MA 109 Tutorial Batch D1 T2
Recap 1

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Hi!

Welcome to your first MA 109 Tutorial.
We will have a weekly tut on Wednesdays 4pm-5pm.

You might wonder, why should I even bother coming to tutorials?
Well, take this from me, but a course is a lot more doable when you aren't speedrunning it the night before the exam.
Taking it in week by week, gives you the time (*that you need*) to digest all this information properly.

It'll be great if you give the questions an attempt beforehand, but even if you don't find the time, that's alright, try to attend the tutorial.
I'll give you a quick recap of the week's material and we'll go ahead with the sums.

I hope I can make MA 109 an enjoyable learning experience for all of you :)

Sets

A set is an unordered collection of distinct objects.

- Unordered: $\{1,2,3\}$ and $\{3,2,1\}$ are the same set.
- Distinct: No two elements can be the same. A set shouldn't look like $\{5,4,3,7,4,4,5\}$. It should be $\{5,4,3,7\}$.

The cardinality of a set gives you an idea of its size. It is finite and equal to n if there is a bijection from $[1,\ldots n]$ to the set S, and infinite, if not.

Maximum, minimum, supremum, infimum

- Maximum: An element that belongs to the set and is greater than everyone else in the set.
- Minimum: An element that belongs to the set and is less than everyone else in the set.

- Upper bound: An element that may or may not belong to the set and is greater than everyone else in the set.
- Lower bound: An element that may or may not belong to the set and is less than everyone else in the set.

- Supremum: The least upper bound
- Infimum: The greatest lower bound

Some examples

Consider the infinite set $\{1, \frac{1}{2}, \frac{1}{3} ...\}$.

The pattern is obvious, right?

What is the...

- Maximum?
- Minimum?
- Supremum?
- Infimum?

How many upper and lower bounds does this set have?

Something interesting which you might have noticed is:
Maximum $\implies$ *Supremum* **but** *Supremum* $\not\Longrightarrow$ *Maximum*

Sequences

A sequence is simply a collection of real numbers, written one after another. One point of difference is that, the **order matters** here, unlike a set.

Think of them as functions which map a natural number to its corresponding output. That output (corresponding to the natural number i, is the i'th member of the sequence.

A finite or infinite sequence is defined accordingly, one from [1,...,n] and the other from $\mathbb{N}$, both mapped to $\mathbb{R}$.

Convergence

Sometimes, a sequence gets *really, really* close to a certain real number.
It is then said to converge to that number.
We put this down formally in the $\epsilon - N_0$ definition of convergence.

Theorem ($\epsilon - N_0$ Definition of Convergence)

A sequence $(a_n)_{n=1}^{\infty}$ is said to converge to a real number c if for every $\epsilon > 0$, there exists an $N_0 \in N$ such that $|c - a_n| < \epsilon$ for every $n \geq N_0$

In simple, not so scary language:

If a sequence converges to a limit c, you can think of any (positive) number ϵ, such that there is always an element in the sequence, after which all elements are within ϵ distance from the limit c.

Some more properties

- Limits add, subtract, multiply and divide (if the denominator is not 0)
- What is a monotonic increasing sequence?
  There exists an element, after which every element is greater than or equal to the previous one.
- A monotonic decreasing sequence is defined similarly.
- Bounded Monotonicity $\implies$ Convergence (why?)
- The Sandwich Theorem
  If there are three sequences a_n, b_n, c_n such that:

$$a_n \leq c_n \leq b_n$$

and

a_n and b_n both converge to the same limit

then

c_n also converges to that same limit

Think about this.

Every convergent sequence is monotonic. True or false?