Assignment 1

Taylor Polynomial

Submission due: On or before 6 Aug, 2025, 4:00 pm.
Viva: 6 Aug, 2025, 4:00 – 6:30 pm.
Venue: F24, 1st floor, Mechanical Engineering Building

Instructions for Submission and Viva

- Please submit your code via Moodle.
- Viva slots have been announced on moodle.
- Plots should have proper axis labels and legends.
- You need to run and demonstrate your code on your laptop during the viva.
- Code written in programming languages other than **Matlab** will **NOT** be considered.

Problem Statement

Using Taylor polynomial, write a **Matlab** code for evaluating the error function for a given n^{th} order expansion about $a = 0$:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

Do not use taylor or erf matlab functions for computing Taylor polynomial. Write your code such that the output includes the following two figures:

- **Figure 1:** Graph of the error function on the interval $[0, 1]$ for 11th and 21st order Taylor polynomial (in the same plot). Also, plot the true value of the function using the `erf` function of Matlab along with these two plots.
- **Figure 2:** Graph of the absolute error E in the interval $[0, 1]$ for 11th and 21st order Taylor polynomial. That is, x-axis represents x and y-axis represents E. Show all three error graphs in the same plot. Use log scale on Y-axis to show the error changes with x.

Hints to derive Taylor polynomial

Recall from class notes (ref: Slide 41), Taylor polynomial of e^{-x} around 0.

$$P_n(x;0) = \sum_{j=0}^{n} \frac{(-1)^j x^j}{j!}$$

We consider the function:

$$F(t) = e^{-t^2}$$

Using variable transformation, $x = t^2$ in Taylor polynomial, we get Taylor polynomial of $F(t)$ as follows,

$$P_n(t;0) = \sum_{j=0}^{n} \frac{(-1)^j t^{2j}}{j!}$$

We integrate term by term from 0 to x to get Taylor polynomial of $erf(x)$:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \left(\sum_{j=0}^{n} \frac{(-1)^j t^{2j}}{j!} \right) dt = \sum_{n=0}^{n} \frac{(-1)^j}{j!} \int_0^x t^{2j}\, dt$$

$$= \frac{2}{\sqrt{\pi}} \sum_{j=0}^{n} \frac{(-1)^j}{j!} \cdot \frac{x^{2j+1}}{2j+1}$$

The order of above Taylor polynomial is 2n+1. In assignment we are asked to plot 11th and 21st order polynomial. So choose $n = 5$ and 10 in above formula to obtain and plot 11th and 21st order polynomial, respectively.