নিচের যোগজগুলির মান নির্ণয় কর ঃ

1.(a) $\int \frac{1}{x}(x+\frac{1}{x})\,dx$ [ঢা.'০৫]

$= \int (1+x^{-2})\,dx = x + \frac{x^{-2+1}}{-2+1} + c$

$= x - \frac{1}{x} + c$

1(b) $\int \frac{(e^x+1)^2}{\sqrt{e^x}}dx$ [ঢা.'০২]

$= \int \frac{e^{2x}+2e^x+1}{e^{\frac{x}{2}}}dx$

$= \int (e^{2x-\frac{x}{2}} + 2e^{x-\frac{x}{2}} + e^{-\frac{x}{2}})dx$

$= \int (e^{\frac{3x}{2}} + 2e^{\frac{x}{2}} + e^{-\frac{x}{2}})dx$

$= \frac{e^{\frac{3x}{2}}}{\frac{3}{2}} + 2\frac{e^{\frac{x}{2}}}{\frac{1}{2}} + \frac{e^{-\frac{x}{2}}}{-\frac{1}{2}} + c$

$= \frac{2}{3}e^{\frac{3x}{2}} + 4e^{\frac{x}{2}} - 2e^{-\frac{x}{2}} + c$

1(c) $\int (1+x^{-1}+x^{-2})dx$ [রা.'০৯]

$= \int (1+\frac{1}{x}+x^{-2})dx$

$= x + \ln x + \frac{x^{-2+1}}{-2+1} + c = x + \ln x - x^{-1} + c$

নিয়ম ঃ হরের অনুবন্ধি রাশি দ্বারা লব ও হরকে গুণ করে হরকে $\sqrt{\ }$ মুক্ত করতে হয়।

2.(a) $\int \frac{1}{\sqrt{x}-\sqrt{x-1}}dx$

$= \int \frac{\sqrt{x}+\sqrt{x-1}}{(\sqrt{x}-\sqrt{x-1})(\sqrt{x}+\sqrt{x-1})}dx$

$= \int \frac{\sqrt{x}+\sqrt{x-1}}{x-(x-1)}dx = \int \frac{\sqrt{x}+\sqrt{x-1}}{x-x+1}dx$

$= \int \{x^{\frac{1}{2}} + (x-1)^{\frac{1}{2}}\}dx$

$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(x-1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= \frac{2}{3}[x^{3/2} + (x-1)^{3/2}] + c$

2(b) $\int \frac{dx}{\sqrt{x+1}+\sqrt{x-1}}$ [রা.'০২; দি.'১০]

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{(\sqrt{x+1}+\sqrt{x-1})(\sqrt{x+1}-\sqrt{x-1})}dx$

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{(x+1)-(x-1)}dx$

$= \frac{1}{2}\int [(x+1)^{1/2} - (x-1)^{1/2}]\,dx$

$= \frac{1}{2}[\frac{(x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(x-1)^{\frac{1}{2}+1}}{\frac{1}{2}+1}] + c$

$= \frac{1}{2}[\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}}] + c$

$= \frac{1}{3}[(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}] + c$ (Ans.)

3.(a) $\int \frac{dx}{1-\sin x}$ [ঢা.'০৭]

$= \int \frac{(1+\sin x)dx}{(1-\sin x)(1+\sin x)}$

$= \int \frac{(1+\sin x)dx}{1-\sin^2 x} = \int \frac{(1+\sin x)dx}{\cos^2 x}$

$= \int (\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x})dx$

$= \int (\sec^2 x + \sec x \tan x)dx$

$= \tan x + \sec x + c$

3(b) $\int \frac{dx}{1+\sin x}$ [য.'০৭,'১৩; চ.'১০ প্র.ভ.প.'০৩]

$= \int \frac{(1-\sin x)dx}{(1+\sin x)(1-\sin x)}$

$= \int \frac{(1-\sin x)dx}{1-\sin^2 x} = \int \frac{(1-\sin x)dx}{\cos^2 x}$

$= \int (\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x})dx$

$= \int (\sec^2 x - \sec x \tan x)dx$

$= \tan x - \sec x + c$

3. (c) $\int \frac{dx}{1+\cos 2x}$ [কু.'০৮]

$= \int \frac{dx}{2\cos^2 x} = \frac{1}{2}\int \sec^2 x dx = \frac{1}{2}\tan x + c$

3(d) $\int \sqrt{1+\cos x}\, dx$ [প্র.ভ.প.'০৪]

$= \int \sqrt{2\cos^2 \frac{x}{2}}\, dx = \int \sqrt{2}\cos\frac{x}{2}\, dx$

$= 2\sqrt{2}\int \cos\frac{x}{2}\, d(\frac{x}{2})$ [$d(\frac{x}{2}) = \frac{1}{2}dx$]

$= 2\sqrt{2}\sin\frac{x}{2} + c$

3(e) $\int \sqrt{1-\cos 2x}\, dx$ [চ.'০৫,'০৯;সি.'০৬; ব.'০৮]

$= \int \sqrt{2\sin^2 x}\, dx = \int \sqrt{2}\sin x\, dx$

$= \sqrt{2}(-\cos x) + c = -\sqrt{2}\cos x + c$

3(f) $\int \sqrt{1-\cos 4x}\, dx$ [চ.'০৭]

$= \int \sqrt{2\sin^2 2x}\, dx = \int \sqrt{2}\sin 2x\, dx$

$= \sqrt{2}(-\frac{\cos 2x}{2}) + c = -\frac{1}{\sqrt{2}}\cos 2x + c$

3(g) $\int \sec x(\sec x - \tan x)dx$ [ব.'১৩]

$= \int (\sec^2 x - \sec x \tan x)dx$

$= \tan x - \sec x + c$

4.(a) $\int \sqrt{1-\sin 2x}\, dx$

$= \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x\cos x}\, dx$

$= \int \sqrt{(\sin x - \cos x)^2}\, dx$

$= \int (\sin x - \cos x)\, dx$ বা $\int (\cos x - \sin x)\, dx$

$= -\cos x - \sin x + c$ বা $\sin x + \cos x + c$

4.(b) $\int \frac{\cos 2x}{\sqrt{1-\sin 2x}} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{\sqrt{\sin^2 x + \cos^2 x - 2\sin x\cos x}} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{\sqrt{(\sin x - \cos x)^2}} dx$

$= \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x} dx$

বা, $\int \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin x - \cos x} dx$

$= \int (\cos x + \sin x)dx$ বা, $-\int (\cos x + \sin x)dx$

$= \sin x - \cos x$ বা, $-(\sin x - \cos x)$

4(c) $\int (\sin x + \cos x)^2 dx$ [প্র.ভ.প. '৯০]

$= \int (\sin^2 x + \cos^2 x + 2\sin x\cos x)dx$

$= \int (1 + \sin 2x)dx = x - \frac{1}{2}\cos 2x + c$

5(a) $\int \sin 5x \sin 3x\, dx$ [রা.'০৮,'১২;ব.'১০; চ.'১২]

$= \int \frac{1}{2}\{\cos(5x-3x) - \cos(5x+3x)\}\, dx$

$= \frac{1}{2}\int (\cos 2x - \cos 8x)\, dx$

$= \frac{1}{2}(\frac{\sin 2x}{2} - \frac{\sin 8x}{8}) + c$

$= \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + c$

5(b) $\int \sin 4x \sin 2x \, dx$ [য.'০৪; রা.'০৫; দি.'১১]

$$= \int \frac{1}{2}\{\cos(4x-2x) - \cos(4x+2x)\}\, dx$$

$$= \frac{1}{2}\int (\cos 2x - \cos 6x)\, dx$$

$$= \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 6x}{6}\right) + c$$

$$= \frac{1}{4}\sin 2x - \frac{1}{12}\sin 6x + c$$

5(c) $\int 3\sin 3x \cos 4x \, dx$ [সি'০২,'০৩; ব.'০৬,'১০]

$$= \int \frac{3}{2}\{\sin(4x+3x) - \sin(4x-3x)\}\, dx$$

$$= \frac{3}{2}\int (\sin 7x - \sin x)\, dx$$

$$= \frac{3}{2}\left(-\frac{1}{7}\cos 7x + \cos x\right) + c$$

$$= \frac{3}{14}(7\cos x - \cos 7x) + c$$

5.(d) $\int \sin 3x \cos 5x \, dx$ [কু.'০৬; সি.,দি.'১২]

$$= \int \frac{1}{2}\{\sin(5x+3x) - \sin(5x-3x)\}\, dx$$

$$= \int \frac{1}{2}(\sin 8x - \sin 2x)dx$$

$$= \frac{1}{2}\left(-\frac{1}{8}\cos 8x + \frac{1}{2}\cos 2x\right) + c$$

$$= \frac{1}{16}(4\cos 2x - \cos 8x) + c$$

5(e) $\int 4\cos 4x \sin 5x dx$ [রা.'০৩]

$$= \int 2\{\sin(5x+4x) + \sin(5x-4x)\}\, dx$$

$$= \int 2(\sin 9x + \sin x)dx$$

$$= 2\left(-\frac{1}{9}\cos 9x - \cos x\right) + c$$

$$= -\frac{2}{9}(\cos 9x + 9\cos x) + c$$

5(f) $\int 5\cos 5x \sin 4x \, dx$ [ঢা.'০৬; দি.,সি.'১৪]

$$= \int \frac{5}{2}\{\sin(5x+4x) - \sin(5x-4x)\}\, dx$$

$$= \int \frac{5}{2}(\sin 9x - \sin x)\, dx$$

$$= \frac{5}{2}\left(-\frac{1}{9}\cos 9x + \cos x\right) + c$$

$$= \frac{5}{18}(9\cos x - \cos 9x) + c$$

5(g) $\int \sin px \cos qx \, dx, \ (p > q)$ [ঢা.'০৩; সি.'০৭]

$$= \int \frac{1}{2}\{\sin(p+q)x + \sin(p-q)x\}\, dx$$

$$= \frac{1}{2}\left\{-\frac{\cos(p+q)x}{p+q} - \frac{\cos(p-q)x}{p-q}\right\} + c$$

$$= -\frac{1}{2}\left\{\frac{\cos(p+q)x}{p+q} + \frac{\cos(p-q)x}{p-q}\right\} + c$$

6.(a) $\int \cos^2 x \, dx$ [চ.'০৮]

$$= \int \frac{1}{2}(1+\cos 2x)\, dx = \frac{1}{2}\left\{\int dx + \int \cos 2x \, dx\right\}$$

$$= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$$

6(b) $\int \cos^2 2x dx$ [ঢা.'০০]

$$= \int \frac{1}{2}(1+\cos 4x)dx = \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right) + c$$

6(c) $\int (2\cos x + \sin x)\cos x \, dx$ [ঢা '০৫]

$$= \int (2\cos^2 x + \sin x \cos x)\, dx$$

$$= \int \left(1 + \cos 2x + \frac{1}{2}\sin 2x\right)\, dx$$

$$= x + \frac{1}{2}\sin 2x + \frac{1}{2}\cdot\left(-\frac{1}{2}\cos 2x\right) + c$$

$$= x + \frac{1}{2}\sin 2x - \frac{1}{4}\cos 2x + c$$

6(d) $\int \sin^3 2x\, dx$ [ঢা.'০১]

$= \int \frac{1}{4}(3\sin 2x - \sin 6x)\, dx$

$= \frac{1}{4}\{3.(-\frac{1}{2}\cos 2x) + \frac{1}{6}\cos 6x\} + c$

$= \frac{1}{8}(-3\cos 2x + \frac{1}{3}\cos 6x) + c$

6.(e) $\int \sin^4 x\, dx$ [কু.'০৯]

$\sin^4 x\, dx = (\sin^2 x)^2 = \{\frac{1}{2}(1-\cos 2x)\}^2$

$= \frac{1}{4}\{1 - 2\cos x + \cos^2 2x\}$

$= \frac{1}{4}\{1 - 2\cos 2x + \frac{1}{2}(1+\cos 4x)\}$

$= \frac{1}{4}[1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x]$

$= \frac{1}{4}[\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x]$

$\int \sin^4 x\, dx$

$= \frac{1}{4}(\frac{3}{2}x - 2.\frac{1}{2}\sin 2x + \frac{1}{2}.\frac{1}{4}\sin 4x) + c$

$= \frac{1}{4}(\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x) + c$

6(f) $\int \cos^4 x\, dx$ [রা.'০৭,'১৪; সি.'০৮; দি.'১৩; য.'১৪]

$\cos^4 x\, dx = (\cos^2 x)^2 = \{\frac{1}{2}(1+\cos 2x)\}^2$

$= \frac{1}{4}\{1 + 2\cos 2x + \cos^2 2x\}$

$= \frac{1}{4}\{1 + 2\cos 2x + \frac{1}{2}(1+\cos 4x)\}$

$= \frac{1}{4}[1 + 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x]$

$= \frac{1}{4}[\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x]$

$\int \cos^4 x\, dx$

$= \int \frac{1}{4}(\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x)\, dx$

$= \frac{1}{4}(\frac{3}{2}x + 2.\frac{1}{2}\sin 2x + \frac{1}{2}.\frac{1}{4}\sin 4x) + c$

$= \frac{1}{4}(\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x) + c$ (Ans.)

অতিরিক্ত প্রশ্ন (সমাধানসহ)

নিচের যোগজগুলি মান নির্ণয় কর ঃ

1(a) $\int \frac{4(\sqrt[3]{x^2}+4)^2}{3\sqrt[3]{x}}dx = \frac{4}{3}\int \frac{(x^{\frac{2}{3}}+4)^2}{x^{\frac{1}{3}}}dx$

$= \frac{4}{3}\int \frac{x^{\frac{4}{3}} + 8x^{\frac{2}{3}} + 16}{x^{\frac{1}{3}}}dx$

$= \frac{4}{3}\int (x^{\frac{4}{3}-\frac{1}{3}} + 8x^{\frac{2}{3}-\frac{1}{3}} + 16x^{-\frac{1}{3}})dx$

$= \frac{4}{3}\int (x + 8x^{\frac{1}{3}} + 16x^{-\frac{1}{3}})dx$

$= \frac{4}{3}(\frac{x^2}{2} + 8\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + 16\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}) + c$

$= \frac{4}{3}(\frac{x^2}{2} + 8\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 16\frac{x^{\frac{2}{3}}}{\frac{2}{3}}) + c$

$= \frac{2}{3}(x^2 + 12x^{4/3} + 48x^{2/3}) + c$

1(b) $\int \frac{a\cot x + b\tan^2 x - c\sin^2 x}{\sin x}dx$

$= \int (a\frac{\cot x}{\sin x} + b\frac{\sin^2 x}{\cos^2 x \sin x} - c\sin x)dx$

$= \int (a\cot x\,\mathrm{cosec}\,x + b\tan x \sec x$

$- c\sin x)dx$

$= -a\,\mathrm{cosec}\,x + b\sec x + c\cos x + c_1$