CS70 Discussion 1b Review

Agnibho Roy

Summer 2020

1 Review

These are just concepts and strategies discussed during discussion section on June 23, 2020. For definitions and formulas, refer to note 2 on the course website.

1.1 Proof Techniques

- Direct proof: Exactly how it sounds, use some property of the hypothesis to derive your conclusion (usually algebraic manipulation of some sort).
- Proof by contraposition: If the direct proof is difficult (like you do not know a property of the hypothesis that you can use to conclude), then try the opposite direction in the form of contraposition ($\neg Q \implies \neg P$).
- Proof by contradition: Assume that the negation of the conclusion, and find an inconsistency in you solution.
- Proof by cases: split the problem into smaller cases that have similar properties, but make sure the cases cover all possibilities of the original problem. For example, let us say that we are proving the proposition $(\forall x)P(x)$, it is may be easier to deal with the subsets of x that are even and odd since they can be written as $2k$ and $2k+1$, respecitvely

1.2 Pigeonhole Principle

- This concept is just formalized logic, but it is more powerful than you think. Whenever you see a question that says "at least n", take it as a hint to try this principle.

2 Extra Problems

These problems are not necessarily in scope. Some may be helpful on exams, but some others are just fun exercises. Reach out to me by email (agnibhoroy@berkeley.edu) if you see any mistakes or have questions about any of the questions.

2.1 Visual Proof

1. Find the sum of the first n odd numbers in terms of a closed-form expression. Start with one dot, and try to build a square by adding more and more dots (add 3 dots, then 5, ...).
2. Now that you know what expression if for finding the sum of the first n odd numbers, find a closed-form expression for the first n even numbers

2.2 Too Many Socks

I love buying socks so let us say that I have 10 pairs of blue socks, 14 pairs of green socks, 3 pairs of yellow socks, and 2 pairs of orange socks. How many socks do I have to pull out of the drawer randomly to guarantee that I get a matching pair for the day?

2.3 Ants on a Checkerboard

There is a game where we have a checkerboard that is 8 by 8 (it is also 8 by 8 inches), and there are a total of 66 ants crawling on it. I have a glass that has a circumference of $\frac{3\pi}{2}$ inches. My friend tells me that I win if I can place this glass on board and capture any two of these ants. Prove that no matter what the positions of the ants are, I can always capture at least 2 ants.