CS70 Discussion 2c Review

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1 Review

These are just concepts and strategies discussed during discussion section on July 1, 2020. For definitions and formulas, refer to note 6 on the course website.

1.1 Planarity and Coloring

- Euler's formula holds for any planar graph, which is $v + f = e + 2$.
- If a graph is planar, then it satisfies the inequality $e < 3v - 6$ but keep in mind that the converse is not true because $K_{3,3}$ is not planar, but satisfies $e < 3v - 6$. For this, we need a tighter bound (which can be proven by counting faces), which is $e < 2v - 4$ for bipartite graphs.
- A graph is non-planar if and only if that graph contains $K_{3,3}$ or K_5.
- Every planar graph can be 5-colored. For any coloring proof, think about taking away a vertex and use induction by adding in the vertex and seeing what color you can give it.

1.2 Hypercubes

- Hypercubes are a recursive geometric sequence. For example, to build the nth dimensional hypercube, you duplicate the $(n-1)th$ hypercube and draw adjacent adjacent edges between the two.
- The vertices of a hypercube can be thought of as bitstrings and they are also very well connected, meaning a large amount of edges have to be removed to disconnect it.

2 Extra Problems

These problems are not necessarily in scope. Some may be helpful on exams, but some others are just fun exercises. Reach out to me by email (agnibhoroy@berkeley.edu) if you see any mistakes or have questions about any of the questions.

2.1 Make It Nonplanar

Consider having two $K_{2,2}$ graphs that are disconnected. At least how many edges do you have to add to make the graph connected and non-planar?

2.2 Weaker Coloring

We have proven in the notes that any graph can be 5-colored. Now come up with a simple proof that it can be 6-colored (should be much simpler).

2.3 Hypercube Edges

1. Prove by induction that the n dimensional hypercube has $n2^{n-1}$ edges.
2. How many edges do you have to remove to make an n dimensional hypercube a tree?