CS70 Discussion 1b Review

Agnibho Roy

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1 Review

These are just concepts and strategies discussed during discussion section on June 24, 2020. For definitions and formulas, refer to note 3 on the course website.

1.1 Induction

- A result of the classic domino effect: if we know that toppling one domino topples the next, then the dominoes will keep toppling indefinitely. Here, the toppling of a domino represents that our hypothesis is true for that domino. In other words, if each domino is an integer, then our hypothesis will be true for each integer by induction.
- Base Case: proving that the first domino falls, which is done directly by plugging in the first value.
- Inductive Hypothesis: here we state what we are trying to prove: that if our hypothesis holds for some k, then it also holds for $k + 1$. We can use this as a fact during our next step.
- Inductive Step: Write out what you are proving by substituting $k + 1$ for k and see if what you are proving still holds.

1.2 Strong Induction

- Strong induction already exists since you do not need to prove anything extra to use it. If basically says that if the kth domino falls, then it must mean that every single domino before it must have fell, rather than just the $(k-1)th$ domino.
- For a lot of purposes, it suffices just to use the "weak" version and just assume the hypothesis holds for $(k-1)$, but sometimes we need the fact that it holds for some or all of $l \leq k$.
- Using strong induction when you only need weak is like using a hammer to press a button. Sure, it works, but you don't need it.

2 Extra Problems

These problems are not necessarily in scope. Some may be helpful on exams, but some others are just fun exercises. Reach out to me by email (agnibhoroy@berkeley.edu) if you see any mistakes or have questions about any of the questions.

2.1 Cubes and Divisors

1. Prove that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 $\forall n \geq 0$.
2. Prove using induction that the sum of the first n cubes $(1^3 + 2^3 \dots n^3)$ is $(\frac{n(n+1)}{2})^2$

2.2 Fibonacci Closed-Form

The nth fibonacci number is represented by the sum of its preceding two values in the sequence: $F_n = F_{n-1} + F_{n-2}$. Prove that we can write the nth Fibonacci number as:

$$F_n = \frac{1}{\sqrt{5}}((\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n)$$

For all $n \geq 2$. (*hint*: do we need strong induction here?)