1. Consider the recursion:

$$U_{i+1} = (U_{i-17} - U_{i-5}).$$

In the event that $U_i < 0$, set $U_i = U_i + 1$.

   (a) Use linear congruence generator to generate the first 17 values of U_i.

   (b) Then generate the values of U_i (for 1000, 10000 and 100000 values).

   (c) For each of the above set of values plot (U_i, U_{i+1}) and also draw a bar diagram. What are your observations ?

2. Consider the exponential distribution with mean θ:

$$F(x) = 1 - e^{-x/\theta} \ , \ x \geq 0.$$

Generate the values of X using the algorithm $X = -\theta \log(1 - U)$, where $U \sim \mathcal{U}[0, 1]$. Plot the distribution function of these generated values (with midpoints of each interval against its frequency, and joined by smooth curve), and the actual distribution function (using the above formula). Do this plot for various values of the number of observations generated. Also, give the corresponding values of the sample mean and variance. Compare the values of mean and variance to see whether they converge to actual values.

3. Consider the Arcsin law with the distribution:

$$F(x) = \frac{2}{\pi} \text{ arc sin } \sqrt{x} \ , \ 0 \leq x \leq 1.$$

Generate the values of X making use of the inverse transformation :

$$X = \sin^2\left(\frac{U\pi}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos(U\pi), \text{ where } U \sim \mathcal{U}[0, 1].$$

Plot the distribution function of these generated values (with midpoints of each interval against its frequency, and joined by smooth curve), and the actual distribution function (using the above formula). Do this plot for various values of the number of observations generated. Also, give the corresponding values of the sample mean and variance.

Submission Deadline: 17th January 2019, 11:59 PM