Central Limit Theorem Experiment Report

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Experiment Parameters:

Population Size: 100,000
Number of Samples per Test: 1,000
Sample Sizes Tested: [5, 10, 15, 20, 25, 30, 50, 100]
Distributions Tested: ['normal', 'bernoulli', 'poisson', 'exponential', 'beta']

Total Experiments Run: 40

Objective:

This experiment tests the Central Limit Theorem by examining how sample size affects the normality of sample means across different population distributions.

Key Questions:
• At what sample size do sample means become normally distributed?
• How does this threshold vary across different distributions?
• How do skewness and kurtosis change with sample size?

Statistical Tests Used:
• Shapiro-Wilk test for normality
• Kolmogorov-Smirnov test for normality
• Descriptive statistics (mean, std, skewness, kurtosis)

NORMAL DISTRIBUTION - RESULTS SUMMARY

Sample Size	Mean	Std Dev	Skewness	Kurtosis	Shapiro p-value	KS p-value	Verdict
5	-0.0192	0.4577	-0.0116	0.054	0.8602	0.8614	Normal
10	-0.0193	0.3141	-0.0333	-0.2155	0.443	0.54	Normal
15	-0.0073	0.2534	-0.0947	-0.143	0.0468	0.4941	Not Normal
20	-0.0057	0.2209	0.0066	-0.1236	0.7583	0.9663	Normal
25	-0.0043	0.203	-0.026	-0.011	0.3591	0.9129	Normal
30	-0.0019	0.1856	0.0152	-0.1495	0.8293	0.9747	Normal
50	-0.0031	0.1414	0.0681	-0.0638	0.9575	0.7739	Normal
100	-0.0027	0.1004	0.0511	0.1239	0.9624	0.9521	Normal

Summary for NORMAL Distribution:

- Normal distributions achieved: 7/8 sample sizes
- Normality threshold: Sample size ≥ 5
- Mean skewness across all sample sizes: -0.0031
- Mean kurtosis across all sample sizes: -0.0661

Interpretation:
The Central Limit Theorem predicts that sample means should become normally distributed as sample size increases, regardless of the population distribution. Green cells indicate normal distributions ($p > 0.05$), red cells indicate non-normal distributions ($p \leq 0.05$).

BERNOULLI DISTRIBUTION - RESULTS SUMMARY

Sample Size	Mean	Std Dev	Skewness	Kurtosis	Shapiro p-value	KS p-value	Verdict
5	0.5	0.2291	-0.0339	-0.4593	0.0	0.0	Not Normal
10	0.5056	0.1573	-0.0971	-0.4048	0.0	0.0	Not Normal
15	0.5028	0.1285	-0.0669	-0.3185	0.0	0.0	Not Normal
20	0.5015	0.113	0.0488	-0.3006	0.0	0.0	Not Normal
25	0.5	0.0993	0.0394	-0.1123	0.0	0.0	Not Normal
30	0.5009	0.09	0.0557	-0.1281	0.0	0.0	Not Normal
50	0.4997	0.0696	0.1631	0.0654	0.0	0.0006	Not Normal
100	0.4993	0.0483	0.0856	-0.0694	0.0031	0.0185	Not Normal

Summary for BERNOULLI Distribution:

- Normal distributions achieved: 0/8 sample sizes
- Normality threshold: Sample size ≥ Not achieved
- Mean skewness across all sample sizes: 0.0243
- Mean kurtosis across all sample sizes: -0.2160

Interpretation:
The Central Limit Theorem predicts that sample means should become normally distributed as sample size increases, regardless of the population distribution. Green cells indicate normal distributions ($p > 0.05$), red cells indicate non-normal distributions ($p \leq 0.05$).

POISSON DISTRIBUTION - RESULTS SUMMARY

Sample Size	Mean	Std Dev	Skewness	Kurtosis	Shapiro p-value	KS p-value	Verdict
5	3.0044	0.7814	0.1409	-0.1172	0.0	0.0002	Not Normal
10	2.978	0.5295	0.177	-0.0957	0.0005	0.0034	Not Normal
15	2.9698	0.442	0.1981	0.0183	0.0006	0.0173	Not Normal
20	2.9788	0.3786	0.1103	0.1433	0.0203	0.0839	Not Normal
25	2.9842	0.3375	0.0514	0.1113	0.0403	0.0299	Not Normal
30	2.9812	0.3104	-0.0109	-0.2313	0.1071	0.0624	Normal
50	2.9856	0.2406	-0.0231	-0.1316	0.2775	0.3797	Normal
100	2.9924	0.1728	-0.0522	-0.1622	0.5087	0.4255	Normal

Summary for POISSON Distribution:

- Normal distributions achieved: 3/8 sample sizes
- Normality threshold: Sample size ≥ 30
- Mean skewness across all sample sizes: 0.0739
- Mean kurtosis across all sample sizes: -0.0581

Interpretation:
The Central Limit Theorem predicts that sample means should become normally distributed as sample size increases, regardless of the population distribution. Green cells indicate normal distributions ($p > 0.05$), red cells indicate non-normal distributions ($p \leq 0.05$).

EXPONENTIAL DISTRIBUTION - RESULTS SUMMARY

Sample Size	Mean	Std Dev	Skewness	Kurtosis	Shapiro p-value	KS p-value	Verdict
5	1.0002	0.4551	0.7915	0.4884	0.0	0.0	Not Normal
10	1.0024	0.3198	0.7351	0.8598	0.0	0.0001	Not Normal
15	1.0016	0.2609	0.5556	0.6992	0.0	0.0448	Not Normal
20	0.9972	0.2227	0.4803	0.7368	0.0	0.6372	Not Normal
25	0.9966	0.1976	0.365	0.356	0.0	0.1338	Not Normal
30	0.9962	0.1826	0.426	0.2673	0.0	0.1055	Not Normal
50	0.9939	0.141	0.2264	-0.2479	0.0007	0.2985	Not Normal
100	0.9951	0.0963	0.1592	-0.2493	0.0365	0.7089	Not Normal

Summary for EXPONENTIAL Distribution:

- Normal distributions achieved: 0/8 sample sizes
- Normality threshold: Sample size ≥ Not achieved
- Mean skewness across all sample sizes: 0.4674
- Mean kurtosis across all sample sizes: 0.3638

Interpretation:
The Central Limit Theorem predicts that sample means should become normally distributed as sample size increases, regardless of the population distribution. Green cells indicate normal distributions ($p > 0.05$), red cells indicate non-normal distributions ($p \leq 0.05$).

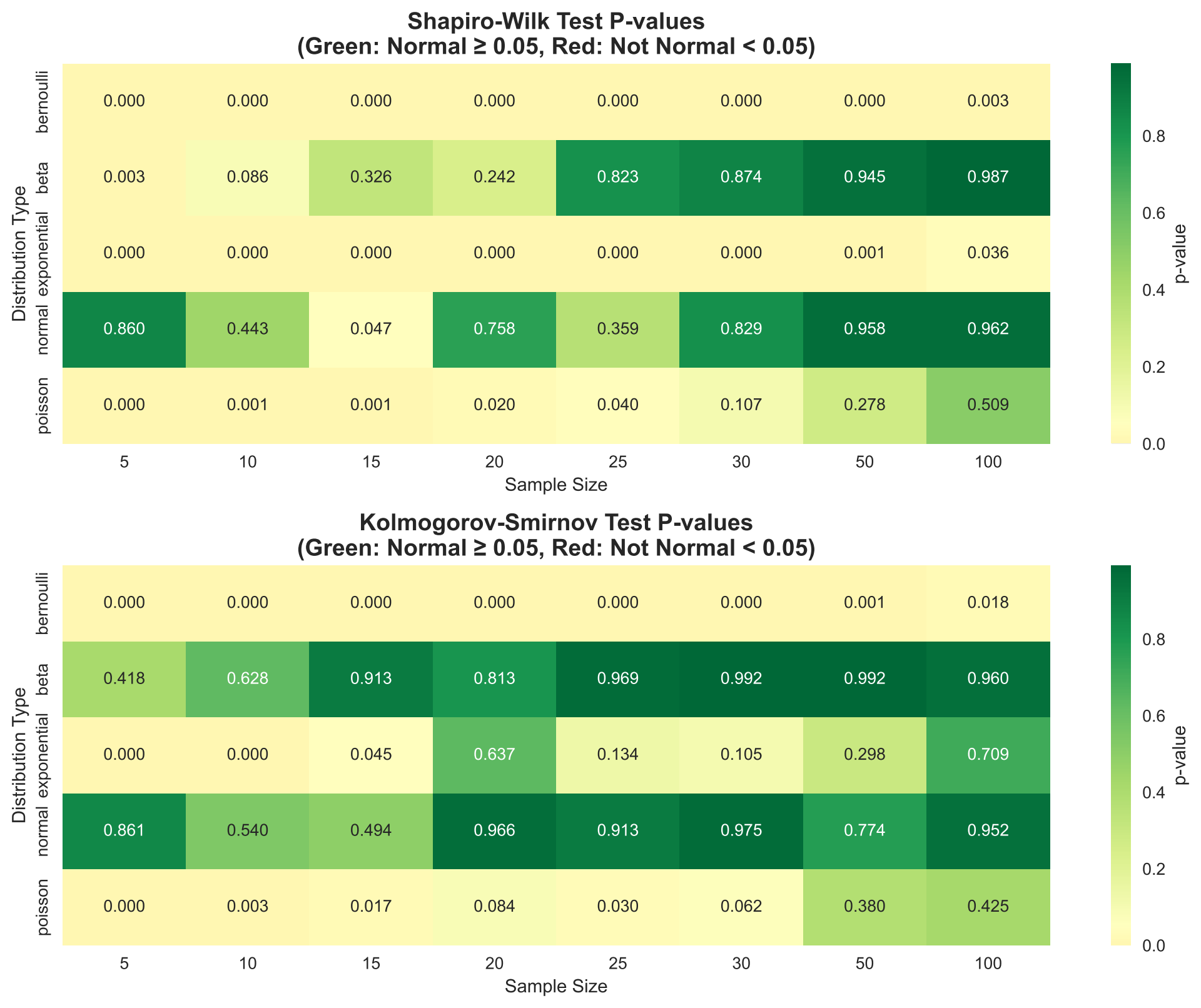
BETA DISTRIBUTION - RESULTS SUMMARY

Sample Size	Mean	Std Dev	Skewness	Kurtosis	Shapiro p-value	KS p-value	Verdict
5	0.2854	0.0723	0.2373	0.1672	0.0025	0.4179	Not Normal
10	0.285	0.0513	0.1781	0.008	0.0863	0.6278	Normal
15	0.2866	0.0412	0.1064	-0.1435	0.3255	0.913	Normal
20	0.2862	0.0363	0.0358	-0.0917	0.2424	0.813	Normal
25	0.2862	0.0327	0.0108	-0.1277	0.8232	0.9689	Normal
30	0.2857	0.0301	0.0117	-0.0689	0.8736	0.9915	Normal
50	0.2852	0.0229	-0.0111	0.0927	0.9449	0.9921	Normal
100	0.285	0.0163	0.0029	0.0649	0.9867	0.9595	Normal

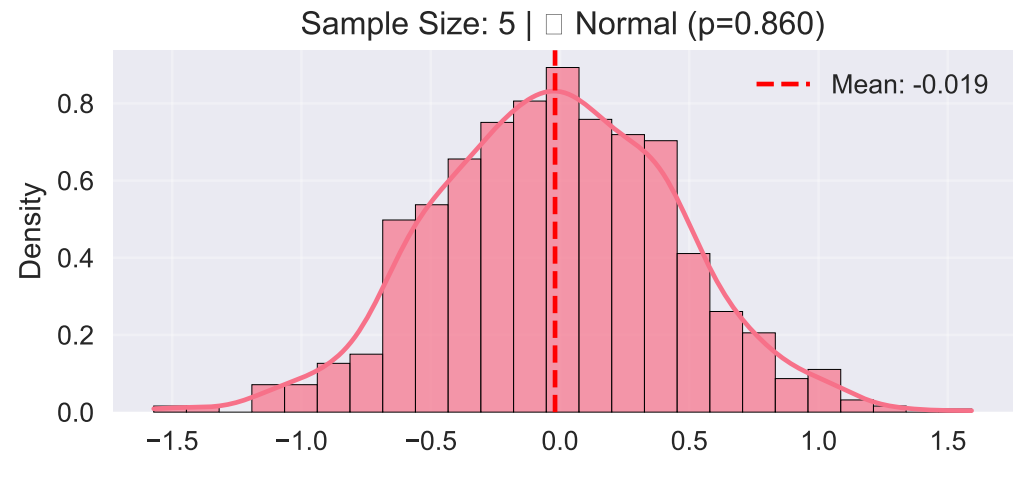
Summary for BETA Distribution:

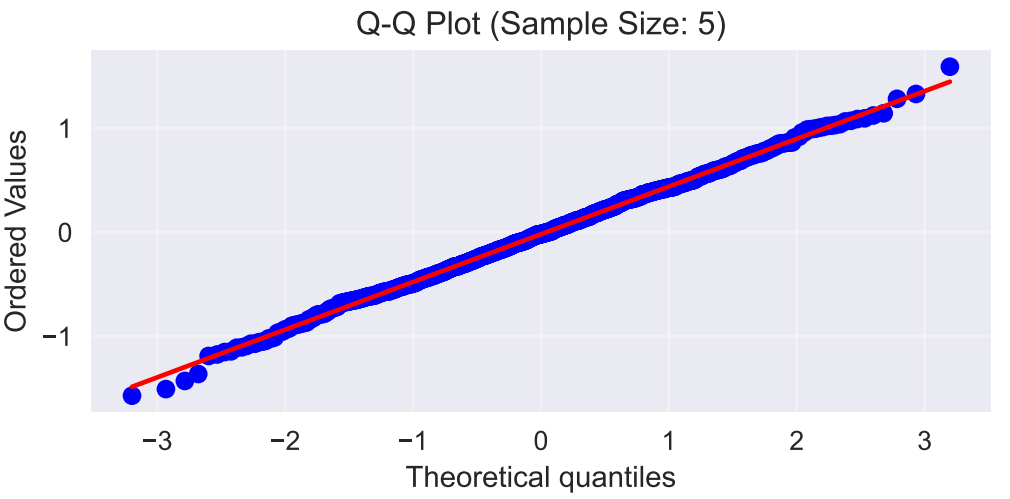
- Normal distributions achieved: 7/8 sample sizes
- Normality threshold: Sample size ≥ 10
- Mean skewness across all sample sizes: 0.0715
- Mean kurtosis across all sample sizes: -0.0124

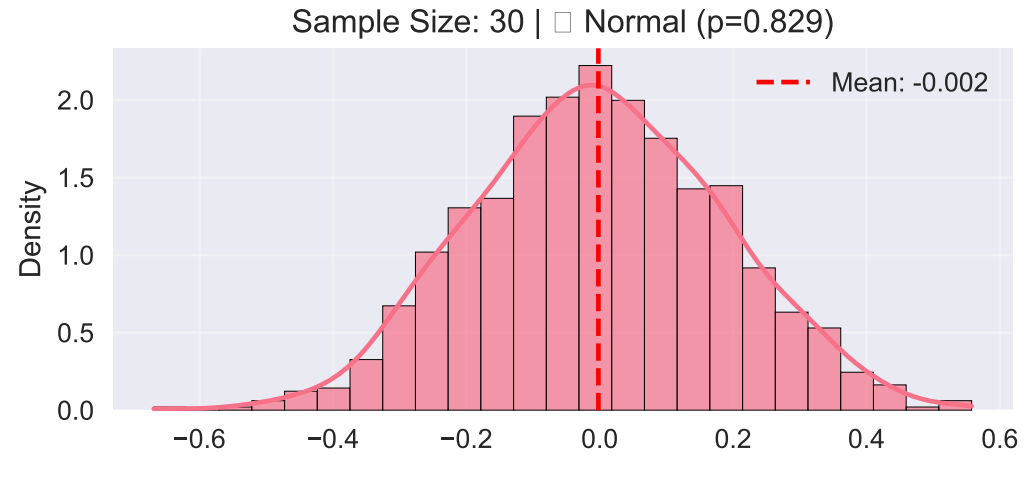
Interpretation:
The Central Limit Theorem predicts that sample means should become normally distributed as sample size increases, regardless of the population distribution. Green cells indicate normal distributions ($p > 0.05$), red cells indicate non-normal distributions ($p \leq 0.05$).

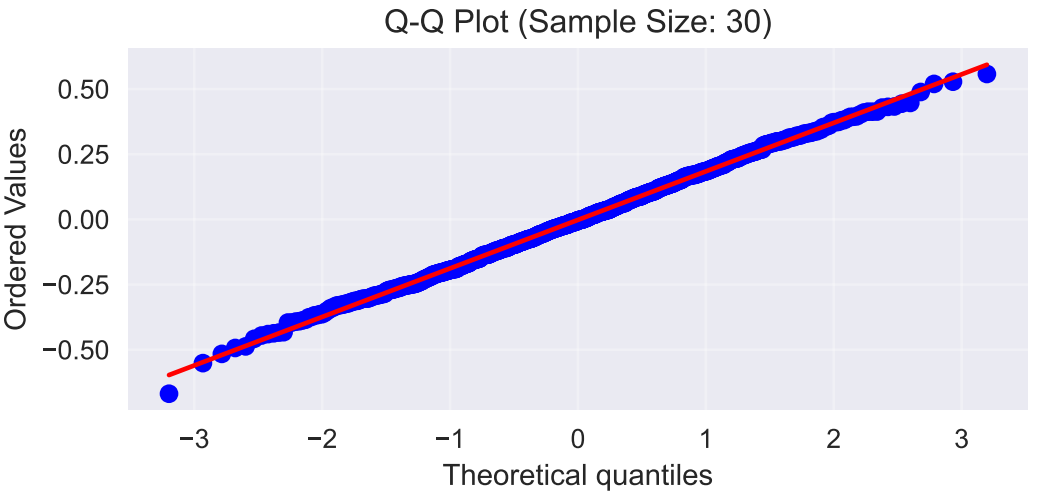

Shapiro-Wilk Test P-values
(Green: Normal ≥ 0.05, Red: Not Normal < 0.05)
Distribution Type
bernoulli
beta
exponential
normal
poisson
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.003
0.003 0.086 0.326 0.242 0.823 0.874 0.945 0.987
0.000 0.000 0.000 0.000 0.000 0.000 0.001 0.036
0.860 0.443 0.047 0.758 0.359 0.829 0.958 0.962
0.000 0.001 0.001 0.020 0.040 0.107 0.278 0.509
5 10 15 20 25 30 50 100
Sample Size
p-value
0.0 0.2 0.4 0.6 0.8
Kolmogorov-Smirnov Test P-values
(Green: Normal ≥ 0.05, Red: Not Normal < 0.05)
Distribution Type
bernoulli
beta
exponential
normal
poisson
0.000 0.000 0.000 0.000 0.000 0.000 0.001 0.018
0.418 0.628 0.913 0.813 0.969 0.992 0.992 0.960
0.000 0.000 0.045 0.637 0.134 0.105 0.298 0.709
0.861 0.540 0.494 0.966 0.913 0.975 0.774 0.952
0.000 0.003 0.017 0.084 0.030 0.062 0.380 0.425
5 10 15 20 25 30 50 100
Sample Size
p-value
0.0 0.2 0.4 0.6 0.8

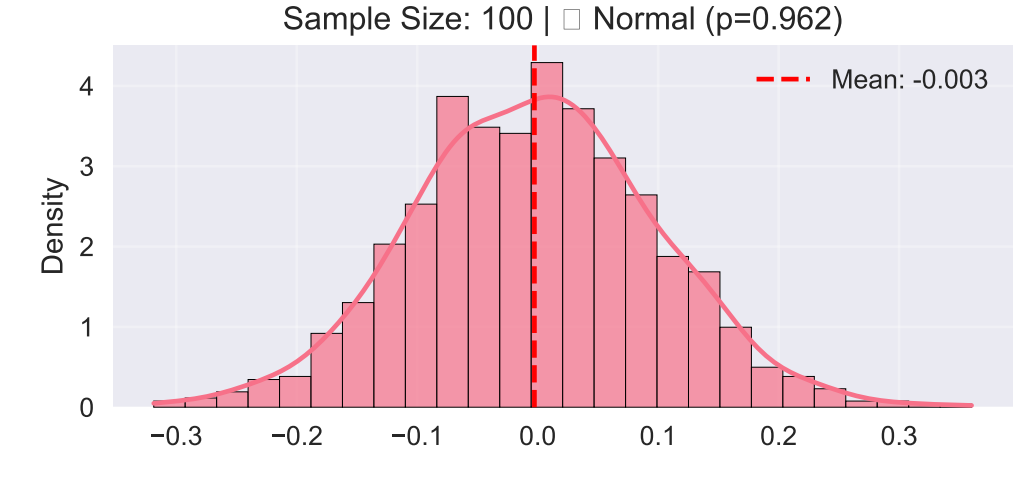
NORMAL Distribution - Sample Means Distribution Evolution

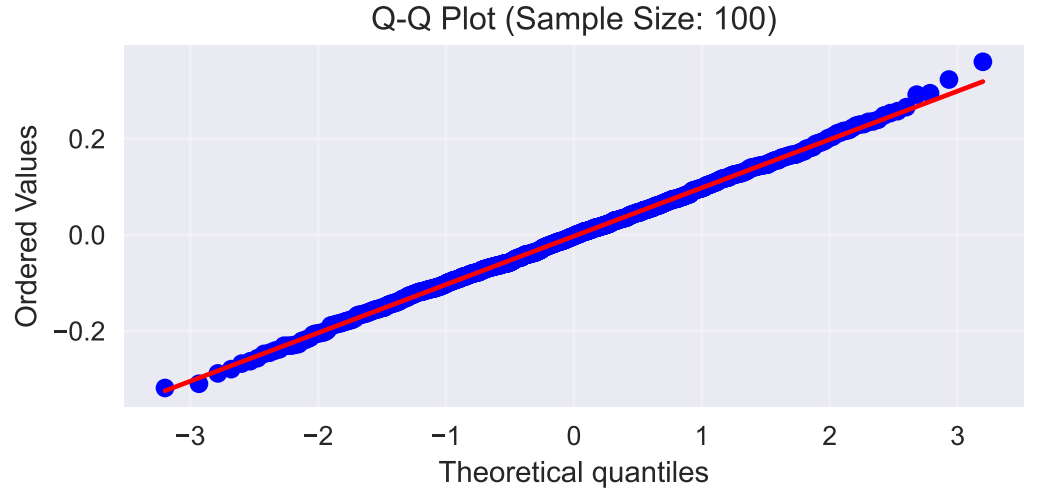




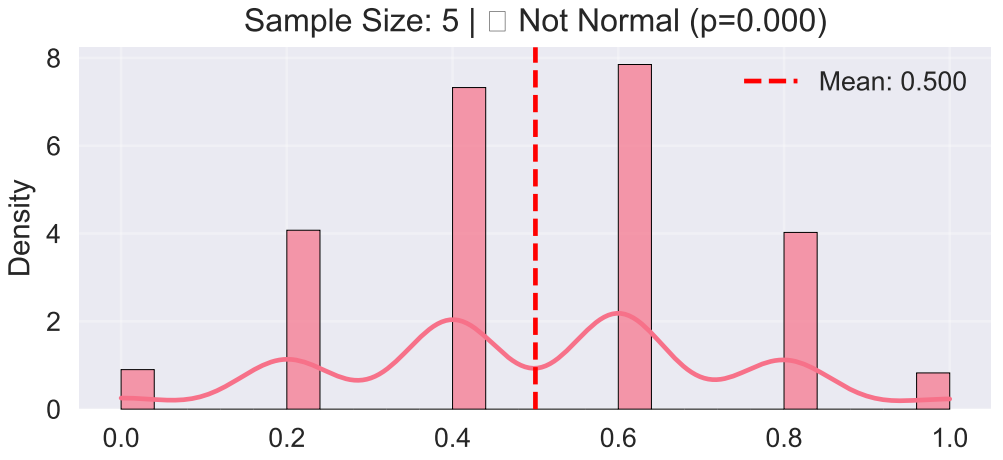


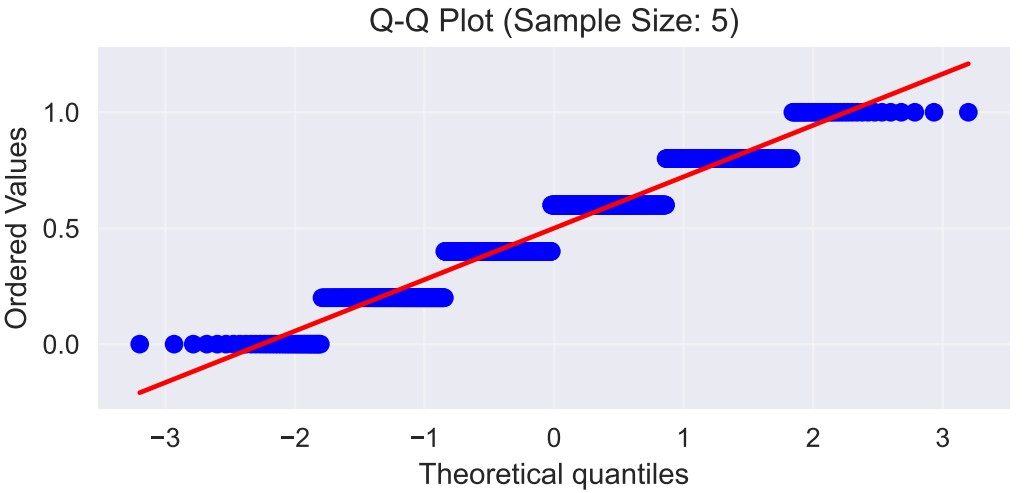


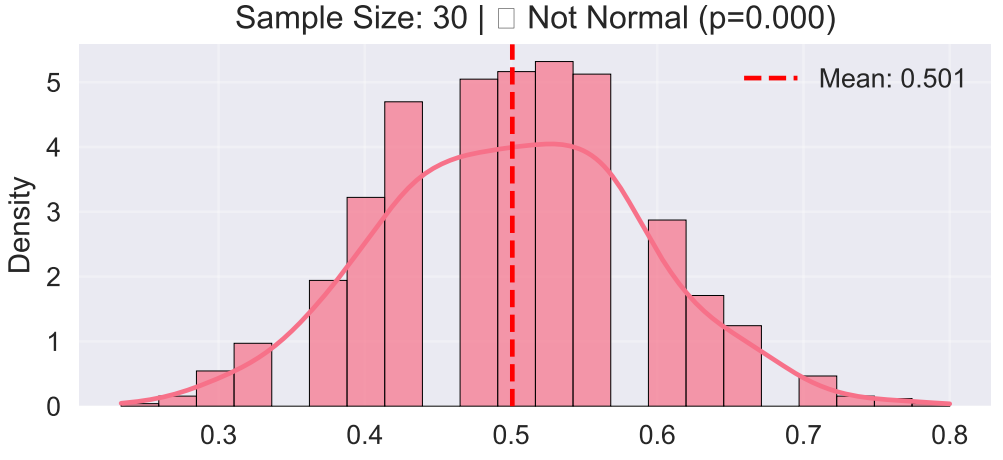


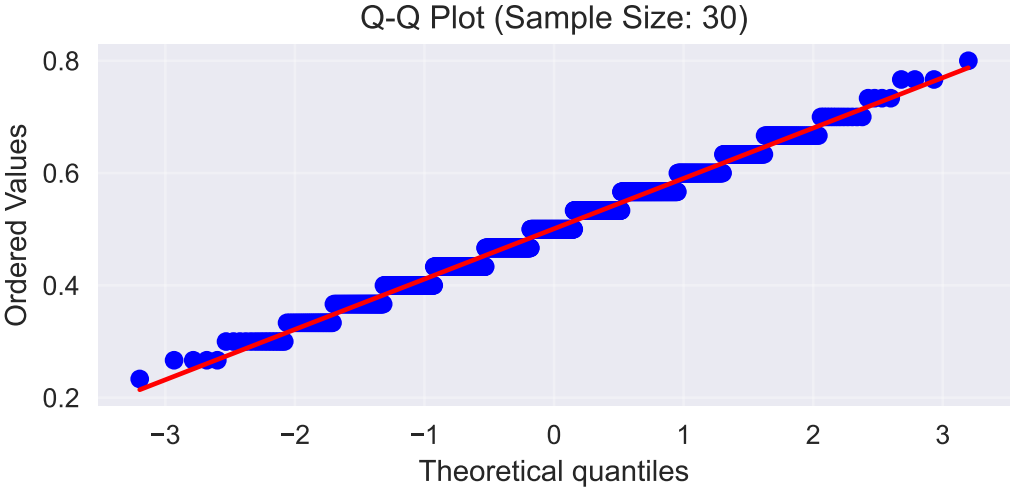


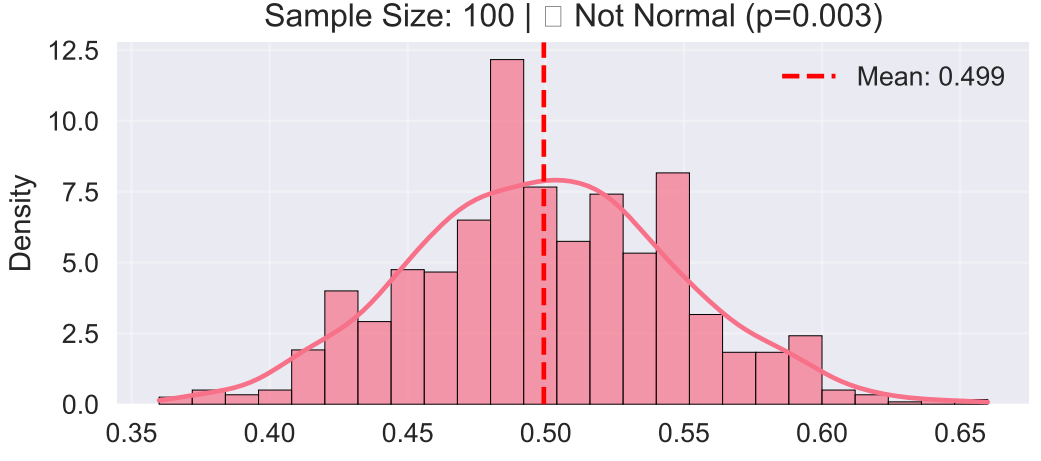
BERNOULLI Distribution - Sample Means Distribution Evolution

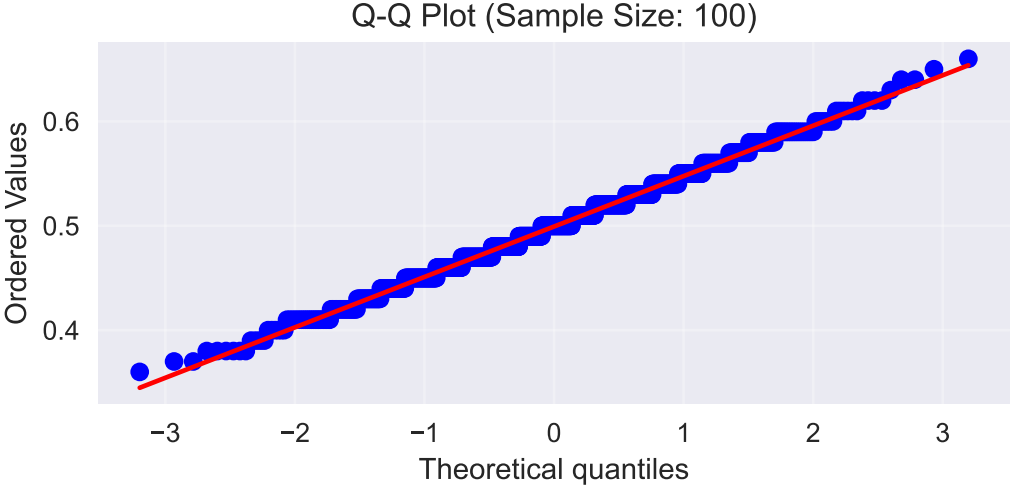




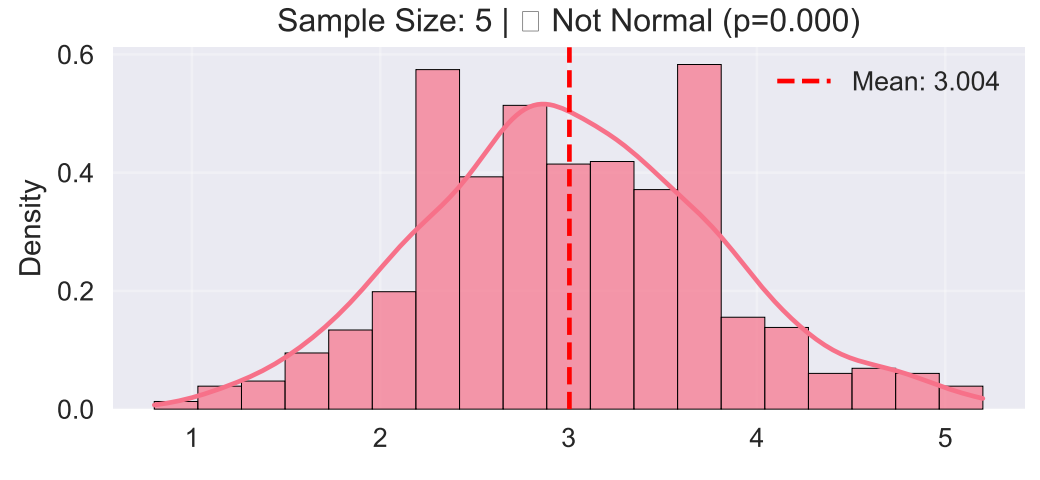


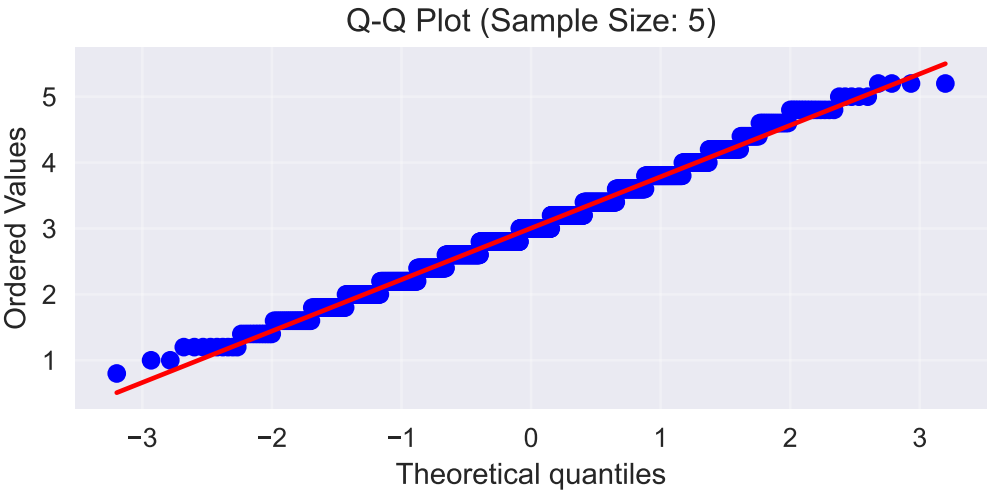


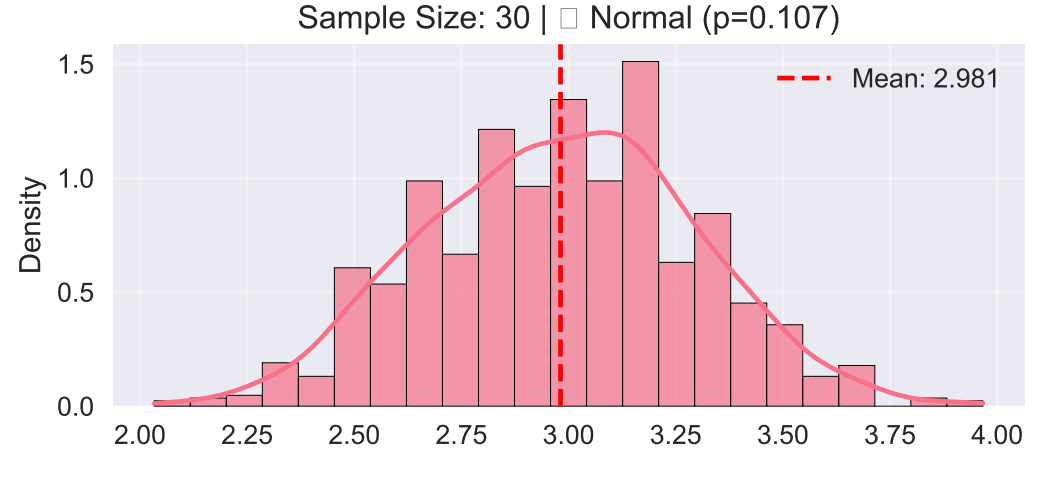


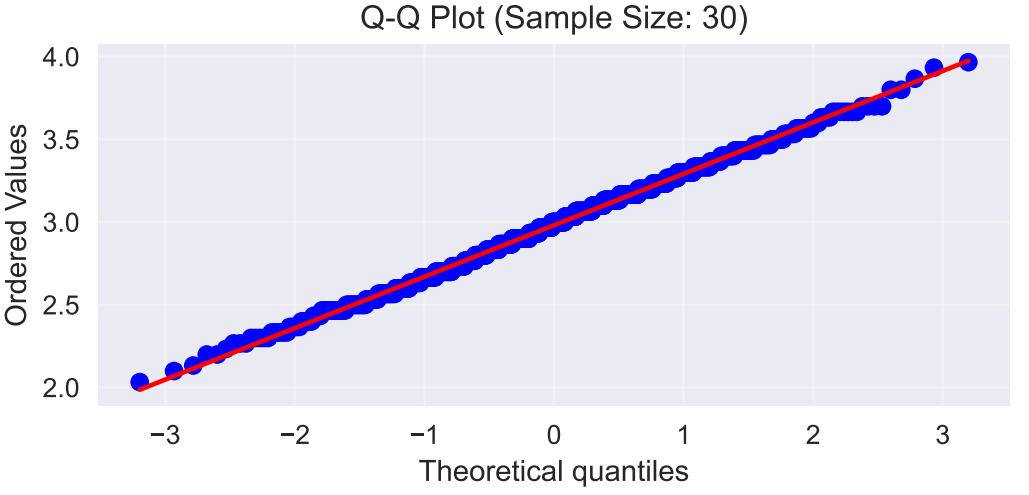


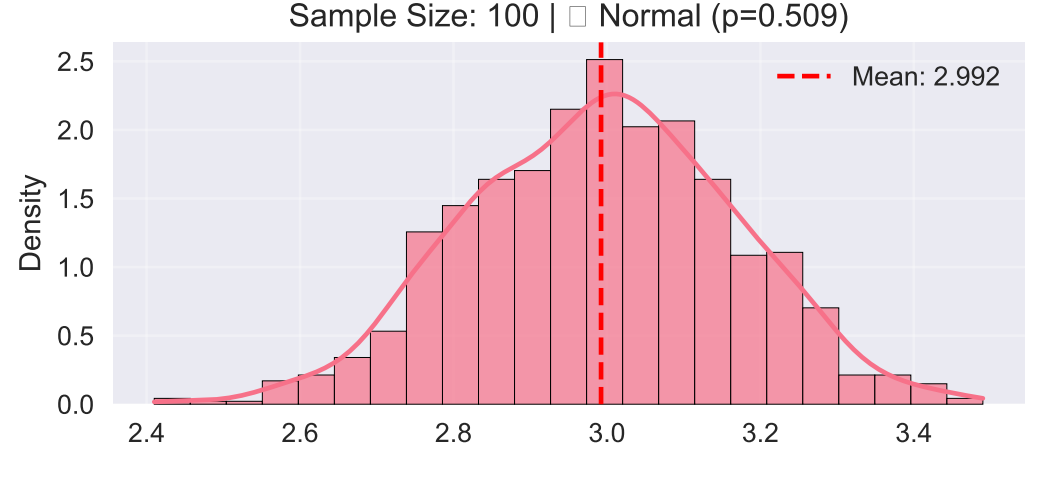
POISSON Distribution - Sample Means Distribution Evolution

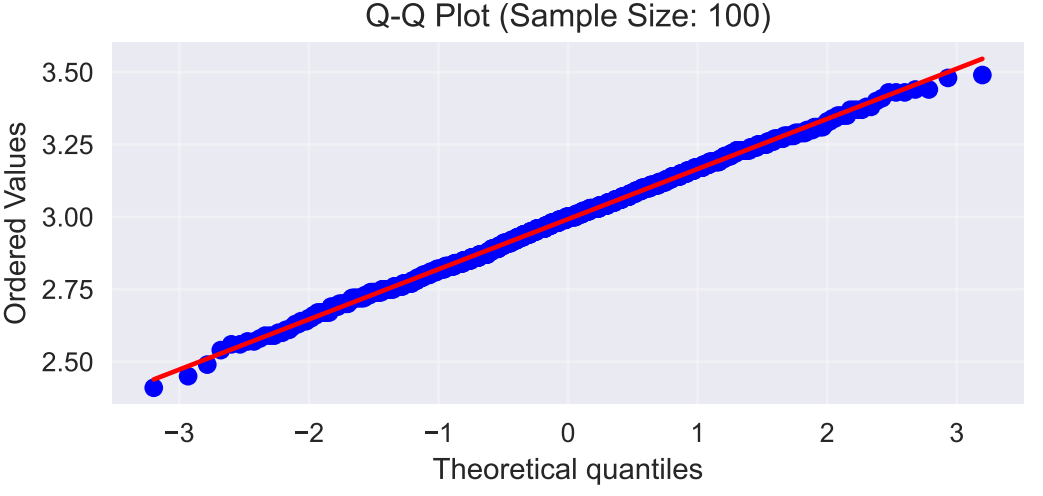




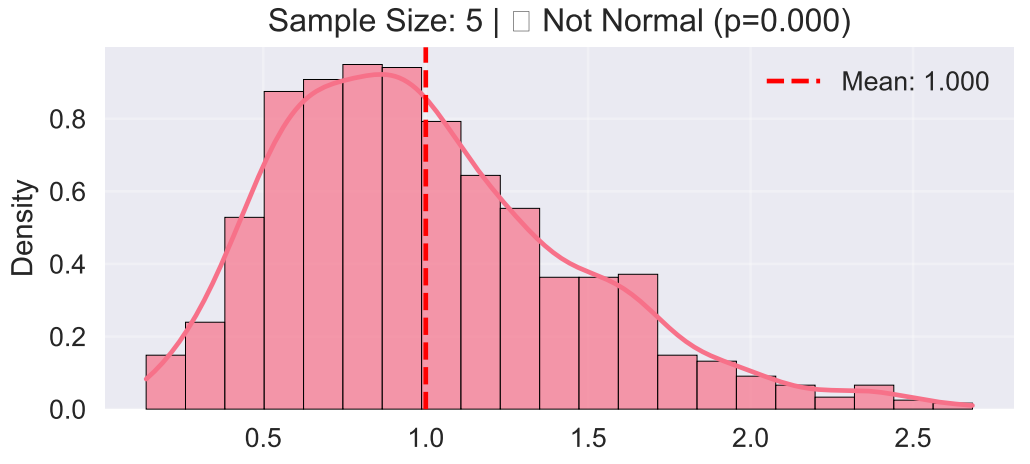


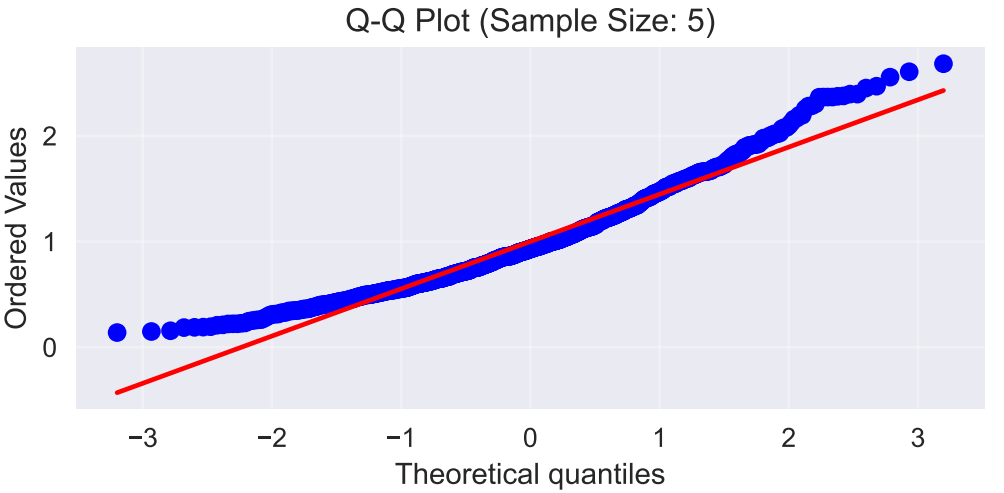


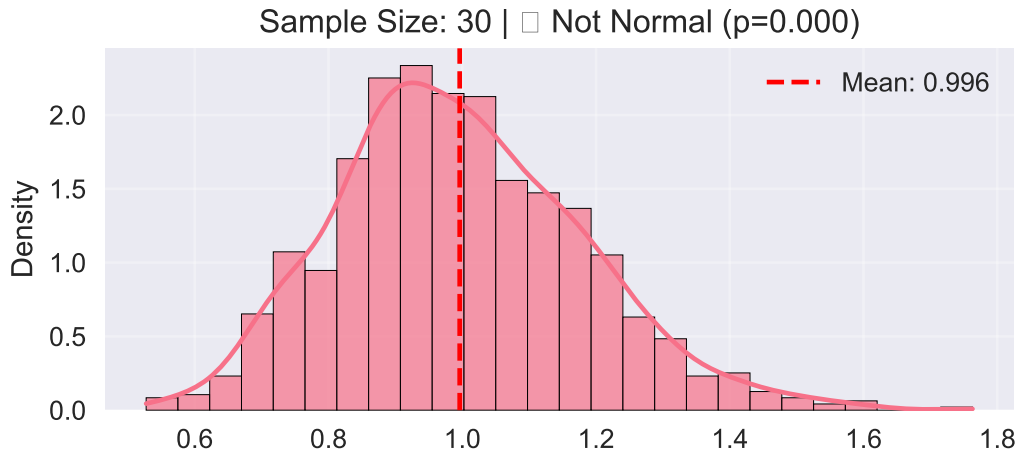


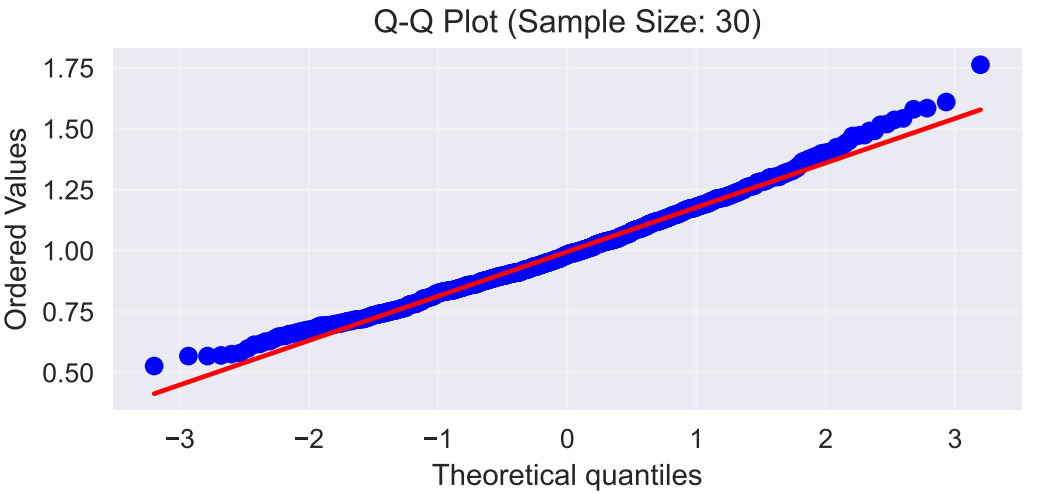


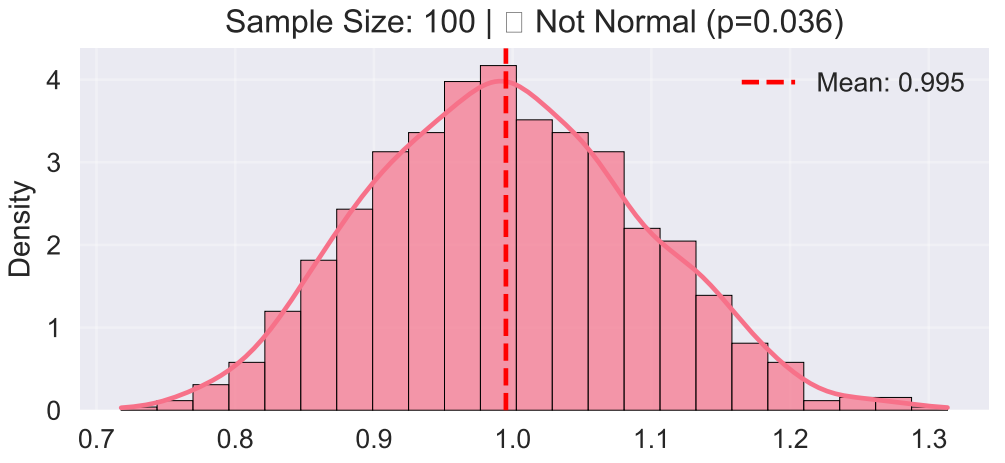
EXPONENTIAL Distribution - Sample Means Distribution Evolution

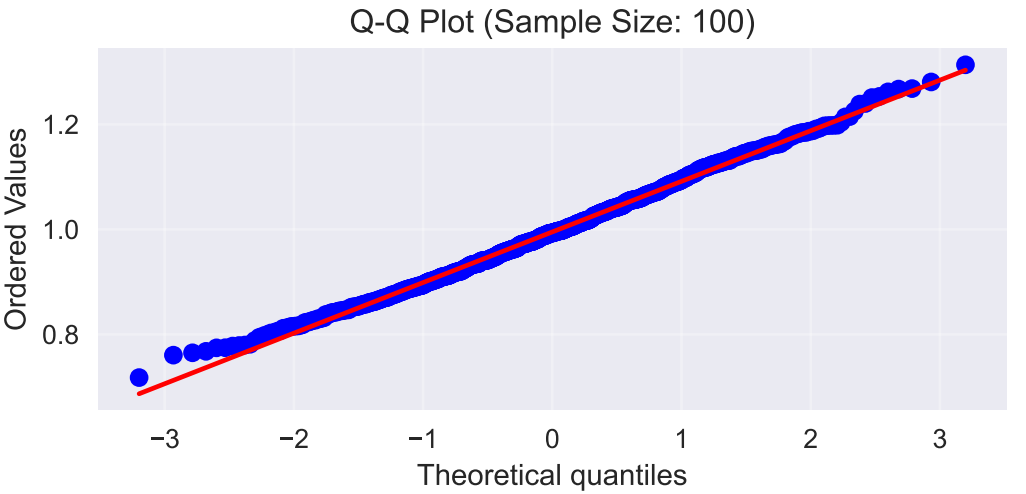




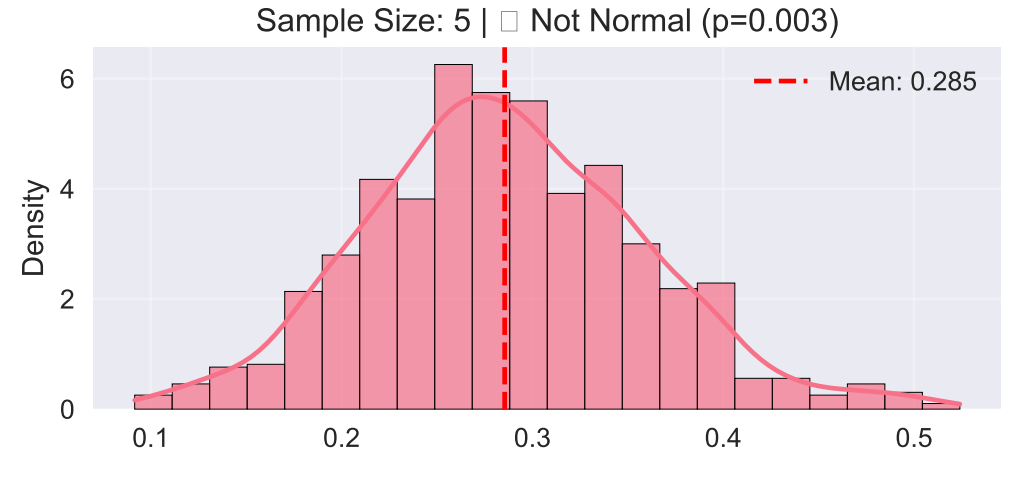


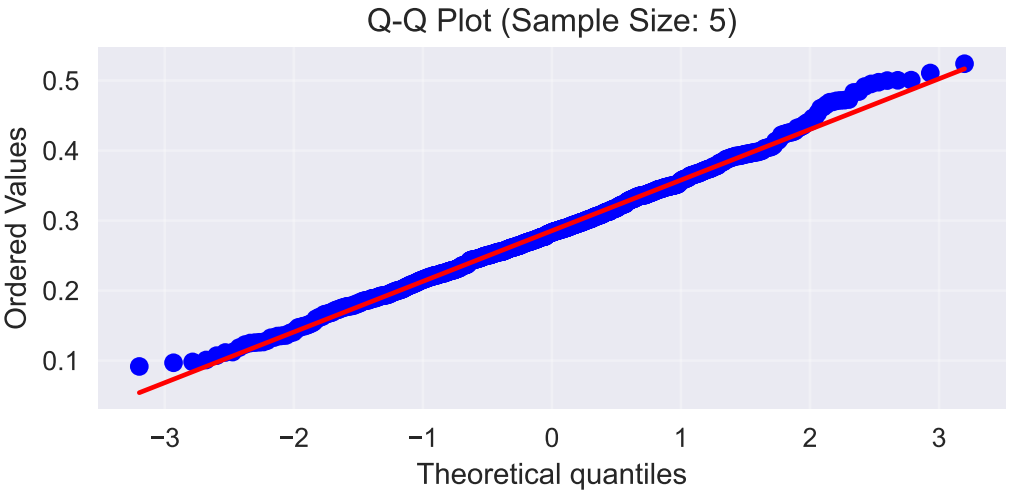


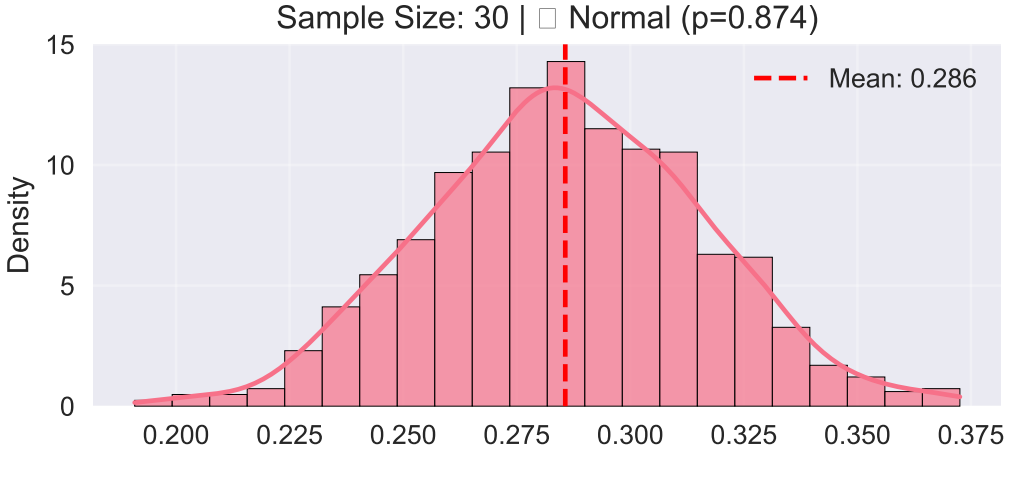


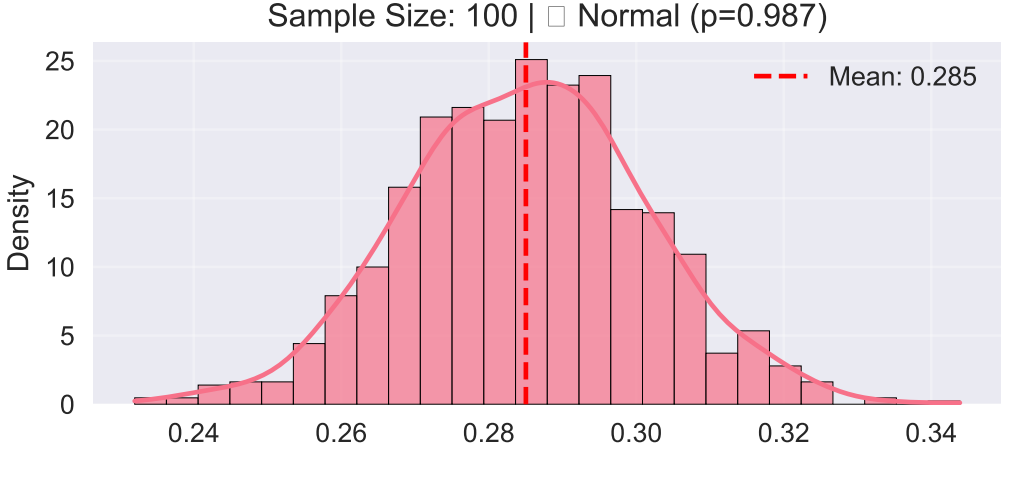


BETA Distribution - Sample Means Distribution Evolution









Evolution of Sample Mean Statistics with Sample Size