1998 AP Calculus BC: Section I, Part A

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

   (A) $-3 < x < 1$
   (B) $-1 < x < 1$
   (C) $x < -3$ or $x > 1$
   (D) $x < -1$ or $x > 3$
   (E) All real numbers

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2. In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

   (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

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3. The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2,-1)$ is

   (A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

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4. $\int \frac{1}{x^2 - 6x + 8}\,dx =$

   (A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

   (B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

   (C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

   (D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

   (E) $\ln\left|(x-2)(x-4)\right| + C$

5. If f and g are twice differentiable and if $h(x) = f\left(g(x)\right)$, then $h''(x) =$

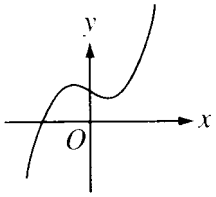
(A) $f''\left(g(x)\right)\left[g'(x)\right]^2 + f'\left(g(x)\right)g''(x)$

(B) $f''\left(g(x)\right)g'(x) + f'\left(g(x)\right)g''(x)$

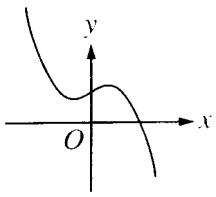
(C) $f''\left(g(x)\right)\left[g'(x)\right]^2$

(D) $f''\left(g(x)\right)g''(x)$

(E) $f''\left(g(x)\right)$



6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

(A)


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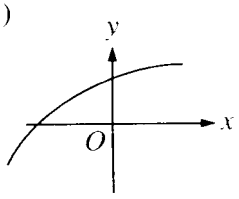

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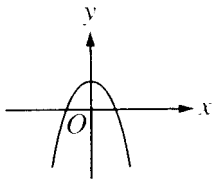

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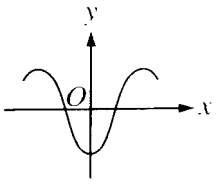
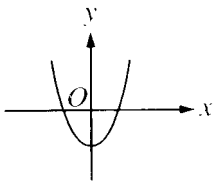


(E)


7. $\displaystyle\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

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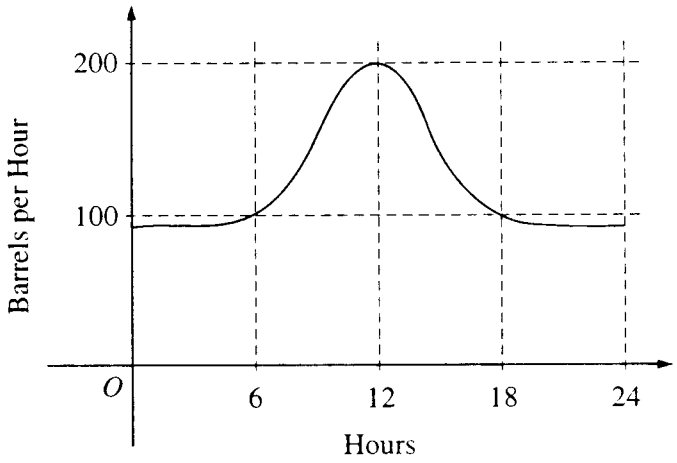
8. If $\frac{dy}{dx}=\sin x\cos^2 x$ and if $y=0$ when $x=\frac{\pi}{2}$, what is the value of y when $x=0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

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10. A particle moves on a plane curve so that at any time $t>0$ its x-coordinate is t^3-t and its y-coordinate is $(2t-1)^3$. The acceleration vector of the particle at $t=1$ is

(A) $(0,1)$ (B) $(2,3)$ (C) $(2,6)$ (D) $(6,12)$ (E) $(6,24)$

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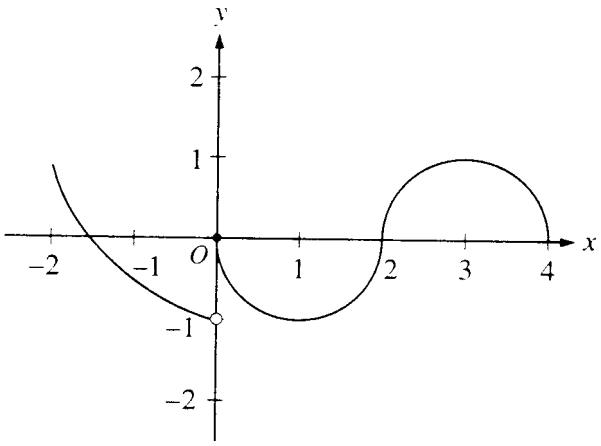
11. If f is a linear function and $0<a<b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

12. If $f(x)=\begin{cases} \ln x & \text{for } 0<x\le 2 \\ x^2\ln 2 & \text{for } 2<x\le 4, \end{cases}$ then $\lim\limits_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

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14. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

(A) $1-\frac{1}{2}+\frac{1}{24}$

(B) $1-\frac{1}{2}+\frac{1}{4}$

(C) $1-\frac{1}{3}+\frac{1}{5}$

(D) $1-\frac{1}{4}+\frac{1}{8}$

(E) $1-\frac{1}{6}+\frac{1}{120}$

15. $\int x\cos x\,dx =$

(A) $x\sin x - \cos x + C$

(B) $x\sin x + \cos x + C$

(C) $-x\sin x + \cos x + C$

(D) $x\sin x + C$

(E) $\frac{1}{2}x^2\sin x + C$

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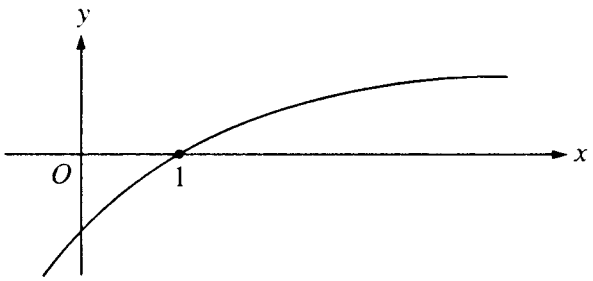
16. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

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19. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (4\sin\theta - 2)^2 \, d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4\sin\theta - 2)^2 \, d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin\theta - 2)^2 \, d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (16\sin^2\theta - 4) \, d\theta$ (E) $\frac{1}{2}\int_0^{\pi} (16\sin^2\theta - 4) \, d\theta$

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20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

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21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1} \, dt$

(B) $\int_0^1 \sqrt{t^2 + t} \, dt$

(C) $\int_0^1 \sqrt{t^4 + t^2} \, dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4 + t^4} \, dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9} \, dt$