ME 683: Computational Gas Dynamics

Quiz 2: 6 PM, April 11^{th}, 2025

Maximum Marks: 20

Godunov Exact Solver

Godunov approach converts every pair of neighbouring cells in the domain as Riemann problems and attempts to calculate the flux at the common face by solving the Riemann problem either exactly or by approximate method. Consider a (discretized) one-dimensional problem such that the conserved variables are u_i and u_{i+1} in an arbitrary i^{th} and its right neghbour $(i+1)^{th}$ cell respectively. As per Godunov approch, we solve the Riemann problem given by

$$u^n(x) = \begin{cases} u_i, & \text{if } x < 0 \\ u_{i+1}, & \text{if } x \geq 0 \end{cases}$$

We solve the Riemann problem (in the present case using exact solutions) and get $u_{i+1/2}$. The corresponding flux at the right face for the i^{th} cell is given by

$$F_{i+1/2} = f(u_{i+1/2}(x=0))$$

Flux at the right face along with the flux calculated at the left face ($f_{i-1/2}$) can then be used to calculate the conserved variable at the next time step given by:

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}(F_{i+1/2}^n - F_{i-1/2}^n)$$

where Δt, Δx are time step and cell size respectively.

Burger Equation

The goal is to write a program that solves the inviscid 1D Burger equation using exact Godunov flux scheme. The inviscid 1D Burger equation is given by

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0 \qquad where \quad f(u) = \frac{1}{2}u^2$$

For the following Riemann problem:

$$u_0(x) = \begin{cases} u_l, & \text{if } x < 0 \\ u_r, & \text{if } x \geq 0 \end{cases}$$

The exact solution can either take a form of a shock wave ($u_l > u_r$) (given shock speed $s = (u_L + u_R)/2$) given by

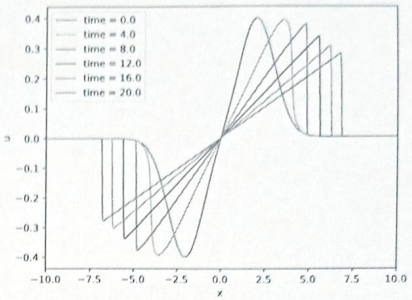
$$u(x,t) = \begin{cases} u_l, & \text{if } x/t < s \\ u_r, & \text{if } x/t \geq s \end{cases}$$

or rarefaction ($u_l < u_r$); given by

$$u(x,t) = \begin{cases} u_l, & \text{if } x/t - u_l \leq 0 \\ x/t, & \text{if } u_l < x/t < u_r \\ u_r, & \text{if } x/t - u_r \geq 0 \end{cases}$$

Simulation Details

- Simulation Type : 1D
- Domain size : 20 m $(-10 \leq x \geq 10)$
- No of cells : 1000
- $\Delta t = 0.001$
- Total Simulation time : $t = 20s$
- Flux Scheme: Godunov Exact Solver



- Initial Condition:

$$u(x, t = 0) = \frac{e^{-\frac{(x-2.0)^2}{2.0}}}{\sqrt{(2\pi)}} - \frac{e^{-\frac{(x+2.0)^2}{2.0}}}{\sqrt{(2\pi)}}$$

- Boundary Condition : Zero gradient at the two sides. (Flux = 0)

Submission

Prepare a folder with your roll number as its name. Add your source code, text file and figure of u vs x at $t = 20.0$ to this folder. Copy this folder in the class pen drive that will be circulated.

Ideally you should be able to finish the quiz in 1 hour. If you cannot submit by 7 PM, you can submit this by today midnight (in which case, maximum marks you can score will be 10 out of 20).

Best of luck!