Econ 512

Fall 2018

Homework 5 – Binary Choice MLE with Random Coefficients

Yinshi Gao
yzg115

Consider the following binary discrete choice model for panel data:

$$Y_{it} = I\left(\beta_i X_{it} + \gamma Z_{it} + u_i + \epsilon_{it} > 0\right),$$

where X_{it} and Z_{it} are scalar regressors for a group of $i = 1, ..., N$ individuals and $t = 1, ..., T$ time periods. $N = 100$ and $T = 20$. The coefficients β_i and u_i are person specific and are modeled as draws from a bivariate normal distribution:

$$\begin{bmatrix}\beta_i \\ u_i\end{bmatrix} \sim N(\mu, \Sigma), \quad \text{where} \quad \mu = \begin{bmatrix}\beta_0 \\ u_0\end{bmatrix}, \quad \Sigma = \begin{bmatrix}\sigma_\beta & \sigma_{\beta u} \\ \sigma_{\beta u} & \sigma_u\end{bmatrix}.$$

Assume ϵ_{it} follows standard logistic distribution, i.e. $F(\epsilon) = \left(1 + e^{-\epsilon}\right)^{-1}$, and then a single contribution to the likelihood function from individual i is

$$L_i(\gamma \mid \beta_i, u_i) = \prod_{t=1}^{T} F(\beta_i X_{it} + \gamma Z_{it} + u_i)^{Y_{it}} \left[1 - F(\beta_i X_{it} + \gamma Z_{it} + u_i)\right]^{1-Y_{it}}.$$

To construct the likelihood function for the data set of NT observations we have to integrate over the joint distribution of (β_i, u_i). The likelihood function for the data set is:

$$L(\gamma, \mu, \Sigma) = \prod_{i=1}^{N} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} L_i(\gamma \mid \beta_i, u_i) \phi(\beta_i, u_i \mid \mu, \Sigma) \, \mathrm{d}\beta_i \, \mathrm{d}u_i,$$

where $\phi(\cdot \mid \mu, \Sigma)$ is the joint density function of bivariate normal distribution $N(\mu, \Sigma)$. Of course, it will be numerically more convenient to work with the log-likelihood function. The data set `hw5.mat` contains 20 x 100 matrices of the variables X, Z, and Y.

1. Assume $u_i = 0 \; \forall \, i$ (ie. take u_i out of the model, so that $u_0 = \sigma_u = \sigma_{u\beta} = 0$). Use Gaussian Quadrature using 20 nodes to calculate the log-likelihood function when $\beta_0 = 0.1$, $\sigma_\beta = 1$, and $\gamma = 0$.

   The value of log-likelihood function is:

   $L = -1.5345e + 03$

2. Now use Monte Carlo Methods using 100 nodes to calculate the log-likelihood function.

   The value of log-likelihood function is:

   $L = -1.2373e + 03$

3. Maximize (or minimize the negative) log-likelihood function with respect to the parameters using both integration techniques above. Use Matlab's fmincon without a supplied derivative to max (min) your objective function.

   The initial values used:

   $[\beta_0, \sigma_\beta, \gamma] = [0.1, 1, 1]$

   - **Minimizing the negative of log-likelihood function from Gaussian Quadrature techniques:** $[\beta_0, \sigma_\beta, \gamma] = [0.4772, 0.0147, 0.0177]$, **the maximized value is**: $1.4534e + 03$.
   - **Minimizing the negative of log-likelihood function from Monte Carlo techniques:** $[\beta_0, \sigma_\beta, \gamma] = [0.6354, 1.0784, 0.0129]$, **the maximized value is**: $1.2202e + 03$.

4. Now allow $u_0 \neq 0$, so allow the parameters σ_u and $\sigma_{\beta u}$ to be non-zero. Maximize the log-likelihood function, estimating all of the parameters, using Monte Carlo methods.

   The initial values used:

   $\mu = [0.5; 0.5]$, $\Sigma = [1, 0.5; 0.5, 1]$, $\gamma = 0.5$

   the estimated parameters:

   $\mu = [0.6484; 0.4733]$, $\Sigma = [1.0327, 0.4656; 0.4656, 0.9970]$, $\gamma = 0.5345$

   the maximized value is:

   $1.3777e + 03$

5. For each estimation, report the starting value, argmax, and maximized value of the log-likelihood function.

(Hint: the matlab function "chol" may come in handy for simulating from the joint density.)