Problem 1)

$$f(x) = f(x_i) + f'(x_i)(x - x_i) + \frac{f''(x_i)}{2}(x - x_i)^2$$

$$+ \frac{f'''(x_i)}{6}(x - x_i)^3 + \frac{f''''(x_i)}{24}(x - x_i)^4$$

$$A_s = \int_{x_{i-1}}^{x_{i+1}} f(x)\,dx = \left(f(x_i)\,x + \frac{f'(x_i)}{2}(x - x_i)^2 \cdot \right.$$

$$\left. + \frac{f''(x_i)}{6}(x - x_i)^3 + \frac{f'''(x_i)}{24}(x - x_i)^4 \right)\Bigg|_{x_{i-1}}^{x_{i+1}} + \int_{x_{i-1}}^{x_{i+1}} \frac{f''''(x_i)}{24}(x - x_i)^4\,dx$$

$h = x_{i+1} - x_i = x_i - x_{i-1}$

$x_{i+1} - x_{i-1}$ ✓ $(x_{i+1} - x_i)^2 - (x_{i-1} - x_i)^2 = h^2 - h^2$

$$A_s = f(x_i)\,2h + f''(x_i)\,h^3\,\frac{1}{3} + \frac{f''''(x_i)}{12}h^4$$

$$+ \left(\frac{f^{(4)}(x_i)}{24}(x - x_i)^5\,\frac{1}{5} \right)\Bigg|_{x_{i-1}}^{x_{i+1}}$$

$(x_{i+1} - x_i)^4 - (x_{i-1} - x_i)^4 = h^4 - h^4 = 0$

$(x_{i+1} - x_i)^3 - (x_{i-1} - x_i)^3 = h^3 - -h^3 = 2h^3$

$$= f(x_i)\,2h + f''(x_i)\,h^3\,\frac{1}{3}$$

$$+ \frac{1}{24} f^{(4)}(x_i)\,\frac{1}{5}\left((x_{i+1} - x_i)^5 - (x_{i-1} - x_i)^5 \right)$$

$$4\text{th term} = \frac{1}{120} f^{(4)}(x_i)\left(h^5 - -h^5 \right) = \frac{1}{60} f^{(4)}(x_i)\,h^5$$

- Error in order of 4th term

ε_{SR} = 4th term over all $i-1$ intervals

One i interval $= \frac{1}{60} f^{(4)}(x_i)\,h^5$ ← $(b-a)^5$

$$\varepsilon_{SR} = \frac{(b-a)^5}{N^4\,60} f^{(4)}(x_i)$$