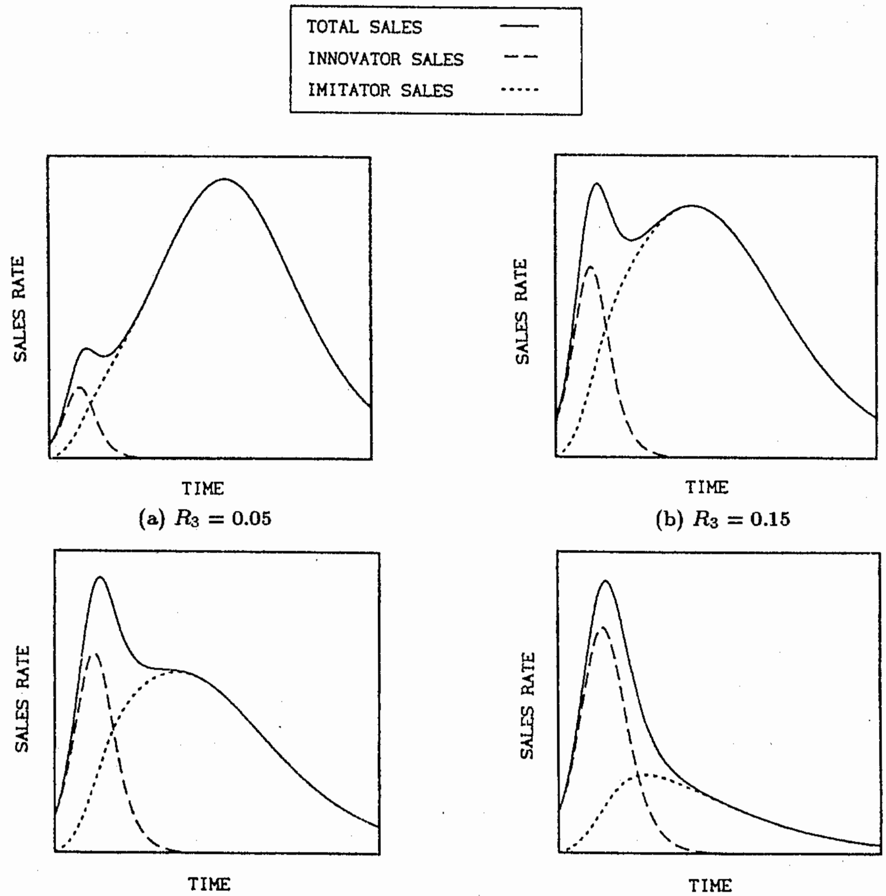
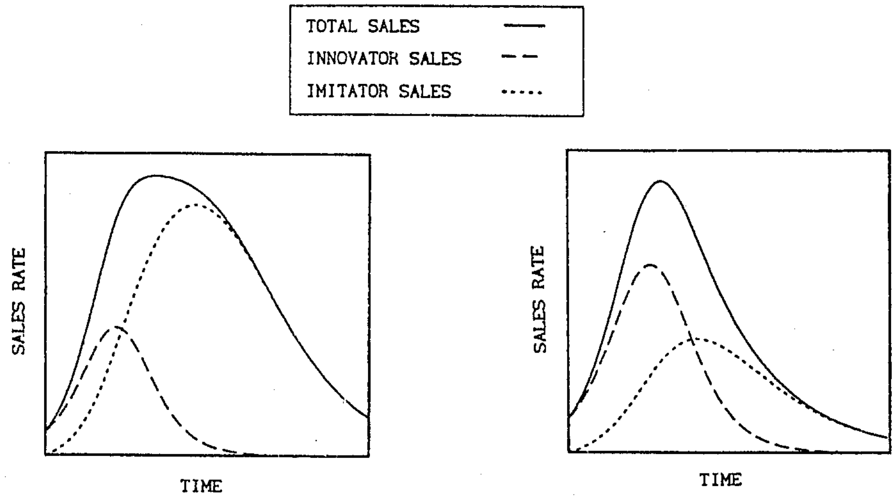


Figure 1. Diffusion patterns for low R_2 ($R_2 = 0.2$) as R_3 varies.

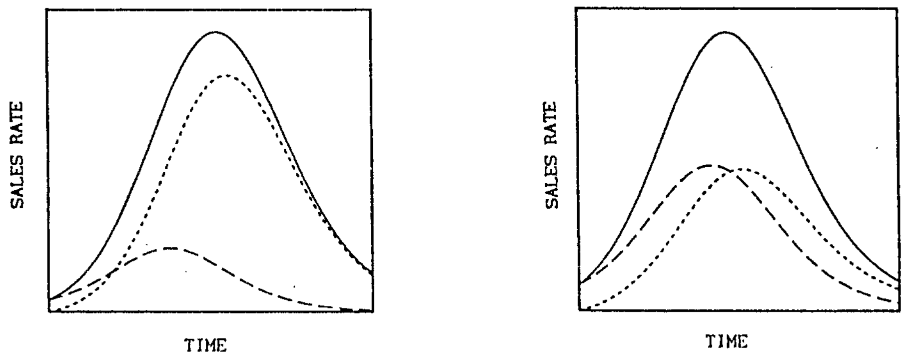
Figure 2 shows two diffusion patterns for a medium value of R_2 (0.4). In this case the shape is unimodal for all values of R_3. Innovators still tend to dominate the initial sales period, while imitators dominate the latter stage. However, these stages are less distinct than for low values of R_2. The period and extent of initial innovator dominance increases as R_3 increases. Note that for higher values of R_3 than shown, the overall shape is very similar, with a greater and longer innovator dominance.

Figure 3 shows typical diffusion patterns for a high value of R_2 (0.8). In this case $T_1 \approx T_2$ and the shape is unimodal for all values of R_3. No distinct stages are evident and the dominance of sales over the entire curve is governed by the value of R_3. When R_3 is small imitators dominate (Figure 3(a)), while for intermediate values of R_3 innovators and imitators share sales



(a) $R_3 = 0.2$ (b) $R_3 = 0.5$

Figure 2. Diffusion patterns for medium R_2 ($R_2 = 0.4$) as R_3 varies.



(a) $R_3 = 0.2$ (b) $R_3 = 0.5$

Figure 3. Diffusion patterns for high R_2 ($R_2 = 0.8$) as R_3 varies.

(Figure 3(b)). For larger values of R_3 (not shown) innovators dominate sales over the entire diffusion curve.

It is postulated that the value of these parameter groups, and hence the shape of the diffusion curve, are dependent on certain product characteristics. Diffusion of innovation literature (see [Gatignon and Robertson, 1985; Rogers, 1983]) proposes that the speed of diffusion is related to product characteristics such as relative advantage, status value, compatibility, observability, complexity and perceived risk.[5] These factors are important in determining the above parameter groups since they are likely to influence innovator and imitative behaviour in different ways. Normally, observability is "the degree to which the results of an innovation are visible to others." In our context, two aspects of observability need to be considered which we will refer to as "search observability" and "experience observability". The former refers to the degree to which the product's benefits are revealed through search activities (as described in Section 3) while the latter refers to ease by which its benefits are observed through product use and communicated to others.

Low R_2 values (tendency towards bimodality) are expected to be positively correlated to high search observability and low experience observability. In addition, high R_2 values are expected to be related to low compatibility and high risk and complexity since imitators would be more concerned about these aspects than innovators. Conversely, status value is expected to be more

[5] In the normal context trialability is also considered important, but it is not important when comparing consumer durables which all have low trialability.

important to innovators, making it negatively related to high R_2 values. High R_3 values (proportion of innovators) would be expected to be related to high relative advantage and low risk.

5. EMPIRICAL VALIDATION

In order to examine the validity of the new model, sales data for six consumer durables were analysed. The new products considered were black and white television, colour television, clothes dryers, room air conditioners, dishwashers and power lawn mowers. The data was collected from the "Statistical Abstracts of the United States". Though this data represents total sales of these products, the erroneous effects of repeat purchases was kept to a minimum by considering only the relatively early years of sales. Periods used were also chosen to coincide with earlier works for easy comparison.

To estimate the model's parameters for each data set a nonlinear least squares method, analogous to that proposed for the Bass model by Srinivasan and Mason [1986], was employed. That is, the theoretical cumulative sales curve was differenced to give the estimated sales for each period, with the parameters selected to minimise the sum of squared error. The cumulative sales curve was calculated from Equations (6) and (7) with the infinite series truncated to give the required accuracy. To assess the relative performance of the new model, the data was also analysed using the Bass model. Further, results tabulated in [Easingwood *et al.*, 1983] for their non-uniform influence model (NUI) are used for comparison. Table 1 summarizes the parameter estimates for each data set, while Table 2 compares the mean absolute deviation and mean squared error for each model. The actual sales and the fitted sales estimates for both the new model and the Bass model are depicted in Figures 4–9. The figures also show the relative importance of innovator and imitator sales throughout the diffusion processes as predicted by the fitted new model. The following comments are warranted:

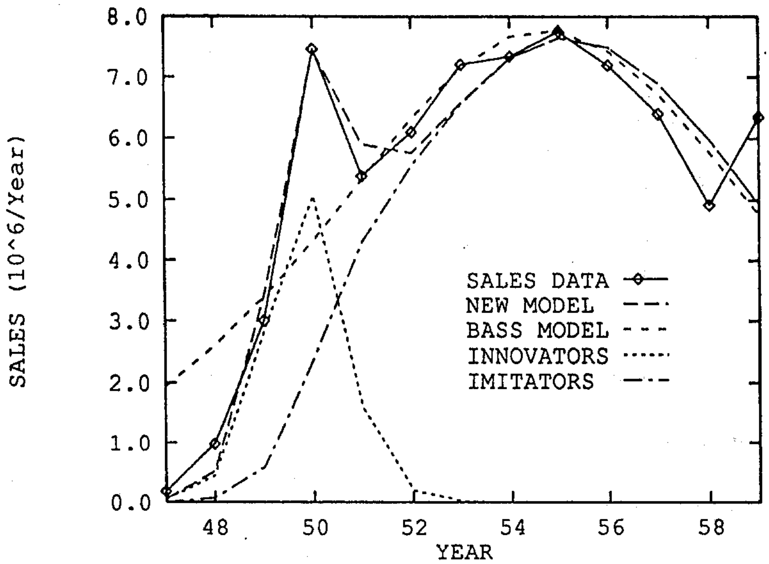


Figure 4. Actual and fitted sales for black & white television.

- Table 2 indicates that the new model consistently provides a better fit to the consumer durable data than either the Bass or NUI models. The new model produced a considerable improvement in the mean squared error, the mean absolute error and the explained variance adjusted for degrees of freedom. The mean squared error for the Bass and NUI models are on average 132% and 51% higher, respectively, than the new model. Similarly, the mean absolute error for the Bass and NUI models are on average 43% and 19% higher, respectively, than the new model.
- Tanny and Derzko [1988] reported that their two compartment model failed to produce a better fit than the Bass model for any of the twelve innovations tested. Each time they achieved convergence with their estimation routine, their model reduced to the Bass model (a special case of their model). They report that their estimation routine fails to locate a

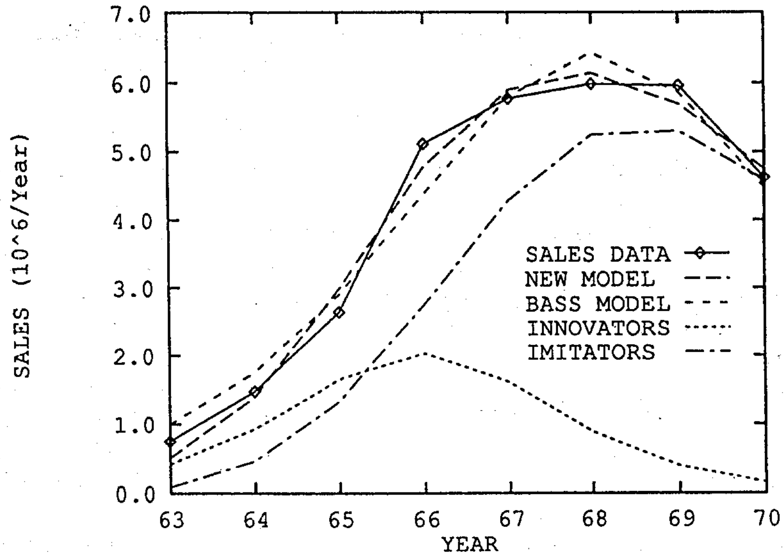


Figure 5. Actual and fitted sales for colour television.

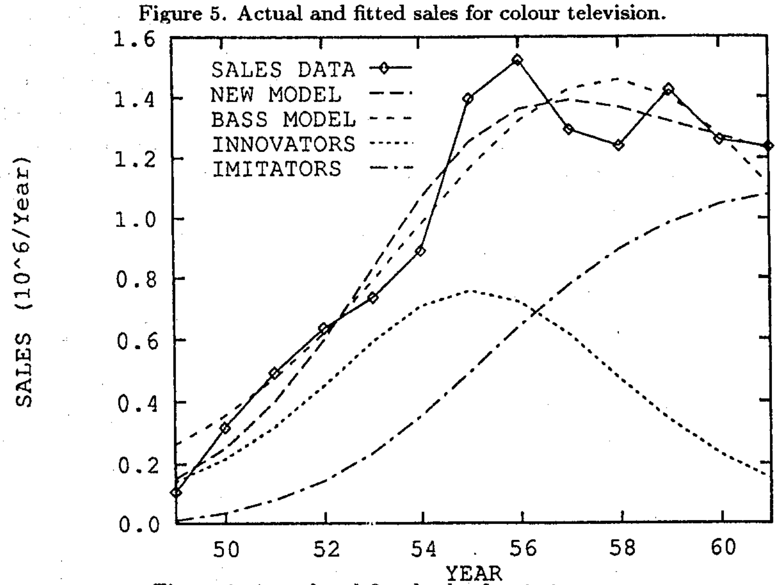


Figure 6. Actual and fitted sales for clothes dryers.

point with a minimum residual sum of squares as low as the Bass fit. They only tested two of the products reported in this paper (room air conditioners and clothes dryers), both of which reduced to the Bass model. In contrast, our model yields a superior fit. We discuss this in the next section.

- Estimation of model parameters based solely on sales data early in the penetration suffers from a problem of parameter identifiability. This is true for our model as well diffusion models of the type developed in the past. As a consequence, one can estimate only certain groupings of parameters instead of the original parameter set.[6] This implies that the model forecasts based solely on early penetration (first purchase sales) data lacks credibility due to the non-uniqueness in the estimates of model parameters. A wide range of variations in individual parameter values with the same value for the groupings will yield similar fits to the initial data but the forecasts can vary significantly. The problem of identifiability persists until the stage where the sales data exhibits a point of inflection and becomes concave downward. We discuss this further in Appendix B.

[6] For the Bass model (or our innovator model), when N is small, $dN/dt \approx aK + bKN(t)$. Hence, only the groupings aK and bK can be estimated, not the three parameters separately. Similarly for our model, for low penetration of imitators, $dN_2/dt \approx \gamma K_2(N_1(t) + N_2(t))$ implying only the grouping γK_2 can be estimated.

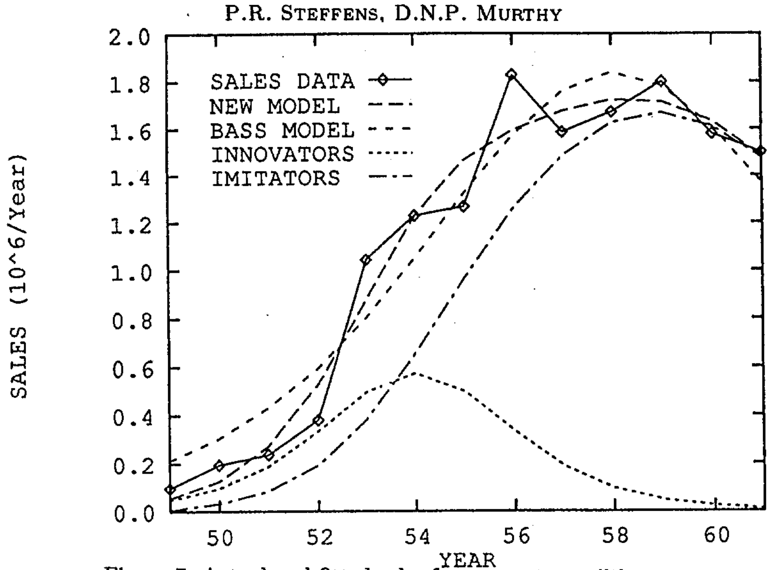


Figure 7. Actual and fitted sales for room air conditioners.

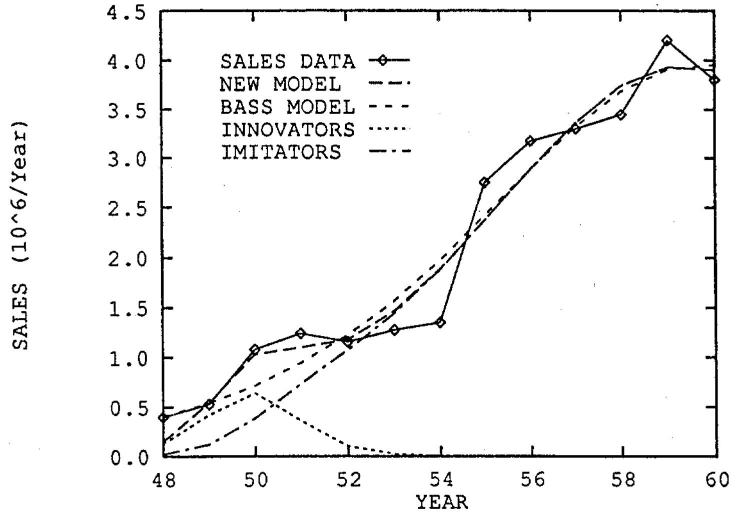


Figure 8. Actual and fitted sales for power lawnmowers.

- The above situation occurs for the case of dishwashers. As seen in Figure 9 the data available for estimation does not include the inflection point and hence the identifiability problem exists and individual parameter estimates suffer from non-uniqueness. The only comparable parameters between the two models are the total adopting population (K for Bass' model and K_3 for the new model). For the case of dishwashers Bass's model estimates a total adopting population of 55 million while our model estimates 11.5 million. As expected, there is a significant difference between the two. This is in contrast to the other five cases where the data available for estimation includes the inflection point. For these cases the population size estimates for the two models were roughly the same (within 30%).
- The data used for model fitting is the total sales data, with no indication of each sub-population (innovator/imitator) sales. For the cases where the total sales exhibit unimodal shapes (colour televisions, clothes dryers, room air conditioners), fits that were reasonably close to optimal were obtained for large variations in K_1 and K_2. However their sum, K_3, remained fairly constant, implying that for the type of data available, only K_3 and not K_1 and K_2 is identifiable and uniquely estimable for unimodal penetration curves.

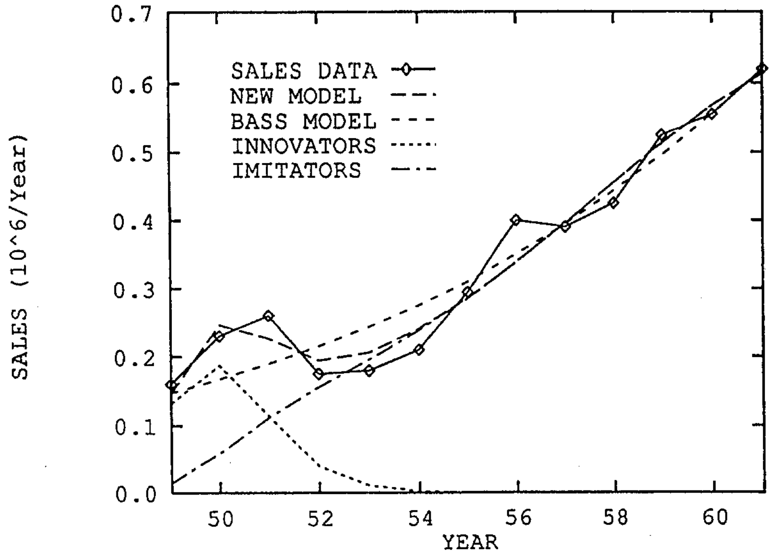


Figure 9. Actual and fitted sales for dishwashers.

Table 1. Parameter estimates for consumer durables.

Product	Period Covered	New Model Parameters K_1^*	K_2^*	α	β	γ	Bass Parameters K^*	a	b
Black & White Television	1947–1959	10.20	74.38	.0012	.2210	.0043	85.99	.0193	.0038
Colour Television	1963–1970	8.191	34.29	.0318	.1156	.0124	39.66	.0185	.0155
Clothes Dryers	1949–1961	5.955	19.19	.0187	.0794	.0070	16.49	.0136	.0198
Room Air Conditioners	1949–1961	2.942	17.31	.0105	.2616	.0164	18.72	.0094	.0200
Power Lawnmowers	1948–1960	1.693	44.53	.0347	.9050	.0074	50.33	.0069	.0060
Dishwashers	1949–1961	0.485	10.96	.1701	2.565	.0203	54.68	.0025	.0024

*Parameter estimates for K_1, K_2, K are in millions.

Table 2. Fit statistics for consumer durables.

Product	Mean Squared Error* Bass	NUI	New	Mean Absolute Error Bass	NUI	New	Explained Variance⊗ Bass	NUI	New
Black & White Television	1487	N.A.	359	814.5	N.A.	454.9	69.3	N.A.	90.7
Colour Television	119.5	75.4	51.7	276.7	228.7	207.7	95.8	96.7	97.0
Clothes Dryers	16.4	12.9	10.6	101.5	92.0	89.3	90.2	91.5	92.1
Room Air Conditioners	26.3	14.9	13.2	144.6	105.6	91.4	92.0	94.9	94.9
Power Lawnmowers	81.0	N.A.	61.6	229.5	N.A.	199.5	94.0	N.A.	94.5
Dishwashers	1.75	1.50	0.67	33.1	31.5	21.2	90.6	91.5	95.7

*Mean Squared Error is in thousands.
⊗ % Adjusted for degrees of freedom.

- For the cases in which the data exhibits a bimodal shape (b&w television, power lawnmowers, dishwashers) both of K_1 and K_2 are identifiable since the innovator sales and the imitator sales must peak in the proximity of the first and second peak respectively of the actual sales in order for our model to replicate this bimodal behaviour. It is the ability of our model to reproduce these bimodal curves which lends support to its structure.

Table 3. Important ratios of parameter estimates.

Product	R_2	R_3
Black & White Television	0.29	0.12
Colour Television	0.57	0.19
Clothes Dryers	0.38	0.24
Room Air Conditioners	0.50	0.14
Power Lawnmowers	0.04	0.04
Dishwashers	0.11	0.04

Table 4. Important product characteristics.

Product	Initial Price (Adjusted)	Functional Category	Product Superseeded
Black & White Television	$380	ENTERTAINMENT	RADIO
Colour Television	$250	ENTERTAINMENT	B & W TELEVISION
Clothes Dryers	$220	CONVENIENCE	NONE
Room Air Conditioners	$410	COMFORT	FAN
Power Lawnmowers	$ 95	LABOUR SAVING	MANUAL MOWERS
Dishwashers	$275	LABOUR SAVING	NONE

- Table 3 shows the estimated values of two important parameter groupings, R_2 and R_3. Firstly we note that both parameters are consistently quite small ($R_2 < 0.6, R_3 < 0.25$). This means that innovators are depicted as an early adopting minority, which is an intuitively appealing result. Further, R_3 is smaller for the bimodal cases, as expected from the discussion in Section 4.
- Table 4 shows some product information useful for determining some product characteristics important in determining the diffusion behaviour. The product price is closely related to perceived risk, while the functional category and related products already on the market will determine the perceived relative advantage, compatibility, complexity and observability. Though from this limited study it is not possible to confirm any of the postulates relating product characteristics to the diffusion pattern, the following comments can be made.
- First we compare black & white television to color television due to the similar nature of the products. The important difference between the two being of course that black and white television had virtually no forerunner, replacing radio as a means of home entertainment, whereas colour television was an advancement on black and white. Therefore black and white television had a higher relative advantage and lower compatibility. As expected the lower compatibility and higher cost of the black and white televisions corresponds to a lower R_2 value. Dishwashers, one of the other products with a low R_2 value, had a high status value following introduction since they were closely associated with entertaining guests. Similarly, lawnmowers may have had a high status value following introduction.

6. DISCUSSION AND CONCLUSIONS

Since the pioneering work of Bass [1969], many models have been proposed to explain the time dependent nature of new product/technology diffusion. This paper suggests a parsimonious model of this diffusion process of a new consumer durable product based on a novel simplification of buyers response to product information and its transmission, which offers more structural flexibility than previous models.

Our new model divides the population into two groups, innovators and imitators, which differ in their response to the new product. Though this innovator/imitator interpretation of the Bass model and many of its derivatives has been prevalent in the literature, no simple diffusion model has previously represented the differing behaviour of innovators and imitators.

In particular, the model considers innovators as those willing to adopt the product based on information pertaining to its search attributes, while imitators require information about the product's experience attributes before adopting. This simple distinction is consistent with the qualitative literature regarding the diffusion of innovations.

Another limiting feature of previous mathematical diffusion models is that they can accommodate only unimodal shapes. However, first purchases of consumer durables often exhibit bimodal characteristics. In particular, we note that such sales often display an early small peak and decline, before continuing with the traditional bell shaped curve. The new model accommodates such situations in addition to the more traditional unimodal curves. Further, the shape of the diffusion curve can, at the theoretical level, be explained in terms of some important time and magnitude scales.

When fitted to sales data for six consumer durables, our model was found to significantly out perform the Bass model, the popular more flexible NUI model [Easingwood *et al.*, 1983] and the two-compartment model of Tanny and Derzko [1988] who found their model to perform no better than the Bass model. In particular, our model significantly out performed the other models when the data exhibited early bimodal characteristics. This improved model performance lends some credibility to the innovator/imitator dichotomy as represented by the model.

From a managerial viewpoint, our model offers an important interpretation of the underlying nature of the diffusion process. It is reasonable to assume that the two groups will also respond differently to marketing variables. Therefore, the inclusion of such variables into the model may offer new insight into market response.

Typically, diffusion models are used to forecast future adoptions from early sales data for long term production, distribution and financial planning. We have shown that unique estimation of model parameters is not possible early in the product's penetration (see Appendix B). Rather, only a smaller number of parameter groupings are estimable until the data exhibits a concave downward trend, and thus long range forecasts are impossible. Once enough data is available, using traditional diffusion curves any decline in sales results in a forecast of a continuing decline in sales. However, our model illustrates that a decline in sales does not necessarily signal the final stages of the product's penetration. Alternatively, a decline may in fact occur towards the beginning of the product's penetration.

It was difficult to ascertain from this preliminary empirical analysis any clear relations between product characteristics and the shape of the diffusion curve. It is hoped that a more extensive investigation will reveal product types or characteristics for which a bimodal diffusion pattern can be expected. Our model would not, in general, be able to correctly predict a bimodal shape solely from sales data prior to the second rise in sales. However, it does indicate that caution should be exercised with diffusion curve forecasts from early sales when sales are declining, especially if they significantly underestimate total sales predictions obtained by other means.

Finally, the potential of the model for forecasting based solely on early sales data is limited. Since the model contains five parameters, it is unlikely to produce significantly improved forecasts than the Bass model which only has three parameters. However, if our interpretation of our models parameters is correct, then some of these could possibly be estimated by other means, such as analogy or direct measurement. This would effectively reduce the number of model parameters to be estimated by the sales data, thus enabling our model to be used more effectively as a predictive tool. Alternatively, if adoption data for each sub-population were collected, only two and three parameters would have to be estimated from the data sets. Furthermore, the three parameters estimates associated with the innovator sales would become stable early in the overall penetration. Such data would also serve to validate (or otherwise) the new model more stringently.

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APPENDIX A

Here we present a solution of the system of differential equations defined by Equations (3) and (4), with initial conditions $N_1(0) = N_2(0) = 0$.

It is easily shown that the solution to (3) is given by:

$$N_1(t) = K_1 \frac{1 - e^{-zt}}{1 + Ce^{-zt}}, \tag{A1}$$

where, $z = \alpha + \beta K_1$, $C = \frac{\beta K_1}{\alpha}$.

Substitution of (A1) into (4) yields upon simplification:

$$\frac{dN_2(t)}{dt} = \gamma K_1 K_2 \frac{1 - e^{-zt}}{1 + Ce^{-zt}} + \left[\gamma K_2 - \gamma K_1 \frac{1 - e^{-zt}}{1 + Ce^{-zt}}\right] N_2(t) - \gamma N_2^2(t). \tag{A2}$$

This is a generalized Riccati equation. It can be transformed to a linear equation using the following transformation:

$$n(t) = \frac{1}{N_2(t) - K_2}. \tag{A3}$$

Differentiating (A3) and using (A2) yields upon simplification:

$$\frac{dn(t)}{dt} = \gamma + g(t)n(t), \tag{A4}$$

where

$$g(t) = \gamma K_2 + \gamma K_1 \frac{1 - e^{-zt}}{1 + Ce^{-zt}}.$$

Equation (A4) is a linear first order differential equation. Its solution is given by:

$$n(t) = L\, e^{\Phi(t)} + e^{\Phi(t)}\, I(t), \tag{A5}$$

where

$$\Phi(t) = \int_0^t g(t)\, dt, \tag{A6}$$

$$I(t) = \int_0^t \gamma\, e^{-\Phi(t)}\, dt, \tag{A7}$$

with L a constant to be determined.

Evaluating the integral of (A6) using $g(t)$ from (A4) yields:

$$\Phi(t) = \gamma K_3\, t + \frac{\gamma}{\beta} \ln(1 + Ce^{-zt}), \tag{A8}$$

where $K_3 = K_1 + K_2$. Substituting (A8) into (A7) yields upon simplification:

$$I(t) = \gamma \int_0^t e^{-\gamma K_3 t}(1 + Ce^{-zt})^{-\gamma/\beta}\, dt. \tag{A9}$$

We now expand the bracketed term as a binomial series to get

$$I(t) = \gamma \int_0^t e^{-\gamma K_3 t} \sum_{j=0}^{\infty} \left\{ \frac{1}{j!}(-Ce^{-zt})^j \prod_{i=0}^{j-1} (\gamma/\beta + i) \right\} dt. \tag{A10}$$

Integrating, after reversing the order of integration and summation, yields:

$$I(t) = \gamma \sum_{j=0}^{\infty} \left\{ \frac{1}{j!} \frac{1}{zj + \beta K_3} [(-C)^j - (-Ce^{-zt})^j\, e^{-\gamma K_3 t}] \prod_{i=0}^{j-1} (\gamma/\beta + i) \right\}. \tag{A11}$$

Substituting (A8) and (A11) into (A5) yields $n(t)$. Using this, we obtain $N_2(t)$ from (A3) with L selected to satisfy the initial condition $N_2(0) = 0$. It is given by:

$$N_2(t) = K_2 + \frac{e^{-\gamma K_3 t}\,(1 + Ce^{-zt})^{-\gamma/\beta}}{I(t) - K_2^{-1}\,(1 + C)^{-\gamma/\beta}}. \tag{A12}$$

APPENDIX B

In this Appendix we discuss the problem of non-identifiability of model parameters in the early stages of product penetration. We confine our attention to the Bass model, though the analysis can be easily extended to include most other diffusion models. The Bass model is given by Equation (1) and for small N (or t) it can be approximated by:

$$\frac{dN(t)}{dt} \approx K\,(a + b\,N(t)). \tag{B1}$$

Equation (B1) can be rewritten as

$$\frac{dN(t)}{dt} \approx (\mu + \nu\,N(t)), \tag{B2}$$

with two parameters μ and ν which are groupings of the original parameter set given by (a, b, K). From initial sales data one can identify uniquely only the parameters of (B2) and not the original set of parameters. For large N (and t) the parameters of the original set can be estimated using (1). Hence, unique estimation of the Bass model parameters from penetration data is not possible, at least until the data extends sufficiently far so that the behaviour of (1) is different from that of (B2). We now discuss the minimum length of data that is needed before we can obtain unique estimates of the original set of parameters. We confine our discussion to the case $a \ll bK$ as empirical studies show that this is true for nearly all consumer durables.

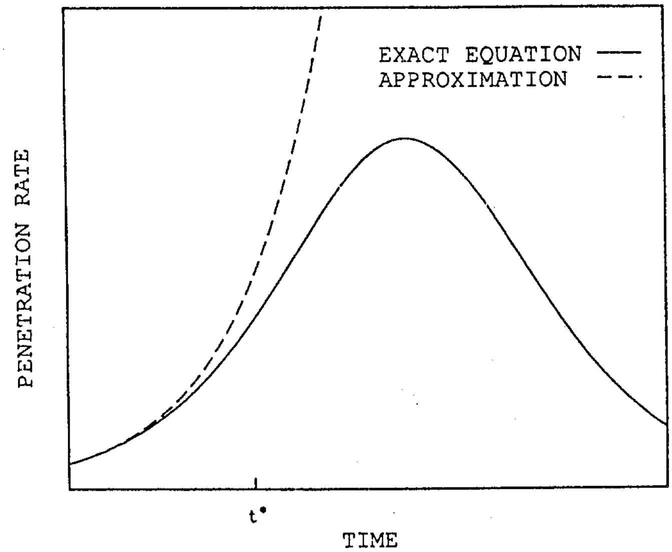

Figure 10. Exact and approximate penetration rate curves.

Of interest are the penetration rate curves since the data used for estimation is normally adoptions per unit time. Figure 10 shows the exact penetration rate, $dN(t)/dt$, obtained from (1) and the approximate penetration rate obtained from (B2) as a function of time for a typical set of parameter values. Qualitatively, these curves will be the same for any other parameter set. Note that the curve for the approximation has a positive slope and is concave upward for all t. The curve for the exact equation (called Bass curve in future), likewise, is concave upward with a positive slope for small t. However, at time t^* the Bass curve undergoes an inflection (i.e., $dN^3(t)/dt^3$ undergoes a sign change) after which it becomes concave downward. It can be seen from the figure that the two curves are very close until time t^*. The flattening of the slope of the Bass curve near t^* represents the first distinguishing feature between the two equations in terms of their behaviour. We note that for the case under consideration (i.e., $a \ll bK$) the inflection point of the Bass curve occurs at t^* given by $N(t^*) \approx 0.2\,K$.

We have established that it is difficult to distinguish between Equations (1) and (B2) before time t^*. This problem becomes more pronounced when using real data since the data (i) are contaminated by errors, and (ii) are in an aggregated form, usually over a year duration. Hence, it is only possible to estimate the two parameter groupings, as opposed to the original set of three parameters, if the data does not extend beyond t^*. Further, we cannot ascertain that the data extends beyond time t^* until it exhibits a concave downward trend. Therefore, unique estimation of the three parameters of the Bass model (Equation (1)) from adoption data is not possible until the data exhibits a concave downward trend. Before this occurs only the two parameter groupings mentioned above can be uniquely estimated.