

- (i) The probability of picking six tickets of the same colour is given by

$$\Pr(\text{six of the same colour}) = 3 \times \frac{6!}{6!0!0!} \left(\frac{1}{3}\right)^6 \left(\frac{1}{3}\right)^0 \left(\frac{1}{3}\right)^0 = \frac{1}{243}.$$

The factor of 3 is present because there are three different colours.

- (ii) The probability of picking five tickets of one colour and one ticket of another colour is

$$\Pr(\text{five of one colour; one of another}) = 3 \times 2 \times \frac{6!}{5!1!0!} \left(\frac{1}{3}\right)^5 \left(\frac{1}{3}\right)^1 \left(\frac{1}{3}\right)^0 = \frac{4}{81}.$$

The factors of 3 and 2 are included because there are three ways to choose the colour of the five matching tickets, and then two ways to choose the colour of the remaining ticket.

- (iii) Finally, the probability of picking two tickets of each colour is

$$\Pr(\text{two of each colour}) = \frac{6!}{2!2!2!} \left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)^2 = \frac{10}{81}.$$

Thus the expected return to any patron was, in pence,

$$100\left(\frac{1}{243} + \frac{4}{81}\right) + \left(40 \times \frac{10}{81}\right) = 10.29.$$

A good time was had by all but the stallholder! ◀

30.15.2 The multivariate Gaussian distribution

A particularly interesting multivariate distribution is provided by the generalisation of the Gaussian distribution to multiple random variables $X_i, i = 1, 2, \dots, n$. If the expectation value of X_i is $E(X_i) = \mu_i$ then the general form of the PDF is given by

$$f(x_1, x_2, \dots, x_n) = N \exp\left[-\frac{1}{2}\right] \sum_i$$