

$$\begin{aligned}
C_1 &= a_1 + b_1 X + c_1 X^2 + d_1 X^3 \\
C_2 &= a_2 + b_2 X + c_2 X^2 + d_2 X^3 \\
C_3 &= a_3 + b_3 X + c_3 X^2 + d_3 X^3 \\
C_4 &= a_4 + b_4 X + c_4 X^2 + d_4 X^3 \\
C_5 &= a_5 + b_5 X + c_5 X^2 + d_5 X^3 \\
C_6 &= a_6 + b_6 X + c_6 X^2 + d_6 X^3 \\
C_7 &= a_7 + b_7 X + c_7 X^2 + d_7 X^3
\end{aligned}$$

These are the 7 intervals, each with 4 unknown values. Thus, there is 28 unknown values.

$$\begin{aligned}
C_1(0) &= 0 & \text{- 1st condition, matches the data points} \\
C_1(1) &= C_2(1) = 50 \\
C_2(2) &= C_3(2) = 106 \\
C_3(3) &= C_4(3) = 204 & \text{- 2nd condition, continuous, the ends of the interval are connected} \\
C_4(4) &= C_5(4) = 258 \\
C_5(5) &= C_6(5) = 292 \\
C_6(6) &= C_7(6) = 320 \\
C_7(7) &= 355
\end{aligned}$$

- This is $8 + 6 = 14$ conditions out of 28 needed.

- 3rd condition, the first derivative is continuous

$$\begin{aligned}
C'_1(1) &= C'_2(1) & C'_5(5) &= C'_6(5) \\
C'_2(2) &= C'_3(2) & C'_6(6) &= C'_7(6) \\
C'_3(3) &= C'_4(3) \\
C'_4(4) &= C'_5(4)
\end{aligned}$$

- Now have 20 conditions

- 4th condition, second derivatives are continuous

$$C_1''(1) = C_2''(1)$$

$$C_5''(5) = C_6''(5)$$

$$C_2''(2) = C_3''(2)$$

$$C_6''(6) = C_7''(6)$$

$$C_3''(3) = C_4''(3)$$

$$C_4''(4) = C_5''(4)$$

- Now have ~~26~~ 26 conditions

- 5th condition, 2nd derivative at the end points = 0

$$C_1''(0) = 0$$

$$C_7''(7) = 0$$

- Now we have 28 conditions for all 28 unknown variables.