

# CS70 Discussion 1d Extra Problems Solutions

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## Injective Polynomials

1. Can you find a set of values  $(a_0, a_1, a_2)$  such that  $a_0x^2 + a_1x + a_2$  is injective?

*Solution.* No, no matter how we set these three values, unless our parameters are 0 or infinite, we cannot eliminate the property that  $y \rightarrow \infty$  as  $x \rightarrow \pm\infty$ . As a result, the parabola is always mirrored around some vertex point, making the points equidistant to this vertex map to the same  $y$  value.

2. Can you find a set of values  $(a_0, a_1, a_2, a_3)$  such that  $a_0x^3 + a_1x^2 + a_2x + a_3$  is injective?

We want to eliminate the "squigglyness" of the center of the structure so that we do not have any duplicate values. Make the coefficients where  $a_0 = 1$  and the rest are 0.

3. For some constants  $a_0 \dots a_n$ , can you find a way for  $a_0x^{2k} + a_1x^{2k-1} \dots a_n$  to be injective?

*Solution.* We are just generalizing the solution derived from part (1). For any even polynomial, we have this symmetric property that makes the polynomial one-to-one. Thus, we cannot find any series of constants that make the polynomial injective.

4. For some constants  $a_0 \dots a_n$ , can you find a way for  $a_0x^{2k+1} + a_1x^{2k} \dots a_n$  to be injective?

*Solution.* Again, we are generalizing our solution to part(2). Just make the first coefficient 1 and the rest 0.