

$$= \frac{1}{2\sqrt{\tan e^{x^2}}} \sec^2 e^x \cdot e^x \cdot 2x = \frac{xe^x \sec^2 e^x}{\sqrt{\tan e^x}}$$

$$5(h) \frac{d}{dx} \{ \sin^2 \log(\sec x) \} \quad [\text{সি. '১২}]$$

$$= 2 \sin \{ \log(\sec x) \} \cdot \cos \{ \log(\sec x) \} \times \frac{d}{dx} \{ \log(\sec x) \}$$

$$= \sin \{ 2 \log(\sec x) \} \times \frac{1}{\sec x \ln 10} \frac{d}{dx} (\sec x)$$

$$= \frac{\sin \{ 2 \log(\sec x) \}}{\sec x \ln 10} \sec x \cdot \tan x$$

$$= \frac{\sin \{ 2 \log(\sec x) \} \cdot \tan x}{\ln 10}$$

$$5(i) \frac{d}{dx} (\sin \sqrt{x}) \quad [\text{সি. '১২; কু. '১৩}]$$

$$= \cos \sqrt{x} \frac{d}{dx} (\sqrt{x})$$

$$= \cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}} = \frac{\cos \sqrt{x}}{2\sqrt{x}}$$

$$6(a) \text{ ধরি, } y = x^2 \sqrt{\frac{1+x}{1-x}} \quad [\text{রা. '০১}]$$

$$\therefore \ln y = 2 \ln x + \frac{1}{2} [\ln(1+x) - \ln(1-x)]$$

ইহাকে এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{y} \frac{dy}{dx} = 2 \cdot \frac{1}{x} + \frac{1}{2} \left[\frac{1}{1+x} - \frac{1}{1-x} (-1) \right]$$

$$\Rightarrow \frac{dy}{dx} = y \left[\frac{2}{x} + \frac{1}{2} \left\{ \frac{1-x+1+x}{(1+x)(1-x)} \right\} \right]$$

$$\Rightarrow \frac{dy}{dx} = x^2 \frac{\sqrt{1+x}}{\sqrt{1-x}} \left[\frac{2}{x} + \frac{1}{2} \left\{ \frac{1-x+1+x}{(1+x)(1-x)} \right\} \right]$$

$$= 2x \sqrt{\frac{1+x}{1-x}} + \frac{x^2}{\sqrt{(1+x)(1-x)^{3/2}}}$$

$$6(b) \sqrt{e^{\sqrt{x}}} \quad [\text{কু. '০৪; ঢা. '০৬, '০৯; য. '১৩}]$$

$$\frac{d}{dx} (\sqrt{e^{\sqrt{x}}}) = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \frac{d}{dx} (e^{\sqrt{x}})$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} e^{\sqrt{x}} \frac{d}{dx} (\sqrt{x})$$

$$= \frac{(e^{\sqrt{x}})^{1-\frac{1}{2}}}{2} \cdot \frac{1}{2\sqrt{x}} = \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}} \quad (\text{Ans.})$$

$$6(c) \frac{1}{\sqrt{x+1} + \sqrt{x+2}} \quad [\text{চ. '০০}]$$

$$= \frac{\sqrt{x+1} - \sqrt{x+2}}{(\sqrt{x+1} + \sqrt{x+2})(\sqrt{x+1} - \sqrt{x+2})}$$

$$= \frac{\sqrt{x+1} - \sqrt{x+2}}{x+1-x-2} = \sqrt{x+2} - \sqrt{x+1}$$

$$\therefore \frac{d}{dx} \left(\frac{1}{\sqrt{x+1} + \sqrt{x+2}} \right) = \frac{1}{2\sqrt{x+2}} - \frac{1}{2\sqrt{x+1}}$$

$$= -\frac{\sqrt{x+2} - \sqrt{x+1}}{2\sqrt{(x+2)(x+1)}} \quad (\text{Ans.})$$

$$6(d) \frac{d}{dx} \left\{ \frac{(x+1)^2 \sqrt{x-1}}{(x+4)^3 e^x} \right\} \quad [\text{কু. '০৯}]$$

$$= \frac{(x+1)^2 \sqrt{x-1}}{(x+4)^3 e^x} \left[\frac{1}{(x+1)^2} \frac{d}{dx} (x+1)^2 + \right.$$

$$\left. \frac{1}{\sqrt{x-1}} \frac{d}{dx} (\sqrt{x-1}) - \frac{1}{(x+4)^3} \frac{d}{dx} (x+4)^3 \right.$$

$$\left. - \frac{1}{e^x} \frac{d}{dx} (e^x) \right]$$

$$= \frac{(x+1)^2 \sqrt{x-1}}{(x+4)^3 e^x} \left[\frac{2(x+1)}{(x+1)^2} + \right.$$

$$\left. \frac{1}{\sqrt{x-1}} \frac{1}{2\sqrt{x-1}} - \frac{3(x+4)^2}{(x+4)^3} - \frac{1}{e^x} (e^x) \right]$$

$$= \frac{(x+1)^2 \sqrt{x-1}}{(x+4)^3 e^x} \left[\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1 \right]$$

$$7(a) \frac{\ln(\cos x)}{x} \quad [\text{ঢা. '০৬; সি. '০৭, '০৯, '১১; য. '১০}]$$

$$\frac{d}{dx} \left\{ \frac{\ln(\cos x)}{x} \right\}$$

$$\begin{aligned}
 &= \frac{x \frac{d}{dx} \{ \ln(\cos x) - \ln(\cos x) \frac{d}{dx} (x) \}}{x^2} \\
 &= \frac{x \frac{1}{\cos x} (-\sin x) - \ln(\cos x) \cdot 1}{x^2} \\
 &= \frac{\{x \tan x + \ln(\cos x)\}}{x^2}
 \end{aligned}$$

7(b) ধরি, $y = \frac{e^{-3x}(3x+5)}{7x-1}$ [স. '০৫]

$$\begin{aligned}
 \ln y &= \ln e^{-3x} + \ln(3x+5) - \ln(7x-1) \\
 &= -3x + \ln(3x+5) - \ln(7x-1)
 \end{aligned}$$

ইহাকে এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\begin{aligned}
 \frac{1}{y} \frac{dy}{dx} &= -3 + \frac{1}{3x+5} (3) - \frac{1}{7x-1} (7) \\
 &= \frac{-3(21x^2 + 32x - 5) + 21x - 3 - 21x - 35}{(3x+5)(7x-1)}
 \end{aligned}$$

$$\Rightarrow \frac{dy}{dx} = y \cdot \frac{-63x^2 - 96x + 15 - 38}{(3x+5)(7x-1)}$$

$$\begin{aligned}
 \Rightarrow \frac{dy}{dx} &= \frac{e^{-3x}(3x+5)}{7x-1} \cdot \frac{-(63x^2 + 96x + 23)}{(3x+5)(7x-1)} \\
 &= \frac{-(63x^2 + 96x + 23)e^{-3x}}{(7x-1)^2}
 \end{aligned}$$

7. (c) $\frac{x^4}{\ln x}$ [স. '০৪]

$$\begin{aligned}
 \frac{d}{dx} \left(\frac{x^4}{\ln x} \right) &= \frac{\ln x \frac{d}{dx} (x^4) - x^4 \frac{d}{dx} (\ln x)}{(\ln x)^2} \\
 &= \frac{\ln x (4x^3) - x^4 \frac{1}{x}}{(\ln x)^2} = \frac{x^3 (4 \ln x - 1)}{(\ln x)^2}
 \end{aligned}$$

8. (a) $\cos x^\circ$ [স. '০৪]

$$\cos x^\circ = \cos \frac{\pi x}{180}$$

$$\begin{aligned}
 \frac{d}{dx} (\cos x^\circ) &= -\sin \frac{\pi x}{180} \cdot \frac{d}{dx} \left(\frac{\pi x}{180} \right) \\
 &= -\sin x^\circ \cdot \frac{\pi}{180} = -\frac{\pi}{180} \sin x^\circ
 \end{aligned}$$

8(b) $e^{5x} \sin x^\circ$ [সি. '০২]

$$= e^{5x} \sin \frac{\pi x}{180}$$

$$\frac{d}{dx} (e^{5x} \sin \frac{\pi x}{180}) = e^{5x} \cdot \cos \frac{\pi x}{180}$$

$$\frac{d}{dx} \left(\frac{\pi x}{180} \right) + \sin \frac{\pi x}{180} \cdot e^{5x} \frac{d}{dx} (5x)$$

$$= e^{5x} \cdot \cos x^\circ \cdot \left(\frac{\pi}{180} \right) + \sin x^\circ \cdot e^{5x} \cdot 5$$

$$= e^{5x} \left(\frac{\pi}{180} \cos x^\circ + 5 \sin x^\circ \right)$$

8(c) $2x^\circ \cos 3x^\circ$ [স. '০৩; স. '০৫; স. '১০, '১৩; সি. '০৬, '০৮, '১১; ব., রা. '০৭, '১৪; সি. '০৯, '১১]

$$2x^\circ \cos 3x^\circ = 2 \frac{\pi x}{180} \cos \frac{3\pi x}{180}$$

$$\frac{d}{dx} (2x^\circ \cos 3x^\circ) = \frac{\pi}{90} [x (-\sin \frac{3\pi x}{180})$$

$$\frac{d}{dx} \left(\frac{3\pi x}{180} \right) + \cos \frac{3\pi x}{180} \frac{d}{dx} (x)]$$

$$= \frac{\pi}{90} [x (-\sin 3x^\circ) \cdot \left(\frac{3\pi}{180} \right) + \cos 3x^\circ \cdot 1]$$

$$= \frac{\pi}{90} (\cos 3x^\circ - \frac{\pi}{60} x \sin 3x^\circ)$$

প্রশ্নমালা IX F

1. (a) $\sqrt{\sin^{-1} x^5}$ [স. '০৪, '০৬]

$$\frac{d}{dx} (\sqrt{\sin^{-1} x^5}) = \frac{1}{2\sqrt{\sin^{-1} x^5}} \frac{d}{dx} (\sin^{-1} x^5)$$

$$= \frac{1}{2\sqrt{\sin^{-1} x^5}} \frac{1}{\sqrt{1-(x^5)^2}} \frac{d}{dx} (x^5)$$

$$= \frac{1}{2\sqrt{\sin^{-1} x^5} \sqrt{1-x^{10}}} (5x^4)$$

$$= \frac{5x^4}{2\sqrt{\sin^{-1} x^5} \sqrt{1-x^{10}}}$$

1.(b) $\tan^{-1}(\sin e^x)$ [স. '০৫; ব. '০৫; স. '০৯]

$$\frac{d}{dx} \{ \tan^{-1}(\sin e^x) \} = \frac{d\{ \tan^{-1}(\sin e^x) \}}{d(\sin e^x)}$$