

# Hierarchy discovery for planning

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## 1 Motivation

People spontaneously organize their environment into clusters of states (state chunks) which constrain planning [1, 2, 3]. Why and how do we do that?

One normative reason is that hierarchical planning is more efficient in time and memory than "flat" planning [4] (e.g. for BFS,  $O(\sqrt[L]{N})$  vs.  $O(N)$ , where  $N$  is the number of states and  $L$  is the hierarchy depth). This is consistent with people's limited working memory capacity [5].

But hierarchies are a form of lossy compression, and some hierarchies might result in inefficient planning.

[example]

Solway et al. 2014 [1] provide a formal definition of an "optimal" hierarchy, but they do not specify how the brain might discover it. In this work, we propose a Bayesian cognitive model of hierarchy discovery and show that it is consistent with human behavior.

## 2 Background

### 2.1 Preliminaries

We assume a 2-layer hierarchy ( $L = 2$ ).  $G = (V, E)$  is the low-level ("flat") graph that is directly observable, where:

- $V$  is the set of vertices (low-level states)
- $E : \{V \times V\}$  is the set of edges

We define  $H = (V', E', c, b)$  as the high-level ("H") graph that is not directly observable, where:

- $V'$  is the set of vertices (high-level states, or state chunks)
- $E' : \{V' \times V'\}$  is the set of edges

- $c : V \rightarrow V'$  are the state chunk assignments
- $b : E' \rightarrow E$  are the bridges

Each low-level state  $i$  is assigned to a state chunk  $k = c_i$ . Each high-level edge  $(k, l)$  has a corresponding low-level edge (the bridge)  $(i, j) = b_{k,l}$ , such that  $c_i = k$  and  $c_j = l$ . For simplicity, we assume both graphs are unweighted and undirected.

## 2.2 Model

Informally, an algorithm that discovers useful hierarchies would satisfy the following desiderata:

1. Favor smaller state chunks,
2. ...but not too many of them;
3. Favor dense connectivity within chunks,
4. ...and sparse connectivity across chunks,
5. ...with the exception of "bridges" that connect chunks

Intuitively, having too few (e.g. one) or too many state chunks (e.g. each state is its own chunk) creates a degenerate hierarchy that reduces the planning problem to the "flat" scenario, and hence medium-sized chunks are best (desiderata 1 and 2). Additionally, the hierarchy ignores transitions across state chunks, which could lead to suboptimal paths generated by the hierarchical planner. It is therefore best to minimize the number of cross-chunk transitions (desiderata 3 and 4). The exception is bridges, which correspond to the links between state chunks (desideratum 5).

These can be formalized into a generative model for "good" hierarchies:

$c \sim CRP(\alpha)$	chunk assignments
$p' \sim Beta(1, 1)$	H graph density
$Pr[(k, l) \in E'] = p'$	H graph edges
$Pr[b_{k,l} = (i, j) \mid (k, l) \in E', c_i = k, c_j = l] = \frac{1}{n_k n_l}$	bridges
$p \sim Beta(1, 1)$	within-chunk density
$q \sim Beta(1, 1)$	cross-chunk density factor
$Pr[(i, j) \in E \mid c_i = c_j] = p$	within-chunk edges
$Pr[(i, j) \in E \mid c_i \neq c_j] = pq$	cross-chunk edges
$Pr[(i, j) \in E \mid b_{c_i, c_j} = (i, j)] = 1$	bridge edges

Hierarchy discovery can then be framed as inverting the generative model (in similar spirit to how k-means clustering can be understood as inference over a Gaussian Mixture model). MCMC implementations of approximate Bayesian inference over this model can also be interpreted as stochastic hill climbing with respect to a utility function defined by the posterior over hierarchies [4].

## Bibliography & References Cited

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