

CS70 Discussion 2a Review

AGNIBHO ROY

SUMMER 2020

1 Review

These are just concepts and strategies discussed during discussion section on June 25, 2020. For definitions and formulas, refer to note 5 on the course website.

1.1 Countability

- General rule of thumb: If you see any concept of "finite" in the question, then the set that is mentioned is going to be countable. You cannot just state this obviously, you have to prove it by either creating a bijection with N or some subset of it.
- Know the general arguments of moving across a diagonal or around a spiral on a coordinate grid to create a bijection between the Cartesian product or rationals. You will often see some variant of this on exam-level questions.
- Using a series of reductions to a set that we already know as countable or uncountable is very helpful and is a valid form of justification. Q2.2 on this sheet is a good example of this. Discussion 2a Q1 is also a very good technique for proving countability or uncountability.
- Cantor's diagonalization can only be used to prove uncountability. Only use this when you can replace the diagonal with another valid element and if the list is infinite. If the list is finite or you cannot change the diagonal DO NOT use this method (and your set is probably countable so try to find a bijection)

1.2 Computability

- There are many problems that are not able to be computed, and this has to do with how you can relate the problem to testhalt. We can take without proof that testhalt does not exist.
- If you are able to create a reduction from testhalt to the subroutine, then you have proven that the subroutine is not possible. If you have some concept of being able to run a program to a finite point or it does not relate to whether the program halts or not, then you cannot make it a subroutine of testhalt. This means that the program is computable.

2 Extra Problems

These problems are not necessarily in scope. Some may be helpful on exams, but some others are just fun exercises. Reach out to me by email (agnibhoroy@berkeley.edu) if you see any mistakes or have questions about any of the questions.

2.1 Bunch of Numbers

1. Is the set of all infinite sequences of integers countable or not?
2. Is the set of all finite ternary strings (strings that are consisted of the set of elements $\{0, 1, 2\}$) countable? What about the set of all infinite ternary strings? Prove your answers.

2.2 I'm Bored of Numbers, Lets Move to Words

This question is going to employ some concepts used in question 1 from discussion 2a. Feel free to review that question before attempting these problems.

1. Prove that the number of finite word sequences from the English alphabet is countable
2. Prove that the number of finite computer programs (such as "for i in range", "hashmap", etc.) is countable.