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Social Influence and the Dynamics of Opinions: The Approach of Statistical Physics

Claudio Castellano[a,b,*]

[a]*Istituto dei Sistemi Complessi (ISC-CNR), Rome, Italy*
[b]*Dipartimento di Fisica, Sapienza Università di Roma, Rome, Italy*

Interacting individuals tend, in general, to increase their similarity. If not influenced by his or her social environment, each individual would choose a personal response to a political question, his or her own unique cultural conventions or special correspondence between objects and words. Still, it is common experience that opinions, cultural features, and languages are shared by large groups of people. We present some dynamical models recently considered by statistical physicists to describe these processes and discuss the main results of this approach. Emphasis is on qualitative aspects such as robustness, size effects, and the role of the topology of the interaction pattern. Copyright © 2012 John Wiley & Sons, Ltd.

INTRODUCTION

Under which conditions a large number of atoms form a solid, a liquid, or a gas, depending on external parameters such as pressure and temperature? The ultimate answer to this fundamental question is provided by statistical physics, a discipline that aims at understanding how the behavior of a large collection of relatively simple microscopic entities generate large-scale thermodynamical properties of matter. The remarkable success of statistical physics in this endeavor naturally raises the question: Can the same concepts and methods be applied in the completely different domain of social sciences, to investigate emergent collective phenomena in groups of interacting agents? The general meaning, consistency, and scope of this approach is a matter of a longstanding debate that will not be entered here (Buchanan, 2007; Helbing, 2010). The focus will be instead on a much more specific question, which has attracted a lot of interest in recent years within the community of statistical physicists (Castellano *et al*., 2009). Under which conditions does a group of individuals reach consensus on a certain issue or persist in their disagreement? Opinion dynamics is the subfield of statistical physics that tries to answer this question. In the past decade, it has attracted the interest of a large number of scholars. In this paper, we present some of the results obtained in this line of research, focusing on the basic conceptual ingredients that enter the models and the main qualitative conclusions that can be learned from them. In these models, individuals are, in general, considered identical, and all possible opinions have equal intrinsic value. We are interested in whether and how consensus emerges; no global state of the system is 'better' than the others. The focus will be on results obtained by statistical physicists. The connection with relevant literature in the

*Correspondence to: Istituto dei Sistemi Complessi (ISC-CNR), Via dei Taurini 19, I-00185 Rome, Italy. E-mail: claudio.castellano@roma1.infn.it

domains of sociology and psychology is very limited; this reflects an insufficient interdisciplinary cross-fertilization that still affects the field. For this reason, we caution the reader that some words might be used in a sense that is not fully correct or is even plainly misleading from the point of view of the socio-economic sciences.

OPINION DYNAMICS

Opinion dynamics deals with the investigation of the basic processes allowing consensus to arise out of an initial situation in which individuals do not agree. The term opinion must be intended here in a broad and abstract sense as any trait of an individual that can be changed with a relatively limited cost: an opinion about a certain issue, a social convention, or a technological standard. Consensus occurs when all members of a group choose the same state or action from a set of two or more possible alternatives.

In all these cases, the choice of each individual is constantly influenced by the repeated interaction with acquaintances, which may have the same or a different opinion: Whether I will buy a specific type of computer will depend on the possibility to share software with my friends; the choice of my family and my friends will influence me in deciding which candidate to support in next elections. Interactions are mutual; the opinion of an individual is influenced by his or her peers but can also influence them so that complex feedback loops are originated. Out of these interactions, occurring at the individual level, macroscopic phenomena are generated: technological standards spread, leading competitors to extinction; social conventions are globally accepted; and some candidates obtain large portions of votes whereas others are much less popular. Opinion dynamics aims at understanding how the microscopic mechanisms translate into phenomena at large scale.

Among the different contexts where statistical physics has been used to approach problems in the socio-economic context, opinion dynamics is probably the most simplified and abstract case. Although inspiration is taken from concrete social phenomena, typically extremely simplified models are considered, with no attempt to model realistically and quantitatively any specific phenomenon occurring in the real world. Individuals are treated as highly idealized entities whose behavior is described by extremely simple laws. Essentially, the ingredients considered are different forms of social influence (acting in a local neighborhood), noise and the topology of the interaction pattern. To a large extent, the variability of personal attitudes or preferences is disregarded.

Opinion dynamics assumes nonstrategic behavior. Agents do not try to optimize any utility function. They adaptively respond to the environment by just imitating (or contradicting) their neighbors: no payoff is associated to their choice. In this respect, it is important to stress that all opinions are assumed equally valid; there is no reason for selecting a priori any of them. Which of the opinions eventually dominate does not depend on any intrinsic fitness. For this reason, we do not deal with any spreading phenomenon (of information, rumors, or gossip), where states of the individuals are intrinsically different and transitions are possible only in one direction, from 'not informed' to 'informed'. We model instead genuine 'fair' competition among similarly plausible ideas. In the context of socio-economic literature, the issue of how conventions are shared among groups of individuals is studied by means of coordination games (Blume, 1993; Ellison, 1993). Here, the setting is similar to a pure coordination game: no opinion is better than others. The focus is not on realistic or nontrivial features of the interaction among two individuals. The aim is to understand genuinely collective effects emerging out of simple iterated interactions among many individuals.

Opinion dynamics can be interpreted as a process by which collective decisions are taken in a completely decentralized setup. In this kind of problems, the classical source of inspiration is the behavior of animal groups (ants, bees, etc.), which, in a self-organized manner, select the best nest site among a number of candidates or choose the shortest path between the nest and a food source (Bonabeau *et al.*, 1999). Some of the models described in the following paragraphs are indeed related to quorum response models, introduced for investigating decision-making in animal groups (Sumpter and Pratt, 2009).

Before starting to present the main models and results, it is useful to summarize what are the typical questions opinion dynamics tries to address. Is consensus reached or diversity of opinions persists indefinitely? In the latter case, what are the features of the asymptotic state? Which opinions do survive? In which proportions? In the former case, which opinion will asymptotically dominate? Is the initial majority always bound to win? And how long does the process last? How do the answers to these questions depend on the system size? How do they depend on the pattern of interactions? Which perturbations to the dynamical

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rules do change the global behavior? Which of them do instead leave the evolution essentially unaltered?

The question about how much time it takes to reach consensus is very important. In some cases (imagine insects deciding between two possible locations for the new nest in the presence of predators), the time needed to reach a collective agreement has a crucial effect. In other cases, it may happen that consensus is, in principle, always reached, but this occurs over a temporal scale growing extremely fast with the number of individuals involved. Hence, the time to consensus becomes so large that, in practice, no consensus can be achieved already for groups of moderate size.

BINARY CHOICE MODELS

Imagine a situation where individuals are confronted with a choice between two mutually exclusive options, yes/no to a referendum, for instance. Binary choice models are used to investigate how different microscopic interaction mechanisms affect the global competition between the two options and eventually lead to consensus. For this kind of situation, a paradigmatic model is the so-called Ising model with Glauber dynamics, which describes, at the most elementary level, the magnetization process of materials.

Adapting to the Majority: Ising–Glauber Dynamics

In some ferromagnets, the magnetic moments of individual atoms (spins) can assume only two positions (along one direction and in the opposite direction, indicated as up/down or ± 1). At high temperature, because of thermal agitation, spins are randomly up or down. When the temperature is rapidly lowered, neighboring spins will align, and over time, this leads to a globally ordered (magnetized) state. In the Ising–Glauber dynamics, the crucial (and only) ingredient is the role played by the local majority (local field in physics jargon): If the majority of neighbors of a spin is in the up state, the spin tends to adapt to them and flip to the up state; if the majority is down, the tendency will be toward the down state; and if there is an equal number of up and down neighbors, there will be no drive toward any of the two states and the spin will flip with probability 1/2.

The translation of this model into a model for opinion dynamics is straightforward: Spins describe individuals, which must choose between two mutually exclusive alternatives. Each of them experiences a pressure, exerted by peers, toward the opinion shared by the local majority. What happens if this rule is iterated time and again? The answer to this question crucially depends on the type of topological pattern mediating the interaction among agents. This is a fundamental fact, true for all models, which must be always kept in mind. A specific microscopic mechanism of interaction can lead to completely different global outcomes (consensus or lack of it) depending on the structure of the network connecting the individuals.

The simplest possible interaction pattern is homogeneous mixing, equivalent to considering agents sitting on the nodes of a fully connected network. For each interaction, a node selects its neighbors at random among all the others. No spatial structure of any kind exists. In this setting, under Ising–Glauber dynamics, an individual selects K other nodes at random and becomes equal to the state held by the majority of them. It is easy to recognize that this dynamics always leads to full consensus. But which type of consensus is established? Do all individuals agree on the ‘yes’ or on the ‘no’ opinion? More formally, if in the initial stage the fraction of agents preferring opinion ‘yes’ is x, what is the probability $E(x)$ (also termed exit probability) that the final state is ‘yes’? And how much time $T(x, N)$ does it take, on average, as a function of the initial value of x and of the number of individuals N? In the case of homogeneous mixing, the answer to these questions can be computed using standard methods (Kolmogorov backward equation) (Crow and Kimura, 1970). For example, for $K = 4$, the exit probability is

$$E(x) = \frac{\int_0^x dtG(t)}{\int_0^1 dtG(t)} \text{ where} \qquad G(t) = [1 + 4t + 4t(1 - t)]^{N/2}, \tag{1}$$

which is displayed in Figure 1a for various system sizes.

The exit probability is therefore a step-like function, with value very close to 1(0) if, in the initial state, x is close to 1(0) and a transition for values around $x = 1/2$. As the system size N grows, the transition in the probability $E(x)$ becomes abrupt. Figure 1 shows that, when opinion dynamics is dictated by peer pressure, consensus is reached almost always on the opinion that was initially shared by the majority of people. There is very little chance for an initially minoritarian opinion to become majority. Therefore, under Ising–Glauber dynamics, a small change in the initial conditions may have a dramatic effect on the final state, completely changing the type of final consensus reached.

The average time to consensus is given by the expression

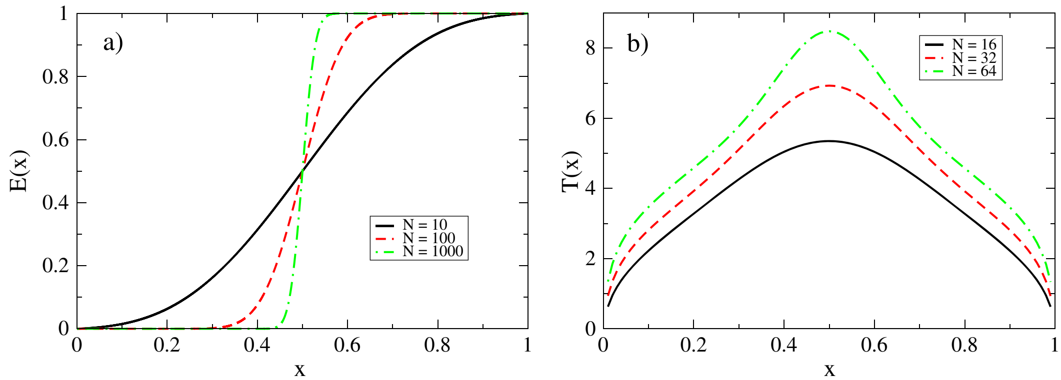

Figure 1. Ising–Glauber dynamics: (a) Probability *E(x)* that, in the final state, all agents have the 'yes' opinion, as a function of the fraction x of agents with 'yes' opinion in the initial state. As the size of the system considered gets larger, the transition becomes more and more abrupt. Asymptotically for large system size, the final state is univocally determined by the majority in the initial state. (b) Average time to consensus $\boldsymbol{T(x,N)}$ as a function of x for various values of the system size N. Time is measured as the number of attempted updates per individual.

$$T(x,N) = \frac{1}{\int_0^1 \mathrm{d}t G(t)} \left\{ [1-E(x)] \int_0^x \mathrm{d}t \frac{E(t)}{D(t)G(t)} + E(x) \int_x^1 \mathrm{d}t \frac{1-E(t)}{D(t)G(t)} \right\}, \tag{2}$$

where

$$D(t) = \frac{[4t(1-t)+1]t(1-t)}{2N}, \tag{3}$$

and it is displayed in Figure 1b. An important message provided by Equation (2) and Figure 1b is that consensus is reached very fast, with a very slow dependence on the system size: $T(x,N) \sim \ln(N)$.

When agents are arranged in a regular square lattice, the temporal evolution under Ising–Glauber dynamics is divided into two distinct regimes (Figure 2). On a short time scale, agents align with their neighbors forming compact domains with all agents locally sharing the same opinion. The shared opinion will be 'yes' in some parts of the system and 'no' in different areas so that the global state will be composed of a mosaic of deeply intertwined homogeneous domains. During the ensuing stage, some of the domains tend to grow, at the expense of others, which shrink. This qualitative picture occurs for any ordered pattern of interaction, that is, any d-dimensional regular lattice.

Under this dynamics, the typical size l of surviving homogeneous domains grows in time as $l(t) \sim t^{1/2}$

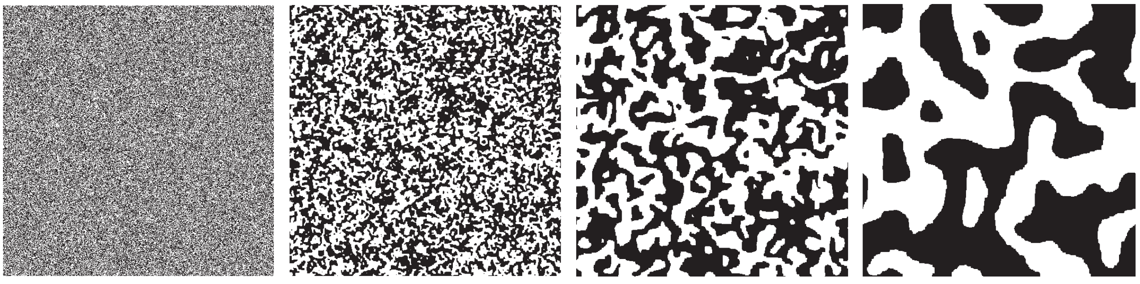

Figure 2. Evolution of the Ising model with Glauber dynamics on a square lattice of side $\boldsymbol{L=512}$. Black dots represent agents with 'yes' opinion and white dots agents with 'no' opinion. The leftmost panel is the initial fully disordered configuration; other panels are snapshots at successive times ($\boldsymbol{t=10, t=100, t=1000}$, respectively). Notice that domain pattern remains the same at successive times, the only difference being the typical domain size, which grows over time as $\boldsymbol{l(t) \sim t^{1/2}}$.

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(Gunton *et al.*, 1983; Bray, 1994), implying that consensus requires a time of the order of $T \sim L^2 \sim N^{2/d}$ to be established (L is the linear size of the system, $N \sim L^d$ is the total number of agents, and d is the spatial dimensionality).[1] The logarithmic growth with N found for homogeneous mixing is recovered in the limit of infinite dimensionality $d \to \infty$. For any $d \geq 2$, the exit probability $E(x)$ is very similar to Figure 1 (details depending on the dimension d).

Among the many variants that can be introduced in the Ising model (some of them to be discussed in the succeeding paragraphs), the most natural is the addition of some 'noise', intended as the possibility for agents to spontaneously change their opinion independently from the state of their peers. If this event is not too frequent, no dramatic change occurs in the global behavior (De Oliveira *et al.*, 1993): If a spin inside a domain of a certain opinion spontaneously flips to the other, it is rapidly reabsorbed by the peer pressure of neighbors. If noise is instead large, spontaneous opinion changes are too frequent to be reabsorbed. They accumulate, and the system enters the so-called disordered phase, a steady state with continuously reshaped domains of opposite opinions competing forever.

Copying a Random Neighbor: Voter Dynamics

A model similar to the Ising–Glauber model, possibly even simpler, has acquired a great relevance in the field of opinion dynamics: the voter model. As the name suggests, the model has been introduced (actually not by physicists but by applied probabilists (Clifford and Sudbury, 1973; Holley and Liggett, 1975)) as a schematization of the behavior of an ensemble of 'voters', which must choose between two alternative choices. Interestingly, the voter model is also studied in other unrelated disciplines such as theoretical ecology or surface catalysis. The underlying idea is that, in order to decide, an agent simply selects at random one of his neighbors and picks up its opinion. Similarly to the Ising–Glauber dynamics, also in this model, an agent tends to become aligned with the local majority: If three out of four neighbors are in a state 'yes', with probability 3/4, the agent will uniform its opinion to the local majority. However, the peer pressure is, in this case, less strict: with probability 1/4, the agent can assume the opinion shared by the minority. In Figure 3, the difference between the microscopic dynamics of the two models is illustrated by the shape of the probability $P(x)$ that an individual, surrounded by a fraction x of agents in the opposite state, changes opinion.

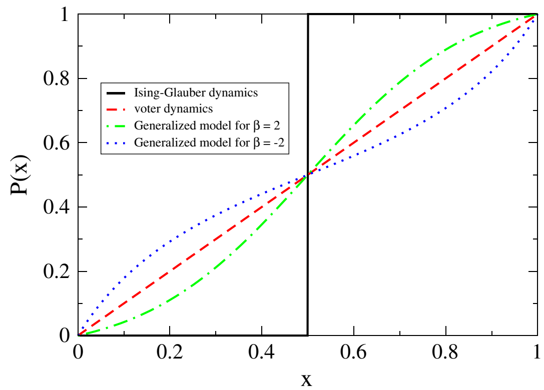

Figure 3. Probability $P(x)$ that an individual, surrounded by a fraction x of individuals in opposite state, changes opinion.

The difference between voter and Ising dynamics is also evident when the macroscopic evolution in two dimensions is visualized (Figure 4). In the voter case, domains tend to grow over time as well, but interfaces are much rougher and noisy. In physical terms, Ising–Glauber ordering is governed by surface tension, that is, the tendency to reduce the length of the interface to reduce the energy associated with them. In the voter case, no surface tension is present and the evolution is dominated instead by fluctuations.

Voter dynamics is rather tolerant toward the minority state at the individual level (Figure 3), and this is reflected macroscopically in the type of consensus reached. The exit probability and the time to consensus (in the homogeneous mixing case) are reported in Equation ((4)) and plotted in Figure 5:

$$\begin{aligned} E(x) &= x \qquad \forall N \\ T(x,N) &= -N\,[x\ln(x) + (1-x)\ln(1-x)]. \end{aligned} \tag{4}$$

For any number of agents N, the probability to end up in a certain state is simply equal to the fraction of agents sharing that opinion in the initial state. For example, consider a system where only 10 agents out of 100 have the 'yes' opinion in the initial stage. It is not very frequent (occurring 10% of the times) but far from impossible that the 10 agents spread their opinion to all the others and consensus is reached on the initially minoritarian opinion. This is in stark contrast with what occurs for Ising–Glauber dynamics, where the possibility that the initial majority is dynamically overturned is negligible. This 'relaxed' attitude toward minorities also affects the time to reach consensus. Under homogeneous mixing and for any regular lattice in dimension $d > 2$, the time needed to reach the final state grows as $T \sim N$ with

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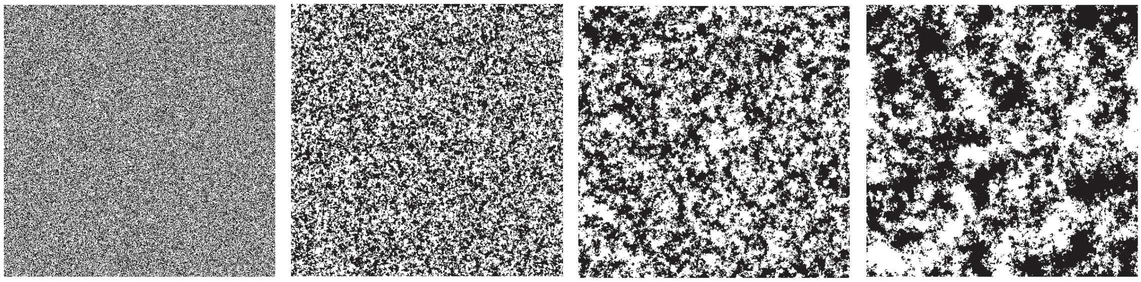

Figure 4. Evolution of the voter dynamics on a square lattice of side $L=512$. Black dots represent agents with 'yes' opinion and white dots agents with 'no' opinion. The leftmost panel is the initial fully disordered configuration; other panels are snapshots at successive times ($t=10, t=100, t=1000$, respectively). Interfaces between domains are much rougher than for the Ising–Glauber case.

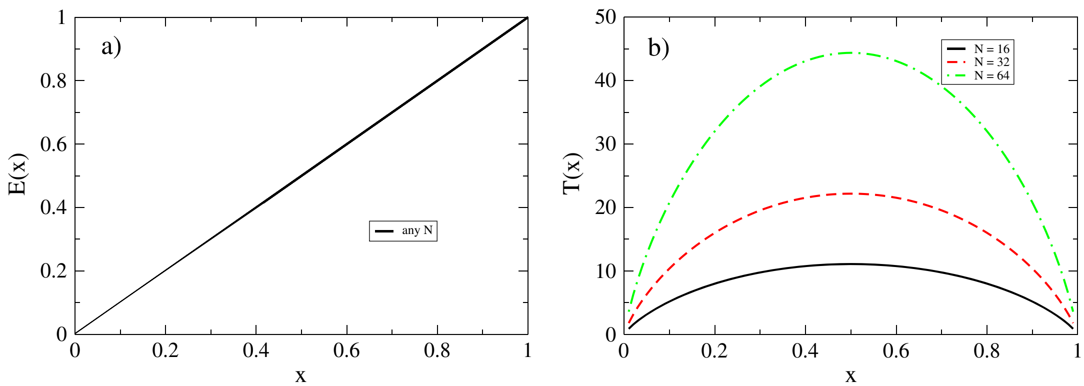

Figure 5. Voter model: (a) Probability $E(x)$ that, in the final state, all agents have the 'yes' opinion, as a function of the fraction x of agents with 'yes' opinion in the initial state. (b) Average time to consensus $T(x,N)$ as a function of x for various values of the system size N. Time is measured as the number of attempted updates per individual.

the number N of agents ($T \sim N \ln N$ in $d=2$), hence more quickly than for the Ising–Glauber dynamics

Other Binary Choice Models

Is voter dynamics robust to perturbations? The generic answer is no. As soon as some details are changed, the voter behavior is destroyed and the dynamics becomes akin to the Ising model, either in the ordered or in the disordered phase. A clear manifestation of this is evident when one considers a generalized form of $P(x)$ (Figure 3). For example, let us consider a function

$$P(x) = \frac{e^{\beta x} - 1}{e^{\beta x} + e^{\beta(1-x)} - 2}. \tag{5}$$

As depicted in Figure 3, $P(x)$ has, for $\beta > 0$, an intermediate shape between the Ising–Glauber case (which is recovered for $\beta \to \infty$) and the voter case (recovered for $\beta = 0$). For negative β, $P(x)$ tends to become flat around its midpoint. The effect of the different shapes on the way consensus is reached is evident in Figure 6.

For $\beta > 0$, the behavior is similar to the Ising–Glauber case and the final state is decided by the initial majority. For $\beta < 0$ instead, the exit probability tends to become equal to 1/2, independent of the initial state. Correspondingly, the average consensus time T grows exponentially with the size N (Figure 7).

In this situation, the tendency is toward coexistence of the opposite opinions. Consensus is reached only because of some large spontaneous fluctuation, an event that becomes exponentially improbable as the system size grows. This result is rather counterintuitive: Even for $\beta < 0$, according to the function $P(x)$, an individual tends to adapt to the opinion held by the majority of neighbors. The stronger the majority, the stronger the chance to follow it. Yet, this is not sufficient to build up full consensus in the system.

This pattern of behavior turns out to be generic (Vázquez and López, 2008). Depending on the shape of the function $P(x)$, the system is either in the ordered

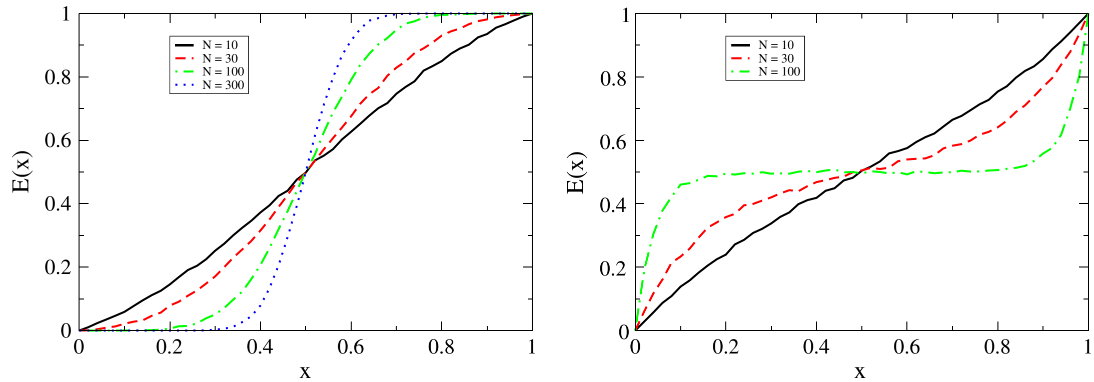

Figure 6. Generalized model. Probability *E(x)* that the final consensus is reached for the 'yes' opinion, as a function of the fraction x of agents with 'yes' opinion in the initial state for the model of Equation (5), for $\boldsymbol{\beta = 2}$ (left) and $\boldsymbol{\beta = -2}$ (right). Curves are the results of numerical simulations averaged over 10^4 realizations.

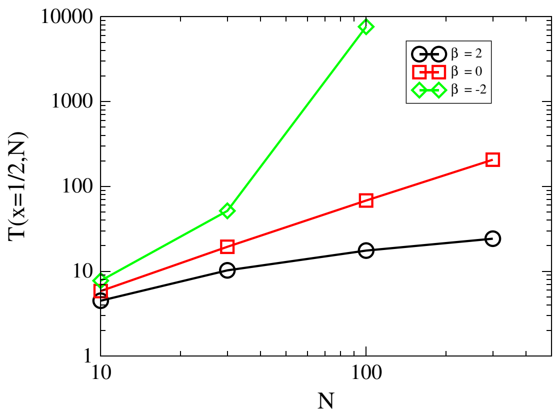

Figure 7. Generalized model. Average time $\boldsymbol{T(x = 1/2, N)}$ for the final consensus to be reached starting with $\boldsymbol{x = 1/2}$, as a function of the system size N.

or in the disordered phase of Ising dynamics. Voter behavior is recovered only for very specific choices of the function. This strong variation depending on little changes of parameters is a nontrivial emergent effect: small changes at the microscopic scale lead to completely different outcomes at a global scale, either consensus or persistent diversity of opinions.

The generic nonlinear form of the probability *P(x)* in Equation (5) is reminiscent of quorum-response models for decision-making in animal groups (Amé *et al*., 2006; Sumpter and Pratt, 2009). In both cases, the positive feedback provided by the nonlinear dependence on the number of other individuals having a certain opinion leads a group of agents to consensus on a common decision.

The fragility of voter behavior and the robustness of Ising's are confirmed by many other variations of the dynamics. One example is the introduction of an intermediate state such that switching from 'yes' to 'no' or vice versa is possible only via an intermediate 'mixed' state. Models of this type have been introduced to describe language competition (Castelló *et al*., 2006) (the intermediate state corresponding to bilingualism): they lead to a behavior of the Ising–Glauber's type, with fast convergence to the state initially preferred by the majority.

Many other interaction rules can be devised to describe other types of microscopic influence mechanisms. In all cases, one finds a behavior of Ising–Glauber's type.

The Sznajd model (Sznajd-Weron and Sznajd, 2000) takes into account the principle of social validation: the convincing power of groups of people is much larger than the one of a single individual (Milgram *et al*., 1969). In this model, a single agent cannot affect the state of its neighbors. On the contrary, if two neighboring individuals agree, they convince their neighbors to accept their opinion. Despite the microscopic difference with respect to Ising–Glauber dynamics, at the aggregate level, the behavior is the same (Castellano and Pastor-Satorras, 2011).

Another model thoroughly investigated is the so-called majority rule (Krapivsky and Redner, 2003). In this case, the microscopic dynamics step deals with groups of r neighboring individuals. The majority opinion within the group is determined, and the opinions of all members are then set equal to the majority. Also this type of dynamics leads the system toward complete consensus on the opinion that was prevalent at the beginning. Also in this case, as for Ising–Glauber dynamics, the time needed scales as T $N^{2/d}$ with the number of individuals involved. Very

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recently, simple modifications of the majority rule model have been used as basis for new mechanisms of collective decision-making (Montes De Oca *et al.*, 2011). In such a case, a population of agents spontaneously converges to the optimal solution of a problem: transporting objects along the shortest among two paths. It constitutes an interesting example of a model inspired by opinion dynamics leading to emergent wisdom of the crowds.

THE EFFECT OF COMPLEX TOPOLOGIES

It is self-evident that regular lattices, the substrate considered in the investigations reported in the previous sections, are not a realistic pattern of interaction for most real social phenomena. Networks are clearly a much more suitable description of the patterns where social interactions take place. Empirical investigations in the past decade have demonstrated that, very often, strong heterogeneity is a crucial ingredient of many networks, including social ones (Albert and Barabási, 2002; Newman, 2003). In a network, each node is characterized by its degree (i.e. the number of other nodes it is connected to). Although in the networks traditionally considered in the past all nodes have about the same number of connections (with small variations), it has become increasingly clear that, in many social networks, there are large disparities in the node connectivity. A quantitative characterization of this is provided by the network degree distribution *P(k)*, which counts what fraction of the nodes have degree k. In many social networks, *P(k)* has a very broad shape, with a tail for large k slowly decaying as $k^{-\gamma}$. Many social networks have $2 < \gamma < 3$ and so they are 'scale-free': The average degree $<k> = \sum_k kP(k)$ is a well-defined value, but degree fluctuations $<k^2> = \sum_k k^2 P(k)$ grow with the number N of nodes in the network and diverge with the system size. Understanding how the dynamics of opinions is affected by the scale-free nature (and other topologically complex features) of interaction patterns is therefore of fundamental importance. It turns out that topology has a crucial role: the same microscopic influence mechanisms can lead to completely different global outcomes depending on how individuals are connected among each other.

Ising–Glauber dynamics is a clear example of this sensitivity to the topology of the substrate. It turns out that, when agents sit on the nodes of a random network, there is a finite probability (Castellano *et al.*, 2005) (even going to 1 as the size N grows (Häggström, 2002)) that consensus is not reached. In such cases, the system remains stuck in a frozen configuration with coexisting 'yes' and 'no' opinions. The origin of these frozen states can be easily understood by considering the situation depicted in Figure 8. Each individual is locally surrounded by a majority agreeing with him or her, but global consensus is not reached because different parts of the network have selected different opinions.

This is in sharp contrast with what happens for the voter model, which instead always reaches consensus, on any graph, on temporal scales growing as power laws of the system size N. We have an interesting effect here. Glauber dynamics, which on regular lattices leads to fast consensus, remains trapped by frozen configurations on networks. Voter dynamics, instead, is much slower on regular lattices but always reaches consensus on networks.

Other interesting phenomena occur for voter dynamics on strongly hetereogeneous networks. Remarkably, the behavior of the model depends on the order in which interacting agents are selected (Castellano, 2005; Sood and Redner, 2005; Suchecki *et al.*, 2005; Sood *et al.*, 2008) In the direct voter model, the first selected node copies the second. In the reverse voter model (also called invasion process), the second copies the first. Although all models converge to consensus in a time linearly scaling with N if the decay of the degree distribution is fast ($\gamma > 3$), in scale-free networks, their behaviors differ. Although the scaling remains linear for the reverse voter, for the direct voter, the time to consensus grows as $N^{(2\gamma-4)/(\gamma-1)}$. Hence, consensus is reached more rapidly as γ gets close to 2.

The heterogeneous topology of scale-free networks has strong consequences also on which type of consensus is reached, that is, on the exit probability *E(x)*. It turns out that, for direct voter dynamics, hubs

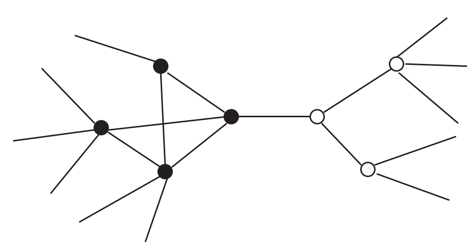

Figure 8. Example of a frozen configuration on a network under Glauber–Ising dynamics. Black (white) circles indicate 'yes' ('no') opinion. No further change can occur because all nodes are in agreement with the majority of their neighbors.

(i.e. nodes with a large number of connections) play a relevant role and the final state tends to be correlated with their initial state. For the invasion process, the opposite is true: nodes with low degree have a strong influence on which final state is reached. This is made very clear by considering what happens when a single 'mutant' opinion is introduced in a system where all other agents agree on the opposite opinion. If the mutant agent has degree k, the probability that its opinion is spread to the whole system is proportional to k for the direct voter dynamics, whereas it is proportional to $1/k$ for the invasion process.

THE AXELROD MODEL FOR CULTURAL DISSEMINATION

The extremely simple models for binary choice presented so far are the basis for more complicated models aimed at describing social interactions at lower level of abstraction. The state of an individual is then described by one or more variables, each assuming a range of discrete or continuous values (Castellano *et al.*, 2009). In this way, additional ingredients can be added and their effect studied. For example, recent works (Page *et al.*, 2007; Bednar *et al.*, 2010) have considered individuals characterized each by M internal discrete binary variables under the effect of two distinct driving forces: the tendency to conform to others (described by a voter-like dynamics) and the need to keep an internal consistency between the traits of each individual. The coupling between these two tendencies leads to a considerable increase in the time required to reach consensus compared with the case when internal consistency does not matter; diversity is much more persistent.

With the goal of investigating how different cultural domains arise and preserve their diversity, another very interesting model was introduced by Axelrod (1997). It includes two mechanisms believed to be fundamental in the understanding of the dynamics of cultural assimilation (and diversity): social influence and homophily. The first is the tendency of individuals to become more similar when they interact. The second is the tendency of likes to attract each other so that they interact more frequently. These two ingredients are generally expected to generate a self-reinforcing dynamics leading to a global convergence to a single culture. It turns out, instead, that the model predicts in some cases the persistence of diversity.

Axelrod model gives rise to a very rich and nontrivial phenomenology, with some genuinely novel behavior. Individuals are located on the nodes of a network (or on the sites of a regular lattice) and are endowed with F integer variables $(\sigma_1, \ldots, \sigma_F)$, representing cultural features such as language and religion, each of which can assume q different values, $\sigma_f = 0, 1, \ldots, q-1$ (cultural traits). In an elementary dynamic step, an individual i and one of his neighbors j are selected. The first picks up a feature at random. If the neighbor has the same trait for that feature, then a real interaction can occur: one of the features for which traits are different is selected and i copies the trait of j. Otherwise, nothing happens. It is immediately clear that the dynamics tends to make interacting individuals more similar, but the interaction is more likely for neighbors already sharing many traits (homophily), and it becomes impossible when no trait is common.

Starting from a disordered initial condition (e.g. with uniform random distribution of the traits), the evolution on any finite system leads unavoidably to a frozen state, where no more evolution is possible. Two types of frozen states are possible, one corresponding to consensus (a single culture) and the other to coexistence of many different cultural regions. It turns out that the number of possible traits q in the initial condition determines which of the two classes is reached (Castellano *et al.*, 2000).

For large q, very few individuals share traits. Few interactions occur, leading to the formation of small cultural domains that are not able to grow: a multicultural state. For small q instead, individuals share many traits with their neighbors, interactions are possible, and quickly, a single culture dominates. There exists then a critical value q_c separating the two types of behavior. Notice that, for $q < q_c$, the model predicts that a single culture invades the whole system, but fortuitous circumstances decide exactly which of the competing cultures wins. Dominance is the result of randomness, not of fitness.

Many nontrivial and interesting phenomena occur when more ingredients are introduced in the Axelrod model (Flache *et al.*, 2006). The possibility of one individual to change spontaneously one of his or her traits (cultural drift) corresponds to the addition of flipping events driven by random noise. The inclusion of noise at rate r has a profound influence on the model (Klemm *et al.*, 2003a), whose evolution becomes essentially independent of q. For small noise, a monocultural state is reached asymptotically because disordered configurations are unstable with respect to the perturbation introduced by the noise: the random variation of a trait unfreezes the boundary between two domains leading to the disappearance of one of

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them in favor of the other. However, when the noise rate is large, the disappearance of domains is compensated by the rapid formation of new ones so that the steady state is disordered. The critical rate separating the two behaviors is approximately $1/N$ so that large systems remain disordered.

Another natural modification of the original Axelrod model concerns the effect of media, represented by some external field or global coupling in the system. One possible way to implement an external field consists in defining a mass media cultural message as a set of fixed variables $M = (\mu_1, \mu_2, \ldots, \mu_F)$ (Gonzalez-Avella, 2005). With probability B, the selected individual interacts with the external field M exactly as if it were a neighbor. With probability $1\text{-}B$, the individual selects instead one of his actual neighbors. Rather interestingly, the external field turns out to favor the multicultural phase. The order–disorder transition point is shifted to smaller values of the control parameter $q_c(B)$. For B larger than a threshold such that $q_c(B^*) = 0$, only the disordered phase is present: A strong external field favors the alignment of some individuals with it, but it simultaneously induces a decoupling from individuals too far from it. Hence we arrive at the paradoxical conclusion that a message broadcasted globally can reduce conformity and increase diversity in the system.

Also for the Axelrod model, the effect of complex interaction topologies has been investigated, uncovering a rich and nontrivial phenomenology (Klemm *et al*., 2003b).

CONCLUSIONS

What lessons can be learned from the investigation of models of opinion dynamics? For binary choice models on regular patterns of interaction, such as lattices or the complete graph (all-to-all connections), most different microscopic mechanisms originate a similar macroscopic evolution: consensus is reached if the level of 'noise' is sufficiently low, whereas diversity persists if spontaneous changes are very frequent. In the case consensus is reached, which of the two fully homogeneous states is selected is dictated by the initial majority: if there are more 'yes' at the beginning, the final opinion will be 'yes' for all. Different rules hold for the voter dynamics, which is an interesting but rather singular exception.

Another conclusion that can be drawn is that spatial and topological effects are crucial. The behavior of the same model can drastically change depending on the type of interaction pattern among agents. Disordered topology may make consensus faster, slower, or even impossible. When opinions can assume many possible values, the new ingredient of homophily (or bounded confidence) plays a crucial role to determine whether consensus is reached or not.

A general thread, common to all these models, is that size matters. The time to reach consensus generally grows as the number of agents in a group increases and can become exceedingly large already for moderate group sizes. In these cases, even if an agreement is reached in principle, it is not in practice.

All the results presented earlier consider opinion dynamics occurring on a fixed interaction pattern. But in general, connections can (and actually do) change over temporal scales comparable with those of the variations of opinions. The co-evolution of opinions and dynamics further enriches the type of behaviors that can potentially occur. For example, incompatible opinions may cause the removal of connections between agents in different states and this can lead to a breakup of the global interaction pattern. The investigation of co-evolving topology and dynamics is still at the beginning but is likely to uncover many new and interesting phenomena (Gross and Blasius, 2007).

Finally, let us briefly come back to how these results should be interpreted and used in the social sciences. Let us stress that, although some of these models have been introduced and used in the physical sciences, the range of possible dynamical rules (types of interaction, connectivity patterns) that make sense for describing physical phenomena is quite limited. For example, physical phenomena for which a network substrate plays any realistic role are extremely rare (not to mention specific structures such as scale-free networks). Complex topologies are expected to be much more relevant for social phenomena than for physical ones. The same applies to most of the interaction rules considered in opinion dynamics. This of course does not mean that, say, the 'majority rule' model on a complex scale-free network has any significance for any real social system. Although it is tempting to derive, from the models considered here, far-reaching conclusions on the real world, most of them should essentially be intended as 'null' models. They are controlled testing grounds to learn what occurs if some generic ingredients are introduced: Is a particular mechanism relevant? Does it affect qualitatively the model behavior? Connections with real systems (and with empirical data) must be operated in successive steps, by a careful introduction of additional complications, leveraging on the knowledge acquired in previous steps. A discussion of these issues from the point of view of statistical physicists can be found in (Helbing, 2010).

NOTES

1. See Spirin *et al*. (2001) for complications that may arise due to striped patterns.

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