

CS70 Discussion 1c Review

AGNIBHO ROY

SUMMER 2020

1 Review

These are just concepts and strategies discussed during discussion section on June 24, 2020. For definitions and formulas, refer to note 3 on the course website.

1.1 Induction

- A result of the classic domino effect: if we know that toppling one domino topples the next, then the dominoes will keep toppling indefinitely. Here, the toppling of a domino represents that our hypothesis is true for that domino. In other words, if each domino is an integer, then our hypothesis will be true for each integer by induction.
- Base Case: proving that the first domino falls, which is done directly by plugging in the first value.
- Inductive Hypothesis: here we state what we are trying to prove: that if our hypothesis holds for some k , then it also holds for $k + 1$. We can use this as a fact during our next step.
- Inductive Step: Write out what you are proving by substituting $k + 1$ for k and see if what you are proving still holds.

1.2 Strong Induction

- Strong induction already exists since you do not need to prove anything extra to use it. It basically says that if the k th domino falls, then it must mean that every single domino before it must have fallen, rather than just the $(k - 1)$ th domino.
- For a lot of purposes, it suffices just to use the "weak" version and just assume the hypothesis holds for $(k - 1)$, but sometimes we need the fact that it holds for some or all of $l \leq k$.
- Using strong induction when you only need weak is like using a hammer to press a button. Sure, it works, but you don't need it.

2 Extra Problems

These problems are not necessarily in scope. Some may be helpful on exams, but some others are just fun exercises. Reach out to me by email (agnibhoroy@berkeley.edu) if you see any mistakes or have questions about any of the questions.

2.1 Cubes and Divisors

1. Prove that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 $\forall n \geq 0$.
2. Prove using induction that the sum of the first n cubes ($1^3 + 2^3 \dots n^3$) is $(\frac{n(n+1)}{2})^2$

2.2 Fibonacci Closed-Form

The n th fibonacci number is represented by the sum of its preceding two values in the sequence: $F_n = F_{n-1} + F_{n-2}$. Prove that we can write the n th Fibonacci number as:

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1 + \sqrt{5}}{2} \right)^n - \left(\frac{1 - \sqrt{5}}{2} \right)^n \right)$$

For all $n \geq 2$. (*hint*: do we need strong induction here?)