Week 2: Understanding Nash equilibrium

2.1 Nash equilibrium and the Prisoner's Dilemma

Simultaneous Move Game

Simultaneous Move Game

"Static" game

Simultaneous Move Game

"Static" game

Players take their actions at the same time,

Simultaneous Move Game

"Static" game

Players take their actions at the same time,
and the game ends immediately

Simultaneous Move Game

"Static" game

Players take their actions at the same time, and the game ends immediately

Simultaneous Move Game

"Static" game

Players take their actions at the same time, and the game ends immediately

Game theory predicts that the outcome is a Nash equilibrium

Prisoner's Dilemma

1		
Cooperate		
Defect		

Prisoner's Dilemma

1 \ 2		
Cooperate		
Defect		

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate		
Defect		

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		

Player 1's payoff

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		

Player 1's payoff Player 2's payoff

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		

Player 1's payoff Player 2's payoff

Payoff Table

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Defection is always the best, irrespective of the opponent's action

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Nash Equilibrium

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Nash equilibrium

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Nash equilibrium

Individual rationality

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Nash equilibrium

Individual rationality

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Nash equilibrium

Group rationality

Individual rationality

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Nash equilibrium

Group rationality $\neq$ Individual rationality

2.2 Coordination game and self-fulfilling prophecy

2.2 Coordination game and self-fulfilling prophecy

Coordination Game

Coordination Game

Two friends collaborate on their PCs.

Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY：

Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY：widely used design

Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY：widely used design

Dvorak：designed for faster typing

Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY : widely used design
Dvorak : designed for faster typing

Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

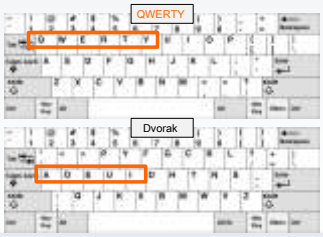
QWERTY : widely used design
Dvorak : designed for faster typing

Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY : widely used design
Dvorak : designed for faster typing

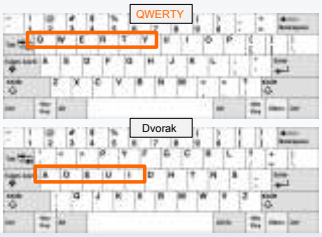


Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY : de facto standard
Dvorak : designed for faster typing

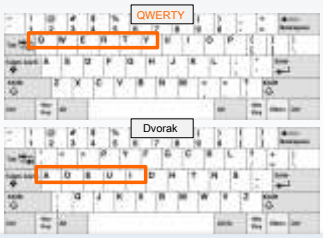


Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY : adopted by the first generation typewriters
Dvorak : designed for faster typing

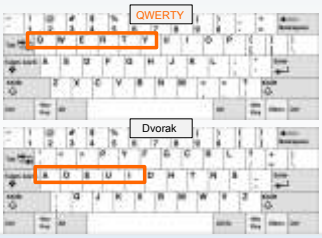


Coordination Game

Two friends collaborate on their PCs.

Choice of keyboard

QWERTY : not explicitly designed for the most efficient typing
Dvorak : designed for faster typing



Coordination Game

QWERTY : not explicitly designed for the most efficient typing
Dvorak : designed for faster typing

Coordination Game

QWERTY : not explicitly designed for the most efficient typing
Dvorak : designed for faster typing

The most efficient design is likely to be different from QWERTY.

Coordination Game

QWERTY : not explicitly designed for the most efficient typing
Dvorak : designed for faster typing

The most efficient design is likely to be different from QWERTY.
Since Dvorak may not be the optimal design, consider a choice between
QWERTY and the Optimal design

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing
Optimal

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing
Optimal

1 \ 2	QWERTY	Optimal
QWERTY		0, 0
Optimal	0, 0	

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing
Optimal

1 \ 2	QWERTY	Optimal
QWERTY	1, 1	0, 0
Optimal	0, 0	

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing

Optimal

1 \ 2	QWERTY	Optimal
QWERTY	1, 1	0, 0
Optimal	0, 0	2, 2

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing

Optimal

1 \ 2	QWERTY	Optimal
QWERTY	(1, 1)	0, 0
Optimal	0, 0	2, 2

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing

Optimal

1 \ 2	QWERTY	Optimal
QWERTY	(1, 1)	0, 0
Optimal	0, 0	(2, 2)

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing

Optimal

1 \ 2	QWERTY	Optimal
QWERTY	(1, 1)	0, 0
Optimal	0, 0	(2, 2)

Nash equilibria

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing

Optimal

1 \ 2	QWERTY	Optimal
QWERTY	(1, 1)	0, 0
Optimal	0, 0	(2, 2)

Nash equilibria

A game may have **many** Nash **equilibria**

Coordination Game

Two friends choose keyboards for their PCs.

QWERTY : not explicitly designed for the most efficient typing

Optimal

1 \ 2	QWERTY	Optimal
QWERTY	(1, 1)	0, 0
Optimal	0, 0	(2, 2)

Nash equilibria

.. and one equilibrium can be better than another for everyone

Society may have many Nash equilibria

Society may have many Nash equilibria

Everyone uses QWERTY

Society may have many Nash equilibria

Everyone uses QWERTY

Everyone uses the optimal design

Society may have many Nash equilibria

Everyone uses QWERTY

Everyone uses the optimal design

Once the society is trapped in a bad equilibrium, it is difficult to get out

Society may have many Nash equilibria

New technology may come in different formats

Society may have many Nash equilibria

New technology may come in different formats

New generation DVD

Society may have many Nash equilibria

New technology may come in different formats

HD DVD

Credit: Cncxbox, CC BY-SA 2.5

New generation DVD

Society may have many Nash equilibria

New technology may come in different formats

HD DVD

Credit: Cncxbox, CC BY-SA 2.5

New generation DVD

Blu-ray

Society may have many Nash equilibria

New technology may come in different formats

Credit: Cncxbox, CC BY-SA 2.5

Society may have many Nash equilibria

New technology may come in different formats

Everyone uses HD DVD

Credit: Cncxbox, CC BY-SA 2.5

Society may have many Nash equilibria

New technology may come in different formats

Everyone uses HD DVD

Credit: Cncxbox, CC BY-SA 2.5

Everyone uses Blu-ray

Society may have many Nash equilibria

New technology may come in different formats

Everyone uses HD DVD

Credit: Cncxbox, CC BY-SA 2.5

Everyone uses Blu-ray

Struggle for de facto standard

Society may have many Nash equilibria

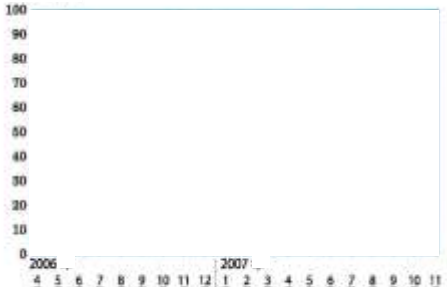
New technology may come in different formats

Everyone uses HD DVD

Credit: Cncxbox, CC BY-SA 2.5

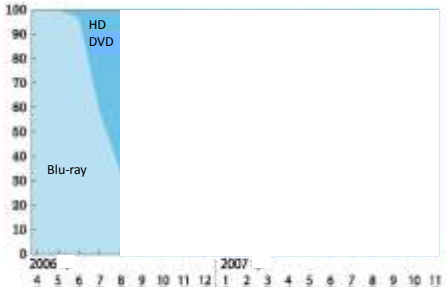
Everyone uses Blu-ray

A format that happens to obtain a large enough market share eventually becomes the de facto standard



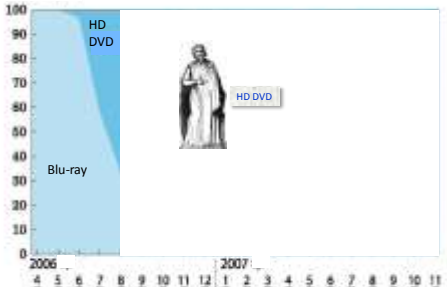
Market shares of DVD recorders in Japan

Kandori (2014), *Mikuro keizaigaku no Chikara*, Nihon-hyoronsha



Market shares of DVD recorders in Japan

Kandori (2014), *Mikuro keizaigaku no Chikara*, Nihon-hyoronsha



Market shares of DVD recorders in Japan

Kandori (2014), *Mikuro keizaigaku no Chikara*, Nihon-hyoronsha

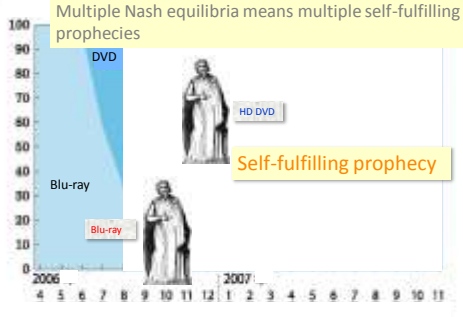
Market shares of DVD recorders in Japan

Kandori (2014), *Mikuro keizaigaku no Chikara*, Nihon-hyoronsha

Market shares of DVD recorders in Japan

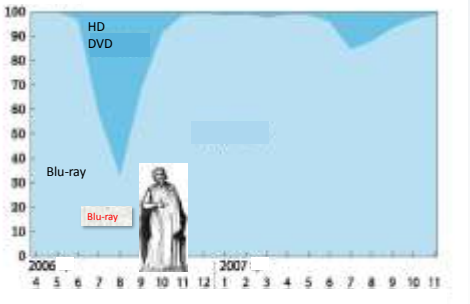
Kandori (2014), *Mikuro keizaigaku no Chikara*, Nihon-hyoronsha

Multiple Nash equilibria means multiple self-fulfilling prophecies



Market shares of DVD recorders in Japan

Kandori (2014), *Mikuro keizaigaku no Chikara*, Nihon-hyoronsha



Market shares of DVD recorders in Japan

Kandori (2014), *Mikuro keizaigaku no Chikara*, Nihon-hyoronsha

Society may have many Nash equilibria

Everyone uses HD DVD

Credit: Cncxbox, CC BY-SA 2.5

Everyone uses Blu-ray

Society may have many Nash equilibria

Everyone uses HD DVD

Credit: Cncxbox, CC BY-SA 2.5

Everyone uses Blu-ray

Which one prevails depends on people's expectations

Society may have many Nash equilibria

Everyone uses HD DVD

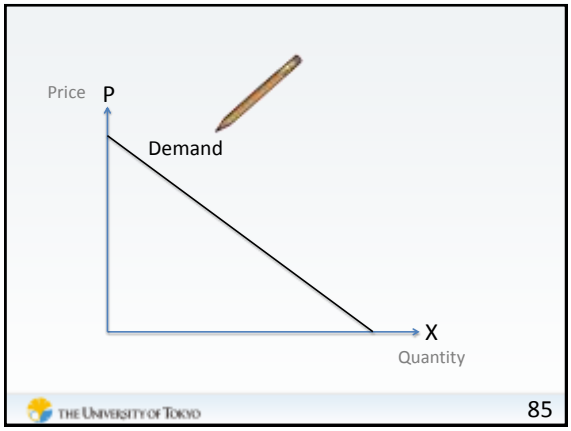
Credit: Cncxbox, CC BY-SA 2.5

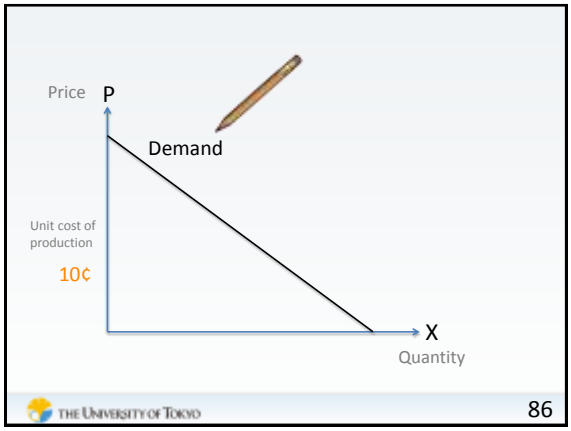
Everyone uses Blu-ray

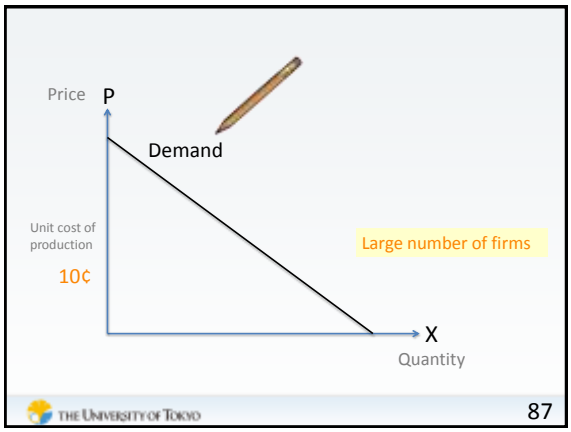
There is no guarantee that the best format becomes the de facto standard

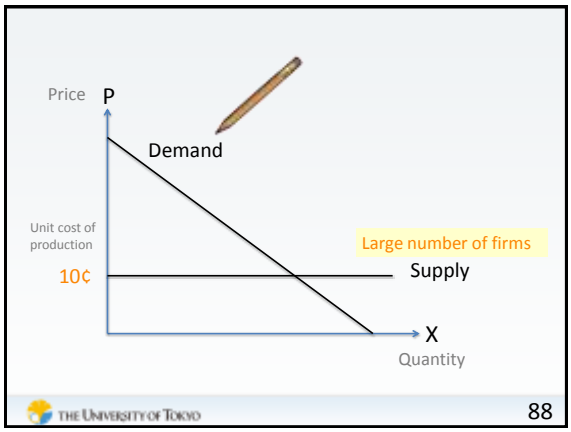
2.3 Market competition

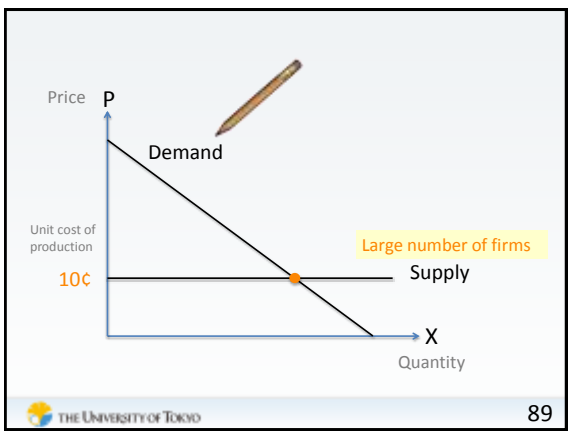
A peek into the power of math

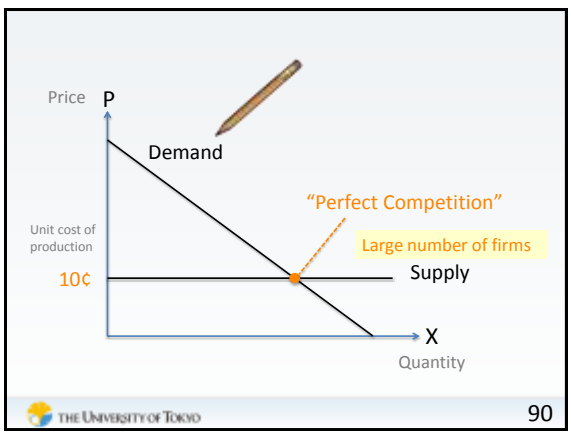

Price
P
Demand
X
Quantity


Price
P
Demand
Unit cost of production
10¢
X
Quantity


Price
P
Demand
Unit cost of production
10¢
Large number of firms
X
Quantity


Price
P
Demand
Unit cost of production
10¢
Large number of firms
Supply
X
Quantity


Price
P
Demand
Unit cost of production
10¢
Large number of firms
Supply
X
Quantity

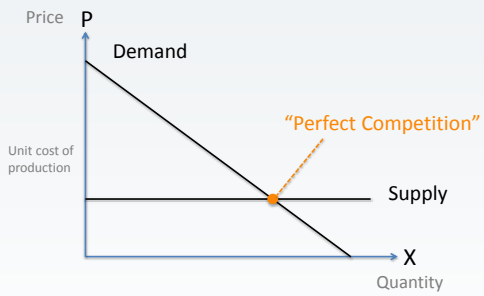

Price
P
Demand
"Perfect Competition"
Unit cost of production
10¢
Large number of firms
Supply
X
Quantity

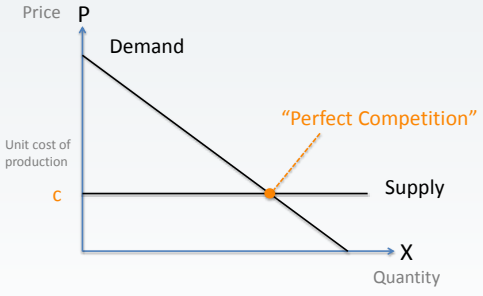
Large number of firms

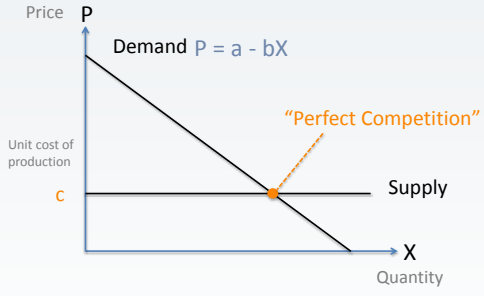
Large number of firms → Perfect Competition

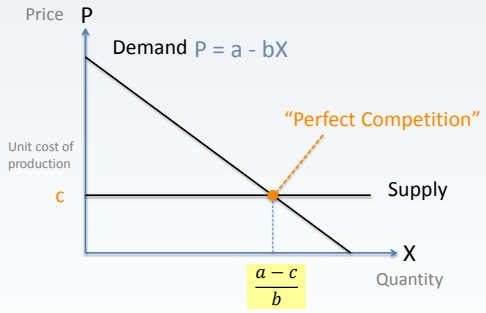
Large number of firms → Perfect Competition

Is this really true?









N firms ➡ Let's find Nash equilibrium

Firms choose their output

N firms ➡ Let's find Nash equilibrium

Firms choose their output

Originally analyzed by French economist Cournot

N firms ➡ Let's find Nash equilibrium

Your profit

N firms ➡ Let's find Nash equilibrium

Your profit $\big(a - b(q + Q)\big) - c$

N firms ➡ Let's find Nash equilibrium

Your output

Your profit $\big(a - b(q + Q)\big) - c$

N firms ➡ Let's find Nash equilibrium

Your output / Other firms' total output

Your profit $(a - b(q + Q)) - c$

N firms ➡ Let's find Nash equilibrium

Your output / Other firms' total output

Your profit $(a - b(q + Q)) - c$

$P = a - bX$ (Demand)

N firms ➡ Let's find Nash equilibrium

Your output / Other firms' total output

Your profit $= \big((a - b(q + Q)) - c\big)\, q$

P

N firms ➡ Let's find Nash equilibrium

Your output / Other firms' total output

Your profit $= \big((a - b(q + Q)) - c\big)\, q$

P

$= b\left(\left[\frac{a-c}{b} - Q\right] - q\right) q$

N firms ➡ Let's find Nash equilibrium

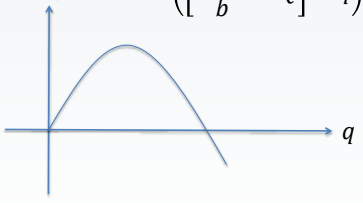
Other firms' total output / Your output

Your profit $= b\left(\left[\frac{a-c}{b} - Q\right] - q\right) q$

N firms ➡ Let's find Nash equilibrium

Other firms' total output / Your output

Your profit $= b\left(\left[\frac{a-c}{b} - Q\right] - q\right) q$

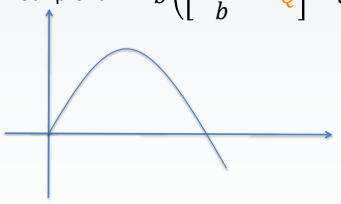


N firms ➡ Let's find Nash equilibrium

Other firms' total output

Your output

Your profit $= b\left(\left[\frac{a-c}{b} - Q\right] - q\right)$ q

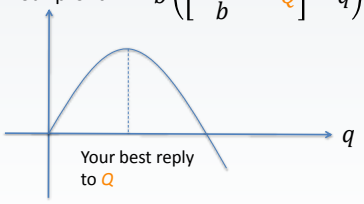


N firms ➡ Let's find Nash equilibrium

Other firms' total output

Your output

Your profit $= b\left(\left[\frac{a-c}{b} - Q\right] - q\right)$ q

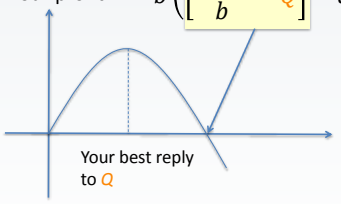


N firms ➡ Let's find Nash equilibrium

Other firms' total output

Your output

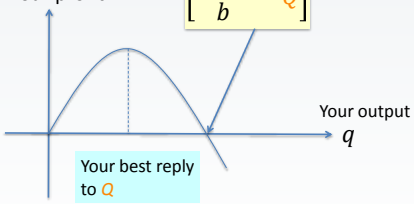
Your profit $= b\left(\left[\frac{a-c}{b} - Q\right] - q\right)$ q



N firms ➡ Let's find Nash equilibrium

Other firms' total output

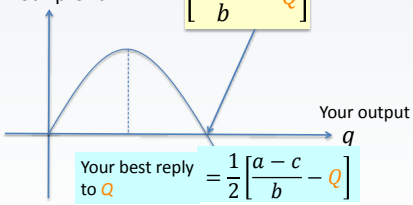
Your profit $\left[\frac{a-c}{b} - Q\right]$



N firms ➡ Let's find Nash equilibrium

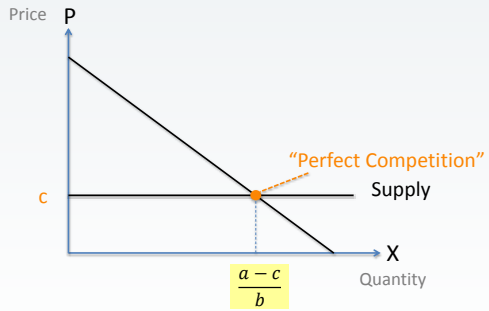
Other firms' total output

Your profit $\left[\frac{a-c}{b} - Q\right]$

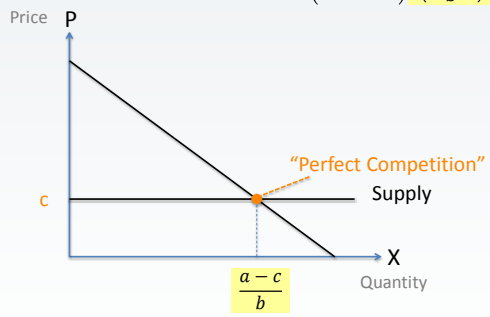


Your best reply to Q $= \frac{1}{2}\left[\frac{a-c}{b} - Q\right]$

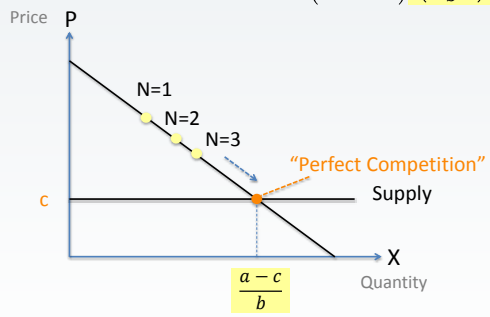
Total output with N firms



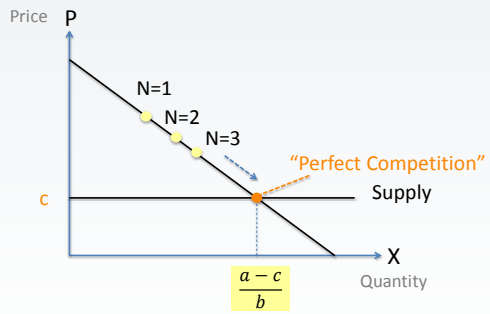
Total output with N firms $\left(1-\frac{1}{N+1}\right)\left(\frac{a-c}{b}\right)$



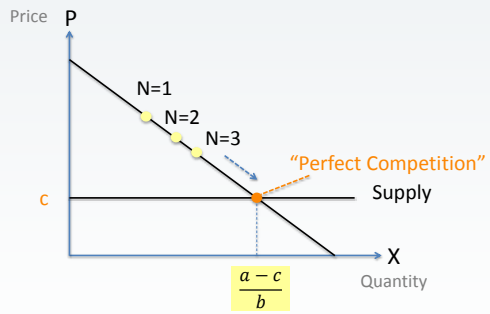
Total output with N firms $\left(1-\frac{1}{N+1}\right)\left(\frac{a-c}{b}\right)$



Large number of firms

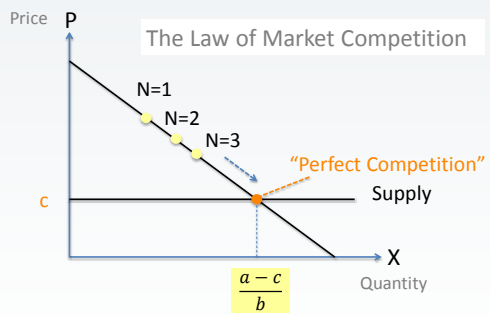


Large number of firms ⇒ Perfect Competition



Large number of firms ⇒ Perfect Competition

The Law of Market Competition



2.4 Why do people come to play Nash equilibrium?

Part I: Rational reasoning

Nash equilibrium

Best Reply

Nash equilibrium

Player 1 Best Reply Player 2

Nash equilibrium

Nash equilibrium

Nash equilibrium

How do players “find out” such a point?

Reasons why players might play a Nash equilibrium

Reasons why players might play a Nash equilibrium

① Rational reasoning

Reasons why players might play a Nash equilibrium

① Rational reasoning

② Pre-play communication

Reasons why players might play a Nash equilibrium

① Rational reasoning

② Pre-play communication

③ Trial-and-error adjustment

Reasons why players might play a Nash equilibrium

① Rational reasoning

② Pre-play communication

③ Trial-and-error adjustment

Which one applies depends on games and contexts.

① Rational reasoning ⇒ Nash equilibrium

① Rational reasoning ⇒ Nash equilibrium

…applies to Prisoner's Dilemma

① Rational reasoning ⇒ Nash equilibrium

…applies to Prisoner's Dilemma

1 \ 2	C	D
C	-1, -1	-15, 0
D	0, -15	-10, -10

① Rational reasoning ⇒ Nash equilibrium

…applies to Prisoner's Dilemma

1 \ 2	C	D
C	-1, -1	-15, 0
D	0, -15	-10, -10

① Rational reasoning ⇒ Nash equilibrium

…applies to Prisoner's Dilemma

1 \ 2	C	D
C	-1, -1	-15, 0
D	0, -15	-10, -10

D is always best, *irrespective of the opponent's strategy*

① Rational reasoning ⇒ Nash equilibrium

…applies to Prisoner's Dilemma

1 \ 2	C	D
C	-1, -1	-15, 0
D	0, -15	-10, -10

D is always best, *irrespective of the opponent's strategy*

Rational players play Nash equilibrium (D,D)

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football		
Shopping		

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football		0, 0
Shopping	0, 0	

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

If my partner goes to Football ⇒ I will go to Football

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

If my partner goes to Football ⇒ I will go to Football

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

If my partner goes to Football ⇒ I will go to Football
If my partner goes Shopping ⇒ I will go Shopping

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

If my partner goes to Football ⇒ I will go to Football
If my partner goes Shopping ⇒ I will go Shopping

① Rational reasoning ⇒ Nash equilibrium

…does NOT apply to the "Battle of the Sexes" game

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

If my partner goes to Football ⇒ I will go to Football
If my partner goes Shopping ⇒ I will go Shopping

Rational reasoning alone does not tell which equilibrium to play

Rationality + ➡ Nash equilibrium

Rationality + Correct beliefs ➡ Nash equilibrium

about others' behavior

Rationality + Correct beliefs ➡ Nash equilibrium

about others' behavior

Rationality + Correct beliefs ➡ Nash equilibrium

Players' reasoning alone is not sufficient to form correct beliefs.

about others' behavior

Rationality + Correct beliefs ➡ Nash equilibrium

about others' behavior

Rationality + Correct beliefs ➡ Nash equilibrium

② Pre-play communication

about others' behavior

Rationality + Correct beliefs ➡ Nash equilibrium

② Pre-play communication

③ Trial-and-error adjustment

2.5 Why do people come to play Nash equilibrium?

Part II: Communication and adjustment

② Pre-play communication ⇒ Nash equilibrium

"Battle of the Sexes" game

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

"Battle of the Sexes" game

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

"Battle of the Sexes" game

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

"Battle of the Sexes" game

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

Man \ Woman	Football	Shopping
Football	3, 2	0, 0
Shopping	0, 0	2, 3

"Battle of the Sexes" game

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

Discussion before playing Battle of the Sexes:

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

Discussion before playing Battle of the Sexes:

Agreeing to go to Football (a Nash equilibrium)

② Pre-play communication ⇒ Nash equilibrium

Correct beliefs

Discussion before playing Battle of the Sexes:

Agreeing to go to Football (a Nash equilibrium)

The promise will be kept without reward or punishment

A promise kept without reward or punishment

A promise kept without reward or punishment

"Self-enforcing Agreement"

A promise kept without reward or punishment

"Self-enforcing Agreement"

⇓

Nash equilibrium

*An agreement cannot be fulfilled, if it is NOT a Nash equilibrium.

*An agreement cannot be fulfilled, if it is NOT a Nash equilibrium.

Ex. "Let us cooperate" in the Prisoner's Dilemma
(NOT a Nash equilibrium)

*An agreement cannot be fulfilled, if it is NOT a Nash equilibrium.

Ex. "Let us cooperate" in the Prisoner's Dilemma
(NOT a Nash equilibrium)

⇒ The promise will be breached.

③ Trial-and-error adjustment ⇒ Nash equilibrium

③ Trial-and-error adjustment ⇒ Nash equilibrium

Accumulating experience in the same game or similar games

③ Trial-and-error adjustment ⇒ Nash equilibrium

Accumulating experience in the same game or similar games

Better idea about what others do

③ Trial-and-error adjustment ⇒ Nash equilibrium

Accumulating experience in the same game or similar games

Better idea about what others do

Finding out and switching to better strategies

③ Trial-and-error adjustment ⇒ Nash equilibrium

Correct beliefs

Accumulating experience in the same game or similar games

Better idea about what others do

Finding out and switching to better strategies

③ Trial-and-error adjustment ⇒ Nash equilibrium

Correct beliefs

Rationality

Accumulating experience in the same game or similar games

Better idea about what others do

Finding out and switching to better strategies

③ Trial-and-error adjustment ⇒ Nash equilibrium

Correct beliefs

Rationality

Example.

③ Trial-and-error adjustment ⇒ Nash equilibrium

Correct beliefs

Rationality

Example.

Traffic around Hamamatsu City

2.6 Why do people come to play Nash equilibrium?

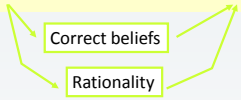
Part III: Experimental evidence

③ Trial-and-error adjustment ⇒ Nash equilibrium

Correct beliefs

Rationality

③ Trial-and-error adjustment ⇒ Nash equilibrium



Experimental evidence

An auction experiment

- An object is for sale.

An auction experiment

- An object is for sale.

  9 buyers

An auction experiment

- An object is for sale.

  9 buyers

  Value of the object = 1000 points

An auction experiment

- An object is for sale.

  9 buyers

  Value of the object = 1000 points

  ≅ $10

An auction experiment

- Buyers **simultaneously** submit bids

An auction experiment

- Buyers simultaneously submit bids

bid = 0, 5,10, ……, 995, 1000

An auction experiment

- The buyer with the highest bid wins and pays his bid

An auction experiment

- The buyer with the highest bid wins and pays his bid

In case of ties, the winner is chosen with an equal probability

Players simultaneously submit their bids

Player 1

Players simultaneously submit their bids

Player 1

500

Players simultaneously submit their bids

Player 1 Player 2

500 990

Players simultaneously submit their bids

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Players simultaneously submit their bids

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

The highest bid

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Player 2 wins the object

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Player 2 wins the object

and pays his bid

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Player 2 wins the object and pays his bid

Winner (Player 2)'s payoff

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Player 2 wins the object and pays his bid

Winner (Player 2)'s payoff

$1000 - 990 = 10$

The value of the object

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Winner (Player 2)'s payoff

$1000 - 990 = 10$

The value of the object

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Not a Nash equilibrium

Winner (Player 2)'s payoff

$1000 - 990 = 10$

The value of the object

Player 1 Player 2 ..... Player 9

500 990 200 970 856 468 778 660 680

Not a Nash equilibrium

bid = 0, 5,10, ..., 995, 1000

Winner (Player 2)'s payoff

$1000 - 990 = 10$

The value of the object

bid = 0, 5,10, ..., 995, 1000

bid = 0, 5,10, ……, 995, 1000

All buyers bid 995

bid = 0, 5,10, ……, 995, 1000

All buyers bid 995

At least two buyers bid 1000

bid = 0, 5,10, ……, 995, 1000

All buyers bid 995

At least two buyers bid 1000

Nash equilibria

bid = 0, 5,10, ……, 995, 1000

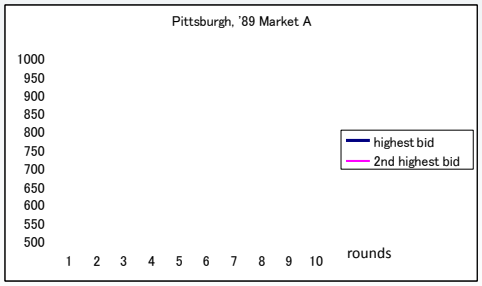
All buyers bid 995

At least two buyers bid 1000

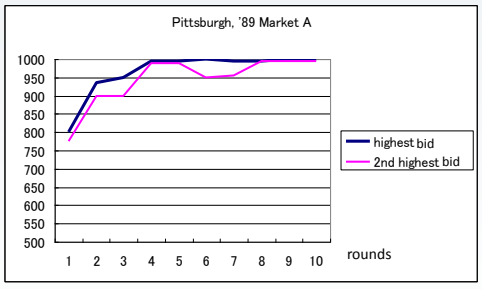
Roth, Prasnikar, Okuno, and Zamir conducted experiments of this game in various countries

American Economic Review 1991

Experimental results

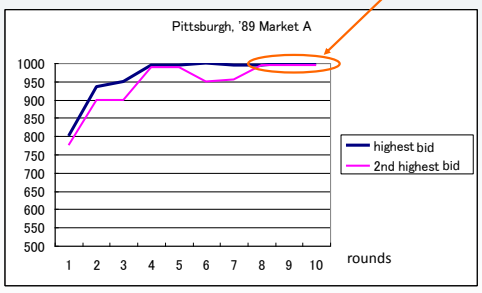


Experimental results

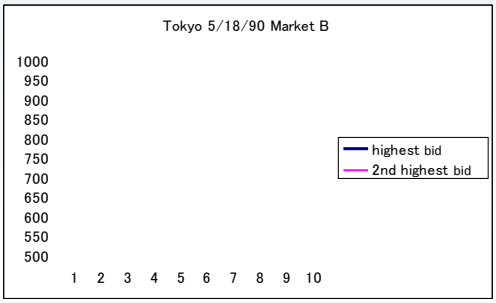


Experimental results

Equilibrium

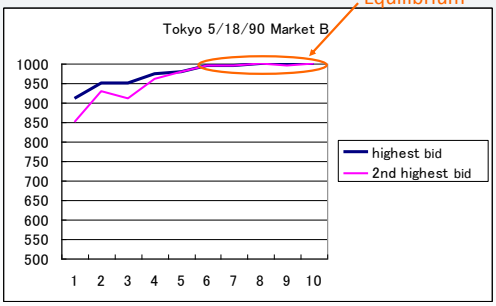


Experimental results

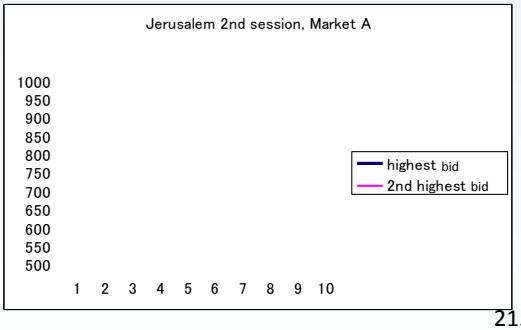


Experimental results

Equilibrium

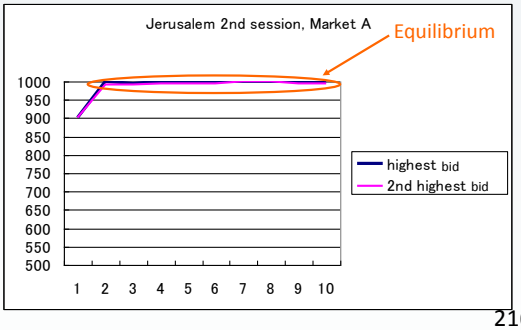


Experimental results



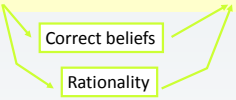
Experimental results

Equilibrium

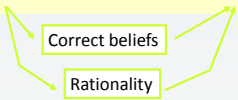


2.7 Stylized facts and Nash equilibrium

Trial-and-error adjustment ⇒ Nash equilibrium

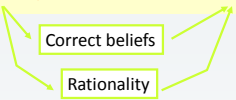


Trial-and-error adjustment ⇒ Nash equilibrium



- **If** an adjustment process ever **converges**, the outcome ought to be a Nash equilibrium.

Trial-and-error adjustment ⇒ Nash equilibrium



- But there is no guarantee that an adjustment process always converges.

If there is no guarantee that a Nash equilibrium always emerges ,

is it really a useful concept?

Stylized fact

Stylized fact

Stable mode of behavior that is repeatedly observed

Stylized fact

Stable mode of behavior that is repeatedly observed

Example:

Stylized fact

Stable mode of behavior that is repeatedly observed

Example: Convention on Tokyo subway escalators

Photo taken in December 2014

Photo taken in December 2014

A "new convention" is being formed in Tokyo subway stations. On the escalators, standing people stay to the left, so that anyone in a hurry can run on the right side. This seems to be a spontaneous order formed by the busy commuters in Tokyo.

Asahi newspaper Feb 24, 1992

Photo taken in December 2014

A stylized fact

Photo taken in December 2014

A stylized fact

Nash equilibrium

Photo taken in December 2014

Stylized facts are likely to be Nash equilibria

Photo taken in December 2014

Stylized facts are likely to be Nash equilibria

If people are not following a Nash equilibrium, there is always someone who can gain by changing his/her behavior.

Photo taken in December 2014

Stylized facts are likely to be Nash equilibria

If people are not following a Nash equilibrium, there is always someone who can gain by changing his/her behavior.

Sooner or later, such a "profitable deviation" is discovered

Photo taken in December 2014

Stylized facts are likely to be Nash equilibria

If people are not following a Nash equilibrium, there is always someone who can gain by changing his/her behavior.

Photo taken in December 2014

Stylized facts are likely to be Nash equilibria

If people are not following a Nash equilibrium, there is always someone who can gain by changing his/her behavior.

The original mode of behavior collapses

Photo taken in December 2014

Stylized facts are likely to be Nash equilibria

If people are not following a Nash equilibrium, there is always someone who can gain by changing his/her behavior.

The original mode of behavior collapses

Not a stable mode of behavior (stylized fact)

An important goal of social sciences is to explain stylized facts.

An important goal of social sciences is to explain stylized facts.

Stylized facts are likely to be Nash equilibria.

An important goal of social sciences is to explain stylized facts.

Stylized facts are likely to be Nash equilibria.

Probably the most important reason why Nash equilibrium is useful

2.8 Make yourself unpredictable:

2.8 Make yourself unpredictable:

Mixed strategy equilibrium

Example:

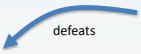
Example: Rock-Paper-Scissors

Example: Rock-Paper-Scissors

Example: Rock-Paper-Scissors

Example: Rock-Paper-Scissors

Example: Rock-Paper-Scissors



Example: Rock-Paper-Scissors

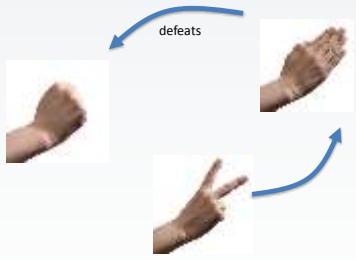


Image Credit: Sertion, CC BY-SA 3.0

Example: Rock-Paper-Scissors

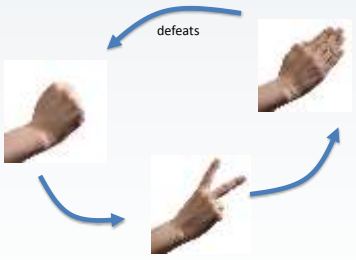


Image Credit: Sertion, CC BY-SA 3.0

No "mutual best replies"

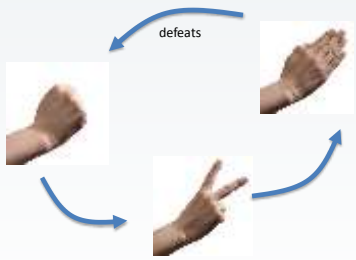


Image Credit: Sertion, CC BY-SA 3.0

No Nash equilibrium?

Image Credit: Sertion, CC BY-SA 3.0

It is important to make yourself unpredictable

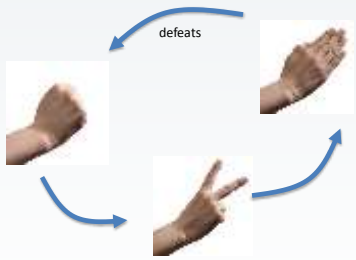


Image Credit: Sertion, CC BY-SA 3.0

You should randomly choose your action

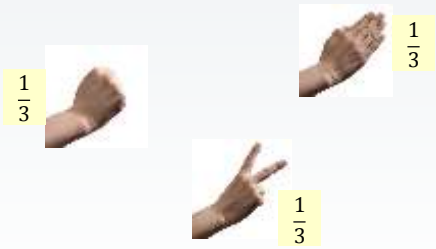
Image Credit: Sertion, CC BY-SA 3.0

Mixed strategy equilibrium

People seem to choose those actions with an **equal probability**

People seem to choose those actions with an **equal probability**

Let's examine what is happening here



	Rock	Paper	Scissors
Rock			
Paper			
Scissors			

	Rock	Paper	Scissors
Rock	0, 0		
Paper		0, 0	
Scissors			0, 0

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	
Paper		0, 0	
Scissors			0, 0

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper		0, 0	
Scissors			0, 0

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	
Scissors			0, 0

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors			0, 0

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1		0, 0

	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors
→ Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors
Rock	0, 0	-1, 1	1, -1
→ Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
→ Scissors	-1, 1	1, -1	0, 0

you \ opponent	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors
→ Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	Rock ($\frac{1}{3}$)	Paper ($\frac{1}{3}$)	Scissors ($\frac{1}{3}$)
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	Rock ($\frac{1}{3}$)	Paper ($\frac{1}{3}$)	Scissors ($\frac{1}{3}$)
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	Rock ($\frac{1}{3}$)	Paper ($\frac{1}{3}$)	Scissors ($\frac{1}{3}$)
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

you \ opponent	Rock ($\frac{1}{3}$)	Paper ($\frac{1}{3}$)	Scissors ($\frac{1}{3}$)
Rock	0, 0	-1, 1	1, -1

you \ opponent	Rock ($\frac{1}{3}$)	Paper ($\frac{1}{3}$)	Scissors ($\frac{1}{3}$)
Rock	0, 0	-1, 1	1, -1

On average, your payoff is

you \ opponent	Rock ($\frac{1}{3}$)	Paper ($\frac{1}{3}$)	Scissors ($\frac{1}{3}$)
Rock	0, 0	-1, 1	1, -1

On average, your payoff is

$$\frac{1}{3}\times 0+\frac{1}{3}\times(-1)+\frac{1}{3}\times 1=0$$

	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors
you \ opponent			
Rock	0, 0	-1, 1	1, -1

Your expected payoff

$$\frac{1}{3}\times 0+\frac{1}{3}\times(-1)+\frac{1}{3}\times 1=0$$

	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	
Paper	1, -1	0, 0	-1, 1	
Scissors	-1, 1	1, -1	0, 0	

	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	
Paper	1, -1	0, 0	-1, 1	
Scissors	-1, 1	1, -1	0, 0	

	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	
Paper	1, -1	0, 0	-1, 1	
Scissors	-1, 1	1, -1	0, 0	

	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	0
Paper	1, -1	0, 0	-1, 1	
Scissors	-1, 1	1, -1	0, 0	

	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	
Paper	1, -1	0, 0	-1, 1	0
Scissors	-1, 1	1, -1	0, 0	

	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	
Paper	1, -1	0, 0	-1, 1	
Scissors	-1, 1	1, -1	0, 0	0

opponent / you	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	0
Paper	1, -1	0, 0	-1, 1	0
Scissors	-1, 1	1, -1	0, 0	0

opponent / you	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	0
Paper	1, -1	0, 0	-1, 1	0
Scissors	-1, 1	1, -1	0, 0	0

If your opponent chooses R, P, and S with an equal probability

opponent / you	$\frac{1}{3}$ Rock	$\frac{1}{3}$ Paper	$\frac{1}{3}$ Scissors	Your expected payoff
Rock	0, 0	-1, 1	1, -1	0
Paper	1, -1	0, 0	-1, 1	0
Scissors	-1, 1	1, -1	0, 0	0

If your opponent chooses R, P, and S with an equal probability

R, P, and S are equally good for you

Opponent choosing R, P and S with an equal probability

Opponent choosing R, P and S with an equal probability

R, P, and S are equally good for you

Opponent choosing R, P and S with an equal probability

Any random choice over R, P, and S are equally good for you

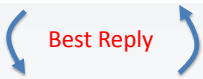
Opponent choosing R, P and S with an equal probability

Choosing R, P and S with an equal probability is a best reply for you

Opponent choosing R, P and S with an equal probability

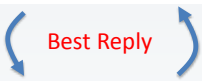
You choosing R, P and S with an equal probability

Opponent choosing R, P and S with an equal probability



You choosing R, P and S with an equal probability

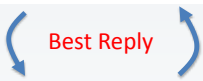
Opponent choosing R, P and S with an equal probability



You choosing R, P and S with an equal probability

Nash equilibrium in random strategies

Opponent choosing R, P and S with an equal probability



You choosing R, P and S with an equal probability

Mixed Strategy Equilibrium

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

Mixed Strategy Equilibrium

2.9 Sports games and game theory

Penalty kick in soccer

Kicker \ Goalie	Left	Right
Left	a	b
Right	c	d

Probabilities that Kicker wins

Kicker \ Goalie	Left	Right
Left	a	b
Right	c	d

Probabilities that Kicker wins

＝Kicker's payoff

Goalie

Kicker	Left	Right
Left	a	b
Right	c	d

Probabilities that Kicker wins

＝Kicker's payoff　　＝－(goalie's payoff)

Goalie

Kicker	Left	Right
Left	a	b
Right	c	d

Probabilities that Kicker wins

a, b, c, and d can be estimated from data.

Data: 1417 penalty kicks in professional soccer games in Spain, Italy, UK and other countries between September 1995 and June 2000.

Data: 1417 penalty kicks in professional soccer games in Spain, Italy, UK and other countries between September 1995 and June 2000.

I. Palacios-Huerta "Professionals Play Minimax", *Review of Economic Studies*, April 2003

Goalie

Kicker	Left	Right
Left (from the point of view of Goalie)	58. 30	94. 97
Right	92. 92	69. 92

Estimated payoff table

Probability that Kicker wins (%)

Computing mixed strategy equilibrium

Goalie

Kicker	Left	Right
Left	58. 30	94. 97
Right	92. 92	69. 92

Computing mixed strategy equilibrium

Goalie

Kicker	p Left	$1-p$ Right
Left	58. 30	94. 97
Right	92. 92	69. 92

Computing mixed strategy equilibrium

Goalie

Kicker	p Left	$1-p$ Right	Kicker's expected payoff
Left	58. 30	94. 97	
Right	92. 92	69. 92	

Computing mixed strategy equilibrium

Goalie

Kicker	p Left	$1-p$ Right	Kicker's expected payoff
Left	58. 30	94. 97	→ 58.30p + 94.97(1-p)
Right	92. 92	69. 92	

Computing mixed strategy equilibrium

Goalie

Kicker	p Left	$1-p$ Right	Kicker's expected payoff
Left	58. 30	94. 97	→ 58.30p + 94.97(1-p)
Right	92. 92	69. 92	→ 92.92p + 69.92(1-p)

Computing mixed strategy equilibrium

Goalie

Kicker	p Left	$1-p$ Right	Kicker's expected payoff
Left	58. 30	94. 97	→ 58.30p + 94.97(1-p) ①
Right	92. 92	69. 92	→ 92.92p + 69.92(1-p) ②

Computing mixed strategy equilibrium

Goalie

Kicker	p Left	$1-p$ Right	Kicker's expected payoff
Left	58. 30	94. 97	→ 58.30p + 94.97(1-p) ①
Right	92. 92	69. 92	→ 92.92p + 69.92(1-p) ②

Equilibrium p is given by ① = ②

	Goalie		Kicker	
	Left	Right	Left	Right
Equilibrium				

	Goalie		Kicker	
	Left	Right	Left	Right
Equilibrium	41.99	58.01	38.54	61.46

	Goalie		Kicker	
	Left	Right	Left	Right
Equilibrium	41.99	58.01	38.54	61.46
Empirical Frequency	42.31	57.69	39.98	60.02

	Goalie		Kicker	
	Left	Right	Left	Right
Equilibrium	41.99	58.01	38.54	61.46
Empirical Frequency	42.31	57.69	39.98	60.02

Game theoretic prediction works surprisingly well in this data set!

Researchers also discovered that game theoretic predictions work well in professional tennis games

2.10 Nash equilibrium exists in all games

Nash discovered that any game has a Nash equilibrium

Nash discovered that any game has a Nash equilibrium

(possibly in mixed strategies),

Nash discovered that any game has a Nash equilibrium

(possibly in mixed strategies), if

Nash discovered that any game has a Nash equilibrium

(possibly in mixed strategies), if

- there are finitely many players

Nash discovered that any game has a Nash equilibrium

(possibly in mixed strategies), if

- there are finitely many players
- each player has finitely many strategies

- there are finitely many players

Always true

- there are finitely many players

- each player has finitely many strategies

- each player has finitely many strategies

It may not be true in some cases

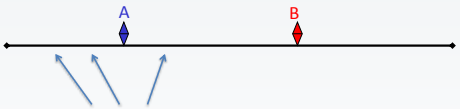
- each player has finitely many strategies

It may not be true in some cases

Ice cream vendors A and B on a street

- each player has finitely many strategies

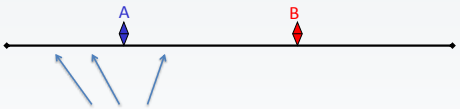
It may not be true in some cases



Infinitely many possible locations

- each player has finitely many strategies

It may not be true in some cases



Infinitely many possible locations

strategies

- each player has finitely many strategies

It may not be true in some cases

- each player has finitely many strategies

It may not be true in some cases

But such a game can be well approximated by

a model with finitely many strategies

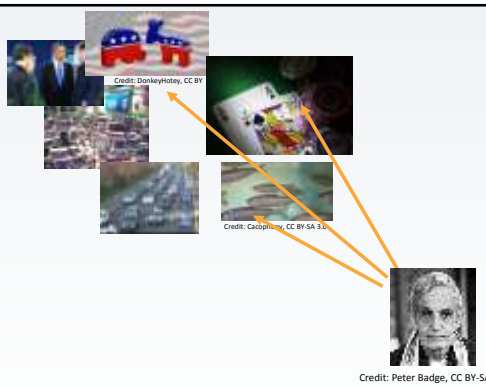
The two conditions are "practically" always satisfied

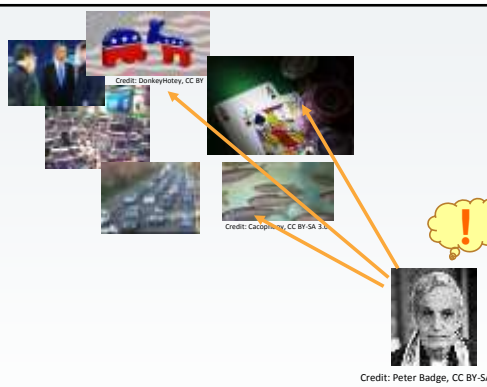
Nash discovered that any game has a Nash equilibrium

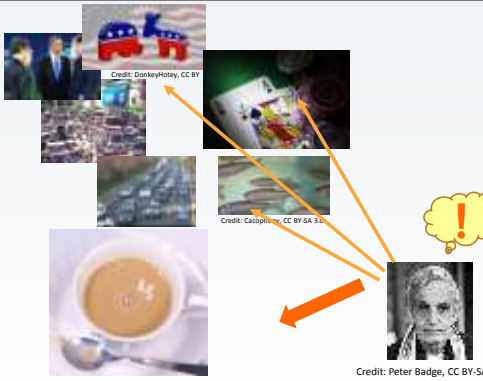
Nash discovered that any game has a Nash equilibrium

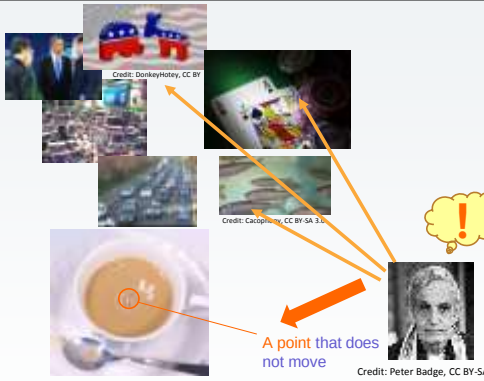
We can analyze any social problem by Nash equilibrium

Nash's insights









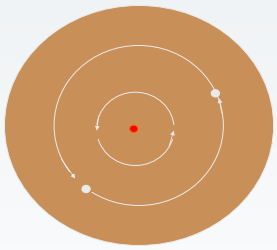
Suppose the surface of coffee represents

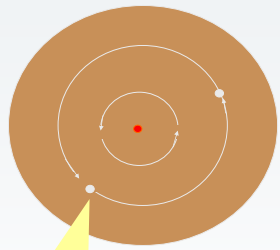
the set of possible human behavior.

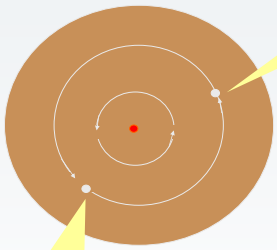
Suppose the surface of coffee represents

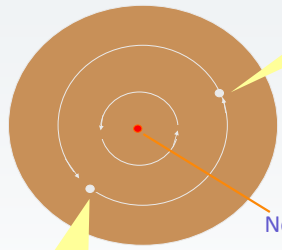
the set of possible human behavior.

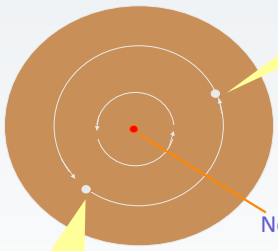
This is really an ingenious idea of Nash!











Let me elaborate on this point, by a simple example

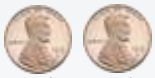
Matching pennies

Matching pennies

Players 1 and 2: each shows one side of a coin

Matching pennies

Players 1 and 2: each shows one side of a coin



Heads Heads

Matching pennies

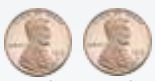
Players 1 and 2: each shows one side of a coin

Heads Heads

Same side

Matching pennies

Players 1 and 2: each shows one side of a coin



Heads Heads

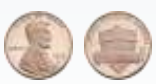
Same side ⇨ Player 1 wins

Matching pennies

Players 1 and 2: each shows one side of a coin

Heads Heads

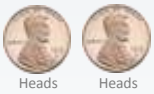
Same side ⇨ Player 1 wins



Heads Tails

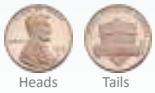
Matching pennies

Players 1 and 2: each shows one side of a coin



Heads Heads

Same side ⇨ Player 1 wins

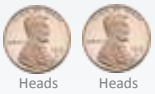


Heads Tails

different sides

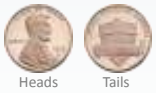
Matching pennies

Players 1 and 2: each shows one side of a coin



Heads Heads

Same side ⇨ Player 1 wins



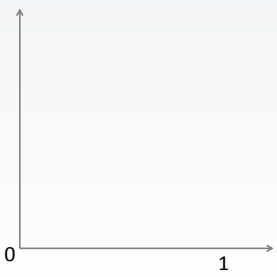
Heads Tails

different sides ⇨ Player 2 wins

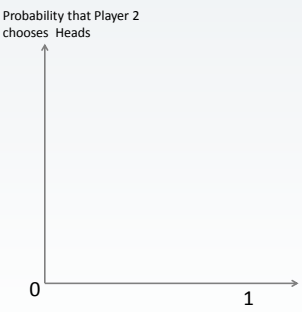
Matching pennies

Matching pennies

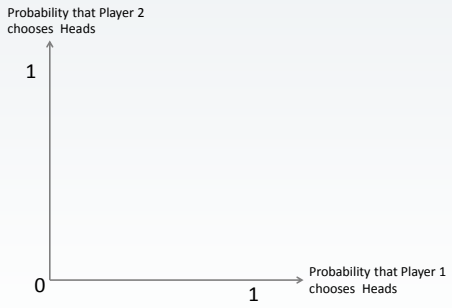
Matching pennies



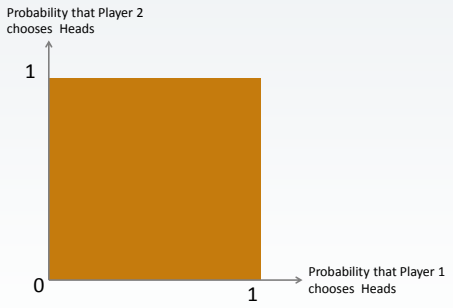
Matching pennies



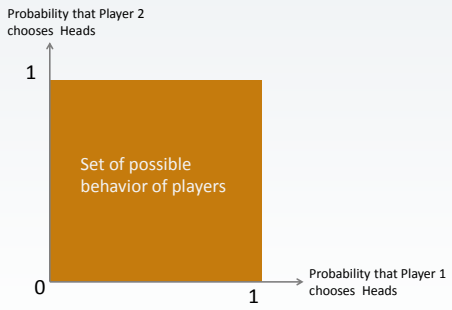
Matching pennies

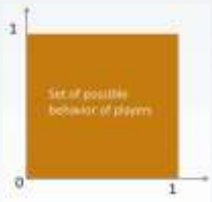


Matching pennies

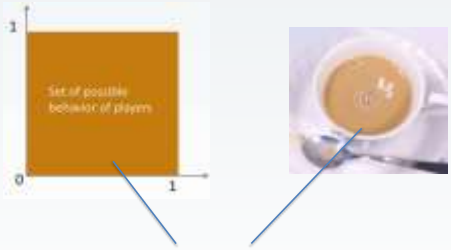


Matching pennies

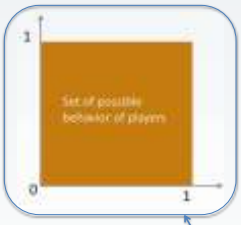




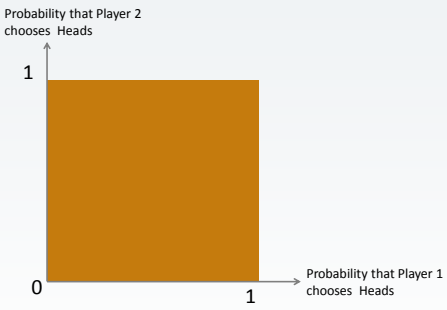
They look similar



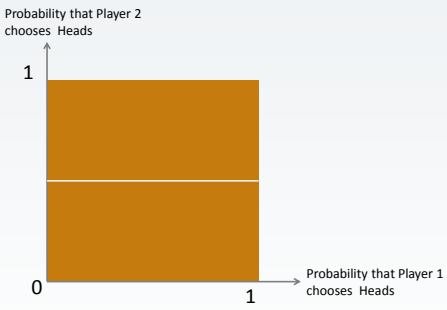
The same mathematical structure



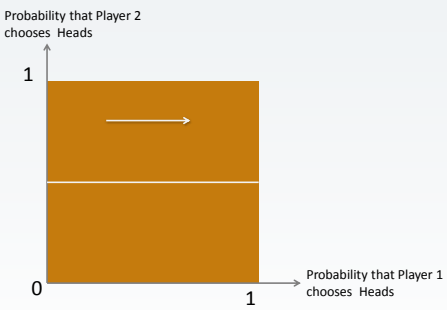
Let us now examine the "law of motion" here



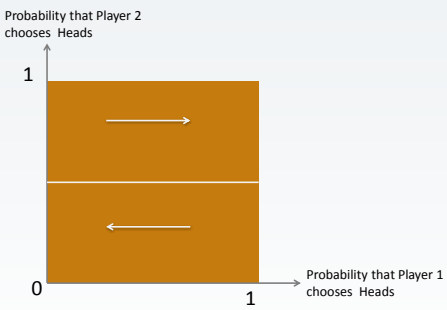
Player 1's better reply



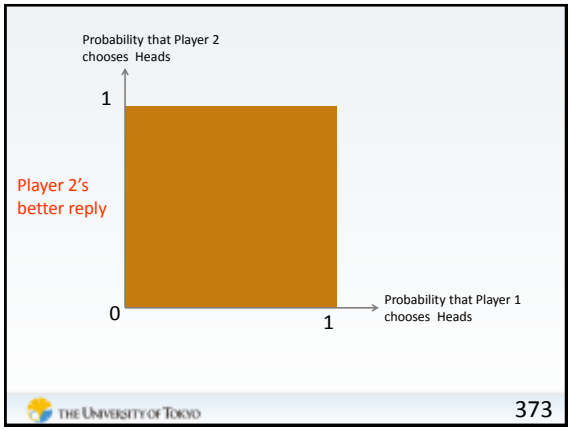
Player 1's better reply

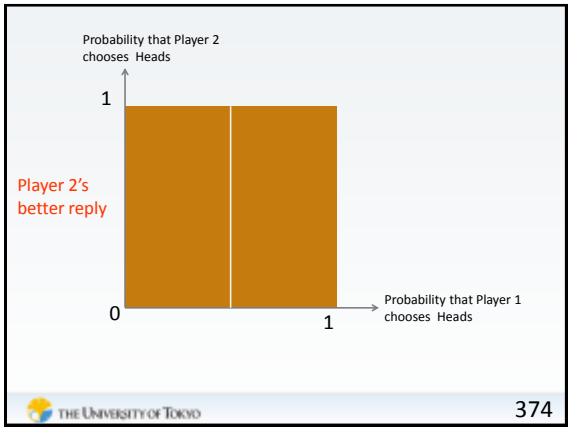


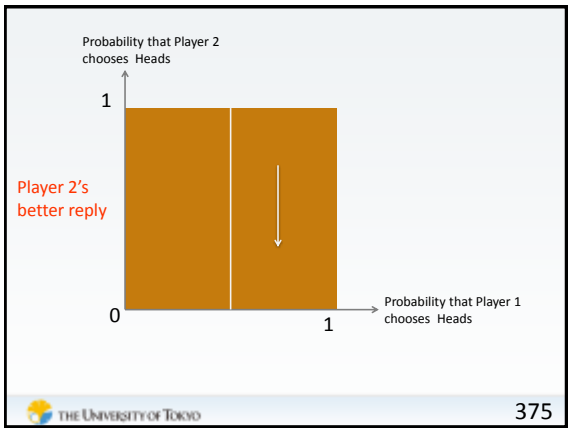
Player 1's better reply

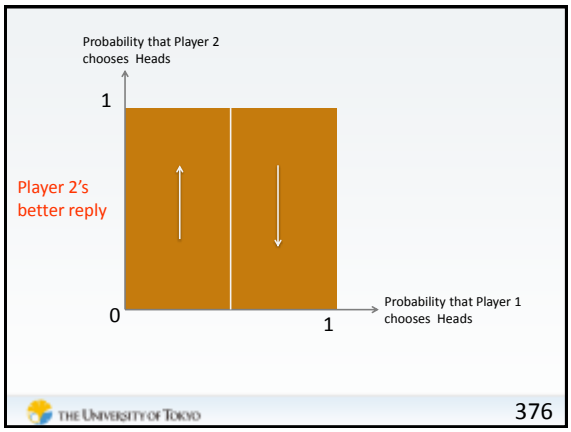


Player 1's better reply

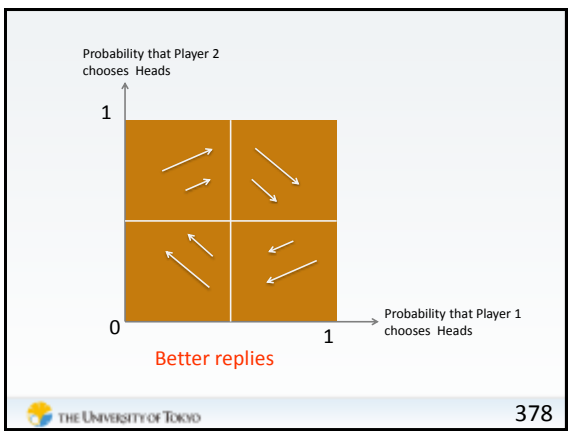

Probability that Player 2 chooses Heads
1
Player 2's better reply
0
1
Probability that Player 1 chooses Heads

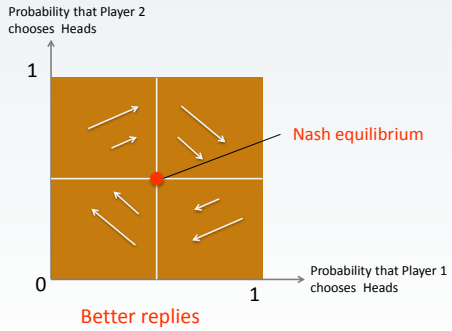

Probability that Player 2 chooses Heads
1
Player 2's better reply
0
1
Probability that Player 1 chooses Heads

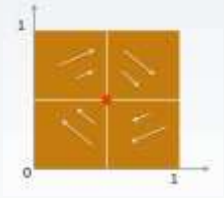

Probability that Player 2 chooses Heads
1
Player 2's better reply
0
1
Probability that Player 1 chooses Heads

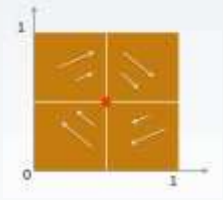

Probability that Player 2 chooses Heads
1
Player 2's better reply
0
1
Probability that Player 1 chooses Heads

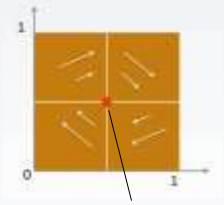

Probability that Player 2 chooses Heads
1
0
1
Probability that Player 1 chooses Heads
Better replies


Probability that Player 2 chooses Heads
1
0
1
Probability that Player 1 chooses Heads
Better replies

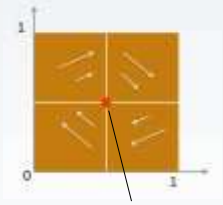






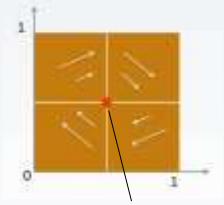


Nash equilibrium exists



Nash equilibrium exists

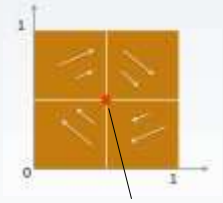
Vortex exists



Nash equilibrium exists

Vortex exists

Both follow from the same mathematical fact



Nash equilibrium exists

Vortex exists

Both follow from the same mathematical fact

Fixed Point Theorem

Nash discovered that any game has a Nash equilibrium

Nash discovered that any game has a Nash equilibrium

We obtained a general "governing principle" that can be applied to any social problem