CS70 Discussion 1a Review

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1 Review

These are just concepts and strategies discussed during discussion section on June 22, 2020. For definitions and formulas, refer to note 1 on the course website.

1.1 Propositional Logic

- Adjacent quantifiers that are of the same type are interchangeable without changing the logic of the expression ($\exists x, \exists, y \equiv \exists y, \exists x$ and $\forall x, \forall, y \equiv \forall y, \forall x$).
- Read propositional statements from left to right: $\forall x \exists y P(x, y)$ means that for every single x ($x = 1, x = 2 \ldots$), can we find a y that makes $P(x, y)$ true. On the other hand, $\exists y \forall x P(x, y)$ means that there is a y, universal to all x, where $P(x, y)$ is true.
- The only sure equality to a statement is its contrapositive. Often times, to prove a statement, proving the contrapositive is actually much simpler (when you have a lot of "not"s in the question)

1.2 Truth Tables

- Truth tables can be transcribed into a propositional statement of the three primitive symbols easily by creating a disjunction of conjunctions. Take all true combinations: $(A \wedge B) \vee (\neg A \wedge B) \vee \ldots$ and combine them with "or" statements.
- Simplify statements using the laws of boolean algebra that we know (negation of a negation, De Morgan's, etc.) to provide a much simpler result.

2 Extra Problems

These problems are not necessarily in scope. Some may be helpful on exams, but some others are just fun exercises. Reach out to me by email (agnibhoroy@berkeley.edu) if you see any mistakes or have questions about any of the questions.

2.1 Simple Propositions

Convert the following sentences into a propositional statement or vice versa. Keep in mind that there may be multiple formulations that are correct. Also, try to reason if these propositions are true or not.

1. There does not exist a positive integer whose square is less than the number itself
2. There exists a real number that lies in between two distinct real numbers

2.2 Set Propositions

A vector space V has the properties of closure for addition, scalar multiplication, and an inverse for every vector $v \in V$. Write a propositional statement that states that if V is a vector space, then it satisfies these three properties.

2.3 Boolean Algebra and Truth Tables

From discussion we looked at an XOR between two boolean variables (question 2). Let an XOR between three boolean variables be represented as $A \oplus B \oplus C$. Write the truth table for this operation and also express XOR using only $(\wedge, \vee, \neg)$ (Hint, think of $A \oplus B \oplus C$ as $(A \oplus B) \oplus C$ for help with the truth table)