Consider what happens to that flow once the system reaches equilibrium. Since the system is in equilibrium, the rate into any state must be equal to the rate of flow out of that state. More generally, the rate of flow into and out of any closed set of states must possess this same property of conservation of probability flow. Let us choose special sets of states that allow us to generate Equation (11.10). Specifically, let us choose the following sequence of sets of states: the first set contains state 0, the second set contains states 0 and 1, the third contains state 0, 1 and 2, and so on, as is illustrated in Figure 11.14. Observe now that equating the flow of across the n^{th} boundary gives us what we need (Equation 11.10), namely

$$\lambda_{n-1} p_{n-1} = \mu_n p_n$$

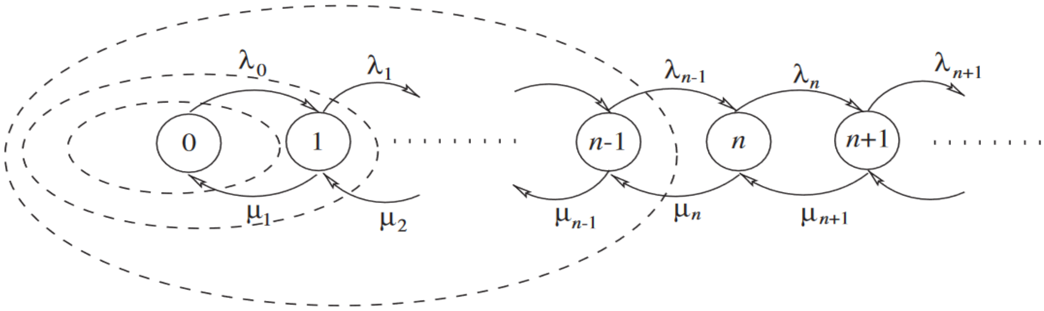

Figure 11.14. Transitions across groups of states.

This method of generating the equilibrium balance equations turns out to be useful in many different circumstances. The reader might wonder why we cluster the states in this fashion and observe the flow into and out of each cluster, rather than simply draw a vertical line to seperate neighboring complex situations, for example, when states are drawn as a two-dimensional grid, seperating states with a signle line becomes more onerous and error-prone.

We now turn our attention to the computation of p_0. We have

$$1 = \sum_{n\geq 0} p_n = p_0 + \sum_{n\geq 1} p_n = p_0 + \sum_{n\geq 1} p_0 \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i} = p_0[1 + \sum n_{\geq} 1 \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i}].$$

Setting $\rho_i = \lambda_{i-1}/\mu_i$, we obtain

$$p_0 = \frac{1}{1 + \sum_{n\geq 1} \prod_{i=1}^{n} \rho_i}.$$

To simplify the notation somewhat, let

$$\xi_k = \begin{cases} 1, & \text{k=0,} \\ \prod_{i=1}^{n} \rho_i, & \text{k>0.} \end{cases}$$

Then

$$p_n = p_0 \xi_n = \frac{\xi_n}{\sum_{k=0}^{\infty} \xi_k} \qquad \text{for } n = 0, 1, 2, \ldots.$$

The limiting distribution $(p_0, p_1, p_2, \ldots)$ is now completely determined. The limiting probabilities are nonzero provided that $p_0 > 0$; otherwise $p_i = 0$ for all i = 0,1,2, ... and there is no steady-state distribution. Thus the existance of a stationary distribution depends on whether of not there exists a finite probability that the queueing system empties itself. If it does then the

Consider what happens to that flow once the system reaches equilibrium. Since the system is in equilibrium, the rate into any state must be equal to the rate of flow out of that state. More generally, the rate of flow into and out of any closed set of states must possess this same property of conservation of probability flow. Let us choose special sets of states that allow us to generate Equation (11.10). Specifically, let us choose the following sequence of sets of states: the first set contains state 0, the second set contains states 0 and 1, the third contains state 0, 1 and 2, and so on, as is illustrated in Figure 11.14. Observe now that equating the flow of across the n^{th} boundary gives us what we need (Equation11.10), namely

$$\lambda_{n-1} p_{n-1} = \mu_n p_n.$$

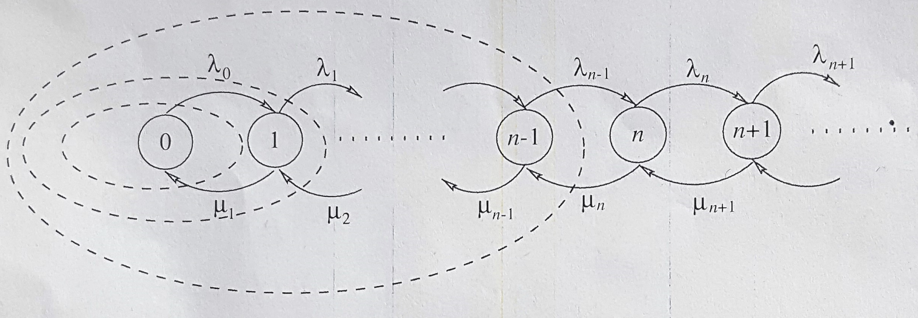

Figure 11.14. Transitions across groups of states.

This method of generating the equilibrium balance equations turns out to be useful in many different circumstances. The reader might wonder why we cluster the states in this fashion and observe the flow into and out of each cluster, rather than simply draw a vertical line to separate neighboring states and equate flow across this line. In this particular case, there is no difference, but in more complex situations, for example, when states are drawn as a two-dimensional grid, separating states with a single line becomes more onerous and error-prone.

We now turn our attention to the computation of p_0. We have

$$1 = \sum_{n\geq 0} p_n = p_0 + \sum_{n\geq 1} p_n = p_0 + \sum_{n\geq 1} p_0 \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i} = p_0 \left[1 + \sum_{n\geq 1} \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i} \right].$$

Setting $\rho_i = \lambda_{i-1}/\mu_i$, we obtain

$$p_0 = \frac{1}{1 + \sum_{n\geq 1} \prod_{i=1}^{n} \rho_i}.$$

To simplify the notation somewhat, let

$$\xi_k = \begin{cases} 1, & k = 0, \\ \prod_{i=1}^{k} \rho_i, & k > 0. \end{cases}$$

Then

$$p_n = p_0 \xi_n = \frac{\xi_n}{\sum_{k=0}^{\infty} \xi_k} \quad \text{for } n = 0, 1, 2, \ldots.$$

The limiting distribution $(p_0, p_1, p_2, \ldots)$ is now completely determined. The limiting probabilities are nonzero provided that $p_0 > 0$; otherwise $p_i = 0$ for all $i = 0, 1, 2, \ldots$ and there is no steady-state distribution. Thus the existence of a stationary distribution depends on whether or not there exists a finite probability that the queueing system empties itself. If it does then the

```
\usepackage{tikz}
\usepackage[shortlabels]{enumitem}
\setlist[enumerate]{nosep}
\usepackage{amssymb}
\usepackage{mathptmx}
\usepackage{ragged2e}
\usepackage{setspace}
\usepackage{amsmath}
\usepackage[T1]{fontenc}
\usepackage{graphicx}

\begin{document}
{\fontfamily{ptm}\selectfont
\begin{flushright}
\textbf{11.3 General Birth-Death Processes \hspace{5mm}    417}
  \par\noindent\rule{\textwidth}{0.1pt}
\end{flushright}
\begin{justify}
Consider what happens to that flow once the system reaches equilibrium. Since the system is
\end{justify}
\begin{center}
$\lambda_{n-1} p_{n-1} = \mu_n p_n$
\includegraphics[width=15cm, height=5cm]{Capture4.PNG}

Figure 11.14. Transitions across groups of states.

\end{center}
\begin{justify}

This method of generating the equilibrium balance equations turns out to be useful in many

We now turn our attention to the computation of $p_0$. We have
 \begin{equation*}
  1 = \sum_{n_\geq0} {p_n} = p_0 + \sum_{n_\geq1} {p_n} = p_0 + \sum_{n_\geq1} {p_0} \prod_
  \end{equation*}
 Setting $\rho_i = \lambda_{i-1}/\mu_i$, we obtain
 $$p_0 = \frac{1}{1+\sum_{n_\geq1}\prod_{i=1}^{n} \rho_i}.$$
 To simplify the notation somewhat, let
 \begin{equation*}
 \xi_k = \begin{cases} 1, & \mbox{k=0,} \\ \prod_{i=1}^{n} \rho_i, & \mbox{k>0.} \end{cases
 \end{equation*}
Then
\begin{equation*}
p_n = p_0 \xi_n = \frac{\xi_n}{\sum_{k=0}^{\infty} {\xi_k}}  \hspace{5mm} \mbox{for } n = 0
\end{equation*}
The limiting distribution ($p_0,p_1,p_2,. . .$) is now completely determined. The limiting
\end{justify}
\begin{center}
    \includegraphics[width=18cm, height=25cm]{20190529_090151.png}
    \end{center}
```