1997 AP Calculus AB: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 (4x^3 - 6x)\,dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

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2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

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3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

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4. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) –1 (D) –3 (E) –5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

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6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

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7. $\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

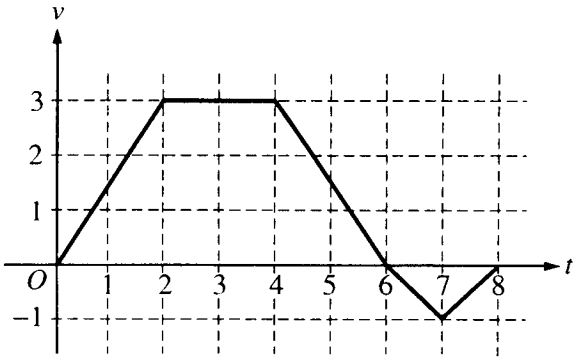
(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

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9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

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10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

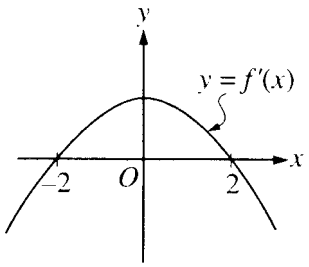
(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

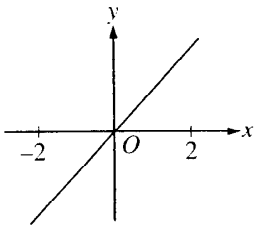
(C) $y = 2\left(x - \frac{\pi}{4}\right)$

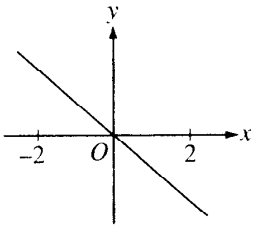
(D) $y = -\left(x - \frac{\pi}{4}\right)$

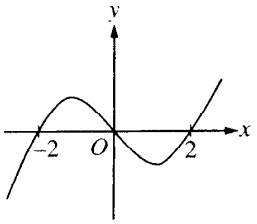
(E) $y = -2\left(x - \frac{\pi}{4}\right)$

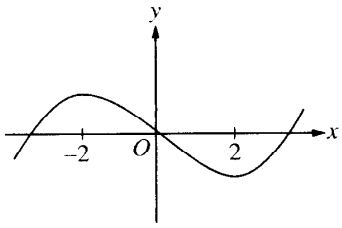


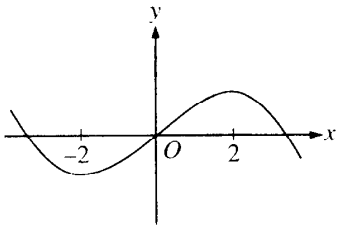
11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)


(B)


(C)


(D)


(E)


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12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

13. Let f be a function defined for all real numbers x. If $f'(x) = \dfrac{\left|4 - x^2\right|}{x - 2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

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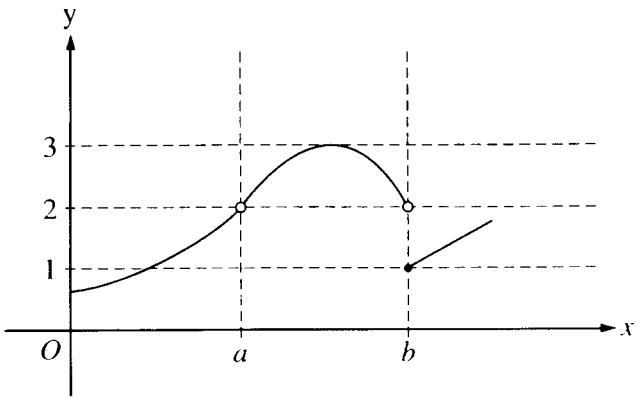
14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

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17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

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18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

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19. If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2-1}\right|$

(B) $\frac{2x}{\left|x^2-1\right|}$

(C) $\frac{2|x|}{x^2-1}$

(D) $\frac{2x}{x^2-1}$

(E) $\frac{1}{x^2-1}$

20. The average value of $\cos x$ on the interval $[-3, 5]$ is

(A) $\dfrac{\sin 5 - \sin 3}{8}$

(B) $\dfrac{\sin 5 - \sin 3}{2}$

(C) $\dfrac{\sin 3 - \sin 5}{2}$

(D) $\dfrac{\sin 3 + \sin 5}{2}$

(E) $\dfrac{\sin 3 + \sin 5}{8}$

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21. $\lim\limits_{x \to 1} \dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

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22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

(E) All values of x

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23. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\dfrac{32\pi}{5}$ (B) $\dfrac{16\pi}{3}$ (C) $\dfrac{16\pi}{5}$ (D) $\dfrac{8\pi}{3}$ (E) π