Second Exam

Agneta Apaļka

May, 2019

Sets

I	set $i \in I$ of demands
J	set $j \in J$ of demand levels
M	set $m \in M$ of demand types
Ω	set $\omega \in \Omega$ of scenarios
K	set $k \in K$ of arcs, k = 1…8
P	set $p \in P$ of paths

Data

$aa_{k,p}$	arc path incidence matrix, where $aa_{k,p}$ = 1 if path p uses arc k
$dd_{i,p}$	demand path incidence matrix, where $dd_{i,p}$=1 if path p can satisfy demand i
b	total budget
c_k	unit installing cost for arc k

Random Variables

$ppDemand_i$	the point to point demand for i
r_k	reliability of arc k

Decision Variables

x_k	first stage capacity allocation by arc k
$y^{\omega}_{i,p}$	second stage assignment of demand to path p under scenario u
z_i	second stage demand short

Formulation

$$\min E[h(x,\tilde{\xi})] \qquad (24)$$

$$\text{s.t.} \quad \sum_K c_k x_k \le b \qquad (25)$$

$$x \in \mathbb{R}^+$$

$$h(x,\tilde{\xi}) = \min \sum_i z_i \qquad (26)$$

$$\sum_{\{p:dd(i,p)=1\}} y_{i,p} + z_i = ppDemand_i \qquad \forall i \in I \qquad (27)$$

PS4-2 Telecommunications Network Planning

Description

(Higle and Sen) Figure 2 depicts a small telecommunications planning network for high-capacity services such as televideo conference calls. We wish to "build" this network by installing capacity on each of the seven arcs. The cost of installing capacity on (A, E), (E, D), or (D, C) is 2.9 per unit while the cost of installing capacity on (A, B), (E, B), (D, B), or (C, B) is 1.0 per unit. We have a total budget of 20.0.

The capacity expansion decisions must be made in the face of uncertainty with respect to the future values of point-to-point demand pairs and arc reliabilities. Arcs (A, E), (E, D), and (D, C) are "100%" reliable while arcs (A, B), (E, B), (D, B), and (C, B) operate at full capacity with probability 0.7 and zero capacity with probability 0.3. The point-to-point demand distributions are specified in Table 7 and Table 8. The respective demands and capacity availabilities are modeled as independent random variables.

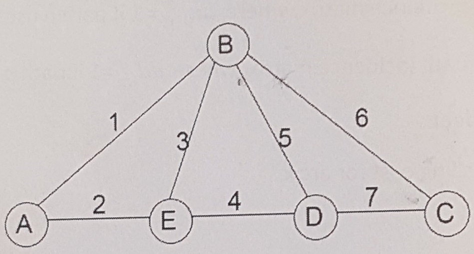

Figure 2: Telecommunications Planning Network

	B	C	D	E
A	type 1	type 2	type 2	type2
B		type 1	type 1	type 1
C			type 2	type 2
D				type 2

Table 7: Point-to-point demand distribution types

Type 1		Type 2	
d^ω	$P(\tilde{d}=d^\omega)$	d^ω	$P(\tilde{d}=d^\omega)$
0	0.05	0	0.1
1	0.2	1	0.4
2	0.5	2	0.4
3	0.2	3	0.1
5	0.05		

Table 8: Demand distribution

PS4-2a

Formulate the telecommunications capacity expansion problem as a two-stage stochastic linear program with an objective that minimizes that expected number of blocked calls, i.e., minimizes unmet demand. Do this for a "generic" problem.

The following notation describes our formulation:

Sets

I set $i \in I$ of demands

J set $j \in J$ of demand levels

M set $m \in M$ of demand types

Ω set $\in \Omega$ of scenarios

K set $k \in K$ of arcs, k = 1...8

P set $p \in P$ of paths

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$aa_{k,p}$ arc path incidence matrix, where $aa_{k,p} = 1$ if path p uses arc k

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z_i second stage demand short

Formulation

min $E[h(x, \xi)]$ (24)

s.t. $\sum_K c_k x_k \geq b$ (25)

$x \in R^+$

$h(x, \xi) = min \sum_i z_i$ (26)

$\sum_{p:dd(i,p)=1} \mathsf{y}_{i,p} + z_i = ppDemand_i \qquad \forall i \in I$ (27)

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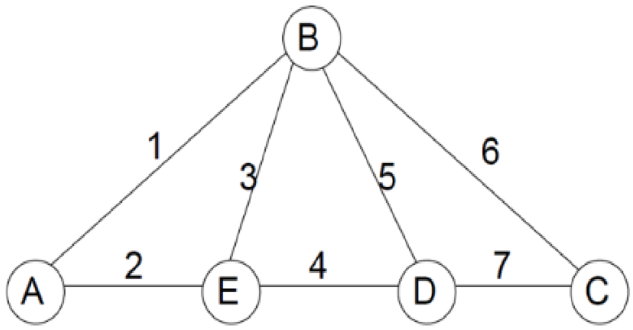

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PS4-2a

Formulate the telecommunications capacity expansion problem as a two-stage stochastic linear program with an objective that minimizes that expected number of blocked calls, i.e., minimizes unmet demand. Do this for a "generic" problem.

The following notation describes our formulation:

```
\documentclass{report}
\usepackage[utf8]{inputenc}
\usepackage{amsmath}
\usepackage[T1]{fontenc}
\usepackage[document]{ragged2e}
\usepackage{graphicx}
\usepackage{geometry}
 \geometry{
 a4paper,
 total={174mm,247mm},
 left=23mm,
 top=18mm,
 right=25mm,
 }
 \title{Second Exam}
\author{Agneta Apaļka}
\date{May, 2019}


\usepackage{lmodern}
\pagenumbering{gobble}
\begin{document}
\maketitle

\newpage
\includegraphics[width=160mm, height=240mm]{a0.png}

\includegraphics[width=160mm, height=240mm]{a1.png}
\newpage
{\fontfamily{lmss}\selectfont
\paragraph{\textit{Sets}}
\hspace{8mm}
\begin{description}

\item \hspace{8mm} I  \hspace{3mm } set $i \in I$ of demands

\item \hspace{8mm}J \hspace{3mm} set $j \in J$ of demand levels

\item \hspace{8mm}M \hspace{3mm}set $m \in M$ of demand types

\item \hspace{8mm}$\Omega$ \hspace{3mm} set $\w \in \Omega$ of scenarios

\vspace{5mm}

\item \hspace{8mm}K \hspace{3mm}set $k \in K$ of arcs, k = 1...8

\item \hspace{8mm}P \hspace{3mm}set $p \in P$ of paths
\end{description}

\vspace{3mm}
\paragraph{\textit{Data}}

\begin{description}

\item \hspace{8mm}$aa_{k,p}$ \hspace{10mm}arc path incidence matrix,
where $aa_{k,p}$ = 1 if path $p$ uses arc $k$

\item \hspace{8mm}$dd_{i,p}$ \hspace{10mm}demand path incidence matrix,
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```latex
where $dd_{i,p}$ =1 if path $p$ can satisfy demand $i$

\item \hspace{8mm}$b$ \hspace{16mm}total budget

\item \hspace{8mm}$c_k$ \hspace{14mm}unit installing cost for arc k

\end{description}

\vspace{3mm}
\paragraph{\textit{Random Variables}}

\begin{description}
\item \hspace{8mm}$ppDemand_{i}$ the point to point demand for $i$

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\vspace{3mm}
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\vspace{3mm}
\paragraph{\textit{Formulation}}

$ $

\vspace{3mm}
min $E[h(x,\xi)]$ \hspace{120mm}(24)

\vspace{5mm}
s.t.\hspace{10mm} $\sum_K c_k x_k \geq b$ \hspace{105mm}(25)

\vspace{5mm}
\hspace{17mm}$x \in R^+$

\vspace{4mm}
$h(x,\xi) = min \sum_i z_i$ \hspace{112mm}(26)

\vspace{7mm}
\hspace{17mm}\sum_{{p:dd(i,p)=1}} $y_{i,p} + z_i = ppDemand_i$
\hspace{32mm}$\forall i \in I$\hspace{31mm}(27)


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\textbf{PS4-2 Telecommunications Network Planning}

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\textbf{Description}
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\begin{justify}
(Higle and Sen) Figure 2 depicts a small telecommunications planning network for high-capacity service
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\begin{justify}
The capacity expansion decisions must be made in the face of uncertainty with respect to the future va
\end{justify}
\begin{center}

\includegraphics[width=10cm, height=5cm]{Capture1.PNG}

Figure 2: Telecommunications Planning Newtork

\includegraphics[width=12cm, height=5cm]{Capture2.PNG}

Table 7: Point-to-point demand distribution types
\vspace{2mm}
\includegraphics[width=10cm, height=4cm]{Capture3.PNG}

Table 8: Demand distribution

\end{center}

\textbf{PS4-2a}

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\begin{justify}
Formulate the telecommunications capacity expansion problem as a two-stage stochastic linear program w
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