CS70 Discussion 8a (Exam Review Session)

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Summer 2020

1 Can You Guess Their Age?

There are three siblings in a family and it is known that sibling A is the youngest, sibling B is the middle child, and sibling C is the oldest (we only know their ages in years). Moreover, we know that the siblings are all at least of age 10 and at most of age 20. Agnibho, Srishti, and Khalil take some guesses at the number of ways that we could assign ages to each of the siblings.

1. Agnibho is the most stupid, and forgets that the siblings have different ages and who the youngest, middle, and oldest are. How many combinations of ages could Agnibho guess from?

2. Srishti is a little less forgetful, but did not listen carefully. She does not know that none of the siblings have the same age as another, but understands that $C \geq B \geq A$. How many combinations of ages could Srishti guess from?

3. Khalil is the most attentive of them all, and knows that the sibling ages must be different and that $C > B > A$. How many combinations of ages could Khalil guess from?

2 Increasing and Decreasing, Nearly

Six cards numbered 1 through 6 are to be lined up in a row. Find the number of arrangements of these six cards where one of the cards can be removed leaving the remaining five cards in either ascending or descending order. (Source: 2020 AIME I)

3   Gardening

1. There is a landscaper who wants to plant flowers in a row, which consist of s sunflowers, b bluebells, and r roses, and wants the roses to be spread out. How many arrangements exists where no two roses are next to each other?

2. How many arrangements exist where at least two other flowers are separating each rose?

4   Fixed Permutations

For each permutation σ of 1 through n, let $\sigma(i)$ denote the value at position i. For example, if the permutation is $2, 4, 1, 3$ we have $\sigma(1) = 2$ and $\sigma(2) = 4$ (Source: CS 70 SP19 MT2)

1. For a fixed $1 \leq k \leq n$, how many permutations σ of 1 through n are there where for all $i < k, \sigma(i) < \sigma(k)$? Express your answers in terms of n and k.

2. How many permutations of 1 through n are there such that for each i, $\sigma(\sigma(i)) = i$ and $\sigma(i) \neq i$? (For example, the permutation $3, 4, 1, 2$ is such a permutation, since for example $\sigma(\sigma(1)) = \sigma(3) = 1$. You may assume n is even.