ME 683: Computational Gas Dynamics

Coding Assignment 2: System of Linear Hyperbolic Equation

March 24^{th}- April 6^{th} 2025

Weightage: 10%

Introduction

The system of linear hyperbolic PDEs have the following form:

$$U_t + AU_x = 0$$

where U is the conserved variable for an m-equation system stored in a column matrix and A is an $m \times m$ square matrix.

In case of Riemann problem, the initial condition takes the following form:

$$U^0(x) = \begin{cases} U_L, & \text{if } x < 0 \\ U_R, & \text{if } x \geq 0 \end{cases}$$

where U_L and U_R are column matrices for the right and the left side values about the origin.

Goal

Write a program that can solve the system of 1D linear hyperbolic PDE with Riemann problem based initial condition using the Upwind Scheme for calculating numerical flux at the cell faces. The user of the code will provide the following details that the code has to read:

- Square Matrix A for m-equation system
- Initial conditions (U_L and U_R)
- Domain size (L) (Your domain will extend from $-L/2$ to $L/2$)
- No of cells (N)
- Total Simulation time (t)

You can use pre-defined library functions (*Eigen/Boost/MKL/LAPACK/BLAS* for C/C++ and Fortran or *linalg* for Python or math library functions for Matlab) or functions taken from other sources for calculating eigenvalues and eigenvectors. Assume $CFL = 0.8$ and find the U profile at the final simulation time.

Using your code, solve the following Riemann problem for a 2-equation system and compare the final U profile with analytical results:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t + \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

with Riemann condition:

$$U_1^0 = \begin{cases} 1, & \text{if } x < 0 \\ -0.5, & \text{if } x \geq 0 \end{cases}$$

$$U_2^0 = \begin{cases} 0.5, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$$

for a domain of length 20 m with 1000 cells after a total simulation time of 0.2 s.

Submission

You have to submit your code(s) and along with two line plots (U vs x at initial condition and at final simulation time). The plots should have proper title, labels and legends. Please add relevant and detailed comments to your code; negative marking for codes with no comments. Mail zip/tar file (RollNo.zip, no other names allowed) of your plots+codes to *tapan.mankodi@iitg.ac.in* and *harshal.srivastava@iitg.ac.in* by 9:00 PM, April 6^{th} 2025 (Sunday). Late submissions have 50% penalty.

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