

**problem set no. 3****Problem 1. Rejection Sampling for the Semi-Circle Distribution.**

Let

$$p(x|R) = \frac{2}{\pi R^2} \sqrt{R^2 - x^2}$$

For  $R = 0.1, 0.2, \dots, 3.0$ , use rejection sampling to sample from  $p(x|R)$ . Plot the sample variance for each value of  $R$ , and show graphically that the variance for the semi-circle distribution is  $\frac{R^2}{4}$ .

**Problem 2. Importance Sampling of the Galenshore distribution.**

Recall that the *Galenshore*( $\alpha, \theta$ ) distribution has density

$$p(y) = \frac{2}{\Gamma(\alpha)} \theta^{2\alpha} y^{2\alpha-1} e^{-\theta^2 y^2}$$

for  $y > 0$ ,  $\theta > 0$  and  $\alpha > 0$ .

a) Assume  $\alpha = 1$  and  $\theta = 0.5$ . For  $k = 10, 100$  and  $1000$ , draw samples of size  $k$  from an Expo(1) distribution. Use these samples as important sampling proposals to estimate the mean of the Galenshore. Compare your estimates for each  $k$  to the true mean:

$$E[Y|\alpha] = \frac{\Gamma(\alpha + 1/2)}{\theta \Gamma(\alpha)}$$

b) Plot a histogram of the log importance weights for  $k = 1000$ .

c) Now draw samples of size  $k = 10, 100$  and  $1000$  from  $Z \sim N(1.5, 1)$ . For each  $k$ , compute the estimated mean. Plot the histogram of log importance weights for  $k = 1000$  (exclude the points for which  $Z < 0$  when plotting this histogram). Based on the two histograms and the comparison between estimates and true mean, which proposal is better?