Week 3: Rationality, knowledge and evolution in games

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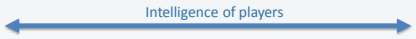
Week 3: Rationality, Knowledge and evolution in games



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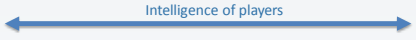

Hyper rationality

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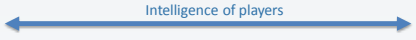

Hyper rationality

Extremely sophisticated reasoning

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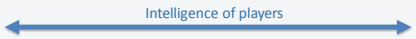

Hyper rationality

Low rationality

Extremely sophisticated reasoning

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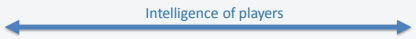
Week 3: Rationality, Knowledge and evolution in games



Hyper rationality

Low rationality

Zero-intelligence

Extremely sophisticated reasoning

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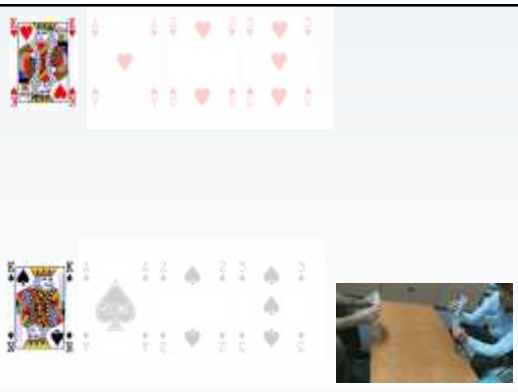
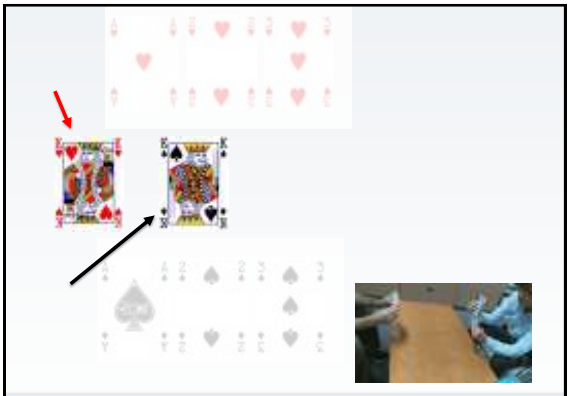
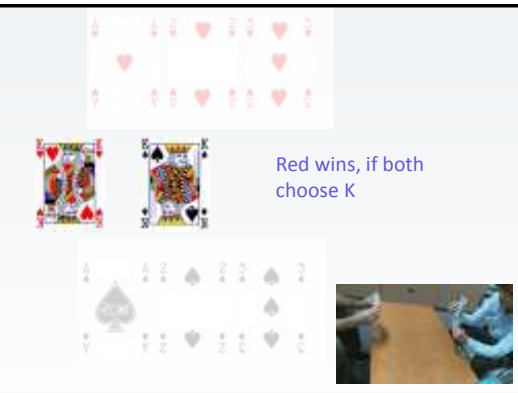
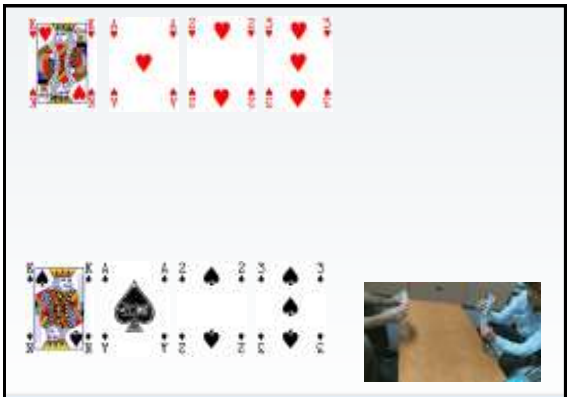
3.1 Digression: The card game revisited

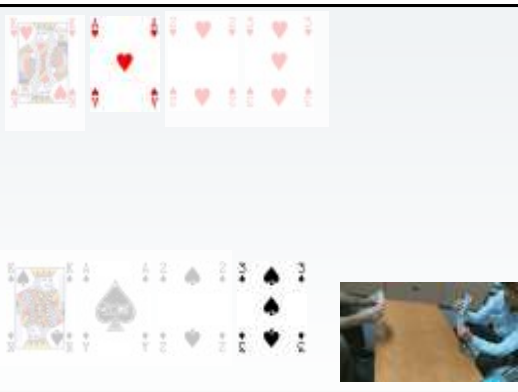
Each player carefully chooses one card and show it to the opponent (simultaneously)

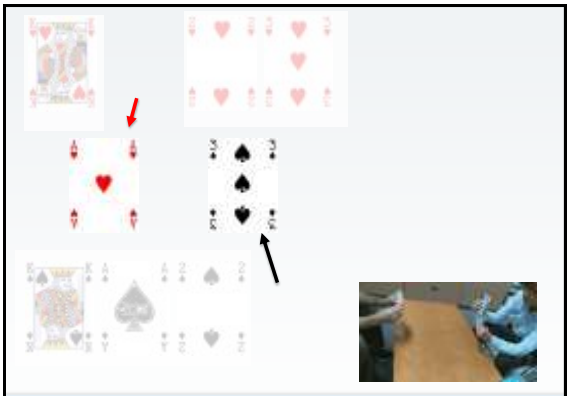


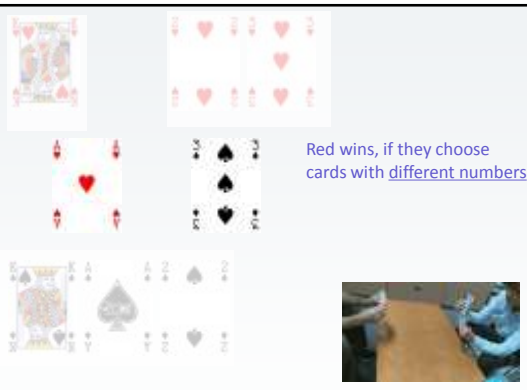



Red wins, if both
choose K

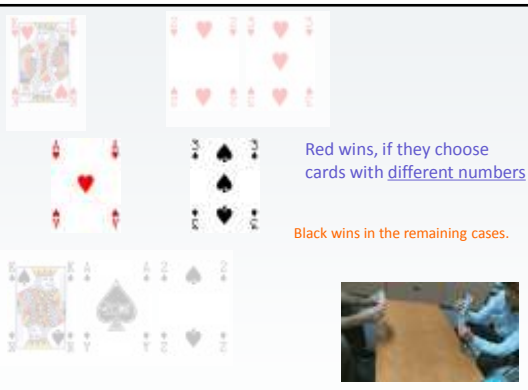








Red wins, if they choose cards with different numbers



Red wins, if they choose cards with different numbers

Black wins in the remaining cases.

Red player wins if …

- both choose K

such as  and 

- players choose cards with different numbers

Red player wins if …

- both choose K

such as  and 

- players choose cards with different numbers

Black player wins if …

- only one player chooses K
- players choose cards with the same number

such as  and 

Who has an advantage, Red or Black?

What is the winning rate of each player?

Any prediction about the cards they choose?

Game theory provides very sharp predictions

In this card game, it is important to make yourself unpredictable

In this card game, it is important to make yourself unpredictable

as in Rock-Paper-Scissors

In this card game, it is important to make yourself unpredictable

as in Rock-Paper-Scissors

Let us find a mixed strategy equilibrium

In this card game, it is important to make yourself unpredictable

as in Rock-Paper-Scissors

Let us find a mixed strategy equilibrium

Four cards are chosen with certain probabilities

	K	1	2	3
K				
1				
2				
3				

Red wins if "both K" or "different numbers"

	K	1	2	3
K				
1				
2				
3				

Red wins if "both K" or "different numbers"

	K	1	2	3
K	**1, 0**			
1				
2				
3				

Red wins if "both K" or "different numbers"

	K	1	2	3
K	**1, 0**			
1			**1, 0**	**1, 0**
2		**1, 0**		**1, 0**
3		**1, 0**	**1, 0**	

Payoff table of the card game

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

Probability distribution of Black player's action

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

p

Probability distribution of Black player's action

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

p q

Probability distribution of Black player's action

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

p q r

Probability distribution of Black player's action

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

1 − p − q − r p q r

Probability distribution of Black player's action

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

Cards 1,2, and 3 have a similar role

	K	1	2	3
K	**1, 0**	**0, 1**	**0, 1**	**0, 1**
1	**0, 1**	**0, 1**	**1, 0**	**1, 0**
2	**0, 1**	**1, 0**	**0, 1**	**1, 0**
3	**0, 1**	**1, 0**	**1, 0**	**0, 1**

Finding mixed strategy equilibrium

Cards 1,2, and 3 have a similar role

Let us **guess** that they are chosen with the same probability

	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

p p p

Cards 1,2, and 3 have a similar role

Let us **guess** that they are chosen with the same probability

	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

1–3p p p p

Cards 1,2, and 3 have a similar role

Let us **guess** that they are chosen with the same probability

	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

➡ 1–3p

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

➡ 1–3p

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

➡ 1–3p

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	**1**, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	**1**, 0	**1**, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

➡ 1–3p

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	**1**, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	**1**, 0	**1**, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

➡ 1–3p

➡ 2p

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	**1**, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	**1**, 0	**1**, 0
2	0, 1	**1**, 0	0, 1	**1**, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

➡ 1–3p

➡ 2p

➡ 2p

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	**1**, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	**1**, 0	**1**, 0
2	0, 1	**1**, 0	0, 1	**1**, 0
3	0, 1	**1**, 0	**1**, 0	0, 1

Winning rate of Red

➡ 1–3p

➡ 2p

➡ 2p

➡ 2p

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	**1**, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	**1**, 0	**1**, 0
2	0, 1	**1**, 0	0, 1	**1**, 0
3	0, 1	**1**, 0	**1**, 0	0, 1

Winning rate of Red

➡ 1–3p

➡ 2p

➡ 2p

➡ 2p

"Red player mixes four cards" implies that those numbers are equal

Finding mixed strategy equilibrium

	1–3p	p	p	p
	K	1	2	3
K	**1**, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	**1**, 0	**1**, 0
2	0, 1	**1**, 0	0, 1	**1**, 0
3	0, 1	**1**, 0	**1**, 0	0, 1

Winning rate of Red

➡ 1–3p

➡ 2p

➡ 2p

➡ 2p

"Red player mixes four cards" implies that those numbers are equal

Finding mixed strategy equilibrium

	1−3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

- 1−3p
- 2p
- 2p
- 2p

Finding mixed strategy equilibrium

1− 3p = 2p

	1−3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

- 1−3p
- 2p
- 2p
- 2p

Finding mixed strategy equilibrium

1− 3p = 2p

p = 1/5 = 0.2

	1−3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

- 1−3p
- 2p
- 2p
- 2p

Finding mixed strategy equilibrium

1− 3p = 2p

p = 1/5 = 0.2

0.4

	1−3p	p	p	p
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

- 1−3p
- 2p
- 2p
- 2p

Finding mixed strategy equilibrium

We have found Black's equilibrium behavior and the winning rate of Red

Probability distribution of Black's action

		0.2	0.2	0.2
	K	1	2	3
K	1, 0	0, 1	0, 1	0, 1
1	0, 1	0, 1	1, 0	1, 0
2	0, 1	1, 0	0, 1	1, 0
3	0, 1	1, 0	1, 0	0, 1

Winning rate of Red

Probability distribution of Black's action

	0.4	0.2	0.2	0.2	
	K	1	2	3	Winning rate of Red
K	1, 0	0, 1	0, 1	0, 1	
1	0, 1	0, 1	1, 0	1, 0	
2	0, 1	1, 0	0, 1	1, 0	
3	0, 1	1, 0	1, 0	0, 1	

Probability distribution of Black's action

	0.4	0.2	0.2	0.2	
	K	1	2	3	Winning rate of Red
K	1, 0	0, 1	0, 1	0, 1	➡ 0.4
1	0, 1	0, 1	1, 0	1, 0	
2	0, 1	1, 0	0, 1	1, 0	
3	0, 1	1, 0	1, 0	0, 1	

Probability distribution of Black's action

	0.4	0.2	0.2	0.2	
	K	1	2	3	Winning rate of Red
K	1, 0	0, 1	0, 1	0, 1	➡ 0.4
1	0, 1	0, 1	1, 0	1, 0	➡ 0.4
2	0, 1	1, 0	0, 1	1, 0	➡ 0.4
3	0, 1	1, 0	1, 0	0, 1	➡ 0.4

Finding mixed strategy equilibrium

Similar calculation shows .....

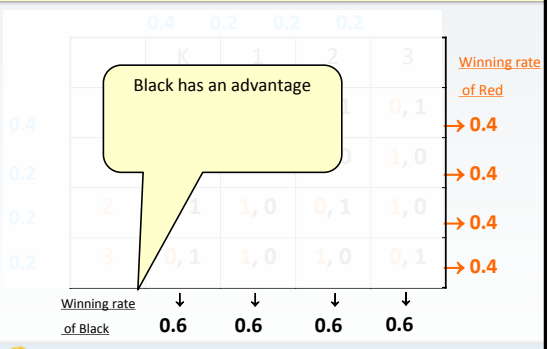
Equilibrium of the card game

		0.4	0.2	0.2	0.2	
		K	1	2	3	Winning rate of Red
0.4	K	1, 0	0, 1	0, 1	0, 1	→ 0.4
0.2	1	0, 1	0, 1	1, 0	1, 0	→ 0.4
0.2	2	0, 1	1, 0	0, 1	1, 0	→ 0.4
0.2	3	0, 1	1, 0	1, 0	0, 1	→ 0.4
Winning rate of Black		↓ 0.6	↓ 0.6	↓ 0.6	↓ 0.6	

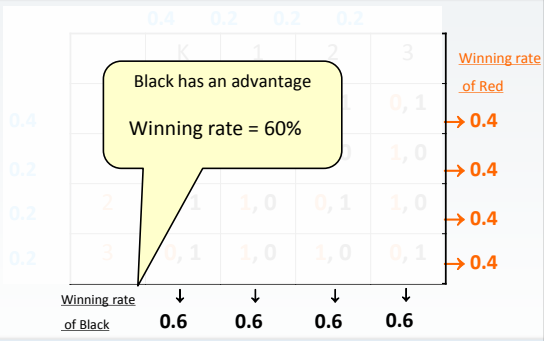
Prediction #1

		0.4	0.2	0.2	0.2	
		K	1	2	3	Winning rate of Red
0.4	K	1, 0	0, 1	0, 1	0, 1	→ 0.4
0.2	1	0, 1	0, 1	1, 0	1, 0	→ 0.4
0.2	2	0, 1	1, 0	0, 1	1, 0	→ 0.4
0.2	3	0, 1	1, 0	1, 0	0, 1	→ 0.4
Winning rate of Black		↓ 0.6	↓ 0.6	↓ 0.6	↓ 0.6	

Prediction #1



Prediction #1



Prediction #2

		0.4 K	0.2 1	0.2 2	0.2 3
0.4	K				
0.2	1				
0.2	2				
0.2	3				

Prediction #2

		0.4 K	0.2 1	0.2 2	0.2 3
0.4	K				
0.2	1				
0.2	2				
0.2	3				

K is played most often (0.4)

3.2 Digression: How you played the card game

3.2 Digression: How you played the card game

and addressing the concerns about game theory

The card game we played is based on

The card game we played is based on

O'Neill, B.(1987)

"Nonmetric Test of the Minimax Theory of Two-Person Zerosum Games",

Proceedings of the National Academy of Sciences, U.S.A. Vol 84, pp. 2106-2109

Do people always behave in the same way in the card game?

Do people always behave in the same way in the card game?

		pairs	repetitions
1987	O'Neil	25	105
2004	Students, The University of Tokyo	29	105
2006a	Students, The University of Tokyo	30	105
2006b	High school students	95	105
2009	Students, The University of Tokyo	102	105
2014a	Students, The University of Tokyo	105	105
2014b	High school students	47	30

Do people always behave in the same way in the card game?

		pairs	repetitions
1987	O'Neil	25	105
2004	Students, The University of Tokyo	29	105
2006a	Students, The University of Tokyo	30	105
2006b	High school students	95	105
2009	Students, The University of Tokyo	102	105
2014a	Students, The University of Tokyo	105	105
2014b	High school students	47	30

Experiments by Kandori

Winning Rates

	Red	Black
Equilibrium	0.4	0.6
1987		
2004		
2006a		
2006b		
2009		
2014a		
2014b		

Winning Rates

	Red	Black
Equilibrium	0.4	0.6
1987	0.408	0.592
2004		
2006a		
2006b		
2009		
2014a		
2014b		

Winning Rates

	Red	Black
Equilibrium	0.4	0.6
1987	0.408	0.592
2004	0.414	0.586
2006a		
2006b		
2009		
2014a		
2014b		

Winning Rates

	Red	Black
Equilibrium	0.4	0.6
1987	0.408	0.592
2004	0.414	0.586
2006a	0.39	0.61
2006b	0.402	0.598
2009	0.409	0.591
2014a	0.414	0.586
2014b	0.416	0.584

Frequencies of cards

	Red				Black			
	K	1	2	3	K	1	2	3
Equilibrium	0.4	0.2	0.2	0.2	0.4	0.2	0.2	0.2
1987								
2004								
2006a								
2006b								
2009								
2014a								
2014b								

Frequencies of cards

	Red				Black			
	K	1	2	3	K	1	2	3
Equilibrium	0.4	0.2	0.2	0.2	0.4	0.2	0.2	0.2
1987	0.36	0.22	0.22	0.2	0.43	0.23	0.18	0.17
2004								
2006a								
2006b								
2009								
2014a								
2014b								

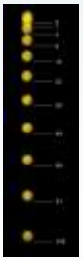
Frequencies of cards

	Red				Black			
	K	1	2	3	K	1	2	3
Equilibrium	0.4	0.2	0.2	0.2	0.4	0.2	0.2	0.2
1987	0.36	0.22	0.22	0.2	0.43	0.23	0.18	0.17
2004	0.32	0.23	0.22	0.24	0.41	0.23	0.18	0.19
2006a	0.39	0.21	0.2	0.21	0.42	0.21	0.17	0.19
2006b	0.32	0.23	0.22	0.23	0.37	0.21	0.2	0.21
2009	0.39	0.21	0.2	0.2	0.42	0.2	0.19	0.19
2014a	0.38	0.21	0.2	0.21	0.44	0.2	0.17	0.18
2014b	0.41	0.2	0.19	0.2	0.42	0.21	0.18	0.19

With those results in the card game, let us address

With those results in the card game, let us address

the concerns about a mathematical theory of human behavior

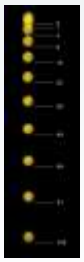


Credit: MichaelMaggs, CC BY-SA 3.0

With those results in the card game, let us address

the concerns about a mathematical theory of human behavior

(Lecture 1.4)



Credit: MichaelMaggs, CC BY-SA 3.0

(Valid) concerns about game theory

#1 Free will defeats any attempt to predict human behavior by a math formula

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#2 You can just ask, “why did you do that?”

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#2 You can just ask, “why did you do that?”

No need for mathematical model

#3 I’ve never heard it works.

(Valid) concerns about game theory

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	Red				Black			
	K	1	2	3	K	1	2	3
Equilibrium	0.4	0.2	0.2	0.2	0.4	0.2	0.2	0.2
data	0.36	0.22	0.22	0.2	0.43	0.23	0.18	0.17

(Valid) concerns about game theory

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	Red				Black			
	K	1	2	3	K	1	2	3
Equilibrium	0.4	0.2	0.2	0.2	0.4	0.2	0.2	0.2
data	0.36	0.22	0.22	0.2	0.43	0.23	0.18	0.17

You are attracted to the behavior that is best for you

(Valid) concerns about game theory

#2 You can just ask, “why did you do that?”

	Red				Black			
	K	1	2	3	K	1	2	3
Equilibrium	0.4	0.2	0.2	0.2	0.4	0.2	0.2	0.2
data	0.36	0.22	0.22	0.2	0.43	0.23	0.18	0.17

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You are often unable to articulate why you took certain behavior

	Red				Black			
	K	1	2	3	K	1	2	3
Equilibrium	0.4	0.2	0.2	0.2	0.4	0.2	0.2	0.2
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data	0.36	0.22	0.22	0.2	0.43	0.23	0.18	0.17

Game theory can uncover the mechanism operating behind your instinctive behavior

(Valid) concerns about game theory

#3 I've never heard it works.

Stan Ulam

(Valid) concerns about game theory

#3 I've never heard it works.

Stan Ulam

Paul Samuelson

Credit: Bender235, CC BY

(Valid) concerns about game theory

#3 I've never heard it works.

Name me one proposition in all of the social sciences which is **both true and non-trivial**.

Stan Ulam

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Credit: Bender235, CC BY

(Valid) concerns about game theory

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Name me one proposition in all of the social sciences which is **both true and non-trivial**.

Stan Ulam

This was a test that I always failed.

Paul Samuelson

Credit: Bender235, CC BY

(Valid) concerns about game theory

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Name me one proposition in all of the social sciences which is **both true and non-trivial**.

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(Valid) concerns about game theory

#3 I've never heard it works.

Name me one proposition in all of the social sciences which is **both true and non-trivial**.

Stan Ulam

Game theory now shows that there are actually such propositions.

3.3 "Payoffs" in a game: what exactly are those numbers?

3, 2	0, 0
0, 0	3, 2

Payoffs in a game

3, 2	0, 0
0, 0	3, 2

Payoffs in a game

3, 2	0, 0
0, 0	3, 2

Prize or profit ($) ?

Payoffs in a game

3, 2	0, 0
0, 0	3, 2

Prize or profit ($) ?
"Benefits" ?

Payoffs in a game

3, 2	0, 0
0, 0	3, 2

Prize or profit ($) ?
"Benefits" ?
"Satisfaction" ?

Payoffs in a game

3, 2	0, 0
0, 0	3, 2

Prize or profit ($) ?
"Benefits" ?
"Satisfaction" ?

I will explain what they are

Payoffs in a game

3, 2	0, 0
0, 0	3, 2

Prize or profit ($) ?
"Benefits" ?
"Satisfaction" ?

I will explain what they are (should be)

When there is no uncertainty

When there is no uncertainty

- No random events

When there is no uncertainty

- No random events (weather, stock price ....)

When there is no uncertainty

- No random events (weather, stock price ….)
- No random behavior

When there is no uncertainty

- No random events (weather, stock price ….)
- No random behavior ("mixed strategies")

When there is no uncertainty

- No random events (weather, stock price ….)
- No random behavior ("mixed strategies")

To determine a player's payoffs,

When there is no uncertainty

- No random events (weather, stock price ….)
- No random behavior ("mixed strategies")

To determine a player's payoffs,
you can assign larger numbers for better outcomes for the player

When there is no uncertainty

- No random events (weather, stock price ….)
- No random behavior ("mixed strategies")

To determine a player's payoffs,
you can assign larger numbers for better outcomes for the player

Best outcome	Worse outcome	Worst outcome

When there is no uncertainty

- No random events (weather, stock price ….)
- No random behavior ("mixed strategies")

To determine a player's payoffs,
you can assign larger numbers for better outcomes for the player

Best outcome	Worse outcome	Worst outcome
3	2	1

When there is no uncertainty

- No random events (weather, stock price ….)
- No random behavior (“mixed strategies”)

To determine a player’s payoffs,
you can assign larger numbers for better outcomes for the player

There are many ways ……

Best outcome	Worse outcome	Worst outcome
3	2	1

When there is no uncertainty

To determine a player’s payoffs,
you can assign larger numbers for better outcomes for the player

There are many ways ……

Best outcome	Worse outcome	Worst outcome
100	8	-1

When there is no uncertainty

To determine a player’s payoffs,
you can assign larger numbers for better outcomes for the player

Best outcome	Worse outcome	Worst outcome
20	19	0

When there is no uncertainty

To determine a player’s payoffs,
you can assign larger numbers for better outcomes for the player

Any such assignment is fine….

Best outcome	Worse outcome	Worst outcome
20	19	0

When there is no uncertainty

To determine a player’s payoffs,
you can assign larger numbers for better outcomes for the player

Any such assignment is fine…. because

Best outcome	Worse outcome	Worst outcome
20	19	0

When there is no uncertainty

Best outcome	Worse outcome	Worst outcome
20	19	0

When there is no uncertainty

Best outcome	Worse outcome	Worst outcome
20	19	0

Payoff maximization means ....

When there is no uncertainty

... choosing better outcome

Best outcome	Worse outcome	Worst outcome
20	19	0

Payoff maximization means ....

When there is no uncertainty

To determine a player's payoffs,
you can assign larger numbers for better outcomes for the player

... choosing better outcome

Best outcome	Worse outcome	Worst outcome
20	19	0

Payoff maximization means ....

When there is some uncertainty,

When there is some uncertainty,
you should assign payoffs more carefully.

When uncertainty is present

When uncertainty is present

- Random events (weather, stock price ….)

When uncertainty is present

- Random events (weather, stock price ….)
- Random behavior (“mixed strategies”)

When uncertainty is present

- Random events (weather, stock price ….)
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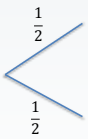

When uncertainty is present

- Random events (weather, stock price ….)
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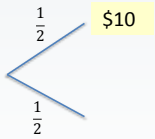

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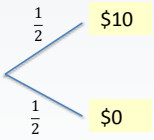

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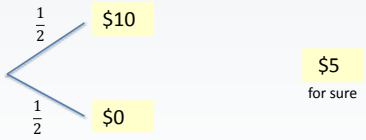

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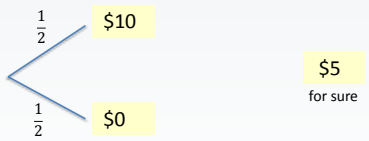
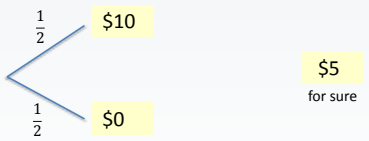


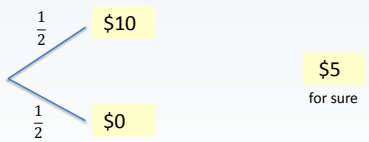
Which do you like?

When uncertainty is present



When uncertainty is present

On average, you can get $5



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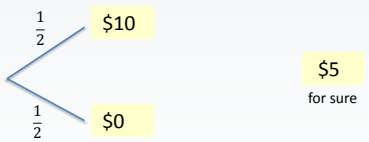
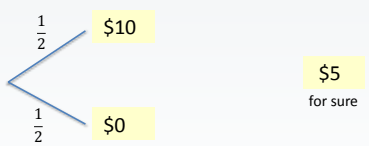


Risky choice

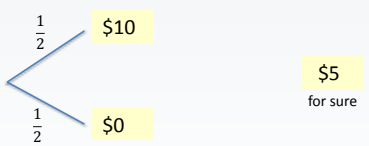
When uncertainty is present

On average, you can get $5



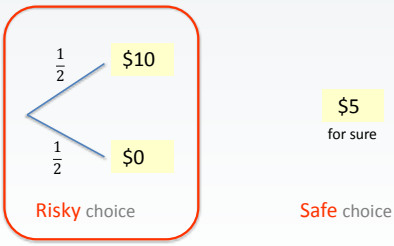
Risky choice Safe choice

When uncertainty is present



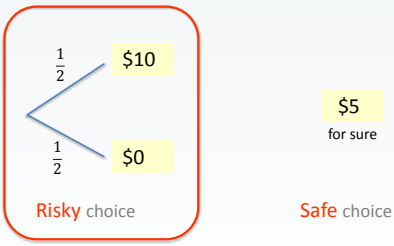
Risky choice Safe choice

When uncertainty is present

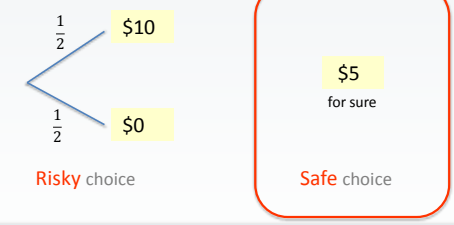


When uncertainty is present

Risk-loving player chooses this

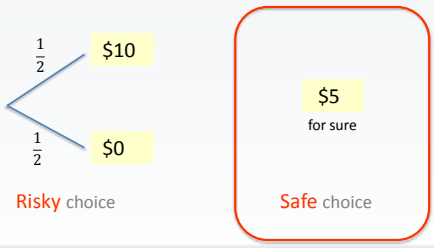


When uncertainty is present



When uncertainty is present

Risk-averse player chooses this



When uncertainty is present

How do we represent a player's attitude towards risk?

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First, you assume player gets "utility" from outcome

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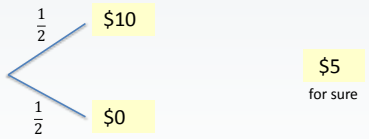

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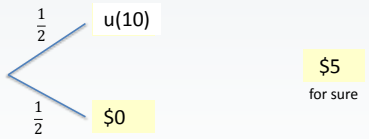

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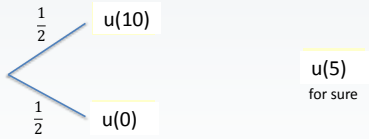

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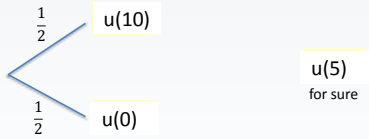
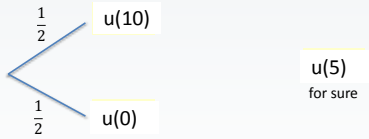


When uncertainty is present

How do we represent a player's attitude towards risk?

Then you assume player maximizes expected utility



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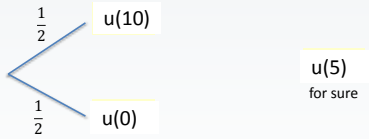
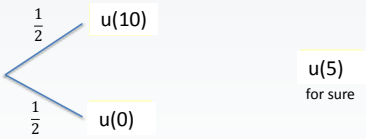


$\frac{1}{2}u(10) + \frac{1}{2}u(0)$

When uncertainty is present

How do we represent a player's attitude towards risk?

Then you assume player maximizes expected utility

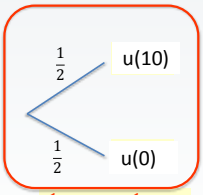


u(5)
for sure

$\frac{1}{2}u(10)+\frac{1}{2}u(0)$ $u(5)$

When uncertainty is present

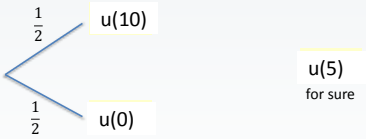
Risk-loving player chooses this



u(5)
for sure

$\frac{1}{2}u(10)+\frac{1}{2}u(0)$ $>$ $u(5)$

When uncertainty is present



u(5)
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$\frac{1}{2}u(10)+\frac{1}{2}u(0)$ $u(5)$

When uncertainty is present

u(5)
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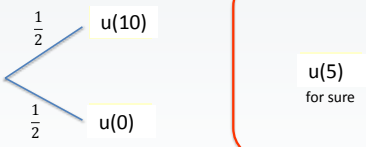
$\frac{1}{2}u(10)+\frac{1}{2}u(0)$ $<$ $u(5)$

When uncertainty is present

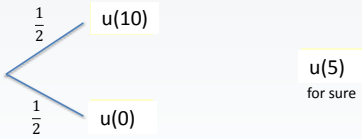
Risk-averse player chooses this



u(5)
for sure

$\frac{1}{2}u(10)+\frac{1}{2}u(0)$ $<$ $u(5)$

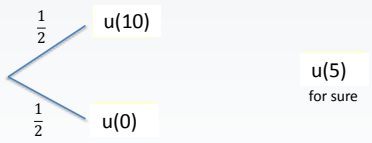
When uncertainty is present



u(5)
for sure

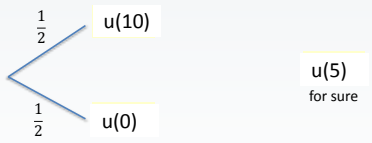
When uncertainty is present

(1) A player's payoff = "utility" of outcome



When uncertainty is present

(1) A player's payoff = "utility" of outcome
(2) We assume players maximize expected utility (payoff)



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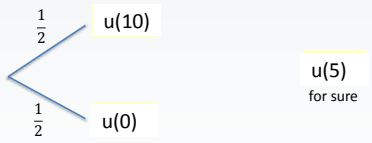
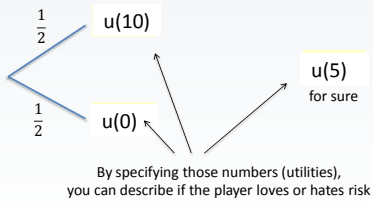


When uncertainty is present

(1) A player's payoff = "utility" of outcome
(2) We assume players maximize expected utility (payoff)



By specifying those numbers (utilities),
you can describe if the player loves or hates risk

3.4 What does it mean that a player is rational?

- "Rationality" is a vague concept

- "Rationality" is a vague concept
  People use this term to mean different things

- "Rationality" is a vague concept
  People use this term to mean different things
- I will present a clear-cut definition of rationality that game theorists use

What does it mean that A is rational?

Roulette  → 
A

What does it mean that A is rational?

Roulette  → 
A

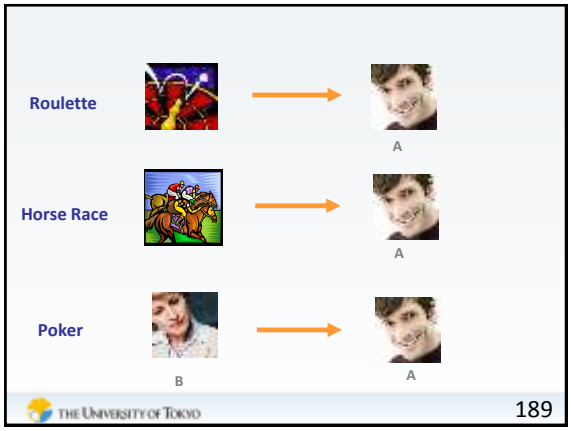
Horse Race  → 
A

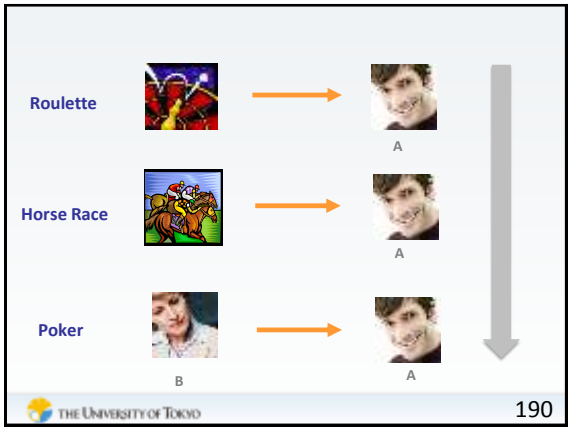
What does it mean that A is rational?

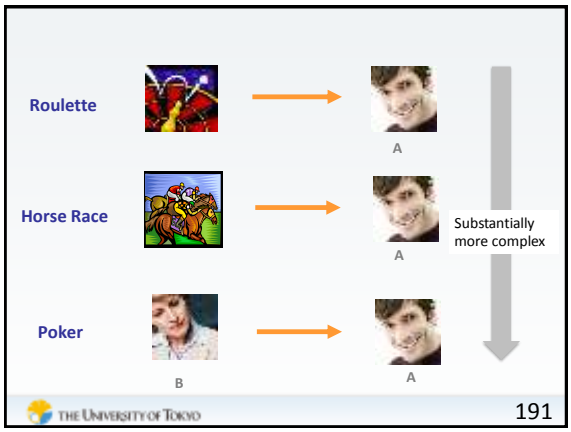
Roulette  → 
A

Horse Race  → 
A

Poker  → 
B A

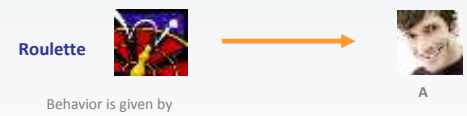






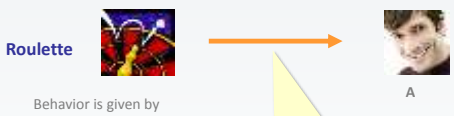
Rationality for roulette

Rationality for roulette



Behavior is given by
objective probabilities

Rationality for roulette



Behavior is given by
objective probabilities

Maximizing
expected utility

Rationality for horse race

Horse Race  

A

Rationality for horse race

Horse Race  

No objective probabilities

A

Rationality for horse race

Horse Race  

No objective probabilities

A

Assign
subjective probabilities

Rationality for horse race

Horse Race  

No objective probabilities

A

Assign
subjective probabilities

Maximizing
expected utility

A Definition of Rationality

An agent is rational if he/she

A Definition of Rationality

An agent is rational if he/she

- is aware of all possible events,

A Definition of Rationality

An agent is rational if he/she

- is aware of **all possible events**,
- assigns (**objective or subjective**) **probabilities** over events, and

A Definition of Rationality

An agent is rational if he/she

- is aware of **all possible events**,
- assigns (**objective or subjective**) **probabilities** over events, and
- maximizes **expected utility.**

Let us apply this definition to human interaction

Let us apply this definition to human interaction

Poker

B

A

Let us apply this definition to human interaction

Poker

B

A

A's rationality

Let us apply this definition to human interaction

Poker

B

A

Behavior is not given

A's rationality

Let us apply this definition to human interaction

Poker

B

A

Behavior is not given

Assign subjective probabilities

A's rationality

Let us apply this definition to human interaction

Poker

B

A

Behavior is not given

Assign subjective probabilities

A's belief

A's rationality

Let us apply this definition to human interaction

Poker

B

A

Behavior is not given

Maximization of expected utility

Assign subjective probabilities

A's belief

A's rationality

B's behavior

A

Maximization of expected utility

A's belief

A's rationality

B's behavior

A

Maximization of expected utility

A's belief

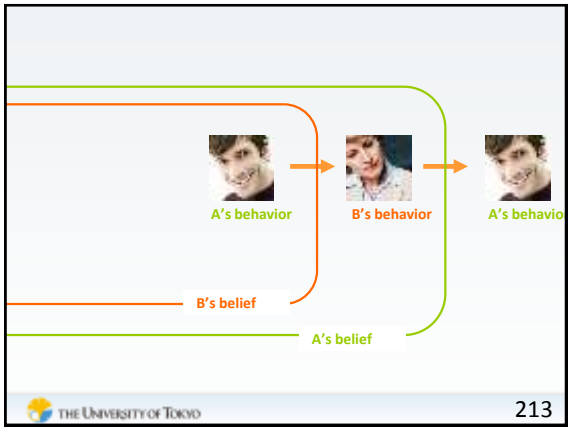
A's rationality

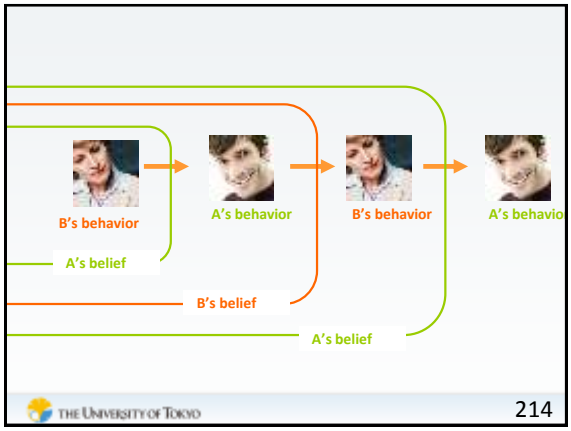
The story does not end here, because B is also rational

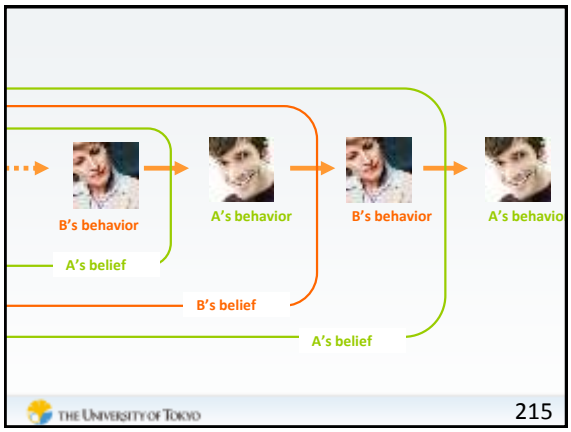
B's behavior

A's behavio

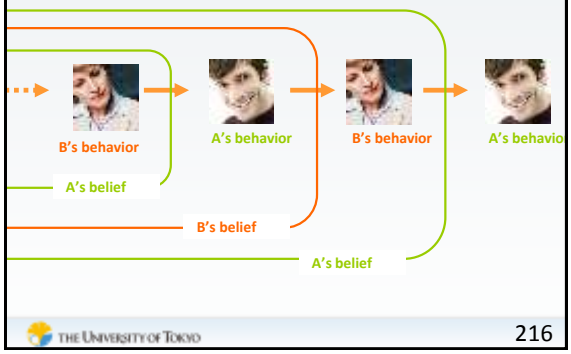
A's belief



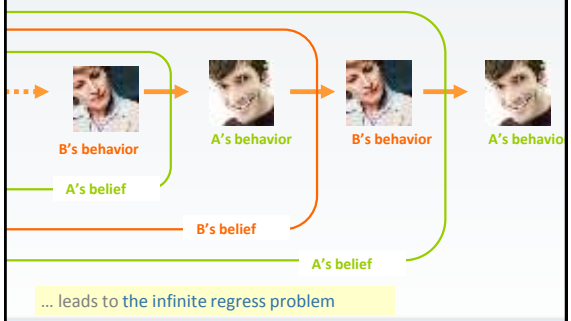




Applying the definition of rationality to human interaction ...



Applying the definition of rationality to human interaction ...



... leads to the infinite regress problem

In this week, we will see that the hierarchy of sophisticated reasoning

In this week, we will see that the hierarchy of sophisticated reasoning

my belief about your belief about my belief….

In this week, we will see that the hierarchy of sophisticated reasoning

my belief about your belief about my belief….

can sometimes leads to a surprising outcome

3.5 Domination: Strategies that are "obviously good or bad"

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate		
Defect		

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
Defect		-10, -10

Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	
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Prisoner's Dilemma

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Cooperate	-1, -1	-15, 0
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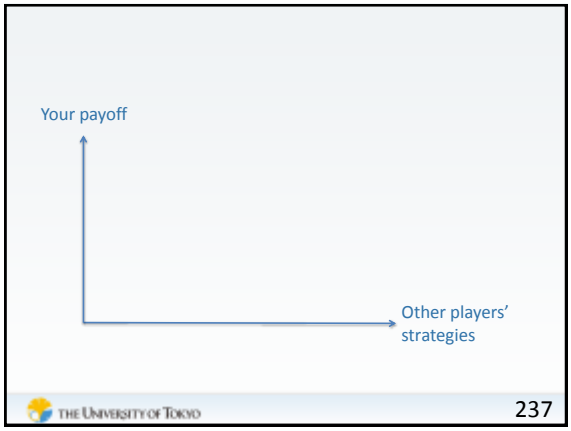
Defection is always the best, irrespective of the opponent's action

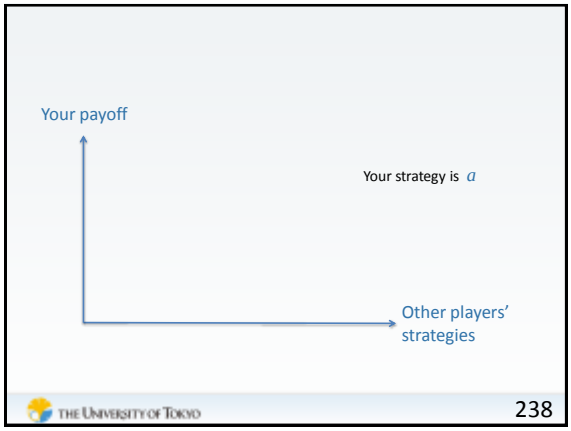
Prisoner's Dilemma

1 \ 2	Cooperate	Defect
Cooperate	-1, -1	-15, 0
Defect	0, -15	-10, -10

Defection is always the best, irrespective of the opponent's action

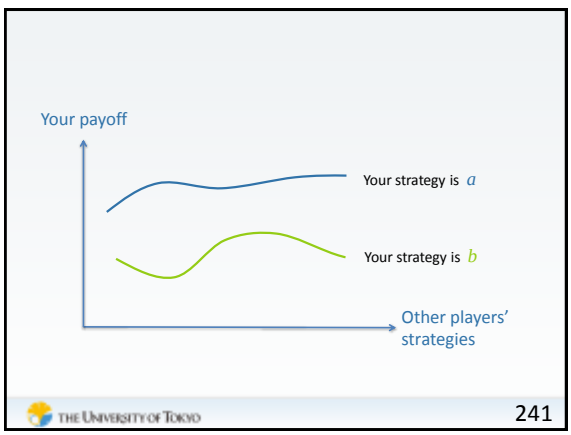
We say that Defection dominates Cooperation


Your payoff
Other players' strategies


Your payoff
Your strategy is a
Other players' strategies


Your payoff
Your strategy is a
Other players' strategies


Your payoff
Your strategy is a
Your strategy is b
Other players' strategies

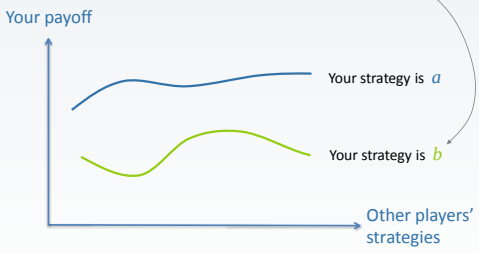

Your payoff
Your strategy is a
Your strategy is b
Other players' strategies


Your payoff
Your strategy is a
Your strategy is b
Other players' strategies

Strategy *a* strictly dominates strategy *b*

Strategy *a* weakly dominates strategy *b*

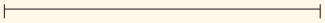
A rational player never chooses a strictly dominated strategy



An example of dominated strategy

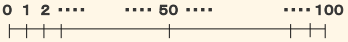
Hotelling's location game

Hotelling's location game

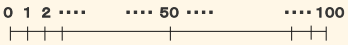


Hotelling's location game (with finitely many locations)

Hotelling's location game (with finitely many locations)

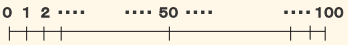


Hotelling's location game (with finitely many locations)



- There are 101 slots for ice cream vendors A and B

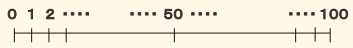
Hotelling's location game (with finitely many locations)



- There are 101 slots for ice cream vendors A and B

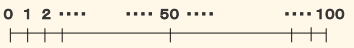
(Vendors can choose the same slot, occupying each side of the street)

Hotelling's location game (with finitely many locations)



- There are 101 slots for ice cream vendors A and B
  (Vendors can choose the same slot, occupying each side of the street)
- Customers are uniformly distributed

Hotelling's location game (with finitely many locations)



- There are 101 slots for ice cream vendors A and B
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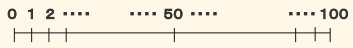
- Customers are uniformly distributed
  Customer goes to the closest vendor

Hotelling's location game (with finitely many locations)



- There are 101 slots for ice cream vendors A and B
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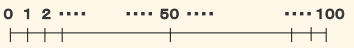
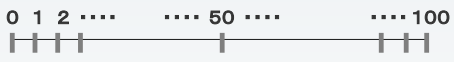
  (The same location → vendors equally split the customers)

Hotelling's location game (with finitely many locations)



- There are 101 slots for ice cream vendors A and B
  (Vendors can choose the same slot, occupying each side of the street)
- Customers are uniformly distributed
  Customer goes to the closest vendor
  (The same location → vendors equally split the customers)
- Payoff = the number of customers



The end-slots (0 and 100) are "bad" locations,

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more precisely …

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more precisely …

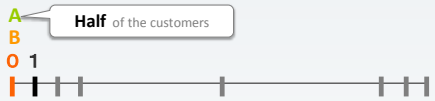
0 is strictly dominated by 1

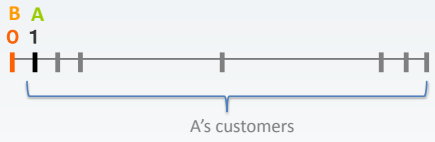
0 is strictly dominated by 1

0 is strictly dominated by 1

No matter where B is located, for A, 1 is always better than 0

First, consider the case where B is at 0



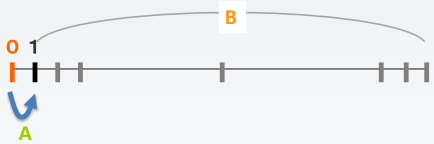


1 is better than 0 for A

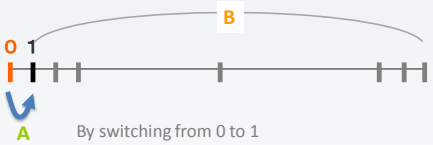
Second, consider the case where B is here

Second, consider the case where B is here

Second, consider the case where B is here

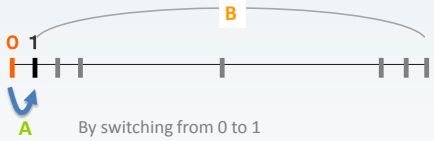


Second, consider the case where B is here



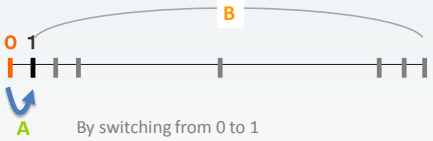
By switching from 0 to 1

Second, consider the case where B is here



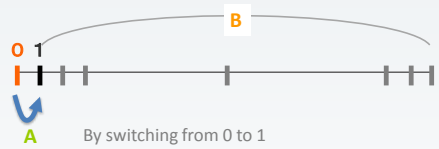
By switching from 0 to 1
A moves closer to B

Second, consider the case where B is here



By switching from 0 to 1
A moves closer to B
and can steal customers from B

Second, consider the case where B is here



By switching from 0 to 1
A moves closer to B
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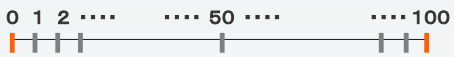
1 is better than 0 for A

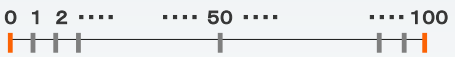
No matter where B is located, for A, 1 is always better than 0

No matter where B is located, for A, 1 is always better than 0

No matter where B is located, for A, 1 is always better than 0

0 is strictly dominated by 1





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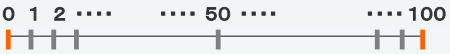
strictly dominated



Rational players never choose those locations

3.6 Common knowledge of rationality

(1) Players are rational.

(1) Players are rational.

(2) Players know that they are rational.

(1) Players are rational.

(2) Players know that they are rational.

(3) Players know that they know that they are rational

(1) Players are rational.

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(1) Players are rational.

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(3) Players know that they know that they are rational

Are there any differences between (2), (3), ... ?

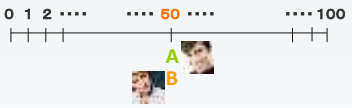
Location game with finitely many slots

Location game with finitely many slots

Nash equilibrium

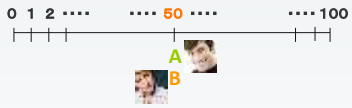
Location game with finitely many slots

Nash equilibrium



Location game with finitely many slots

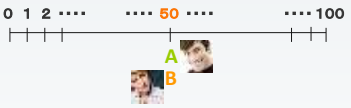
Nash equilibrium



"Hyper rational situation"

Location game with finitely many slots

Nash equilibrium

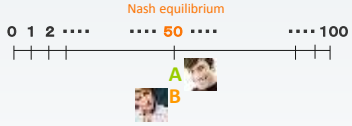


"Hyper rational situation"

- Unlimited ability of sophisticated reasoning

Location game with finitely many slots

Nash equilibrium

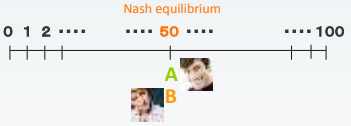


"Hyper rational situation"

- Unlimited ability of sophisticated reasoning
- Players' rationality is "completely known"

Location game with finitely many slots

Nash equilibrium



"Hyper rational situation"

- Unlimited ability of sophisticated reasoning
- Players' rationality is "completely known"

⇓

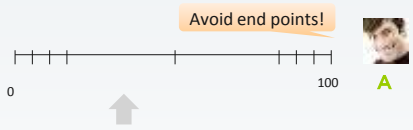
A and B choose the Nash equilibrium location

Previous lecture showed ....

Previous lecture showed ....

If A is rational...

Previous lecture showed ....



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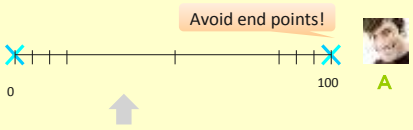

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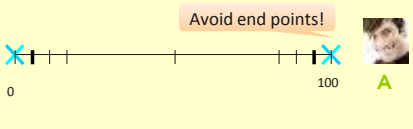
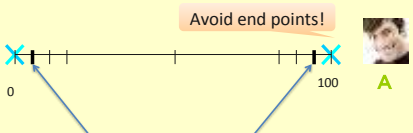
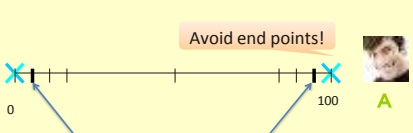




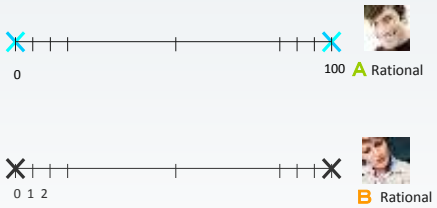
However, those points (1 and 99) might be a good choice for A,

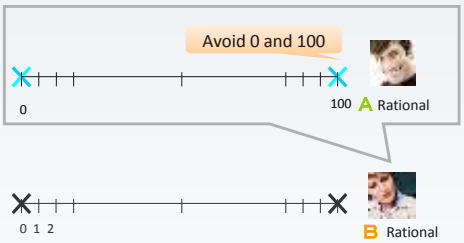


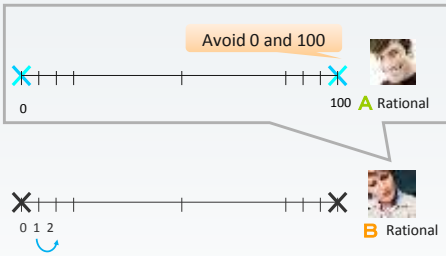
However, those points (1 and 99) might be a good choice for A,

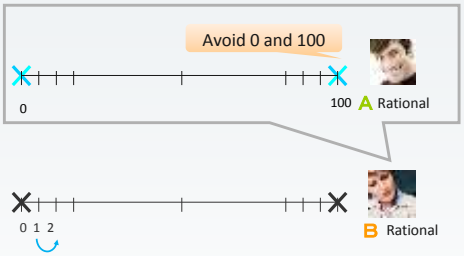
If B is stupid and chooses 0 or 100

Now suppose B knows that A is rational.

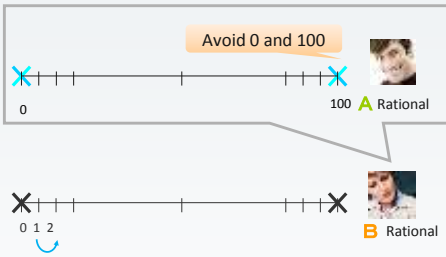








B knows that A is rational and therefore A won't choose 0 or 100



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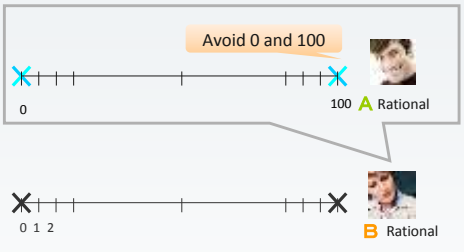
Location 2 strictly dominates location 1



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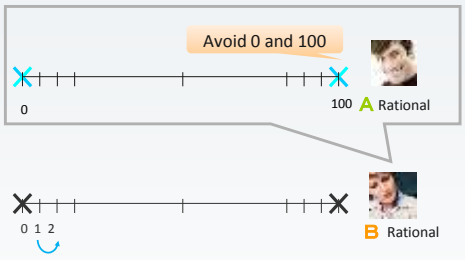
Location 2 strictly dominates location 1
No matter where A is located between 1 and 99, 2 is always better than 1



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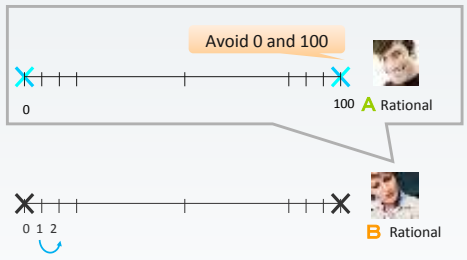

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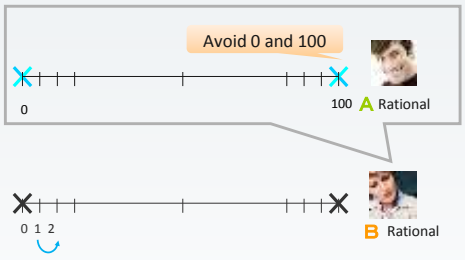
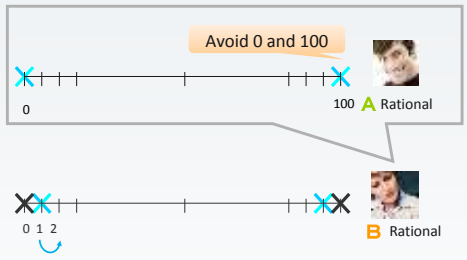
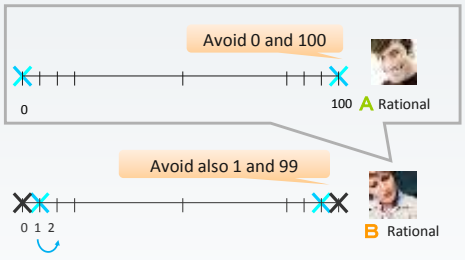
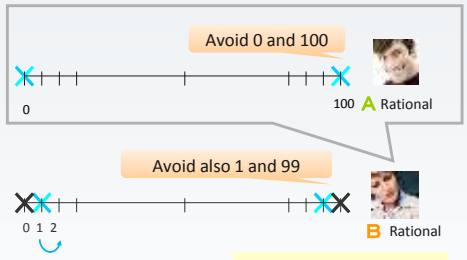
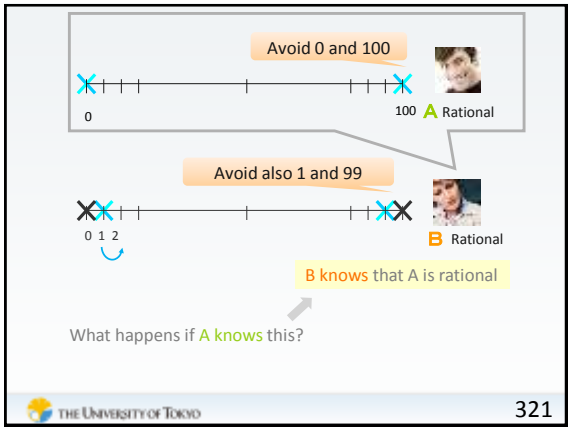
Location 2 strictly dominates location 1
No matter where A is located between 1 and 99, 2 is always better than 1
Same logic as in the previous lecture

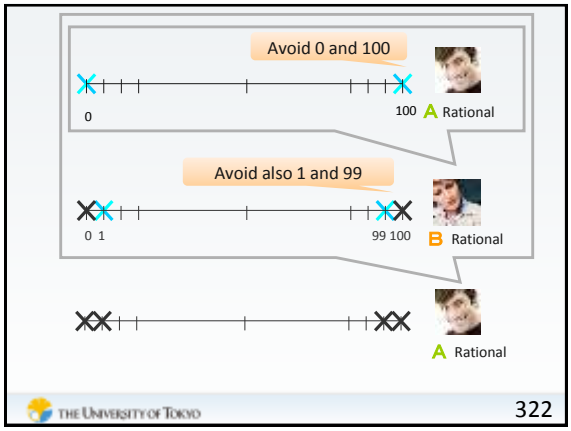









Avoid 0 and 100
0
100
A Rational
Avoid also 1 and 99
0 1 2
B Rational
B knows that A is rational
What happens if A knows this?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99 100
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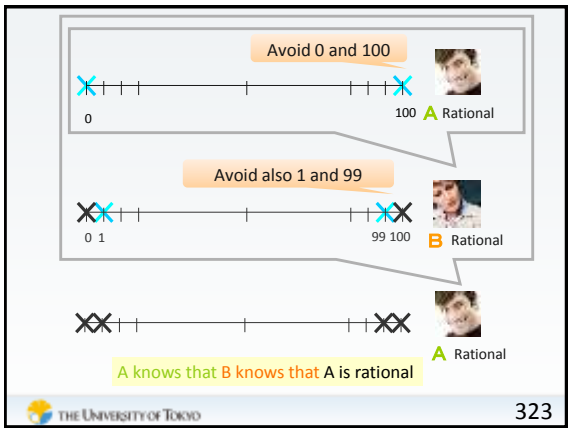
Avoid 0 and 100
0
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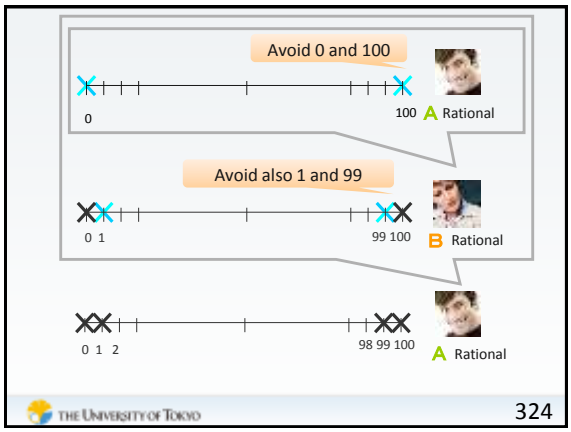
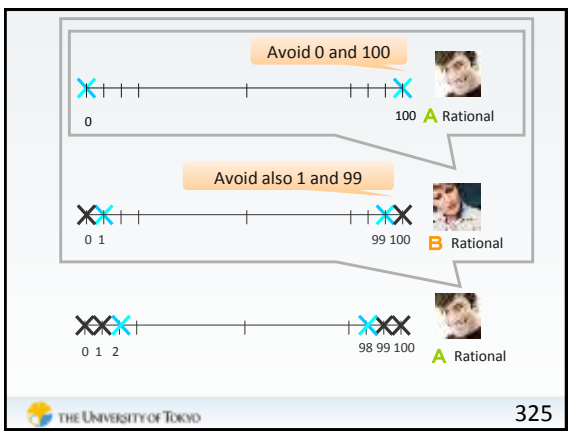
0 1
99 100
B Rational
A Rational
A knows that B knows that A is rational


Avoid 0 and 100
0
100
A Rational
Avoid also 1 and 99
0 1
99 100
B Rational
0 1 2
98 99 100
A Rational


Avoid 0 and 100
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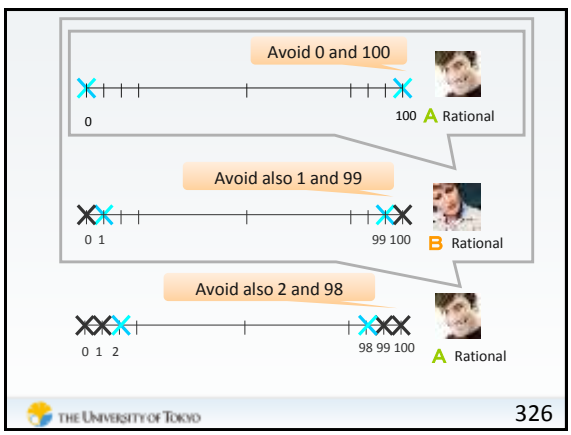

Avoid 0 and 100
0
100
A Rational
Avoid also 1 and 99
0 1
99 100
B Rational
Avoid also 2 and 98
0 1 2
98 99 100
A Rational

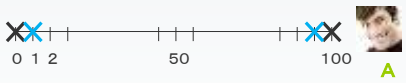
If players know that they know that … they are rational

If players know that they know that … they are rational

? times

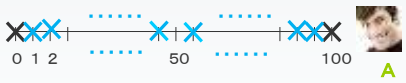
If players know that they know that … they are rational

? times



If players know that they know that … they are rational

? times



If players know that they know that … they are rational

? times

The choice is 50!

If players know that they know that … they are rational

49 times

The choice is 50!

Summary

Summary

If players know that they know that … they are rational

many times

Summary

If players know that they know that … they are rational

many times

Summary

If players know that they know that … they are rational

many times

Players play Nash equilibrium in the location game

Players know that they know that … they are rational

Players know that they know that … they are rational

Any number of times

Players know that they know that … they are rational

Any number of times

"Common knowledge of rationality"

Players know that they know that … they are rational

Any number of times

"Common knowledge of rationality"

Sometimes common knowledge of rationality leads to a Nash equilibrium

3.7 Low rationality:

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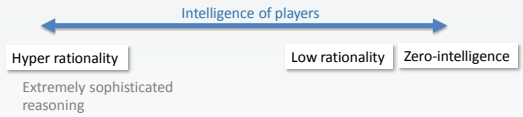
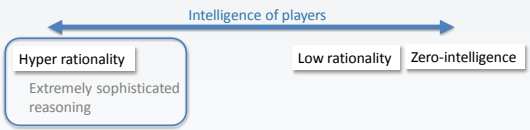


Extremely sophisticated reasoning

3.7 Low rationality:



Extremely sophisticated reasoning

3.7 Low rationality:

Extremely sophisticated reasoning

3.7 Low rationality: What happens if players are not very smart?

Extremely sophisticated reasoning

Outcome may be chaotic

Outcome may be chaotic

or

Outcome may be chaotic

or

Humans tend to make similar “mistakes”

Outcome may be chaotic

or

Humans tend to make similar “mistakes”

- Finding patterns of low-rationality behavior

What happens if players are not very smart?

Outcome may be chaotic

or

Humans tend to make similar "mistakes"

- Finding patterns of low-rationality behavior
  - Field data

What happens if players are not very smart?

Outcome may be chaotic

or

Humans tend to make similar "mistakes"

- Finding patterns of low-rationality behavior
  - Field data
  - Lab experiments

What happens if players are not very smart?

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- Finding patterns of low-rationality behavior
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Behavioral Game Theory/Economics

What happens if players are not very smart?

Outcome may be chaotic

or

Humans tend to make similar "mistakes"

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⬇

Over time, players may find better ways to play the game

What happens if players are not very smart?

Over time, players may find better ways to play the game

What happens if players are not very smart?

Over time, players may find better ways to play the game

- Accumulating experience in the same game or similar games

What happens if players are not very smart?

Over time, players may find better ways to play the game

- Accumulating experience in the same game or similar games
- Observing how others play in the same game or similar games

What happens if players are not very smart?

Over time, players may find better ways to play the game

What happens if players are not very smart?

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- Trying various actions

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"Trial and error adjustment"

Let us see how it works in the location game

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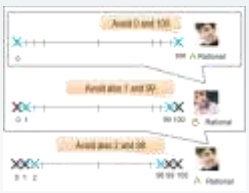

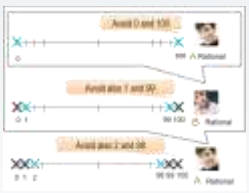

Previous lecture showed that players come to play Nash equilibrium by "hyper rational" reasoning

Now I show that low-rationality adjustment provides the same outcome

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Eventually, he finds out that **end points are bad locations**

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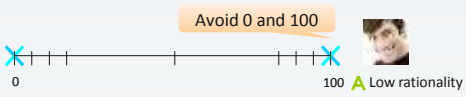
Strictly dominated strategies



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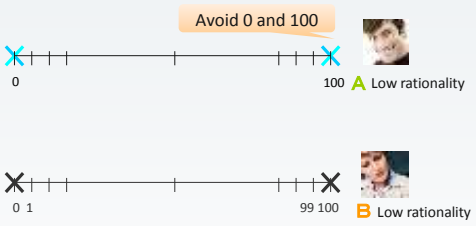
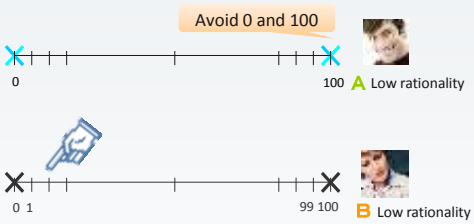
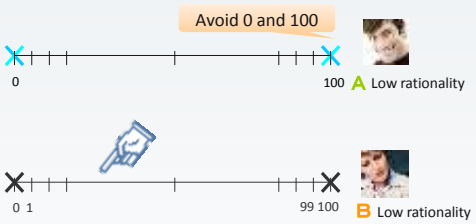
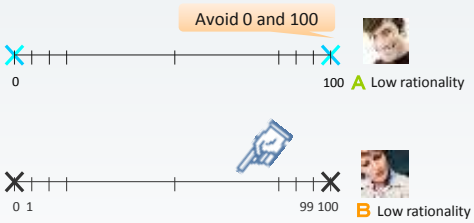
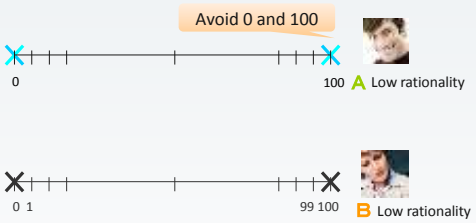
Strictly dominated strategies

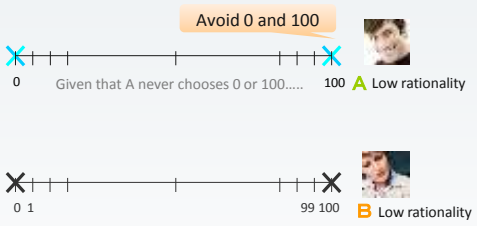


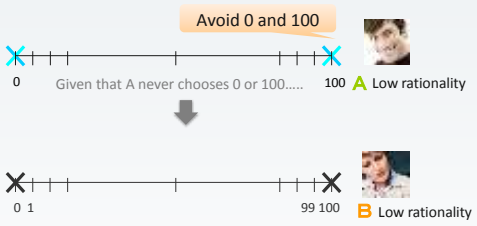












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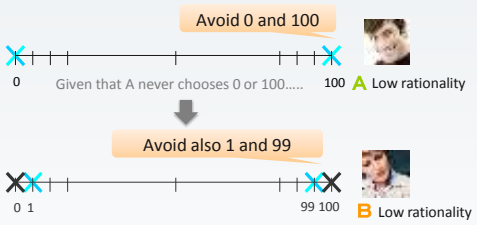

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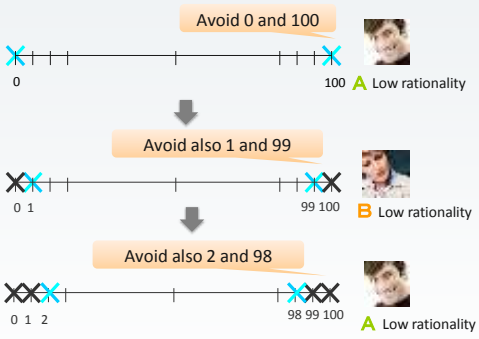

Eventually, players learn to play Nash equilibrium

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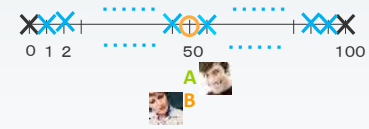

Summary

Summary

- In some games, repeated elimination of **obviously bad strategies**

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Hyper rationality — Low rationality — Zero-intelligence

Extremely sophisticated reasoning

Summary

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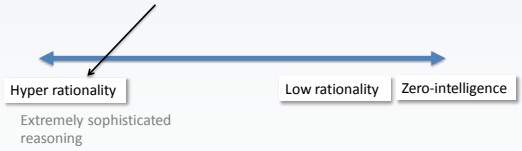

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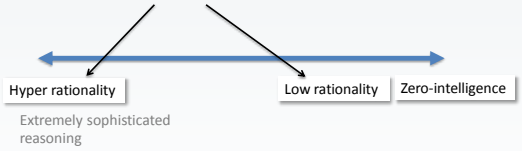

Extremely sophisticated reasoning

3.8 Game theory under zero-intelligence:

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Biological evolution

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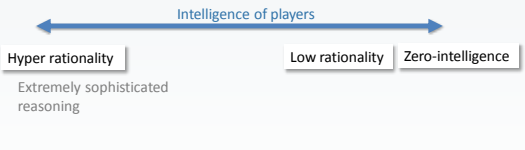

Extremely sophisticated reasoning

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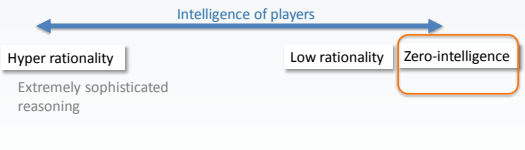

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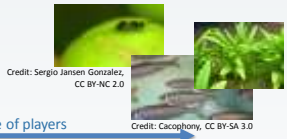

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Credit: Cacophony, CC BY-SA 3.0

Intelligence of players

Hyper rationality

Extremely sophisticated reasoning

Low rationality

Zero-intelligence

Basic Idea

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- Switching to better action

Basic Idea

Over time, players may find better ways to play the game

- Trying various actions
- Switching to better action
- Imitating successful behavior

Basic Idea

Over time, players may find better ways to play the game

- Trying various actions
- Switching to better action
- Imitating successful behavior

Trial and error adjustment

Basic Idea

Over time, players may find better ways to play the game

- Trying various actions
- Switching to better action
- Imitating successful behavior

Trial and error adjustment

Biological evolution

Basic Idea

Over time, players may find better ways to play the game

- Trying various actions
- Switching to better action
- Imitating successful behavior

→ Survival of the fittest

Trial and error adjustment

Biological evolution

Basic Idea

Over time, players may find better ways to play the game

- Trying various actions → Mutation
- Switching to better action
- Imitating successful behavior → Survival of the fittest

Trial and error adjustment

Biological evolution

Basic Idea

Over time, players may find better ways to play the game

- Trying various actions → Mutation
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Trial and error adjustment

Biological evolution

Basic Idea

Over time, players may find better ways to play the game

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Credit: Cacophony, CC BY-SA 3.0

Trial and error adjustment

Biological evolution

Evolutionary Game Theory

Evolutionary Game Theory

John Maynard Smith

Credit: Web of Stories, CC BY-SA 3.0

Strategies

Strategies

- Physical characteristics

Strategies

- Physical characteristics
  - Body size

Strategies

- Physical characteristics
  - Body size
  - Body color

Strategies

- Physical characteristics
  - Body size
  - Body color
- Behavior

Strategies

- Physical characteristics
  - Body size
  - Body color
- Behavior
  - Aggression/defence

Strategies

- Physical characteristics
  - Body size
  - Body color
- Behavior
  - Aggression/defence
  - Parental care

Strategies

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Strategies

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- Behavior
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  - Mating

Genetically programmed characteristics

Strategies

Genetically programmed characteristics

Strategies ⟷ Genes

Genetically programmed characteristics

Strategies ⟷ Genes

Genetically programmed characteristics

Players

Strategies ⟷ Genes

Genetically programmed characteristics

Players

Players are individuals

Strategies ⟷ Genes

Genetically programmed characteristics

Players

Players are individuals

Each individual carries a particular gene

Strategies ⟷ Genes

Genetically programmed characteristics

Players

Players are individuals

Each individual carries a particular gene

Therefore, we can regard player = gene

Strategies ⟷ Genes

Genetically programmed characteristics

Players

Players are individuals

Each individual carries a particular gene

Therefore, we can regard player = gene

Payoff of a player

Payoff of a player

Player

Genes

Payoff of a player

Player

Genes

strategy

Payoff of a player

= Payoff to a strategy

Player

Genes

strategy

Payoff of a player

= Payoff to a strategy

Number of offspring

Player

Genes

strategy

Payoff of a player

= Payoff to a strategy

Number of offspring

Player

Genes

strategy

Successful strategy (gene) has more offsprings

Payoff of a player

= Payoff to a strategy

Number of offspring

Player

Genes

strategy

Successful strategy (gene) has more offsprings

Survival of the fittest

Payoff of a player

= Payoff to a strategy

Number of offspring

Player

Genes

strategy

Successful strategy (gene) has more offsprings

Survival of the fittest

Natural selection

Payoff of a player

= Payoff to a strategy

Number of offspring

Evolutionary "fitness"

Player

Genes

strategy

Successful strategy (gene) has more offsprings

Survival of the fittest

Natural selection

Successful strategy (gene) has more offsprings

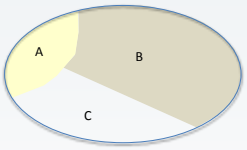
Natural selection

Three strategies (genes)
A,B, and C

Successful strategy (gene) has more offsprings

Natural selection

Evolutionary Game Theory



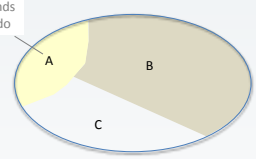
Three strategies (genes) A,B, and C

Successful strategy (gene) has more offsprings

Natural selection

Evolutionary Game Theory

A's fitness depends on what others do



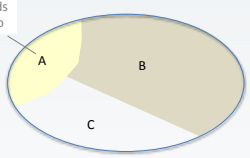
Successful strategy (gene) has more offsprings

Natural selection

Evolutionary Game Theory

A's fitness depends on what others do

Biological game

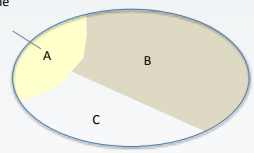


Successful strategy (gene) has more offsprings

Natural selection

Evolutionary Game Theory

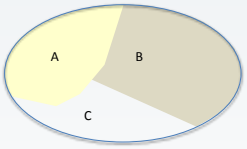
Suppose A has the highest fitness



Successful strategy (gene) has more offsprings

Natural selection

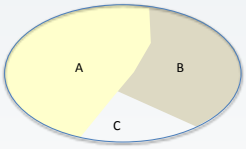
Evolutionary Game Theory



Successful strategy (gene) has more offsprings

Natural selection

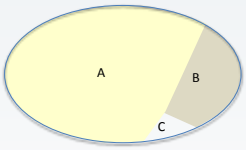
Evolutionary Game Theory



Successful strategy (gene) has more offsprings

Natural selection

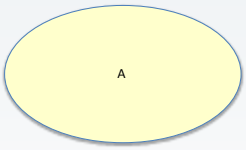
Evolutionary Game Theory



Successful strategy (gene) has more offsprings

Natural selection

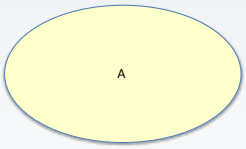
Evolutionary Game Theory



Successful strategy (gene) has more offsprings

Natural selection

Evolutionary Game Theory

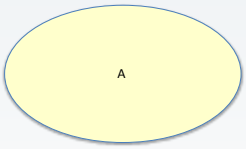


Survival of the fittest

Successful strategy (gene) has more offsprings

Natural selection

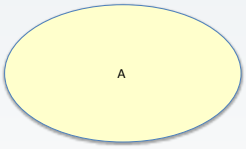
Evolutionary Game Theory



Survival of the fittest

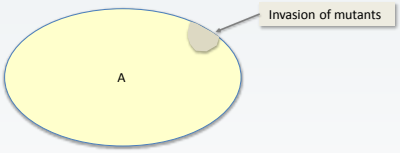
The outcome of evolution

Evolutionary Game Theory



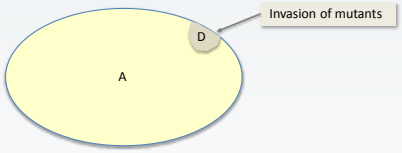
The outcome of evolution

Evolutionary Game Theory



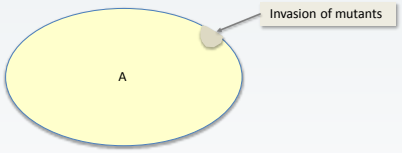
The outcome of evolution

Evolutionary Game Theory



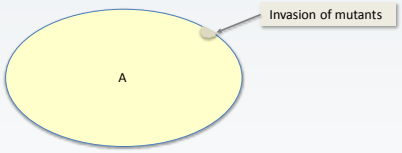
The outcome of evolution

Evolutionary Game Theory



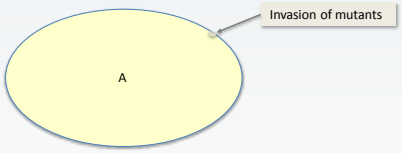
The outcome of evolution

Evolutionary Game Theory



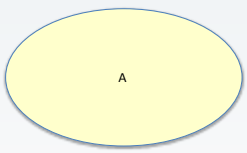
The outcome of evolution

Evolutionary Game Theory



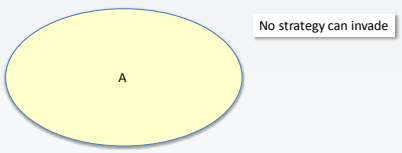
The outcome of evolution

Evolutionary Game Theory

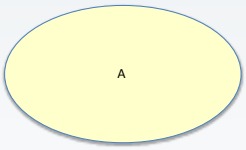


The outcome of evolution

Evolutionary Game Theory



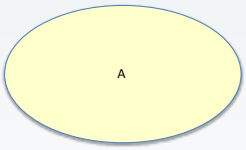
The outcome of evolution



No strategy can invade

No profitable deviation to other strategies

The outcome of evolution



No strategy can invade

No profitable deviation to other strategies

The outcome of evolution Nash equilibrium

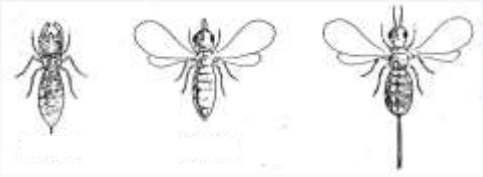
The outcome of biological evolution is Nash equilibrium of a game played by genes.

3.9 Fig wasps play a Nash equilibrium

The outcome of biological evolution is Nash equilibrium of a game played by genes.

Male fig wasps have two “strategies”

Male Female



Male fig wasps have two “strategies”

Male Female

“Staying”

Male fig wasps have two “strategies”

Male Female

“Staying” “Flying”

A certain fraction of females stay in figs

A certain fraction of females stay in figs

A certain fraction of females fly out

A certain fraction of females stay in figs

What happens if **flying males are rare**?

Staying males

Flying males

Staying males have a small chance of mating with a female

Staying males have a small chance of mating with a female

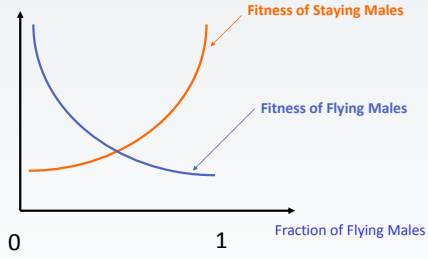
Flying males have a high chance of mating with a female

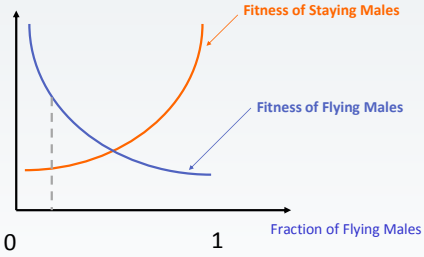
Staying males have a small chance of mating with a female

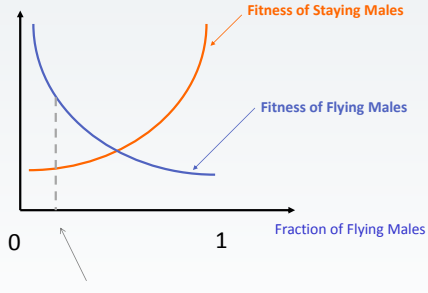
When the fraction of flying males is small .....

When the fraction of flying males is small .....

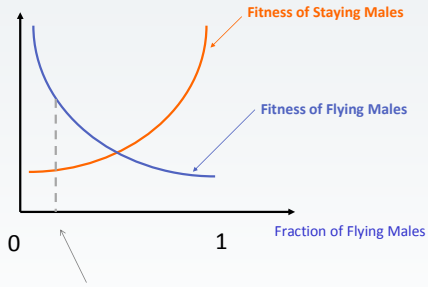
...flying males have more offsprings



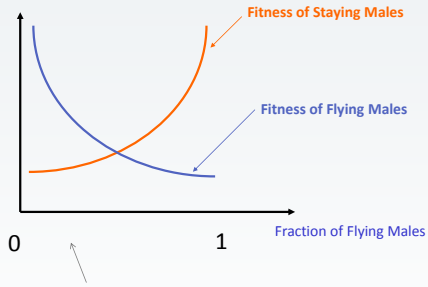




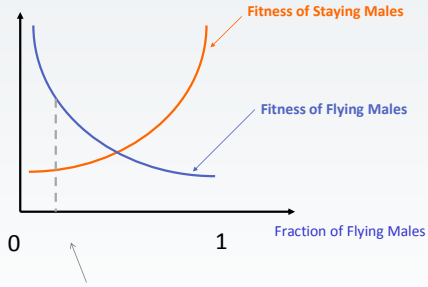
When flying males are rare, they have high fitness



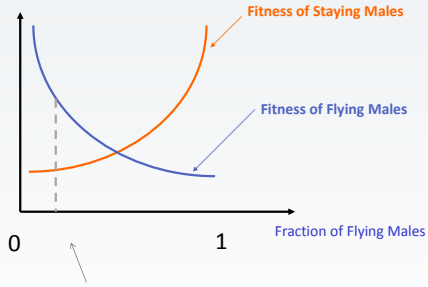
When flying males are rare, they have high fitness
a large number of offsprings



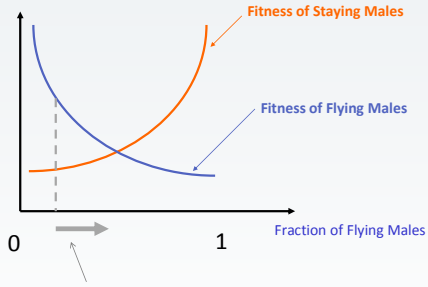
Flying males have a lot of offsprings,



Flying males have a lot of offsprings,



Flying males have a lot of offsprings, so their fraction increases



Flying males have a lot of offsprings, so their fraction increases

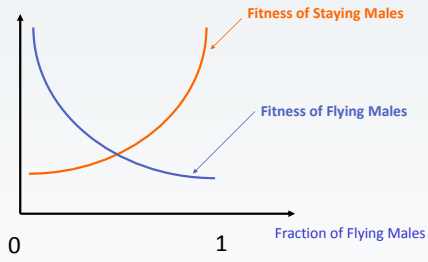
When the fraction of flying males is large .....

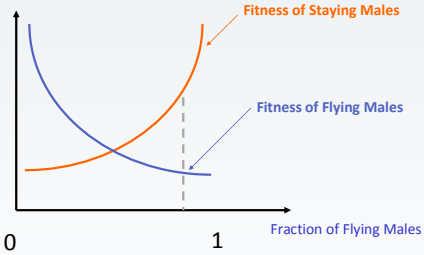
When the fraction of flying males is large .....

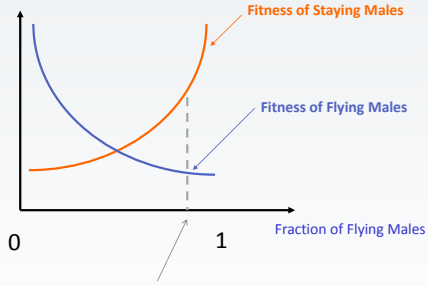
When the fraction of flying males is large .....

When the fraction of flying males is large .....

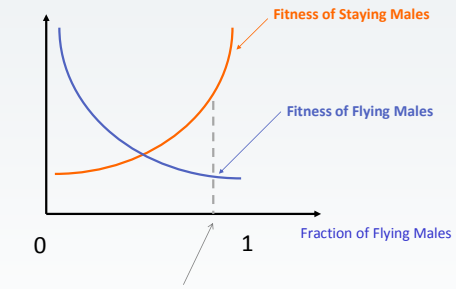
...flying males have less offsprings



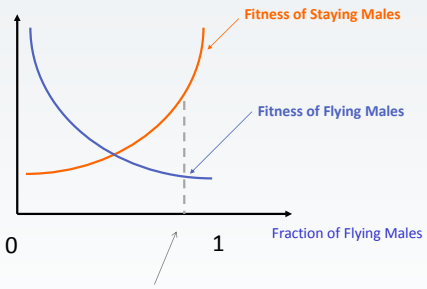




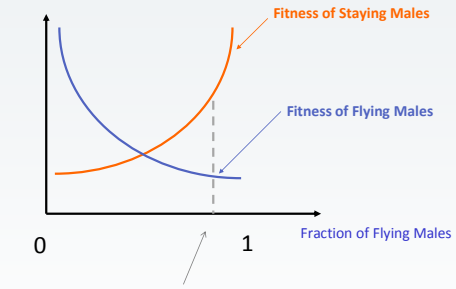
When flying male fraction is high, a flying male has a low fitness



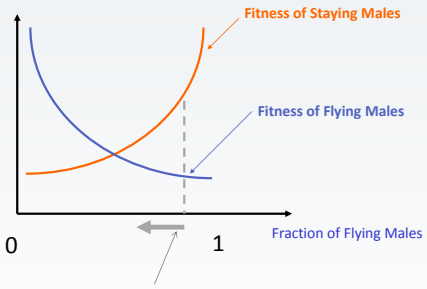
When flying male fraction is high, a flying male has a low fitness
a small number of offsprings



Flying males have few offsprings,



Flying males have few offsprings, so their fraction decreases



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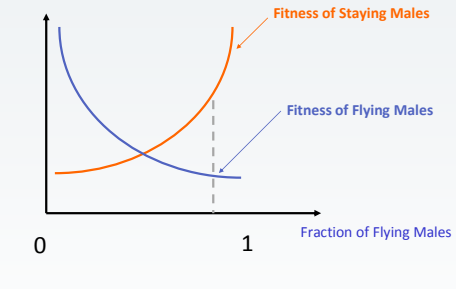
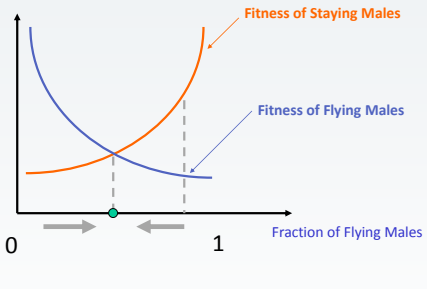
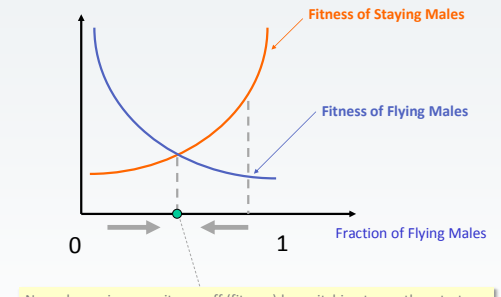
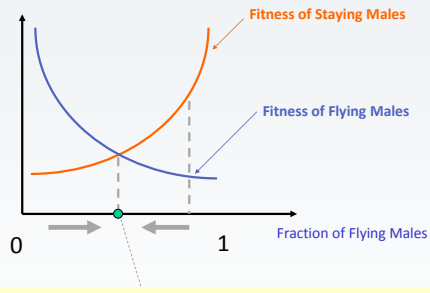




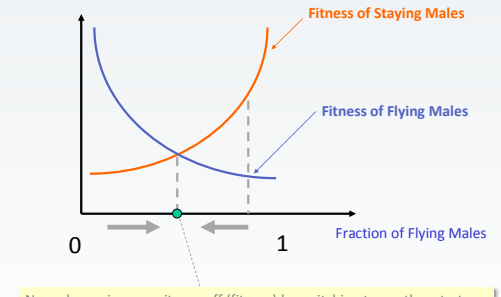


No male can increase its payoff (fitness) by switching to another strategy



No male can increase its payoff (fitness) by switching to another strategy

Nash equilibrium



No male can increase its payoff (fitness) by switching to another strategy

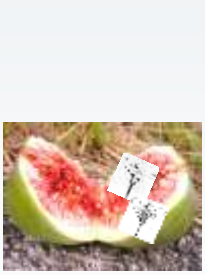
Nash equilibrium = outcome of biological evolution

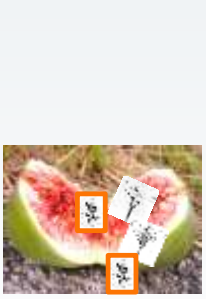
When the fraction of flying males is small .....

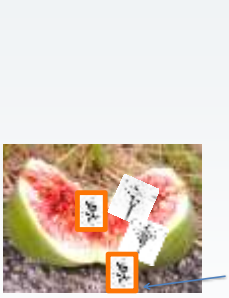
When the fraction of flying males is small .....

...flying males have more offsprings

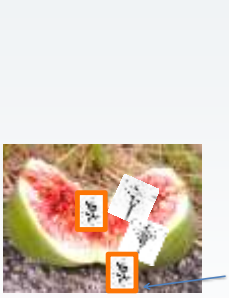
What is the property of Nash equilibrium?







They have equal chance of mating



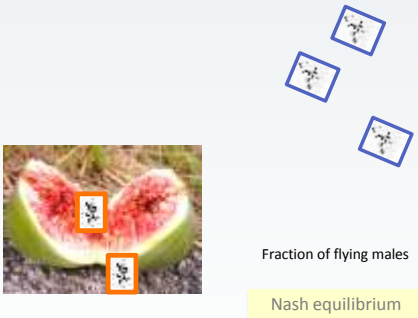
They have equal chance of mating

Nash equilibrium

Nash equilibrium

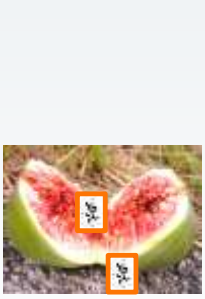
Fraction of females flying out

Nash equilibrium



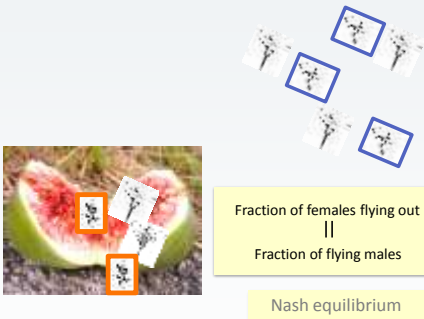
Fraction of flying males

Nash equilibrium



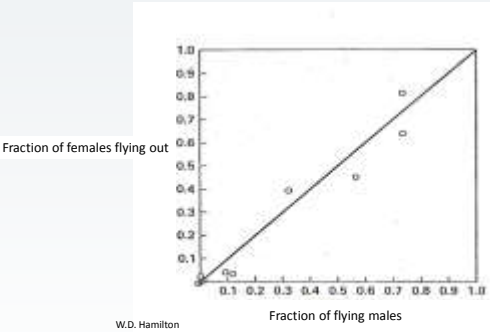
Fraction of females flying out
||
Fraction of flying males

Nash equilibrium



Fraction of females flying out
||
Fraction of flying males

Nash equilibrium



W.D. Hamilton
"Wingless and fighting males in fig wasps and other insects "
in "Sexual Selection and Reproductive Competition in Insects"
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Nash equilibrium predicts that data points should be on the 45 degree line

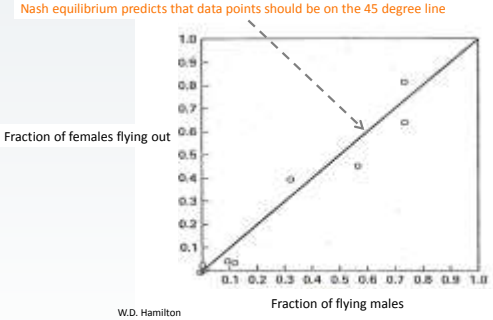

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Fig wasps play a Nash equilibrium

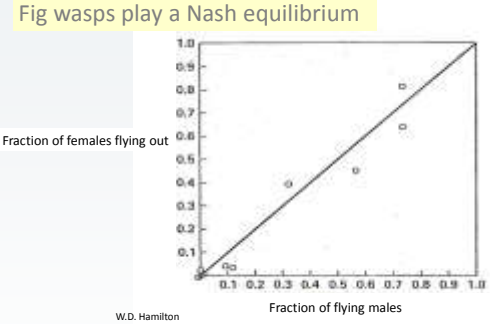

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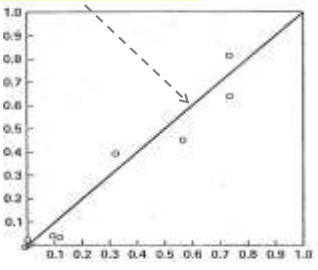
Fig wasps play a Nash equilibrium



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