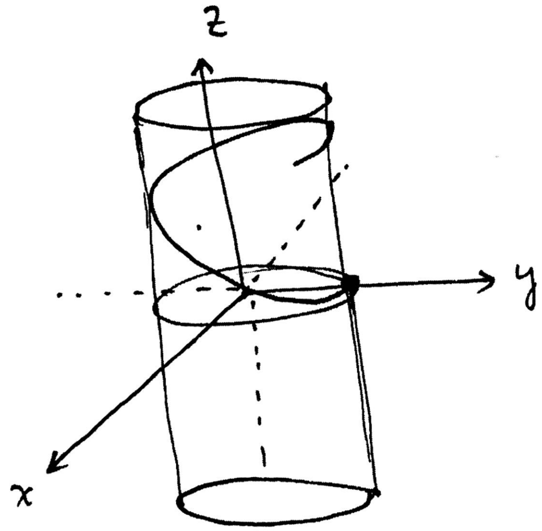
Ex $\quad r(t) = \langle 3\sin t,\ 3\cos t,\ 4t \rangle, \qquad 0 \le t \le 2\pi$




Find $T,\ N,\ B,\ \kappa,\ \tau.\ \ \forall t.$

$r'(t) = \langle 3\cos t,\ -3\sin t,\ 4 \rangle$

$|r'(t)| = \sqrt{9\cos^2 t + 9\sin^2 t + 16}$

$= \sqrt{9+16} = 5$

$$\boxed{T = \frac{r'(t)}{|r'(t)|} = \left\langle \tfrac{3}{5}\cos t,\ -\tfrac{3}{5}\sin t,\ \tfrac{4}{5} \right\rangle}$$

$N = \dfrac{T'(t)}{|T'(t)|}$

$T'(t) = \left\langle -\tfrac{3}{5}\sin t,\ -\tfrac{3}{5}\cos t,\ 0 \right\rangle$

$|T'(t)| = \sqrt{\tfrac{9}{25}\sin^2 t + \tfrac{9}{25}\cos^2 t}$

$= \sqrt{\tfrac{9}{25}} = \tfrac{3}{5}$

$$\boxed{N = \langle -\sin t,\ -\cos t,\ 0 \rangle}$$

$$B = T \times N = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \tfrac{3}{5}\cos t & -\tfrac{3}{5}\sin t & \tfrac{4}{5} \\ -\sin t & -\cos t & 0 \end{vmatrix} = \hat{\imath}\left(\tfrac{4}{5}\cos t\right) - \hat{\jmath}\left(\tfrac{4}{5}\sin t\right) + \hat{k}\left(-\tfrac{3}{5}\cos^2 t - \tfrac{3}{5}\sin^2 t\right)$$

$$\therefore \quad \boxed{B = \left\langle \tfrac{4}{5}\cos t,\ -\tfrac{4}{5}\sin t,\ -\tfrac{3}{5} \right\rangle}$$

$$\kappa = \frac{|T'(t)|}{|r'(t)|} = \frac{\frac{3}{5}}{5} = \frac{3}{25}$$

$$\boxed{\kappa = \frac{3}{25}}$$ constant curvature $\forall t$

$$\tau = -\frac{B'(t)}{|r'(t)|} \cdot N$$

$$B'(t) = \langle -\tfrac{4}{5}\sin t, -\tfrac{4}{5}\cos t, 0\rangle$$

$$\therefore \tau = -\frac{\langle -\frac{4}{5}\sin t, -\frac{4}{5}\cos t, 0\rangle}{5} \cdot \langle -\sin t, -\cos t, 0\rangle$$

$$\tau = -\frac{4}{25}\sin^2 t - \frac{4}{25}\cos^2 t = -\frac{4}{25}$$

$$\boxed{\tau = -\frac{4}{25}}$$ constant torsion $\forall t$

$r(t)$ represents a circular helix, the torsion and curvature are constant; this special property of circular helices means that the curve turns about its axis at a constant rate and rises vertically at a constant rate.