Bounds Review

Srishti Agarwal, Agnibho Roy

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1 Inequalities Practice

A biased coin with probability of getting a heads equal to 0.2 is flipped 20 times.

(1) Find a bound for the probability that the coin lands on heads at least 16 times using Markov's Inequality. Is this a good estimate?

(2) Tighten this bound using Chebyshev's Inequality

(3) Find a probability distribution for X and a value a such that Markov's inequality is tight

Source: https://math.dartmouth.edu/ m20x18/markov

2 Lightbulbs

Let $\{X_1, ..., X_n\}$ be the lifetimes of lightbulbs. We know that they are exponentially distributed with parameter λ and we know that $\lambda >= 10$. Recall that $E[X_m] = \lambda^{-1}$ and $var[X_m] = \lambda^{-2}$. **Use Chebyshev's inequality to construct a 95%-confidence interval for λ^{-1} based on $\{X_1, ..., X_n\}$**

Source: https://www.eecs70.org/static/resources/final/final_fa15_sol.pdf

3 Estimating π

One can estimate π by playing darts with a special dartboard shown in figure 1. Assume every time you throw a dart, the dart will always be inside the square. The probability that your dart lands inside the circle is equal to the ratio of the area of the circle to the area of the square, i.e., $\pi/4$. Let X_i be the random variable denoting whether your dart is within the circle after your i-th throw.

Suppose we throw the dart n times in order to estimate π. **Show that you can estimate π by $M_n = \frac{4}{n}\sum_{i=1}^{n} X_i$. How many times should you throw to ensure your estimation error is within 0.01 with probability at least 95%?** (You can just leave the numerical expression of the number of times but not compute the exact value.)

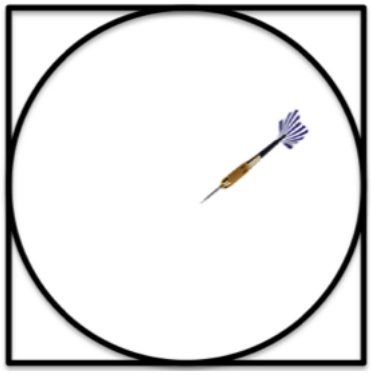

Figure 1: Dartboard

Source: https://www.eecs70.org/static/resources/final/final_fa14_sol.pdf