

# Linking Structure to the Dynamics of Collective Learning Through Diffusion Maps<sup>☆</sup>

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## Abstract

Collective know-how accumulates as capabilities diffuse from group to group, and from generation to generation. Lack of standard formalisms to study how this accumulation occurs precludes the development of a unified view of human development that is both insightful and predictive. We present one such formalism based on Diffusion Maps, which provide a low dimensional representation of the complex process of collective learning. We empirically find that collective learning depends on the amount of collective know-how, but also on the relative position in knowledge space with respect to other populations. The latter is important because it determines the available knowledge that can be transferred from one society to another. Hence, these low-dimensional spaces contain metrics that can be used to quantify how much societies know, and predict future paths of development. As a case study we consider data on trade, which provides consistent data on the technological diversification of hundreds of countries across more than 50 years. Our results are relevant for anthropologists, sociologists, and economists interested in the role of collective know-how as the main determinant of the success and welfare of a society.

*Keywords:* collective learning, economic complexity, cultural evolution, economic development

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## 1. Notes on literature

- Coifman and Lafon (2006)

P: random graphs have proven to be very efficient at finding relevant structures in complex geometries.

5 P: the field of application [of eigenvectors of markov chains] then goes beyond the ideas of clustering and ranking, as using multiple eigenvectors allows to speak of *parametrization* of data sets. A great deal of attention has been recently paid to the so-called “kernel eigenmap methods” such as local linear embedding [21], Laplacian eigenmaps  
10 [11], Hessian eigenmaps [18] and local tangent space alignment [25]. The remarkable idea emerging from these papers is that eigenvectors of Markov matrices can be thought of as coordinates on the data set. Therefore, the data, originally modeled as a graph, can be represented (embedded) as a cloud of points in a Euclidean space.

15 P: These algorithms exhibit two major advantages over classical dimensionality reduction methods (such as principal component analysis or classical multidimensional scaling): they are nonlinear, and they preserve local structures.

20 P: The approach that we present generalizes the classical Newtonian paradigm in which local infinitesimal rules of transition of a system lead to global macroscopic descriptions by integration.

M: Since development occurs very slowly [citation?], and the process of diversification/trait accumulation is a structured process [ProductSpace], we need a representation of the data that preserves the local structures  
25 and the local geometry.

- Lafon and Lee (2006)

## 2. Introduction

## 3. Background

## 30 4. Analytic Results

### *4.1. Elasticity for a single city*

## 5. Simulations

## 6. Discussion and Conclusions

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