Simple Regression Linear

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```
library(tidyverse)
```

INTRODUCTION

In this program we will learn about how to make Simple Linear Regression, by breaking down to each formula.

First, we will start from making the dataset dummy

```
# define x value as predictor
predictor <- c(15, 20, 25, 37, 40, 45, 48, 50, 55, 61, 64, 67, 70)

# define y value as target
target <- c(100, 135, 135, 150, 250, 270, 290, 360, 375, 400, 500, 600,
700)

# set as dataframe
df <- data.frame(x = predictor, y = target)
df
```

```
##     x   y
## 1  15 100
## 2  20 135
## 3  25 135
## 4  37 150
## 5  40 250
## 6  45 270
## 7  48 290
## 8  50 360
## 9  55 375
## 10 61 400
## 11 64 500
## 12 67 600
## 13 70 700
```

PART 1 : FIND THE REGRESSION FORMULA

In Linear Regression, the first information we have to check is the prediction formula : y = a + bx

```
# Start with supporting variable
df <- df %>%
  mutate(xy = x * y,
         x_sq = x ** 2,
         y_sq = y ** 2)
```

```r
n <- nrow(df) # amount of predictor

# assign `a` value
a <- (sum(df$y) * sum(df$x_sq) - sum(df$x) * sum(df$xy)) /
  (n * sum(df$x_sq) - (sum(df$x))**2)


# assign `b` value
b <- (n * sum(df$xy) - sum(df$x) * sum(df$y)) /
  (n * sum(df$x_sq) - (sum(df$x))**2)

paste(sprintf("The formula is y = %.3f + %.3fx", a, b))

## [1] "The formula is y = -118.420 + 9.723x"
```

We have got the formula: y = -118.420 + 9.723x. Now we calculate the predicted y using this formula

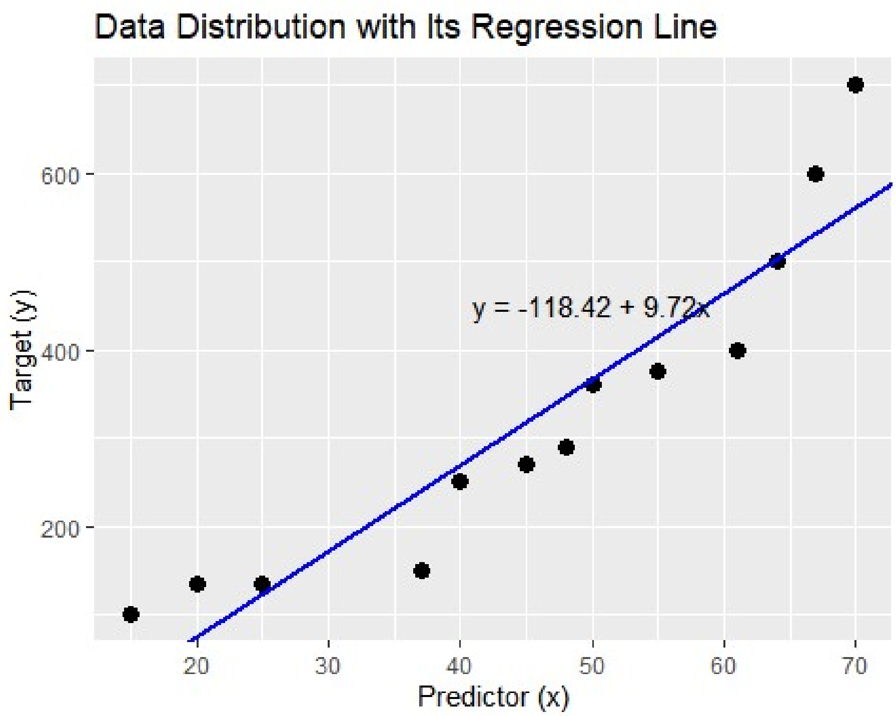
```r
df$y_pred <- a + (b * df$x)
df

##     x   y    xy x_sq   y_sq    y_pred
## 1  15 100  1500  225  10000  27.42085
## 2  20 135  2700  400  18225  76.03439
## 3  25 135  3375  625  18225 124.64794
## 4  37 150  5550 1369  22500 241.32044
## 5  40 250 10000 1600  62500 270.48857
## 6  45 270 12150 2025  72900 319.10211
## 7  48 290 13920 2304  84100 348.27024
## 8  50 360 18000 2500 129600 367.71566
## 9  55 375 20625 3025 140625 416.32920
## 10 61 400 24400 3721 160000 474.66546
## 11 64 500 32000 4096 250000 503.83359
## 12 67 600 40200 4489 360000 533.00171
## 13 70 700 49000 4900 490000 562.16984
```

We will check the R-squared value. This is to check whether the linear regression model will be the good fit for the data

```r
r <- (n * sum(df$xy) - sum(df$x) * sum(df$y)) /
  sqrt ((n * sum(df$x_sq) - (sum(df$x))**2) * (n * sum(df$y_sq) -
(sum(df$y))**2))

r_sq <- r ** 2 # this value is called Multiple R Squared

# Meanwhile, adjusted R-squared will be as follow
k <- 1 # we only have one independent variable
adjusted_r_sq <- 1 - (((1 - r_sq) * (n - 1)) / (n-k-1))

paste(sprintf("The model fits the data with percentage %.2f%%",
adjusted_r_sq*100))

## [1] "The model fits the data with percentage 85.89%"
```

As the end of PART 1, let's see how the distribution of data, include with the regression line (predicted value line)

```
df %>%
  ggplot(aes(x = x, y = y)) +
  geom_point(size = 3) +
  geom_abline(intercept = a, slope = b, size = 1, color = "blue") +
  labs(title = "Data Distribution with Its Regression Line",
       x = "Predictor (x)",
       y = "Target (y)") +
  annotate(geom = "text", x = 50, y = 450, label = "y = -118.42 + 9.72x")
```



PART 2 : FIND THE RESIDUAL

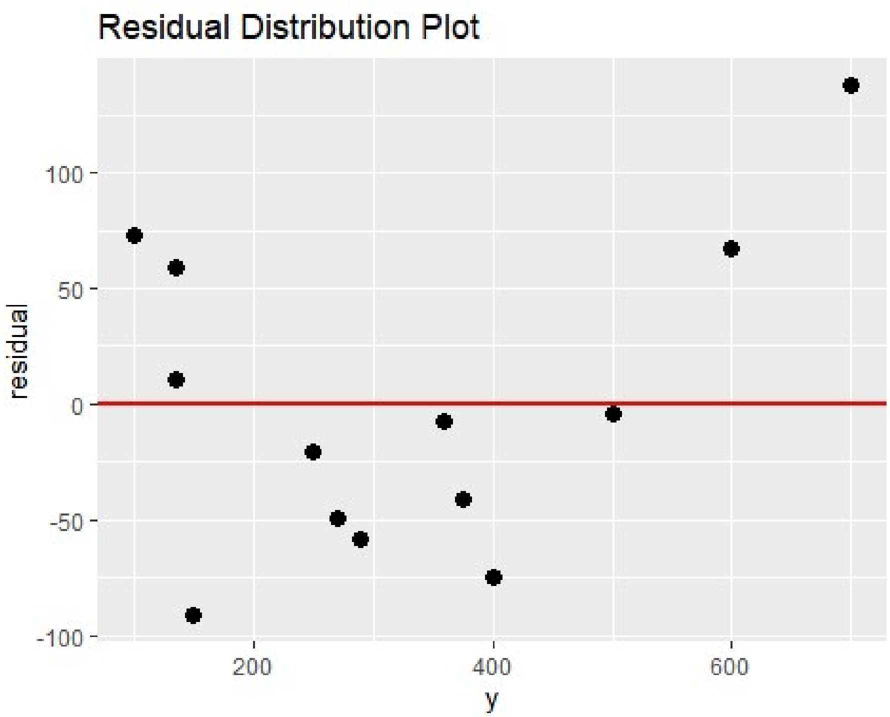
Residual is the discrepancy between actual y value with predicted y value

```
df$residual <- df$y - df$y_pred
df

##     x   y    xy x_sq   y_sq    y_pred   residual
## 1  15 100  1500  225  10000  27.42085  72.579155
## 2  20 135  2700  400  18225  76.03439  58.965610
## 3  25 135  3375  625  18225 124.64794  10.352065
## 4  37 150  5550 1369  22500 241.32044 -91.320443
## 5  40 250 10000 1600  62500 270.48857 -20.488570
## 6  45 270 12150 2025  72900 319.10211 -49.102115
## 7  48 290 13920 2304  84100 348.27024 -58.270242
## 8  50 360 18000 2500 129600 367.71566  -7.715660
## 9  55 375 20625 3025 140625 416.32920 -41.329205
## 10 61 400 24400 3721 160000 474.66546 -74.665459
## 11 64 500 32000 4096 250000 503.83359  -3.833586
```

```
## 12 67 600 40200 4489 360000 533.00171  66.998287
## 13 70 700 49000 4900 490000 562.16984 137.830161
```

In Linear Regression analysis, we will check the distribution of Residual itself

```
df %>%
  ggplot(aes(x = y, y = residual)) +
  geom_point(size = 3) +
  geom_hline(yintercept = 0, size = 1, color = "red") +
  labs(title = "Residual Distribution Plot")
```



PART 3 : FIND THE STANDARDIZED RESIDUAL

Before we check the standardized residual, there are several supporting variable we need to make

```
df$residual_sq <- df$residual ** 2

predictor_mean <- mean(df$x)
df$predictor_sd <- (df$x - predictor_mean) ** 2 # deviation of predictor
data
predictor_ssdev <- sum(df$predictor_sd) # sum of square of predictor
standard deviation

RSE <- sqrt(sum(df$residual_sq) / (n-k-1))
```

We start by counting the leverage; which by definition, is how far an observation value, from those of the other observations.

```
df$leverage <- (1/n) + (((df$x - predictor_mean) ** 2) / predictor_ssdev)
df
```

```
##     x   y    xy x_sq  y_sq    y_pred   residual residual_sq predictor_sd
## 1  15 100  1500  225  10000  27.42085  72.579155  5267.73372   956.236686
## 2  20 135  2700  400  18225  76.03439  58.965610  3476.94316   672.005917
## 3  25 135  3375  625  18225 124.64794  10.352065   107.16525   437.775148
## 4  37 150  5550 1369  22500 241.32044 -91.320443  8339.42329    79.621302
## 5  40 250 10000 1600  62500 270.48857 -20.488570   419.78149    35.082840
## 6  45 270 12150 2025  72900 319.10211 -49.102115  2411.01768     0.852071
## 7  48 290 13920 2304  84100 348.27024 -58.270242  3395.42107     4.313609
## 8  50 360 18000 2500 129600 367.71566  -7.715660    59.53140    16.621302
## 9  55 375 20625 3025 140625 416.32920 -41.329205  1708.10316    82.390533
## 10 61 400 24400 3721 160000 474.66546 -74.665459  5574.93071   227.313609
## 11 64 500 32000 4096 250000 503.83359  -3.833586    14.69638   326.775148
## 12 67 600 40200 4489 360000 533.00171  66.998287  4488.77052   444.236686
## 13 70 700 49000 4900 490000 562.16984 137.830161 18997.15314   579.698225
##      leverage
## 1  0.32446533
## 2  0.25088614
## 3  0.19025051
## 4  0.09753475
## 5  0.08600502
## 6  0.07714365
## 7  0.07803975
## 8  0.08122586
## 9  0.09825162
## 10 0.13576805
## 11 0.16151579
## 12 0.19192321
## 13 0.22699032
```

Then, we calculate the Standardized Residuals

```
df$residual_std <- df$residual / (RSE * sqrt(1 - df$leverage))
df
```

```
##     x   y    xy x_sq  y_sq    y_pred   residual residual_sq predictor_sd
## 1  15 100  1500  225  10000  27.42085  72.579155  5267.73372   956.236686
## 2  20 135  2700  400  18225  76.03439  58.965610  3476.94316   672.005917
## 3  25 135  3375  625  18225 124.64794  10.352065   107.16525   437.775148
## 4  37 150  5550 1369  22500 241.32044 -91.320443  8339.42329    79.621302
## 5  40 250 10000 1600  62500 270.48857 -20.488570   419.78149    35.082840
## 6  45 270 12150 2025  72900 319.10211 -49.102115  2411.01768     0.852071
## 7  48 290 13920 2304  84100 348.27024 -58.270242  3395.42107     4.313609
## 8  50 360 18000 2500 129600 367.71566  -7.715660    59.53140    16.621302
## 9  55 375 20625 3025 140625 416.32920 -41.329205  1708.10316    82.390533
## 10 61 400 24400 3721 160000 474.66546 -74.665459  5574.93071   227.313609
## 11 64 500 32000 4096 250000 503.83359  -3.833586    14.69638   326.775148
## 12 67 600 40200 4489 360000 533.00171  66.998287  4488.77052   444.236686
## 13 70 700 49000 4900 490000 562.16984 137.830161 18997.15314   579.698225
##      leverage residual_std
## 1  0.32446533   1.25730849
## 2  0.25088614   0.97001543
## 3  0.19025051   0.16379680
## 4  0.09753475  -1.36869452
## 5  0.08600502  -0.30513604
## 6  0.07714365  -0.72775787
## 7  0.07803975  -0.86406116
```

```
## 8  0.08122586  -0.11460998
## 9  0.09825162  -0.61968092
## 10 0.13576805  -1.14355838
## 11 0.16151579  -0.05960895
## 12 0.19192321   1.06118511
## 13 0.22699032   2.23205843
```

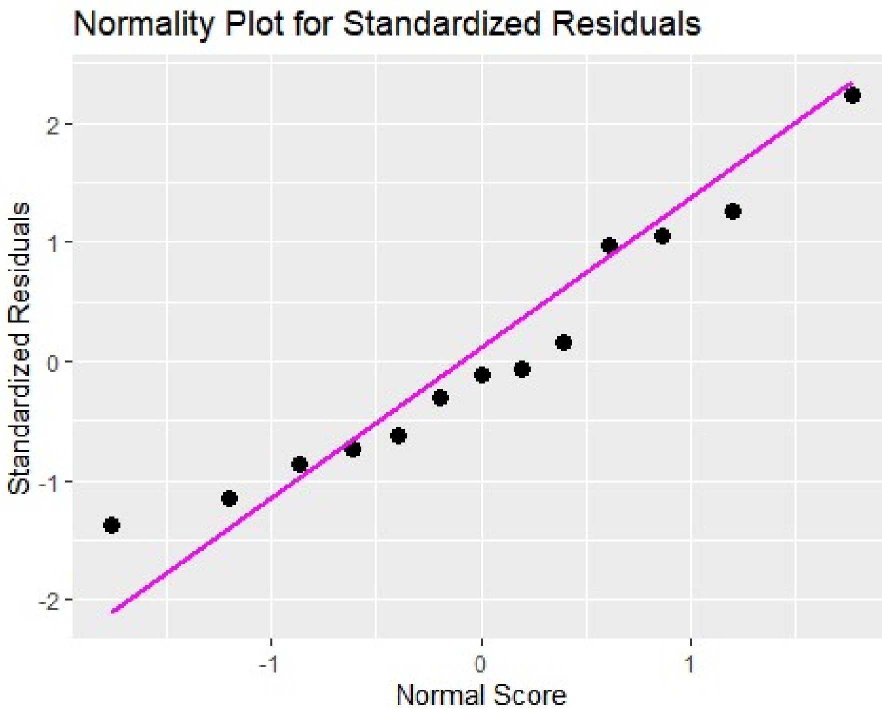
In R, we can plot standardized residual with qqplot directly. However, here we want to know where the calculation is from. Let's start by making another dataset.

```
# sort the value of Standardized Residuals
qq_df <- data.frame(residual_std = sort(df$residual_std))

# add rank -> start with 1 for the smallest value
qq_df$rank <- c(1:n)

# check percentile or quantile -> show the percentage of rank among
overall
qq_df$quantile <- (qq_df$rank - 0.5) / n

# check qnorm of each quantile
qq_df$qnorm <- qnorm(qq_df$quantile)

qq_df
```

```
##    residual_std rank   quantile      qnorm
## 1   -1.36869452    1 0.03846154 -1.7688250
## 2   -1.14355838    2 0.11538462 -1.1983797
## 3   -0.86406116    3 0.19230769 -0.8694238
## 4   -0.72775787    4 0.26923077 -0.6151411
## 5   -0.61968092    5 0.34615385 -0.3957253
## 6   -0.30513604    6 0.42307692 -0.1940281
## 7   -0.11460998    7 0.50000000  0.0000000
## 8   -0.05960895    8 0.57692308  0.1940281
## 9    0.16379680    9 0.65384615  0.3957253
## 10   0.97001543   10 0.73076923  0.6151411
## 11   1.06118511   11 0.80769231  0.8694238
## 12   1.25730849   12 0.88461538  1.1983797
## 13   2.23205843   13 0.96153846  1.7688250
```

Plot the data to check normality

```
qq_df %>%
  ggplot(aes(x = qnorm, y = residual_std)) +
  geom_point(size = 3) +
  geom_qq_line(aes(sample = residual_std), line.p = c(0.25, 0.75), size =
1, color = "magenta") +
  labs(title = "Normality Plot for Standardized Residuals",
       x = "Normal Score",
       y = "Standardized Residuals")
```


Normality Plot for Standardized Residuals
Standardized Residuals
2
1
0
-1
-2
-1
0
1
Normal Score