CS70 Discussion 1a Extra Problems Solutions

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Summer 2020

Simple Propositions

Convert the following sentences into a propositional statement or vice versa. Keep in mind that there may be multiple formulations that are correct. Also, try to reason if these propositions are true or not.

1. There does not exist a positive integer whose square is less than the number itself

   Solution. $\neg\exists x P(x) \equiv \forall x \neg P(x)$. Thus, we can write it as $(\forall x \in \mathcal{Z}^{+})\ x^2 \geq x$

2. There exists a real number that lies in between two distinct real numbers

   Solution. $(\forall x, y \in \mathcal{R})(x \neq y)(\exists z \in \mathcal{R})(y < z < x) \vee (x < z < y)$

Set Propositions

A vector space V has the properties of closure for addition, scalar multiplication, and an inverse for every vector $v \in V$. Write a propositional statement that states that if V is a vector space, then it satisfies these three properties.
Solution. $(\forall x, y \in V)(x + y \in V) \wedge (\forall c \in \mathcal{R})(cx \in V) \wedge (\exists u \in V)(u + v = \vec{0})$

Boolean Algebra and Truth Tables

From discussion we looked at an XOR between two boolean variables (question 2). Let an XOR between three boolean variables be represented as $A \oplus B \oplus C$. Write the truth table for this operation and also express XOR using only $(\wedge, \vee, \neg)$ (Hint, think of $A \oplus B \oplus C$ as $(A \oplus B) \oplus C$ for help with the truth table)
Solution.

A	B	C	$(A \oplus B) \oplus C$
T	T	T	F
T	T	F	F
T	F	T	F
F	T	T	F
F	F	T	T
T	F	F	T
F	T	F	T
F	F	F	F

We know that from this truth table, we can just write a disjuction of conjunctions of the three different True possibilities, which is:

$$(A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C)$$

This can also be simplified to other possibilities that are equivalent in logic to the one above.