

# CS70 Discussion 2a Review

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## 1 Review

These are just concepts and strategies discussed during discussion section on June 30, 2020. For definitions and formulas, refer to note 6 on the course website.

### 1.1 Trees and Graphs

- A cycle and a path do not contain repeated vertices other than the starting and ending vertex for a cycle. A walk is just a series of edges, and a tour is just a walk with no repeated edges and starts and ends on the same vertex.
- A tree on  $n$  vertices is a connected graph that always has  $n - 1$  edges and contains no cycles. Removing an edge from a tree makes it disconnected (any graph with  $< n - 1$  edges is disconnected).
- To prove theorems that apply to "any graph" either induct on the number of edges or vertices, not the degree. Often times way will be easier than another in terms of the number of cases to consider.
- An Eulerian tour requires all vertices to have even degree, but by letting two of the vertices have odd degrees, we can designate those as the start and end vertices to create an Eulerian walk.

## 2 Extra Problems

These problems are not necessarily in scope. Some may be helpful on exams, but some others are just fun exercises. Reach out to me by email (agnibhoroy@berkeley.edu) if you see any mistakes or have questions about any of the questions.

### 2.1 Degree Bounds

Consider an  $n$ -ary tree where the maximum number of children for any one node is  $n$ . Prove that the average degree of the tree is always less than  $n$ .

### 2.2 Graph Complement

Let  $\bar{G}$  be the complement of  $G$  that contains all the edges that  $G$  does not have, and does not contain any of the same edges as  $G$ . Prove that for any graph, at least one of  $G$ , or  $\bar{G}$  is connected