Economics 512 – Homework 6

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This assignment extends Problem 7.4 in in Miranda & Fackler. Consider competitive price-taking firm that maximizes discounted sum of expected future profits from harvesting a non-renewable resource. For example, suppose a lumber company is deciding how many trees to harvest in a forest where trees do not grow back. The discount factor of our lumber company is $\delta = 0.95$. The firm earns revenue of $p \cdot x$ per period if it has harvested amount x and market price was p. To harvest x the firm incurs a convex cost $0.2 \cdot x^{1.5}$. The firm is small relative to the market, and has rational expectations that the price of lumber will follow an AR(1) process:

$$p_t = p_0 + \rho \cdot p_{t-1} + u \tag{1}$$

Where $p_0 = 0.5$, $\rho = 0.5$, and u is a mean-zero normal disturbance with standard deviation $\sigma_u = 0.1$. The initial stock of lumber may be anything from 0 to 100.

Q.1

Formulate firm's dynamic optimization problem. Specifically, formulate the Bellman equation, identify state and policy variables, their spaces and transition probabilities. Assume initial stock is between 0 and 100.

This is a infinite horizon model with time $t = \{1, 2, , \ldots, \infty\}$ measured in periods.

The state variables:

$s_t =$ number of lumbers left to be harvested at the beginning of period t

$s_t \in [0, 100]$

$p_t =$ price in period t

The policy variable:

$$x_t = \text{how much to harvest in period } t$$

The transition functions:

$$s_{t+1} = s_t - x_t$$
$$p_{t+1} = p_0 + \rho p_t + u$$

The reward function:

$$f(p_t, , x_t) = p_t x_t - 0.2 x_t^{1.5}$$

The value function

$$V(p_t, s_t) = \text{maximum expected revenue could get from period } t$$

and **the Bellman's equation:**

$$V(p_t, s_t) = \max_{0 \leq x_t \leq s_t} \{f(p_t, x_t) + \delta V(p_{t+1}, s_{t+1})\}$$

Q.2

Take a look at `tauchen.m` *in the repository (you should know where), use it to generate grid that approximates process for* p_t *with 21 grid points.*

AR(1) process:

$$p_t = p_0 + \rho p_{t-1} + u$$

could be reformulate as:

$$p_t = \rho p_{t-1} + u'$$

where $u' = u + p_0$, has mean p_0 and standard deviation $\sigma_{u'} = 0.1$.

Thus, use tauchen's method to generate grid:

$[prob, grid] = tauchen(21, 0.5, 0.5, 0.1)$;

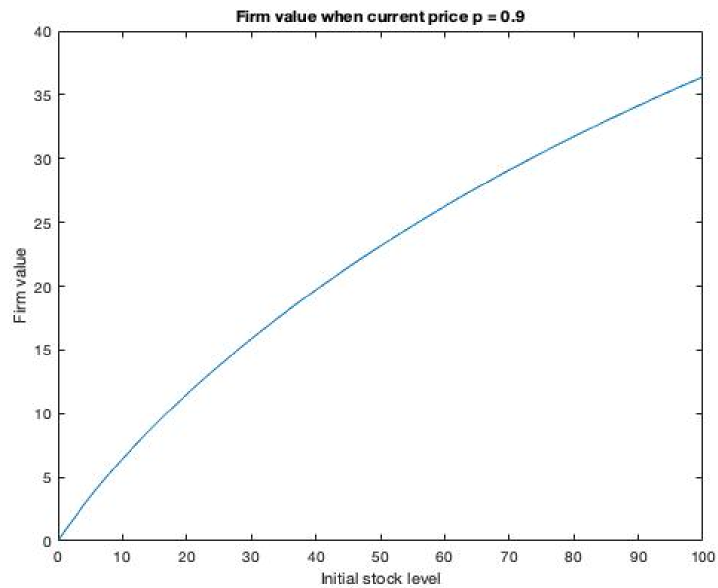
The grid we get is:

$[0.6536, 0.6882, 0.7229, 0.7575, 0.7922, 0.8268, 0.8614, 0.8961, 0.9307, 0.9654, 1.0000, 1.0346, 1.0693, 1.1039, 1.1386, 1.1732, 1.2078, 1.2425, 1.2771, 1.3118, 1.3464]$

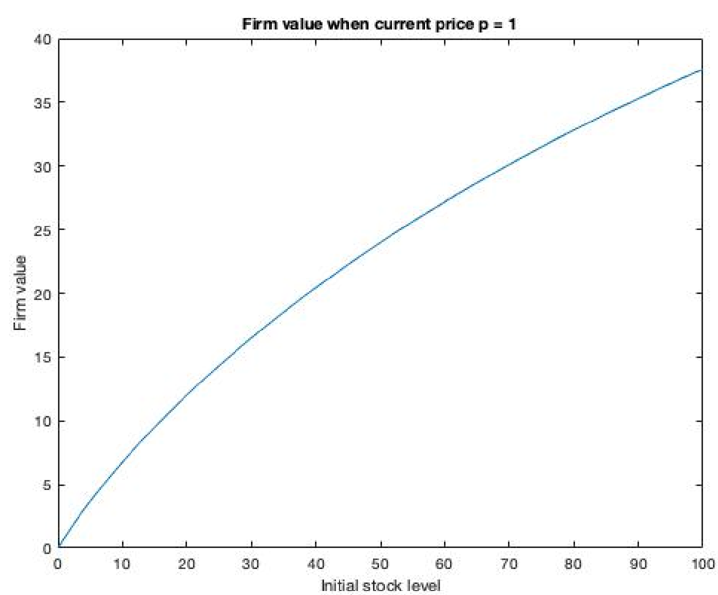
Q.3

Solve the firm's problem using value function iteration. Plot the value of the firm depending on its initial stock (x-axis) and the current price of lumber, for $p \in 0.9, 1, 1.1$.

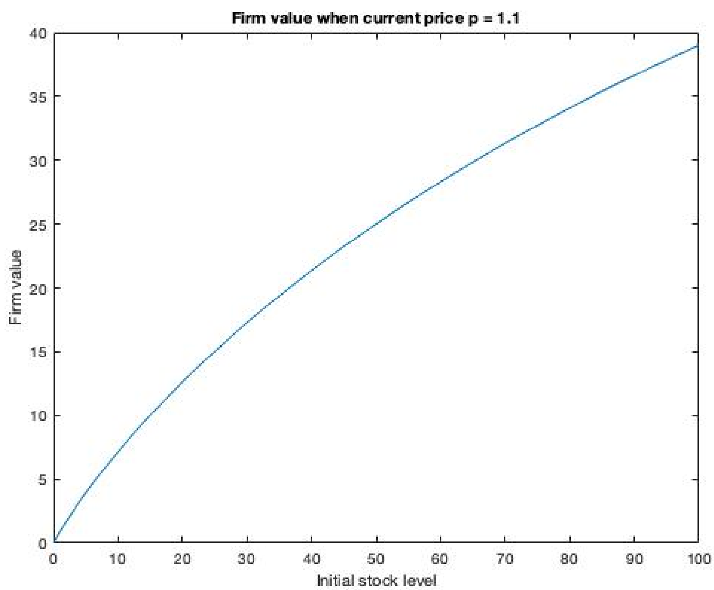
When $p = 0.9$,



When $p = 1$,

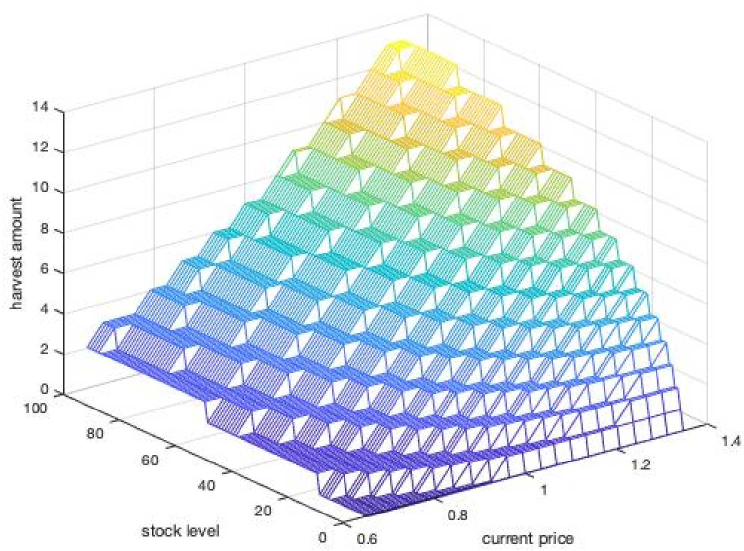


When $p = 1.1$,



Q.4

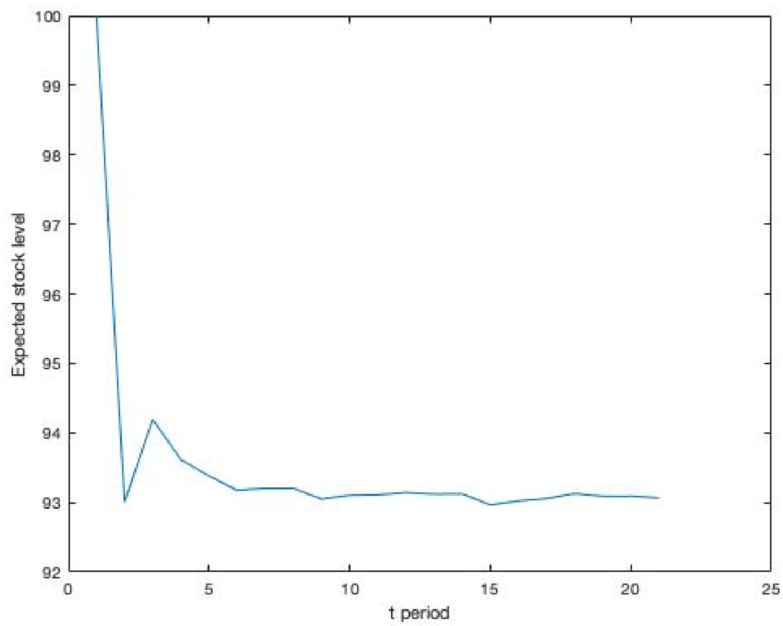
Plot next period optimal stock (or harvest amount if you prefer) as a function of today's price for different amount of lumber left in stock.



Q.5

Assume firm starts with stock of 100 and today's price is 1. Plot expected stock over time for 20 periods ahead. Include the 90 percent confidence interval.

The expected stock over time,



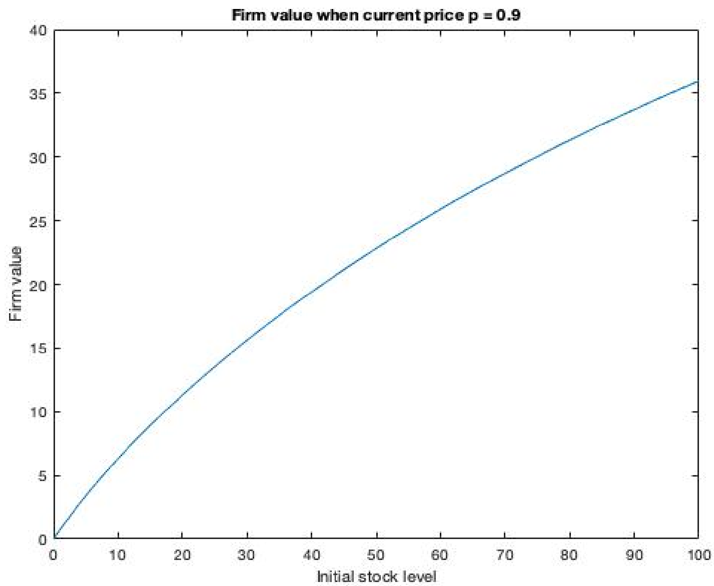
Q.6

Redo the 2-4 for coarse grid of 5 points in Tauchen's representation.

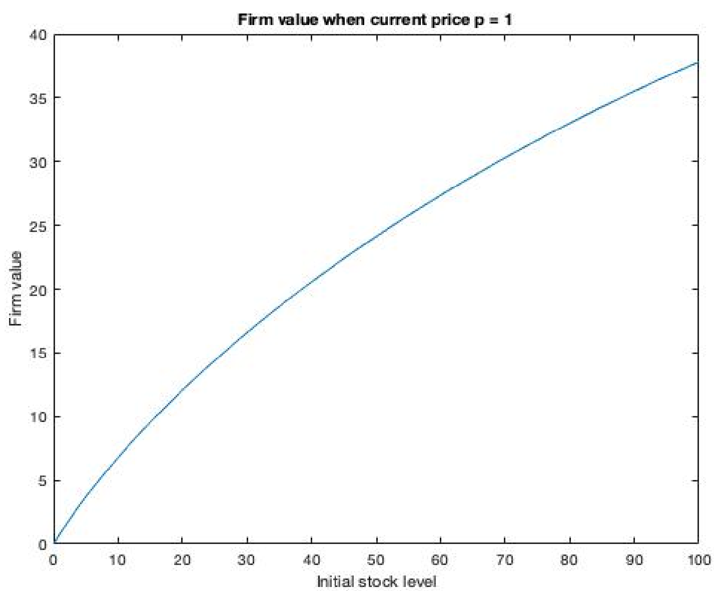
Using tauchen's method, the grid we get is:

$[0.6536, 0.8268, 1.0000, 1.1732, 1.3464]$

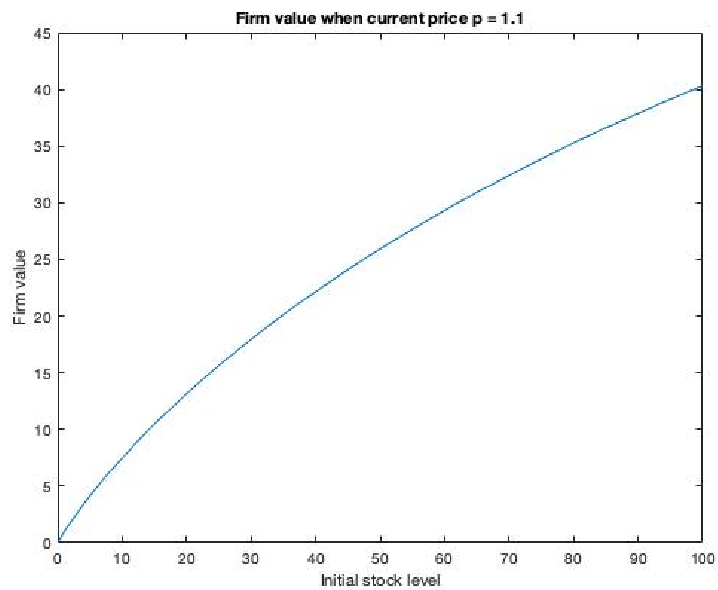
When $p = 0.9$,



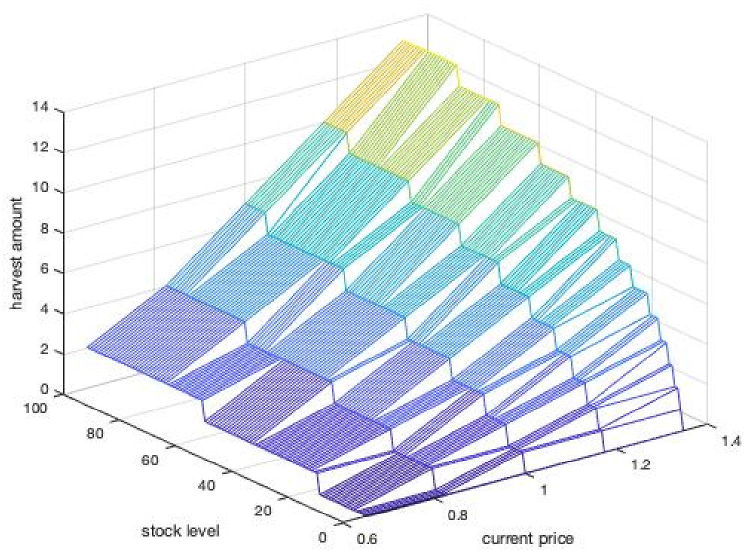
When $p = 1$,



When $p = 1.1$,



And the harvest amount as a function of today's price for different amount of lumber left,



Code

```
%% Q.2 Use tauchen.m to generate 21 grids for p_t
N = 21;
```

```
[prob,grid] = tauchen(N,0.5,0.5,0.1)

%% Q.3 Plot the value of firm depending on the initial stock and current price p = 0.9, 1, 1.1
% Use value function iteration
S = 100;
delta = 0.95;
V_ini = zeros(N, S + 1);
prob = prob';


V = zeros(N, S + 1); % V(i,j) maximum firm value when price is grid(i), amount of harvest left is j+1
X_temp = zeros(N, S + 1); % X(i,j) optimal harvest policy when price is grid(i), amount of harvest left is j+1
diff = 1;
tol = 1e-6;

while diff > tol
    for i = 1 : N
        for s = 1 : (S+1)
            V_temp = [];
            for x = 1 : s
                V_temp(x) = grid(i) * (x - 1) - 0.2 * ((x - 1).^1.5) ...
                    + delta * (V_ini(:, s - x + 1))' * prob(:, i);
            end
            [V(i,s), X_temp(i,s)] = max(V_temp);
        end
    end
    diff = norm(V - V_ini);
    V_ini = V;
end

X = X_temp - 1; % adjust the optimal harvest amount

```

```
% plot the value of the firm depending on the initial stock
% when current p = 0.9, plot V(8,:), since grid(8) = 0.8961 close to 0.9

x = 0 : S;
plot(x, V(8,:))
xlabel('Initial stock level')
ylabel('Firm value')
title('Firm value when current price p = 0.9')

% when current p = 1
x = 0 : S;
plot(x, V(11,:))
xlabel('Initial stock level')
ylabel('Firm value')
title('Firm value when current price p = 1')

% when current p = 1.1, plot V(14,:), since grid(14) = 1.1039 close to 1.1
x = 0 : S;
plot(x, V(14,:))
xlabel('Initial stock level')
ylabel('Firm value')
title('Firm value when current price p = 1.1')

%% Q.4 Plot optimal harvest amount as a function of today's price and different stock level
[m, n] = meshgrid(grid, 0 : S);
mesh(m,n,X')
xlabel('current price')
ylabel('stock level')
zlabel('harvest amount')

%% Q.5 Firms starts with stock 100 and price 1, expected stock over time for 20 periods

```

```
times = 1000; % use 1000 samples to calculate the expected value
Stock = zeros(times, 21);
Stock(:, 1) = 100;
Stock(:, 2) = 100 - X(find(grid == 1), 100);
p = [];
p(1) = 1;
p(2) = grid(randsample(21,1,true,prob(:,find(grid == 1))));
for i = 1 : times
    for j = 3 : 21
        p(j) = grid(randsample(21,1,true,prob(:,find(grid == p(j
            -1)))));
        Stock(i,j) = 100 - X(find(grid == p(j)), 100);
    end
end
Stock_exp = mean(Stock, 1);

% plot expected stock level against 20 periods ahead
t = 1:21;
plot(t, Stock_exp)
xlabel('t period')
ylabel('Expected stock level')



%% Q.6 Redo 2-4 for coarse grid of 5 points in Tauchen's
    representation

N = 5;

% and use the code in 2-4
```