EECS 126 Notes

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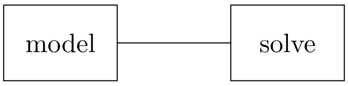
The following is a compilation of my lectures notes for the spring rendition of EECS 126 taught by professor Kannan Ramachandran. I only kept up typing up my notes in the first couple lectures, and got a little lazy and wrote the rest by other means. Currently working on transcribing them and should be done in a couple of weeks.

Contents

1 Lecture 1 - 1/21/2020

1.1 Introduction

This class is a class that deals with uncertainty, and has roots in predictions, strategy and decision making, and design. We can set up any problem in this class in the following way:



Here, we use the model to attempt to understand the problem at hand, and then use a set of tools to solve the question using the model that we built. An example of this is answering the question of how many customers we expect to arrive within an hour at the front of a store. We first model the arrivals to the store as a Poisson Process, determining the rate λ of arrivals based on experimental observations. Then we can attempt to solve the question using our model by calculating the expectation of $E[\text{Poisson} \sim \lambda(1)]$

1.2 Sample Spaces

In order to understand probability, we introduce the notion of a **sample space (Ω)**, which is the set of all possible outcomes of an experiment. Two important properties of sample spaces are they are **mutually exclusive** and **collectively exhaustive**, which means that no two samples in the sample space can occur at the same time in one iteration of the experiment, and the sample space sufficiently represents all possible outcomes of the experiment, respectively. Ex. A single toss of two fair coins gives has a sample space $\Omega = \{HH, HT, TH, TT\}$, representing all possible outcomes for the single experiment. Sample spaces can be infinite at times; take for example tossing a coin until the first heads, which has has a countably infinite sample space $\Omega = \{H, TH, TTH \dots\}$ or the amount of time you have to wait for a bus to come at a bus stop that will come at latest at some time T, which has a uncountably infinite sample space $\Omega = (0, T)$.

An **event** is defined as any allowable subset of Ω that combines. The probability of these events is the sum of the individual probabilities of the elements in the sample space that are in the event. Another way to think about it is that the events are mapped to a probability that is proportional to Ω in its weight. Note that this means that the total possible events is $2^{|\omega|}$.

Example 1.2.1: Consider the experiment of a single toss of two fair coins. Let the event A = We get at least 1 head. Then:

$$P(A) = \frac{P(\{HT, TH, HH\})}{P(\{TT, HT, TH, HH\})} = \frac{3}{4}$$

All of these concepts together form a summarized notion of what a probability space is for an experiment and can essentially be treated as the "cheat sheet" for any question pertaining to the experiment. In general, we obtain the following definition:

Definition 1.2.1: The **probability space** for an experiment can be concisely summarized as (Ω, F, P) where Ω denotes the sample space, F denotes the set of all possible events, which are subsets of Ω, and P denotes the probability of each $\zeta \in F$; an injective mapping of $F \to (0, 1)$

1.3 Basic Probability Concepts

there are some basic rules that all probability spaces should follow, and they are defined by Kolmomgorov as the axioms of probability:

Definition 1.3.1: The **axioms of probability**:

1. $P(\emptyset) = 0$
2. $P(\Omega) = 1$
3. For mutually exclusive events $A_1 \dots A_n$, $P(\bigcup\limits_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$

These axioms should be followed like a religion, but there are other fundamental probability facts that are derived from the ones above that should be known, one will find that they are quite useful in problem solving.

Definition 1.3.2: Fundamental Facts of Probability:

1. Complement: $P(A^c) = 1 - P(A)$
2. Union of Events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
3. Union Bound: $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$
4. Inclusion/Exclusion Principle: (insert formula)

The most important of these facts is the **Union Bound**, which we can derive by argument with a visual argument using Venn Diagrams. Consider two Venn diagrams representing two separate events, A_1 and A_2. We know that when we take the sum of the two events, we count the middle twice, essentially over counting the intersection by a factor of 2. We can generalize this to n events, where there are a various number of intersections now, namely 2^n possible intersections that could be double counted (can you see why?). Of course, when we have mutual exclusivity of events (intersection is $\emptyset$), then the definition 1.3.2 (3) turns into the equality in definition 1.3.1 (3). The Union Bound is quite useful when it becomes difficult to properly calculate intersections. Here is an example of using the union bound as well as the notion of the considering the complement of an event.

Example 1.3.1: Consider a sphere that has $\frac{1}{10}$th of its surface colored blue, and the rest is colored red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red

1.4 Conditional Probability

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2.1 Birthday Paradox

2.2 Independence

3 Lecture 3: 1/28/2020

3.1 Discrete Random Variables

3.2 Expectation and Variance

3.3 Popular RVs

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5 Lecture 5: 2/04/2020

6 Lecture 6: 2/06/2020

7 Lecture 7: 2/11/2020

8 Lecture 8: 2/13/2020

9 Lecture 9: 2/18/2020

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