$= e^x \sin e^x + \cos e^x + c$

(d) ধরি, $I = \int \sin\sqrt{x}\, dx$ এবং $\sqrt{x} = z$

তাহলে $\frac{1}{2\sqrt{x}} dx = dz \Rightarrow dx = 2z\, dz$ এবং

$I = \int 2z \sin z\, dz$

$= 2[\, z\int \sin z\, dz - \int \{\frac{d}{dz}(z)\int \sin z\, dz\}\, dz\,]$

$= 2[\, z(-\cos z) - \int 1.(-\cos z)dz\,]$

$= -2z\cos z + 2\sin z + c$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + c$

3 (a) $\int x\sin^2\frac{x}{2}dx$ [য.বো.'০২]

$= \int x\frac{1}{2}(1-\cos x)dx$

$= \frac{1}{2}\int xdx - \frac{1}{2}\int x\cos x\, dx$

$= \frac{1}{2}.\frac{x^2}{2} - \frac{1}{2}[x\int \cos x\, dx - \int\{\frac{d}{dx}(x)\int \cos xdx\}dx]$

$= \frac{x^2}{4} - \frac{1}{2}[x\sin x - \int 1.\sin x\, dx]$

$= \frac{x^2}{4} - \frac{1}{2}[x\sin x - (-\cos x)] + c$

$= \frac{x^2}{4} - \frac{1}{2}x\sin x - \frac{1}{2}\cos x + c$

(b) $\int x^2\cos^2\frac{x}{2}dx = \int x^2\frac{1}{2}(1+\cos x)dx$

$= \frac{1}{2}[\int x^2 dx + \int x^2\cos xdx]$

$= \frac{1}{2}[\frac{x^3}{3} + x^2(\sin x) - (2x)(-\cos x) + (2)(-\sin x)] + c$

$= \frac{1}{2}[\frac{x^3}{3} + x^2\sin x + 2x\cos x - 2\sin x] + c$

(c) $\int x\cos 2x\cos 3xdx$

$= \int x\frac{1}{2}(\cos 5x - \cos x)dx$

$= \frac{1}{2}[x\int \cos 5xdx - \int\{\frac{d}{dx}(x)\int \cos 5xdx\}dx + x\int \cos xdx - \int\{\frac{d}{dx}(x)\int \cos xdx\}dx]$

$= \frac{1}{2}[x(\frac{\sin 5x}{5}) - \int 1.(\frac{\sin 5x}{5})dx + x\sin x - \int 1.\sin xdx]$

$= \frac{1}{2}[\frac{1}{5}x\sin 5x + \frac{\cos 5x}{25} + x\sin x + \cos x] + c$

4. (a) $\int x\sec^2 xdx$ [ঢা.'০১,'১৪]

$= x\int \sec^2 xdx - \int\{\frac{d}{dx}(x)\int \sec^2 xdx\}dx$

$= x\tan x - \int 1.\tan xdx$

$= x\tan x + \ln|\cos x| + c$

4.(b) $\int x\sec^2 3x\, dx$ [ঢা.'০১]

$= x\int \sec^2 3xdx - \int\{\frac{d}{dx}(x)\int \sec^2 3xdx\}dx$

$= x\frac{\tan 3x}{3} - \int 1.\frac{\tan 3x}{3}dx$

$= \frac{x}{3}\tan 3x + \frac{1}{9}\ln|\cos 3x| + c$

(c) $\int x\tan^2 xdx$ [রা.'০৫; সি.'০৫]

$= \int x(\sec^2 x - 1)dx = \int x\sec^2 xdx - \int xdx$

$= x\int \sec^2 xdx - \int\{\frac{d}{dx}(x)\int \sec^2 xdx\}dx - \frac{x^2}{2}$

$= = x\tan x - \int 1.\tan xdx - \frac{x^2}{2}$

$= x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$

(d) ধরি, $I = \int \cos ec^3 x\, dx$

$= \int \cos ec^2 x\cos ecx\, dx$

$= \cos ecx\int \cos ec^2 x\, dx - \int\{\frac{d}{dx}(\cos ecx)\int \cos ec^2 xdx\}dx$

$= -\operatorname{cosec} x \cot x - \int (-\operatorname{cosec} x \cot x).(-\cot x)dx =$

$-\operatorname{cosec} x \cot x - \int \operatorname{cosec} x(\operatorname{cosec}^2 x - 1)dx$

$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec} x dx$

$\Rightarrow I = -\operatorname{cosec} x \cot x - I + \ln|\tan\frac{x}{2}| + c_1$

$\Rightarrow 2I = -\operatorname{cosec} x \cot x + \ln|\tan\frac{x}{2}| + c_1$

$\Rightarrow I = -\frac{1}{2}\operatorname{cosec} x \cot x + \frac{1}{2}\ln|\tan\frac{\pi}{2}| + \frac{1}{2}c_1$

$\Rightarrow I = -\frac{1}{2}\operatorname{cosec} x \cot x + \frac{1}{2}\ln|\tan\frac{\pi}{2}| + c$

5. সূত্র (MCQ এর জন্য):

$\int x^n \ln x dx = \frac{x^{n+1}}{n+1}(\ln x - \frac{1}{n+1})$

(a) $\int x \ln x dx$ [য.'০৩; ঢা.'০৬; ব.'০৮]

$= \ln x \int x dx - \int \{\frac{d}{dx}(\ln x)\int x dx\}dx$

$= \ln x.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}.\frac{x^2}{2} + c = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$

(b) $\int x^n \ln x dx$ [প্র.ভ.প.'১৩]

$= \ln x \int x^n dx - \int \{\frac{d}{dx}(\ln x)\int x^n dx\}dx$

$= \ln x.\frac{x^{n+1}}{n+1} - \int \frac{1}{x}.\frac{x^{n+1}}{n+1}dx$

$= \frac{x^{n+1}}{n+1}\ln x - \frac{1}{n+1}\int x^n dx$

$= \frac{x^{n+1}}{n+1}\ln x - \frac{1}{n+1}.\frac{x^{n+1}}{n+1} + c$

$= \frac{x^{n+1}}{n+1}\ln x - \frac{x^{n+1}}{(n+1)^2} + c$

(c) $\int x^2(\ln x)^2 dx$ [প্র.ভ.প.'০৫]

$= (\ln x)^2 \int x^2 dx - \int \{\frac{d}{dx}(\ln x)^2 \int x^2 dx\}dx$

$= (\ln x)^2 \frac{x^3}{3} - \int 2\ln x.\frac{1}{x}.\frac{x^3}{3}dx$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}\int x^2 \ln x dx$

$= \frac{x^3}{3}(\ln x)^2 -$

$\frac{2}{3}[\ln x \int x^2 dx - \int \{\frac{d}{dx}(\ln x)\int x^2 dx\}dx]$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\ln x.\frac{x^3}{3} - \int \frac{1}{x}.\frac{x^3}{3}dx]$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx]$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\frac{x^3}{3}\ln x - \frac{1}{3}.\frac{x^3}{3}] + c$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}[\frac{x^3}{3}\ln x - \frac{x^3}{9}] + c$

$= \frac{x^3}{27}[9(\ln x)^2 - 6\ln x + 2] + c$

(d) $\int (\ln x)^2 dx$ [য.'০৫; চ.'০৭; প্র.ভ.প. '১০]

$= (\ln x)^2 \int dx - \int \{\frac{d}{dx}(\ln x)^2 \int dx\}dx$

$= (\ln x)^2.x - \int 2\ln x.\frac{1}{x}.x dx$

$= x(\ln x)^2 - 2\int \ln x dx$

$= x(\ln x)^2 - 2[\ln x \int dx - \int \{\frac{d}{dx}(\ln x)\int dx\}dx] =$

$x(\ln x)^2 - 2[\ln x.x - \int \frac{1}{x}.x dx]$

$= x(\ln x)^2 - 2[x\ln x - \int dx]$

$= x(\ln x)^2 - 2[x\ln x - x] + c$

$= x\{(\ln x)^2 - 2\ln x + 2\} + c$

(e) ধরি, $I = \int \frac{\ln(\ln x)dx}{x}$ এবং $\ln x = z$.

তাহলে $\frac{1}{x}dx = dz$ এবং $I = \int \ln z\, dz$

$\Rightarrow I = \ln z\int dz - \int\{\frac{d}{dz}(\ln z)\int dz\}dz$

$= \ln z . z - \int\frac{1}{z}.zdz = z\ln z - \int dz$

$= z\ln z - z + c = \ln x\{\ln(\ln x) - 1\} + c$

(f) ধরি, $I = \int\frac{\ln\sec^{-1}x}{x\sqrt{x^2-1}}dx$ [ঢা.'০৮; সি.'১৪]

এবং $\sec^{-1}x = z \Rightarrow \frac{dx}{x\sqrt{x^2-1}} = dz$

$\therefore I = \int\ln z\, dz$

$= \ln z\int dz - \int\{\frac{d}{dz}(\ln z)\int dz\}dz$

$= \ln z . z - \int\frac{1}{z}.zdz = z\ln z - \int dz$

$= z\ln z - z + c$

$= \{\ln(\sec^{-1}x) - 1\}\sec^{-1}x + c$

6.(a) $\int\tan^{-1}xdx$ [কু.'০২; ঢা.'০৪; ব.'১০]

$= \tan^{-1}x\int dx - \int\{\frac{d}{dx}(\tan^{-1}x)\int dx\}dx$

$= x\tan^{-1}x - \int\frac{x}{1+x^2}dx$

$= x\tan^{-1}x - \frac{1}{2}\int\frac{(0+2x)dx}{1+x^2}$

$= x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + c$

(b) $\int x\sin^{-1}xdx$ [ঢা.'০৭]

$= \sin^{-1}x\int xdx - \int\{\frac{d}{dx}(\sin^{-1}x)\int xdx\}dx$

$= \sin^{-1}x.\frac{x^2}{2} - \int\frac{1}{\sqrt{1-x^2}}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$

$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}[\int\sqrt{1-x^2}dx - \int\frac{1}{\sqrt{1-x^2}}dx]$

$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x - \sin^{-1}x] + c$

$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}[\frac{x\sqrt{1-x^2}}{2} - \frac{1}{2}\sin^{-1}x] + c$

(c) $\int\sin^{-1}x\,dx$ [সি.'০৩; য.'১০; ঢা.'১৪]

$= \sin^{-1}x\int dx - \int\{\frac{d}{dx}(\sin^{-1}x)\int dx\}dx$

$= x\sin^{-1}x - \int\frac{x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1}x - (-\frac{1}{2})\int\frac{(0-2x)dx}{\sqrt{1-x^2}}$

$= x\sin^{-1}x + \frac{1}{2}.2\sqrt{1-x^2} + c$

$= x\sin^{-1}x + \sqrt{1-x^2} + c$

(d) $\int\cos^{-1}xdx$ [কু. '০৫,'১৪; চ.'০৬; য.'০৮; রা.'১০]

$= \cos^{-1}x\int dx - \int\{\frac{d}{dx}(\cos^{-1}x)\int dx\}dx$

$= x\cos^{-1}x + \int\frac{x}{\sqrt{1-x^2}}dx$

$= x\cos^{-1}x + (-\frac{1}{2})\int\frac{(0-2x)dx}{\sqrt{1-x^2}}$

$= x\cos^{-1}x - \frac{1}{2}.2\sqrt{1-x^2} + c$

$= x\cos^{-1}x - \sqrt{1-x^2} + c$

(e) $\int x\sin^{-1}x^2dx$

[ঢা.'০৫; রা.'০৬; প্র.ভ.প. '০৪,'০৬]

$= \sin^{-1}x^2\int xdx - \int\{\frac{d}{dx}(\sin^{-1}x^2)\int xdx\}dx$

$= \sin^{-1}x^2.\frac{x^2}{2} - \int\frac{2x}{\sqrt{1-x^4}}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\sin^{-1}x^2 - \int\frac{x^3}{\sqrt{1-x^4}}dx$

$= \frac{x^2}{2}\sin^{-1}x^2 - (-\frac{1}{4})\int\frac{d(1-x^4)}{\sqrt{1-x^4}}$

$$= \frac{x^2}{2}\sin^{-1}x^2 + \frac{1}{4}.2\sqrt{1-x^4} + c$$

$$= \frac{x^2}{2}\sin^{-1}x^2 + \frac{1}{2}\sqrt{1-x^4} + c$$

6.(f) $\int x\tan^{-1}x dx$

[য.'০৬;সি. '০৪,'০৮; রা.'০৬ ; কু.'১০ ; ব.'১১]

$$= \tan^{-1}x\int x\,dx - \int\{\frac{d}{dx}(\tan^{-1}x)\int x\,dx\}\,dx$$

$$= \tan^{-1}x.\frac{x^2}{2} - \int\frac{1}{1+x^2}.\frac{x^2}{2}\,dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\frac{1+x^2-1}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int(1-\frac{1}{1+x^2})\,dx$$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}(x-\tan^{-1}x) + c$$

$$= \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + c \text{ (Ans.)}$$

7.(a) $\int e^x\cos x\,dx$ [ঢা.'০২;প্র.ভ.প.'০৪,'০৬]

ধরি, $I = \int e^x\cos x\,dx$

$$= e^x\int\cos x\,dx - \int\{\frac{d}{dx}(e^x)\int\cos x\,dx\}\,dx$$

$$= e^x\sin x - \int e^x\sin dx$$

$$= e^x\sin x - e^x\int\sin x\,dx + \int\{\frac{d}{dx}(e^x)\int\sin x\,dx\}\,dx$$

$$= e^x\sin x - e^x(-\cos x) + \int e^x(-\cos x)\,dx$$

$$= e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$$

$$= e^x\sin x + e^x\cos x - I + c_1$$

$$\Rightarrow 2I = e^x\sin x + e^x\cos x + c_1$$

$$\Rightarrow I = \frac{1}{2}e^x(\sin x + \cos x) + \frac{1}{2}c_1$$

$$\int e^x\cos x\,dx = \frac{1}{2}e^x(\sin x + \cos x) + c$$

7(b) $\int e^x\sin x dx$ [কু.'০৮,'১৩; মা.'০৯; রা.,দি.'১৪]

ধরি, $I = \int e^x\sin x dx$

$$= e^x\int\sin x\,dx - \int\{\frac{d}{dx}(e^x)\int\sin x\,dx\}\,dx$$

$$= e^x(-\cos x) - \int e^x(-\cos x)\,dx$$

$$= -e^x\cos x + e^x\int\cos x\,dx - \int\{\frac{d}{dx}(e^x)\int\cos x\,dx\}\,dx$$

$$= -e^x\cos x + e^x\sin x - \int e^x\sin x dx$$

$$= e^x(\sin x - \cos x) - I + c_1$$

$$\Rightarrow 2I = e^x(\sin x - \cos x) + c_1$$

$$\Rightarrow I = \frac{1}{2}e^x(\sin x - \cos x) + \frac{1}{2}c_1$$

$$\int e^x\sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + c$$

7(c) $\int e^{2x}\sin x dx$ [সি.'০২]

ধরি, $I = \int e^{2x}\sin x dx$

$$= e^{2x}\int\sin x\,dx - \int\{\frac{d}{dx}(e^{2x})\int\sin x\,dx\}\,dx$$

$$= e^{2x}(-\cos x) - \int 2e^{2x}(-\cos x)\,dx$$

$$= -e^{2x}\cos x + 2e^{2x}\int\cos x\,dx - 2\int\{\frac{d}{dx}(e^{2x})\int\cos x\,dx\}\,dx$$

$$= -e^{2x}\cos x + 2e^{2x}\sin x - 2\int 2e^{2x}\sin x dx$$

$$= -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x dx$$

$$= e^{2x}(2\sin x - \cos x) - 4I + c_1$$

$$\Rightarrow 5I = e^{2x}(2\sin x - \cos x) + c_1$$

$$\Rightarrow I = \frac{e^{2x}}{5}(2\sin x - \cos x) + \frac{1}{5}c_1$$

$$\therefore I = \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(2\sin x - \cos x) + c$$

7(d) $\int e^{2x}\cos^2 x\,dx = \int e^{2x}\frac{1}{2}(1+\cos 2x)\,dx$

$$= \frac{1}{2}[\int e^{2x}dx + \int e^{2x}\cos 2x\,dx]$$

$$=\frac{1}{2}[\frac{1}{2}e^{2x}+\frac{e^{2x}}{2^2+2^2}(2\cos 2x+2\sin 2x)]+c$$

$$=\frac{1}{2}[\frac{1}{2}e^{2x}+\frac{e^{2x}}{8}(2\cos 2x+2\sin 2x)]+c$$

$$=\frac{1}{8}(2+\cos 2x+\sin 2x)e^{2x}+c$$

8.(a) $\int e^x(\sin x+\cos x)dx$

[সি.'০৫,'১১; ঢা.'১০; কু.'১১]

$$=\int e^x\sin xdx+\int e^x\cos xdx$$

$$=\int e^x\sin xdx+e^x\int\cos xdx-\int\{\frac{d}{dx}(e^x)\int\cos xdx\}dx$$

$$=\int e^x\sin xdx+e^x\sin x-\int e^x\sin xdx$$

$$=e^x\sin x+c$$

বিকল্প পদ্ধতি ঃ

ধরি, $f(x)=\sin x$. $\quad f'(x)=\cos x$ এবং

$$\int e^x(\sin x+\cos x)dx=\int e^x\{f(x)+f'(x)\}dx$$

$$=e^xf(x)+c=e^x\sin x+c$$

8(b) ধরি, $I=\int e^x\sec x(1+\tan x)dx$

[রা.'০৩; য.'১১;চ.'১৩; প্র.ভ.প.'০৪]

এবং $f(x)=\sec x$. $\quad f'(x)=\sec x\tan x$ এবং

$$I=\int e^x(\sec x+\sec x\tan x)dx$$

$$=\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+c$$

$$\int e^x\sec x(1+\tan x)dx=e^x\sec x+c$$

8.(c) ধরি, $I=\int e^x(\tan^{-1}x+\frac{1}{1+x^2})$ এবং

$f(x)=\tan^{-1}x \qquad f'(x)=\frac{1}{1+x^2}$ এবং

$$I=\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+c$$

$$\int e^x(\tan^{-1}x+\frac{1}{1+x^2})=e^x\tan^{-1}x+c$$

8(d) $\int e^x\{\tan x-\ln(\cos x)\}dx$ [প্র.ভ.প.'৯২]

ধরি, $I=\int e^x\{\tan x-\ln(\cos x)\}dx$ এবং [সূত্র প্রয়োগ করে।]

$f(x)=-\ln(\cos x)$

$\therefore f'(x)=-\frac{-\sin x}{\cos x}=\tan x$ এবং

$$I=\int e^x\{-\ln(\cos x)+\tan x\}dx$$

$$=\int e^x\{f(x)+f'(x)\}dx=e^xf(x)+c$$

$$\therefore\int e^x\{\tan x+\ln(\sec x)\}dx=-e^x\ln(\cos x)+c$$

9.(a) $\int\frac{e^x}{x}(1+x\ln x)dx$ [ব. '০১; য.'০৭; দি.'১৩]

ধরি, $I=\int\frac{e^x}{x}(1+x\ln x)dx=\int e^x(\frac{1}{x}+\ln x)dx$

এবং $f(x)=\ln x$. তাহলে $f'(x)=\frac{1}{x}$ এবং

$$I=\int e^x(\ln x+\frac{1}{x})dx=\int e^x\{f(x)+f'(x)\}dx$$

$$=e^xf(x)+c=e^x\ln x+c$$

$$\int\frac{e^x}{x}(1+x\ln x)dx=e^x\ln x+c$$

9(b) $\int e^{-2x}(\frac{1}{x}-2\ln x)dx$ [কু..'০২]

$$=\int e^{-2x}\frac{1}{x}dx-2\int e^{-2x}\ln x\,dx$$

$$=e^{-2x}\int\frac{1}{x}dx-\int\{\frac{d}{dx}(e^{-2x})\int\frac{1}{x}dx\}dx-2\int e^{-2x}\ln x\,dx$$

$$=e^{-2x}\ln x-\int(-2e^{-2x})\ln xdx-2\int e^{-2x}\ln x\,dx$$

$$=e^{-2x}\ln x+2\int e^{-2x}\ln xdx-2\int e^{-2x}\ln xdx$$

$$\therefore\int e^{-2x}(\frac{1}{x}-2\ln x)dx=e^{-2x}\ln x+c$$

9(c) $\int e^{5x}\left\{5\ln x+\frac{1}{x}\right\}dx$ [চ.'০৯; প্র.ভ.প.'৯৯]

$$=\int 5e^{5x}\ln xdx+\int e^{5x}\frac{1}{x}dx$$

$$=\int 5e^{5x}\ln xdx+$$

$$e^{5x}\int\frac{1}{x}dx-\int\{\frac{d}{dx}(e^{5x})\int\frac{1}{x}dx\}dx$$

$$=\int 5e^{5x}\ln x dx+e^{5x}\ln x-\int 5e^{5x}\ln x dx$$

$$\int e^{5x}\left\{5\ln x+\frac{1}{x}\right\}dx = e^{5x}\ln x+\text{c}$$

10.(a) $\int\frac{dx}{x^2+x}$ [ব.'০৩]

$$=\int\frac{dx}{x(x+1)}=\int\{\frac{1}{x(0+1)}+\frac{1}{(x+1)(-1)}\}dx$$

$$=\int(\frac{1}{x}-\frac{1}{x+1})dx = \ln|x|-\ln|x+1|+c$$

10(b) $\int\frac{x+35}{x^2-25}dx$ [চ.'০৪]

$$=\int\frac{x+35}{(x-5)(x+5)}dx$$

$$=\int\{\frac{5+35}{(x-5)(5+5)}+\frac{-5+35}{(-5-5)(x+5)}\}dx$$

$$=\int\{\frac{40}{10(x-5)}-\frac{30}{10(x+5)}\}dx$$

$$=\int\{\frac{4}{x-5}-\frac{3}{x+5}\}dx$$

$$=4\ln|x-5|-3\ln|x+5|+c$$

10(c) $\int\frac{2x-1}{x(x-1)(x-2)}dx$ [ঢা.'০৯]

$$=\int\{\frac{2.0-1}{x(0-1)(0-2)}+\frac{2.1-1}{1(x-1)(1-2)}+\frac{2.2-1}{2(2-1)(x-2)}\}dx$$

$$=\int\{-\frac{1}{2}\frac{1}{x}-\frac{1}{x-1}+\frac{3}{2(x-2)}\}dx$$

$$=-\frac{1}{2}\ln|x|-\ln|x-1|+\frac{3}{2}\ln|x-2|+c$$

10(d) $\int\frac{x^2dx}{x^4-1}$ [রা.'১১; প্র.ভ.প.'৯১]

$$=\int\frac{x^2dx}{(x^2-1)(x^2+1)}$$

$$=\int\{\frac{1}{(x^2-1)(1+1)}+\frac{-1}{(-1-1)(x^2+1)}\}dx$$

$$=\frac{1}{2}\int\frac{1}{x^2-1^2}dx+\frac{1}{2}\int\frac{1}{1+x^2}dx$$

$$=\frac{1}{2}.\frac{1}{2.1}\ln|\frac{x-1}{x+1}|+\frac{1}{2}\tan^{-1}x+c$$

$$=\frac{1}{4}\ln|\frac{x-1}{x+1}|+\frac{1}{2}\tan^{-1}x+c$$

10(e) ধরি, $I=\int\frac{dx}{e^{2x}-3e^x}$ [প্র.ভ.প.'০৪]

এবং $e^x=z$. তাহলে $e^xdx=dz\Rightarrow dx=\frac{dz}{z}$ এবং

$$I=\int\frac{1}{z^2-3z}\frac{dz}{z}=\int\frac{dz}{z^2(z-3)}$$

এখন ধরি, $\frac{1}{z^2(z-3)}\equiv\frac{A}{z}+\frac{B}{z^2}+\frac{C}{z-3}$

$\therefore\ 1\equiv Az(z-3)+B(z-3)+Cz^2\cdots(1)$

(1) এ $z=3$ বসিয়ে পাই, $1=9C\Rightarrow C=\frac{1}{9}$

(1) এ $z=0$ বসিয়ে পাই, $1=-3B\Rightarrow B=-\frac{1}{3}$

(1) এর উভয়পক্ষ থেকে z^2 এর সহগ সমীকৃত করে পাই,

$0=A+C\Rightarrow A=-C=-\frac{1}{9}$

$$\therefore I=\int\{-\frac{1}{9}\frac{1}{z}-\frac{1}{3}\frac{1}{z^2}+\frac{1}{9(z-3)}\}dz$$

$$=-\frac{1}{9}\ln|z|-\frac{1}{3}(-\frac{1}{z})+\frac{1}{9}\ln|z-3|+c$$

$$=\frac{1}{9}\ln|\frac{z-3}{z}|+\frac{1}{3z}+c$$

$$\therefore\int\frac{dx}{e^{2x}-3e^x}=\frac{1}{9}\ln|\frac{e^x-3}{e^x}|+\frac{1}{3e^x}+c$$

11. $\int\frac{1}{x^2(x-1)}dx$ [কু.,রা. '০২; ব.'০৫,'১০]

ধরি, $\frac{1}{x^2(x-1)}\equiv\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$

$\Rightarrow 1=Ax(x-1)+B(x-1)+Cx^2\cdots(1)$

(1) এ $x=0$ বসিয়ে পাই, $1=-B\Rightarrow B=-1$

(1) এ $x=1$ বসিয়ে পাই, $1= C \Rightarrow C=1$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0=A+C \Rightarrow A=-C=-1$

$$\int \frac{1}{x^2(x-1)}dx = \int \{-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x-1}\}dz$$

$$= -\ln|x|-(-\frac{1}{x})+\ln|x-1|+c$$

$$= \ln|\frac{x-1}{x}|+\frac{1}{x}+c$$

12 ধরি, $I=\int \frac{x+2}{(1-x)(x^2+4)}dx$ এবং

$$\frac{x+2}{(1-x)(x^2+4)} \equiv \frac{A}{1-x}+\frac{Bx+C}{x^2+4}$$

$$\Rightarrow x+2 = A(x^2+4)+(Bx+C)(1-x) \cdots (1)$$

(1) এ $x=1$ বসিয়ে পাই, $1+2=5A \Rightarrow A=\frac{3}{5}$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0=A-B \Rightarrow B=A=\frac{3}{5}$

(1) এর উভয়পক্ষ থেকে ধ্রুবপদ সমীকৃত করে পাই,

$2=4A+C \Rightarrow C=2-\frac{12}{5}=-\frac{2}{5}$

$$\therefore I = \frac{3}{5}\int \frac{1}{1-x}dx + \int \frac{\frac{3}{5}x-\frac{2}{5}}{x^2+4}dx$$

$$= -\frac{3}{5}\ln|1-x|+\frac{3}{10}\int \frac{2xdx}{x^2+4}-\frac{2}{5}\int \frac{dx}{x^2+2^2}$$

$$= -\frac{3}{5}\ln|1-x|+\frac{3}{10}\ln(x^2+4)-\frac{2}{5.2}\tan^{-1}\frac{x}{2}+c$$

$$= -\frac{3}{5}\ln|1-x|+\frac{3}{10}\ln(x^2+4)-\frac{1}{5}\tan^{-1}\frac{x}{2}+c$$

13(a) $\int \frac{x^7}{(1-x^4)^2}dx = \int \frac{-x^3(1-x^4)+x^3}{(1-x^4)^2}dx$

$$= \int \{\frac{-x^3}{1-x^4}+\frac{x^3}{(1-x^4)^2}\}dx$$

$$= \frac{1}{4}\int \frac{d(1-x^4)}{1-x^4}-\frac{1}{4}\int \frac{d(1-x^4)}{(1-x^4)^2}$$

$$= \frac{1}{4}\ln|1-x^4|-\frac{1}{4}(-\frac{1}{1-x^4})+c$$

$$= \frac{1}{4}(\ln|1-x^4|+\frac{1}{1-x^4})+c$$

13(b) ধরি, $I=\int \frac{(x-2)^2}{(x+1)^2}dx = \int \frac{x^2-4x+4}{x^2+2x+2}dx$

$$= \int \frac{(x^2+2x+2)-6x+2}{x^2+2x+2}dx$$

$$= \int \{1-\frac{6x-2}{(x+1)^2}\}dx$$ এবং

$$\frac{6x-2}{(x+1)^2} \equiv \frac{A}{x+1}+\frac{B}{(x+1)^2}$$

$$\Rightarrow 6x-2 = A(x+1)+B \cdots (1)$$

(1) এ $x=-1$ বসিয়ে পাই, $B=-6-2=-8$

(1) এর উভয়পক্ষ থেকে x এর সহগ সমীকৃত করে পাই,

$6=A \Rightarrow A=6$

$$\therefore I = \int \{1-\frac{6}{x+1}+\frac{8}{(x+1)^2}\}dx$$

$$= x-6\ln|x+1|-\frac{8}{x+1}+c$$

13(c) ধরি, $I=\int \frac{\sin 2x\, dx}{3+5\cos x} = \int \frac{2\sin x\cos x\, dx}{3+5\cos x}$

এবং $\cos x = z$. তাহলে $-\sin x dx = dz$ এবং

$$I = \int \frac{-2z\,dz}{3+5z} = -\frac{2}{5}\int \frac{3+5z-3}{3+5z}dz$$

$$= -\frac{2}{5}\int (1-\frac{3}{3+5z})dz$$

$$= -\frac{2}{5}(z-\frac{3}{5}\ln|3+5z|)+c$$

$$= \frac{2}{25}(3\ln|3+5z|-5z)+c$$

$$= \frac{2}{25}(3\ln|3+5\cos x|-5\cos x)+c$$

অতিরিক্ত প্রশ্ন (সমাধানসহ)

1. $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$

$$=\int\frac{(\sqrt{x+a}-\sqrt{x+b})dx}{(\sqrt{x+a}+\sqrt{x+b})(\sqrt{x+a}-\sqrt{x+b})}$$

$$=\int\frac{(\sqrt{x+a}-\sqrt{x+b})dx}{(x+a)-(x+b)}$$

$$=\int\frac{(x+a)^{1/2}-(x+b)^{1/2}}{a-b}dx$$

$$=\frac{1}{a-b}\left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{(x+b)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]+c$$

$$=\frac{2}{3(a-b)}[(x+a)^{3/2}-(x+b)^{3/2}]+c$$

2. $\int 3\sin x\cos x dx$

$$=\int\frac{3}{2}(2\sin x\cos x)dx=\frac{3}{2}\int\sin 2x dx$$

$$=\frac{3}{2}(-\frac{1}{2}\cos 2x)+c=-\frac{3}{4}\cos 2x+c$$

3. (a) $\int 3\cos 3x\cos x\,dx$

$$=\int\frac{3}{2}\{\cos(3x+x)+\cos(3x-x)\}\,dx$$

$$=\int\frac{3}{2}(\cos 4x+\cos 2x)\,dx$$

$$=\frac{3}{2}(\frac{1}{4}\sin 4x+\frac{1}{2}\sin 2x)+c$$

$$=\frac{3}{8}(\sin 4x+2\sin 2x)+c$$

3(b) $\int\cos^2\frac{x}{2}dx=\int\frac{1}{2}(1+\cos x)dx$

$$=\frac{1}{2}(x+\sin x)+c$$

4(a) $\int\cos x\cos(\sin x)dx$

$$=\int\cos(\sin x)\,d(\sin x)=\cos(\sin x)+c$$

4(b) ধরি, $I=\int(e^x+\frac{1}{x})(e^x+\ln x)\,dx$ [রা.'০১]

এবং $e^x+\ln x=z$. তাহলে $(e^x+\frac{1}{x})dx=dz$ এবং

$$I=\int z\,dz=\frac{1}{2}z^2+c=\frac{1}{2}(e^x+\ln x)^2+c$$

5 $\int e^{3\cos 2x}\sin 2x\,dx$

$$=-\frac{1}{6}\int e^{3\cos 2x}(-6\sin 3x dx)$$

$$=-\frac{1}{6}e^{3\cos 2x}+c$$

6(a) ধরি, $I=\int\sin^3 x\cos x dx$

এবং $\sin x=z$. তাহলে, $\cos x\,dx=dz$ এবং

$$I=\int z^3\,dz=\frac{1}{4}z^4+c=\frac{1}{4}\sin^4 x+c$$

6(b) ধরি, $I=\int\tan^3 x\sec^2 x dx$ এবং $\tan x=z$

তাহলে, $\sec^2 x\,dx=dz$ এবং

$$I=\int z^3dz=\frac{z^{3+1}}{3+1}+c=\frac{1}{4}\tan^4 x+c$$

6(c) $\int\sin^2(3x+2)dx$

$$=\int\frac{1}{2}\{1-\cos 2(3x+2)\}dx$$

$$=\frac{1}{2}\{\int dx-\int\cos(6x+4)dx\}$$

$$=\frac{1}{2}\{x-\frac{\sin(6x+4)}{6}\}+c$$

$$=\frac{1}{2}x-\frac{1}{12}\sin(6x+4)+c$$

7.(a) $\int\frac{(\ln x)^2}{x}dx=\int(\ln x)^2 d(\ln x)$

$$=\frac{(\ln x)^{2+1}}{2+1}+c=\frac{1}{3}(\ln x)^3+c$$

7(b) $\int\frac{\sqrt{1+\ln x}}{x}dx$

$$=\int(1+\ln x)^{\frac{1}{2}}d(1+\ln x)$$

$$= \frac{(1+\ln x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{2}{3}(1+\ln x)^{3/2} + c$$

7(c) $\int \frac{\cos(\ln x)}{x} dx = \int \cos(\ln x)\, d(\ln x)$

$= \sin(\ln x) + c$

8. $\int \frac{e^{-x}dx}{(5+e^{-x})^2}$

$= \int (5+e^{-x})^{-2}\, d(5+e^{-x}).(-1)$

$$= -\frac{(5+e^{-x})^{-2+1}}{-2+1} + c = \frac{1}{5+e^{-x}} + c$$

9. $\int \frac{e^x(1+x)dx}{\cos^2(xe^x)}$

ধরি, $xe^x = z \qquad e^x(x+1)dx = dz$

$$\int \frac{e^x(1+x)dx}{\cos^2(xe^x)} = \int \frac{dz}{\cos^2 z} = \int \sec^2 z dz$$

$= \tan z + c = \tan(xe^x) + c$

10(a) ধরি, $I = \int \frac{\sin(2+5\ln x)}{x} dx$ এবং

$2 + 5\ln x = z$. তাহলে, $\frac{5}{x}dx = dz$ এবং

$I = \frac{1}{5}\int \sin z\, dz = \frac{1}{5}(-\cos z) + c$

$= -\frac{1}{5}\cos(2+5\ln x) + c$

10(b) $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

$$= \int \frac{\sin\{(x-b)-(x-a)\}dx}{\sin(a-b)\sin(x-a)\sin(x-b)}$$

$$= \int \frac{\{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)\}dx}{\sin(a-b)\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(a-b)}\int \{\cot(x-a) - \cot(x-b)\}dx$$

$$= \frac{\ln|\sin(x-a)| - \ln|\sin(x-b)|}{\sin(a-b)} + c$$

$$= \frac{1}{\sin(a-b)}\ln\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + c$$

11 (a) $\int \frac{\sec^2 x dx}{\sqrt{1+\tan x}} = \int \frac{d(1+\tan x)}{\sqrt{1+\tan x}}$

$= 2\sqrt{1+\tan x} + c$

11(b) $\int \frac{dx}{\sqrt{(\sin^{-1}x)}\sqrt{1-x^2}} = \int \frac{d(\sin^{-1}x)}{\sqrt{(\sin^{-1}x)}}$

$\left[\because d(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}dx\right]$

$= 2\sqrt{\sin^{-1}x} + c \qquad \left[\because \int \frac{dx}{\sqrt{x}} = 2\sqrt{x}\right]$

11(c) ধরি, $I = \int \frac{dx}{(1+x^2)\sqrt{\tan^{-1}x+3}}$

এবং $\tan^{-1}x + 3 = z$. তাহলে, $\frac{dx}{1+x^2} = dz$ এবং

$I = \int \frac{dz}{\sqrt{z}} = 2\sqrt{z} + c \qquad \left[\because \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}\right]$

$$\therefore \int \frac{dx}{(1+x^2)\sqrt{\tan^{-1}x+3}} = 2\sqrt{\tan^{-1}x+3} + c$$

11(d) $\int \frac{\tan(\ln|x|)}{x}dx = \int \tan(\ln|x|)\, d(\ln|x|)$

$= \ln\{\sec(\ln|x|)\} + c$

12(a) $\int \frac{\sec^2 x dx}{\sqrt{1-\tan^2 x}} = \int \frac{d(\tan x)}{\sqrt{1-\tan^2 x}}$

$= \sin^{-1}(\tan x) + c$

12(b) $\int \frac{dx}{\sqrt{15-4x-4x^2}}$

$$= \int \frac{dx}{\sqrt{16-\{(2x)^2+2.2x.1+1^2\}}}$$

$$= \frac{1}{2}\int \frac{d(2x+1)}{\sqrt{4^2-(2x+1)^2}} = \frac{1}{2}\sin^{-1}\left(\frac{2x+1}{4}\right) + c$$

12(c) $\int \frac{dx}{\sqrt{x(4-x)}} = \int \frac{dx}{\sqrt{4x-x^2}}$

$= \int \frac{dx}{\sqrt{2^2-(x^2-4x+2^2)}}$

$= \int \frac{d(x-2)}{\sqrt{2^2-(x-2)^2}} = \sin^{-1}(\frac{x-2}{2}) + c$

12(d) $\int \frac{dx}{\sqrt{a^2-b^2(1-x)^2}}$

$= -\frac{1}{b}\int \frac{d(b-bx)}{\sqrt{a^2-(b-bx)^2}}$

$= -\frac{1}{b}\sin^{-1}(\frac{b-bx}{a}) + c$

12(e) ধরি, $I = \int \sqrt{\tan x}dx$ এবং $\tan x = z^2$

তাহলে, $\sec^2 x\, dx = 2z\, dz$

$\Rightarrow dx = \frac{2zdz}{1+\tan^2 x} = \frac{2z}{1+z^4}$ এবং

$I = \int \frac{2z^2 dz}{1+z^4} = \int \frac{(z^2+1)-(z^2-1)}{1+z^4}dz$

$= \int [\frac{z^2+1}{z^4+1} + \frac{z^2-1}{z^4+1}]dz$

$= \int [\frac{1+\frac{1}{z^2}}{z^2+\frac{1}{z^2}} + \frac{1-\frac{1}{z^2}}{z^2+\frac{1}{z^2}}]dz$

$= \int [\frac{1+\frac{1}{z^2}}{(z-\frac{1}{z})^2+2} + \frac{1-\frac{1}{z^2}}{(z+\frac{1}{z})^2-2}]dz$

$= \int \frac{d(z-\frac{1}{z})}{(z-\frac{1}{z})^2+(\sqrt{2})^2} + \int \frac{d(z+\frac{1}{z})}{(z+\frac{1}{z})^2-(\sqrt{2})^2}$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{z-\frac{1}{z}}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\ln|\frac{z-\frac{1}{z}-\sqrt{2}}{z-\frac{1}{z}+\sqrt{2}}| + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{z^2-1}{\sqrt{2}z} + \frac{1}{2\sqrt{2}}\ln|\frac{z^2-1-\sqrt{2}z}{z^2-1+\sqrt{2}z}| + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{\tan x-1}{\sqrt{2\tan x}} +$

$\frac{1}{2\sqrt{2}}\ln|\frac{\tan x-\sqrt{2\tan x}-1}{\tan x+\sqrt{2\tan x}-1}| + c$

13. ধরি, $I = \int 3\cos^3 x\cos 2x\, dx$

$\cos^3 x\cos 2x = \frac{1}{4}(3\cos x+\cos 3x)\cos 2x$

$= \frac{1}{4}[3\cos x\cos 2x + \cos 3x\cos 2x]$

$= \frac{1}{4}[3.\frac{1}{2}(\cos 3x+\cos x) + \frac{1}{2}(\cos 5x+\cos x)] =$

$\frac{1}{8}(3\cos 3x + 4\cos x + \cos 5x)$

$\therefore I = \frac{3}{8}\int (3\cos 3x + 4\cos x + \cos 5x)\, dx$

$= \frac{3}{8}(3.\frac{1}{3}\sin 3x + 4\sin x + \frac{1}{5}\sin 5x) + c$

14(a) ধরি, $I = \int e^{2x}\cos x dx$

$= e^{2x}\int \cos x\, dx - \int \{\frac{d}{dx}(e^{2x})\int \cos x\, dx\}\, dx$

$= e^{2x}\sin x - \int 2e^{2x}\sin x\, dx$

$= e^{2x}\sin x - 2e^{2x}\int \sin x dx +$

$2\int \{\frac{d}{dx}(e^{2x})\int \sin x\, dx\}\, dx$

$= e^{2x}\sin x - 2e^{2x}(-\cos x) + 2\int 2e^{2x}(-\cos x)dx$

$= e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x dx$

$= e^{2x}(\sin x + 2\cos x) - 4I + c_1$

$\Rightarrow 5I = e^{2x}(\sin x + 2\cos x) + c_1$

$\Rightarrow I = \frac{e^{2x}}{5}(\sin x + 2\cos x) + \frac{1}{5}c_1$

$\therefore I = \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(\sin x + 2\cos x) + c$

14.(b) $\int e^{-3x}\cos 4x\,dx$

$= \dfrac{e^{-3x}}{3^2+4^2}(-3\cos 4x + 4\sin 4x) + c$

[সূত্র প্রয়োগ করে।]

$= \dfrac{e^{-3x}}{25}(-3\cos 4x + 4\sin 4x) + c$

15(a) ধরি, $I = \int e^x \dfrac{1+\sin x}{1+\cos x}dx$

$= \int e^x\{\dfrac{1}{1+\cos x} + \dfrac{\sin x}{1+\cos x}\}dx$

এবং $f(x) = \dfrac{\sin x}{1+\cos x}$

$f'(x) = \dfrac{(1+\cos x)\cos x - \sin x(0-\sin x)}{(1+\cos x)^2}$

$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \dfrac{1+\cos x}{(1+\cos x)^2} = \dfrac{1}{1+\cos x}$ এবং

$I = \int e^x\{\dfrac{\sin x}{1+\cos x} + \dfrac{1}{1+\cos x}\}dx$

$= \int e^x\{f(x)+f'(x)\}dx = e^x f(x) + c$

$\therefore I = \int e^x \dfrac{1+\sin x}{1+\cos x}dx = e^x \dfrac{\sin x}{1+\cos x} + c$

15(b) $\int e^{ax}(a\sin bx + b\cos bx)dx$

$= \int ae^{ax}\sin bx dx + \int be^{ax}\cos bx dx$

$= a\sin bx\int e^{ax}dx - \int\{\dfrac{d}{dx}(a\sin bx)\int e^{ax}dx\}dx + \int be^{ax}\cos bx dx$

$= a\sin bx.(\dfrac{e^{ax}}{a}) - \int(ab\cos bx)(\dfrac{e^{ax}}{a})dx + \int be^{ax}\cos bx dx$

$= e^{ax}\sin bx - \int be^{ax}\cos bx dx + \int be^{ax}\cos bx dx$

$\therefore \int e^{ax}(a\sin bx + b\cos bx)dx = e^{ax}\sin bx + c$

16(a) $\int \dfrac{x-3}{(1-2x)(1+x)}dx$

$= \int[\dfrac{\frac{1}{2}-3}{(1-2x)(1+\frac{1}{2})} + \dfrac{-1-3}{\{1-2(-1)\}(1+x)}]dx$

$= \int[\dfrac{-\frac{5}{2}}{\frac{3}{2}(1-2x)} + \dfrac{-4}{3(1+x)}]dx$

$= -\dfrac{5}{3}(-\dfrac{1}{2})\int\dfrac{d(1-2x)}{(1-2x)} - \dfrac{4}{3}\int\dfrac{1}{1+x}dx$

$= \dfrac{5}{6}\ln|1-2x| - \dfrac{4}{3}\ln|1+x| + c$

16(b) $\int\dfrac{dx}{x^4-1} = \int\dfrac{dx}{(x^2-1)(x^2+1)}$

$= \int\{\dfrac{1}{(x^2-1)(1+1)} + \dfrac{1}{(-1-1)(x^2+1)}\}dx$

$= \dfrac{1}{2}\int\dfrac{dx}{x^2-1^2} - \dfrac{1}{2}\int\dfrac{1}{1+x^2}dx$

$= \dfrac{1}{2}.\dfrac{1}{2.1}\ln|\dfrac{x-1}{x+1}| - \dfrac{1}{2}\tan^{-1}x + c$

$= \dfrac{1}{4}\ln|\dfrac{x-1}{x+1}| - \dfrac{1}{2}\tan^{-1}x + c$

17(a) $\int\dfrac{1}{x(x+1)^2}dx$

ধরি, $\dfrac{1}{x(x+1)^2} \equiv \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$\Rightarrow 1 = A(x+1)^2 + Bx(x+1) + Cx \cdots(1)$

(1) এ $x=0$ বসিয়ে পাই, $1 = A \Rightarrow A = 1$

(1) এ $x=-1$ বসিয়ে পাই, $1 = -C \Rightarrow C = -1$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই

$0 = A + B \Rightarrow B = -A = -1$

$\therefore \int\dfrac{1}{x(x+1)^2}dx = \int\{\dfrac{1}{x} - \dfrac{1}{x+1} - \dfrac{1}{(x+1)^2}\}dz$

$$= \ln|x| - \ln|x+1| - (-\frac{1}{x+1}) + c$$

$$= \ln|\frac{x}{x+1}| + \frac{1}{x+1} + c$$

17(b) $\int \frac{3x+1}{(x+1)^2}dx = \int \frac{3(x+1)-2}{(x+1)^2}dx$

$$= \int \{\frac{3(x+1)}{(x+1)^2} - \frac{2}{(x+1)^2}\}dx$$

$$= \int \{\frac{3}{x+1} - \frac{2}{(x+1)^2}\}dx$$

$$= 3\ln|x+1| - 2(-\frac{1}{x+1}) + c$$

$$= 3\ln|x+1| + \frac{2}{x+1} + c$$

18. (a) $\int \frac{dx}{x(x^2+1)} \int \frac{(x^2+1)-x^2dx}{x(x^2+1)}$

$$= \int \{\frac{1}{x} - \frac{x}{x^2+1}\}dx$$

$$= \int \frac{1}{x}dx - \frac{1}{2}\int \frac{(2x+0)dx}{x^2+1}$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + c$$

18(b) ধরি, $I = \int \frac{xdx}{(x-1)(x^2+4)}$ এবং

$$\frac{x}{(x-1)(x^2+4)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow x = A(x^2+4) + (Bx+C)(x-1) \cdots (1)$$

(1) এ $x=1$ বসিয়ে পাই, $1 = 5A \Rightarrow A = \frac{1}{5}$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$$0 = A + B \Rightarrow B = -A = -\frac{1}{5}$$

(1) এর উভয়পক্ষ থেকে ধ্রুবপদ সমীকৃত করে পাই,

$$0 = 4A - C \Rightarrow C = 4A = \frac{4}{5}$$

$$I = \frac{1}{5}\int \frac{1}{x-1}dx + \int \frac{-\frac{1}{5}x + \frac{4}{5}}{x^2+4}dx$$

$$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\int \frac{2xdx}{x^2+4} + \frac{4}{5}\int \frac{dx}{x^2+2^2}$$

$$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\ln(x^2+4) + \frac{4}{5}.\frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

$$= \frac{1}{5}\ln|x-1| - \frac{1}{10}\ln(x^2+4) + \frac{2}{5}\tan^{-1}\frac{x}{2} + c$$

19.(a) $\int xe^{-x}dx$

$$= x\int e^{-x}dx - \int \{\frac{d}{dx}(x)\int e^{-x}dx\}dx$$

$$= -xe^{-x} - \int 1.(-e^{-x})dx = -xe^{x} - e^{-x} + c$$

19(b) $\int xe^{ax}dx$

$$= x\int e^{ax}dx - \int \{\frac{d}{dx}(x)\int e^{ax}dx\}dx$$

$$= x.\frac{1}{a}e^{ax} - \int 1.(\frac{1}{a}e^{ax})dx = \frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax} + c$$

$$= \frac{1}{a^2}(ax-1)e^{ax} + c$$

19(c) $\int x^3e^{2x}dx$

$$= x^3\int e^{2x}dx - \int \{\frac{d}{dx}(x^3)\int e^{2x}dx\}dx$$

$$= x^3(\frac{1}{2}e^{2x}) - \int (3x^2)(\frac{1}{2}e^{2x})dx$$

$$= \frac{1}{2}x^3e^{2x} - \frac{3}{2}[x^2\int e^{2x} - \int \{\frac{d}{dx}(x^2)\int e^{2x}dx\}dx]$$

$$= \frac{1}{2}x^3e^{2x} - \frac{3}{2}[x^2.\frac{1}{2}e^{2x} - \int (2x).\frac{1}{2}e^{2x}dx]$$

$$= \frac{1}{2}x^3e^{2x} - \frac{3}{2}[\frac{x^2e^{2x}}{2} - \{x\int e^{2x} - \int 1.\frac{e^{2x}}{2}dx\}]$$

$$= \frac{1}{2}x^3e^{2x} - \frac{3}{2}[\frac{x^2e^{2x}}{2} - \{x\frac{e^{2x}}{2} - \frac{e^{2x}}{4}\}] + c$$

$$= (\frac{1}{2}x^3 - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8})e^{2x} + c$$

[MCQ এর ক্ষেত্রেঃ

$$\int x^3e^{2x}dx = \{\frac{1}{2}x^3 - \frac{1}{2^2}(3x^2) + \frac{1}{2^3}(6x) - \frac{1}{2^4}.6\}e^{2x} = \{\frac{1}{2}x^3 - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8}\}e^{2x}]$$

20. (a) $\int x\sin x dx$

$= x\int \sin x dx - \int\{\frac{d}{dx}(x)\int \sin x\, dx\}\, dx$

$= x(-\cos x) - \int 1.(-\cos x)dx$

$= -x\cos x + \sin x + c$

20. (b) $\int x\cos x dx$

$= x\int \cos x dx - \int\{\frac{d}{dx}(x)\int \cos x\, dx\}\, dx$

$= x\sin x - \int 1.\sin x dx$

$= x\sin x + \cos x + c$

20(c) $\int x^2 \sin x dx$

$= x^2\int \sin x dx - \int\{\frac{d}{dx}(x^2)\int \sin x\, dx\}\, dx$

$= x^2(-\cos x) - \int 2x(-\cos x)dx$

$= -x^2\cos x + 2[x\int \cos x -$

$\int\{\frac{d}{dx}(x)\int \cos x\, dx\}\, dx]$

$= -x^2\cos x + 2[x\sin x - \int 1.\sin x\, dx]$

$= -x^2\cos x + 2[x\sin x - (-\cos x)] + c$

$= -x^2\cos x + 2x\sin x + 2\cos x + c$

20(d) ধরি, $I = \int \cos\sqrt{x}\, dx$ এবং $\sqrt{x} = z$

তাহলে $\frac{1}{2\sqrt{x}}dx = dz \Rightarrow dx = 2z\, dz$ এবং

$I = \int 2z\cos z\, dz$

$= 2[\, z\int \cos z\, dz - \int\{\frac{d}{dz}(z)\int \cos z\, dz\}\, dz\,]$

$= 2[\, z\sin z) - \int 1.\sin z dz\,]$

$= 2z\sin z - 2(-\cos z) + c$

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + c$

21.(a) $\int x^2\cos^2 x\, dx$ [প্র.ভ.প.'৮৫,'৯৬]

$= \int x^2\frac{1}{2}(1+\cos 2x)\, dx$

$= \frac{1}{2}[\int x^2\, dx + \int x^2\cos 2x\, dx]$

$= \frac{1}{2}[\frac{x^3}{3} + x^2(\frac{1}{2}\sin 2x) - (2x)(-\frac{1}{2^2}\cos 2x)$

$+ 2(-\frac{1}{2^3}\sin 2x)] + c$

$= \frac{1}{2}[\frac{x^3}{3} + \frac{x^2}{2}\sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x] + c$

21(b) $\int x\sin x\cos x dx = \frac{1}{2}\int x\sin 2x dx$

$= \frac{1}{2}[x\int \sin 2x dx - \int\{\frac{d}{dx}(x)\int \sin 2x dx\}dx]$

$= \frac{1}{2}[x(-\frac{\cos 2x}{2}) - \int 1.(-\frac{\cos 2x}{2})dx]$

$= \frac{1}{4}[-x\cos 2x + \frac{\sin 2x}{2}] + c$

21(c) $\int x\sin x\sin 2x\, dx$

$= \int x\frac{1}{2}(\cos x - \cos 3x)\, dx$

$= \frac{1}{2}[x\int \cos x dx - \int\{\frac{d}{dx}(x)\int \cos x dx\}dx$

$-x\int \cos 3x dx + \int\{\frac{d}{dx}(x)\int \cos 3x dx\}dx]$

$= \frac{1}{2}[x\sin x - \int 1.\sin x dx$

$- x\frac{\sin 3x}{3} + \int 1.\frac{\sin 3x}{3}dx]$

$= \frac{1}{2}[x\sin x + \cos x - \frac{x\sin 3x}{3} - \frac{\cos 3x}{9}] + c$

4. (c) $\int \frac{x}{\sin^2 x}dx = \int x\operatorname{cosec}^2 x\, dx$

$= x\int \operatorname{cosec}^2 x dx - \int\{\frac{d}{dx}(x)\int \operatorname{cosec}^2 x dx\}dx$

$= x(-\cot x) - \int 1.(-\cot x)dx$

$= -x\cot x + \ln|\sin x| + c$

21(d) ধরি, $I=\int \sec^3 x\,dx=\int \sec^2 x \sec x\,dx$

$=\sec x\int \sec^2 x\;dx-\int\{\frac{d}{dx}(\sec x)\int \sec^2 x dx\}dx$

$=\sec x\tan x-\int \sec x\tan x.\tan x dx$

$=\sec x\tan x-\int \sec x(\sec^2 x-1)dx$

$=\sec x\tan x-\int \sec^3 x dx+\int \sec x dx$

$\Rightarrow I=\sec x\tan x-I+\ln|\tan(\frac{\pi}{4}+\frac{x}{2})+c_1$

$\Rightarrow 2I=\sec x\tan x+\ln|\tan(\frac{\pi}{4}+\frac{x}{2})+c_1$

$\Rightarrow I=\frac{1}{2}\sec x\tan x+\frac{1}{2}\ln|\tan(\frac{\pi}{4}+\frac{x}{2})+\frac{1}{2}c_1$

$\Rightarrow I=\frac{1}{2}\sec x\tan x+\frac{1}{2}\ln|\tan(\frac{\pi}{4}+\frac{x}{2})|+c$

22(a) $\int x^2\ln x dx$

$=\ln x\int x^2 dx-\int\{\frac{d}{dx}(\ln x)\int x^2 dx\}dx$

$=\ln x.\frac{x^3}{3}-\int\frac{1}{x}.\frac{x^3}{3}dx=\frac{x^3}{3}\ln x-\frac{1}{3}\int x^2 dx$

$=\frac{x^3}{3}\ln x-\frac{1}{3}.\frac{x^3}{3}+c=\frac{x^3}{3}\ln x-\frac{x^3}{9}+c$

22(b) $\int x^3\ln x dx$

$=\ln x\int x^3 dx-\int\{\frac{d}{dx}(\ln x)\int x^3 dx\}dx$

$=\ln x.\frac{x^4}{4}-\int\frac{1}{x}.\frac{x^4}{4}dx=\frac{x^4}{4}\ln x-\frac{1}{4}\int x^3 dx$

$=\frac{x^4}{4}\ln x-\frac{1}{4}.\frac{x^4}{4}+c=\frac{x^4}{4}\ln x-\frac{x^4}{16}+c$

22(c) $\int\frac{\ln x}{x^2}dx$

$=\ln x\int\frac{1}{x^2}dx-\int\{\frac{d}{dx}(\ln x)\int\frac{1}{x^2}dx\}dx$

$=\ln x.(-\frac{1}{x})-\int\frac{1}{x}.(-\frac{1}{x})dx$

$=-\frac{1}{x}\ln x+\int\frac{1}{x^2}dx=-\frac{1}{x}\ln x+(-\frac{1}{x})+c$

$=-\frac{1}{x}\ln x-\frac{1}{x}+c$

23(a) $\int 2^x\sin x\,dx=\int e^{x\ln 2}\sin x\,dx$

$=\frac{e^{x\ln 2}}{(\ln 2)^2+1^2}[\ln 2.\sin x-1.\cos x]+c$

[সূত্র প্রয়োগ করে।]

$=\frac{2^x}{(\ln 2)^2+1}[\ln 2.\sin x-\cos x]+c$

23(b) $\int(3^x e^x+\ln x)dx$ **[প্র.ভ.প.৮৪]**

$=\int(3e)^x dx+\int\ln x dx$

$=\frac{(3e)^x}{\ln(3e)}+\frac{1}{x}+c=\frac{3^x e^x}{\ln 3+\ln e}+\frac{1}{x}+c$

$=\frac{3^x e^x}{\ln 3+1}+\frac{1}{x}+c$

8(c) ধরি, $I=\int e^x\left\{\frac{1}{1-x}+\frac{1}{(x-1)^2}\right\}dx$ এবং

$f(x)=\frac{1}{1-x}=(1-x)^{-1}$

$\therefore f'(x)=-(1-x)^{-1-1}(-1)=\frac{1}{(1-x)^2}$ এবং

$I=\int e^x\left\{\frac{1}{1-x}+\frac{1}{(1-x)^2}\right\}dx$

$=\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+c$

$\therefore\int e^x\left\{\frac{1}{1-x}+\frac{1}{(x-1)^2}\right\}dx=\frac{e^x}{1-x}+c$

24(a) $\int e^{-x}\left\{\frac{1}{x}+\frac{1}{x^2}\right\}dx=\int\frac{e^{-x}}{x}dx+\int\frac{e^{-x}}{x^2}dx$

$=\frac{1}{x}\int e^{-x}dx-\int\{\frac{d}{dx}(\frac{1}{x})\int e^{-x}dx\}dx+\int\frac{e^{-x}}{x^2}dx$

$=\frac{1}{x}(-e^{-x})-\int(-\frac{1}{x^2})(-e^{-x})dx+\int\frac{e^{-x}}{x^2}dx$

$$= -\frac{e^{-x}}{x} - \int \frac{e^{-x}}{x^2}dx + \int \frac{e^{-x}}{x^2}dx$$

$$\int e^{-x}\left\{\frac{1}{x} + \frac{1}{x^2}\right\}dx = -\frac{e^{-x}}{x} + c$$

24(b) $\int e^x\{\tan x + \ln(\sec x)\}dx$ [প্র.ভ.প. '১১]

ধরি, $I = \int e^x\{\tan x + \ln(\sec x)\}dx$ এবং

$f(x) = \ln(\sec x)$

$\therefore f'(x) = \frac{\sec x \tan x}{\sec x} = \tan x$ এবং

$I = \int e^x\{\ln(\sec x) + \tan x\}dx$

$= \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

$\therefore \int e^x\{\tan x + \ln(\sec x)\}dx = e^x \ln(\sec x) + c$

25(a) ধরি, $I = \int e^x \frac{x^2+1}{(x+1)^2}dx$ [প্র.ভ.প.'০২]

$= \int e^x \frac{x^2-1+2}{(x+1)^2}dx$

$= \int e^x\{\frac{(x-1)(x+1)}{(x+1)^2} + \frac{2}{(x+1)^2}\}dx$

$= \int e^x\{\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\}dx$ এবং $f(x) = \frac{x-1}{x+1}$

$f'(x) = \frac{(x+1).1-(x-1).1}{(x+1)^2}$

$= \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$ এবং

$I = \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

$\int e^x \frac{x^2+1}{(x+1)^2}dx = e^x(\frac{x-1}{x+1}) + c$

25(b) ধরি, $I = \int [\frac{1}{\ln x} - \frac{1}{(\ln x)^2}]dx$ এবং $\ln x = y$.

তাহলে, $x = e^y \Rightarrow dx = e^y dy$ এবং

$I = \int e^y[\frac{1}{y} - \frac{1}{y^2}]dy = \int e^y[\frac{1}{y} + D(\frac{1}{y})]dy$

$[\because D(\frac{1}{y}) = \frac{d}{dx}(\frac{1}{y}) = -\frac{1}{y^2}]$

$= \frac{e^y}{y} + c = \frac{x}{\ln x} + c$

26. $\int \frac{x}{(x-1)^2(x+2)}dx$

ধরি, $\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$

$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2 \cdots(1)$

(1) এ $x = 1$ বসিয়ে পাই, $1 = 3B \Rightarrow B = 1/3$

(1) এ $x = -2$ বসিয়ে পাই, $-2 = 9C \Rightarrow C = -2/9$

(1) এর উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে পাই,

$0 = A + C \Rightarrow A = -C = 2/9$

$\therefore \int \frac{x}{(x-1)^2(x+2)}dx$

$= \int \{\frac{2/9}{x-1} + \frac{1/3}{(x-1)^2} + \frac{-2/9}{x+2}\}dz$

$= \frac{2}{9}\ln|x-1| + \frac{1}{3}(-\frac{1}{x-1}) - \frac{2}{9}\ln|x+2| + c$

$= \frac{2}{9}\ln|\frac{x-1}{x+2}| - \frac{1}{3(x-1)} + c$

27(a) ধরি, $I = \int \frac{x^2+1}{(x+2)^2}dx$

$= \int \frac{x^2+4x+4-(4x+3)}{(x+2)^2}dx$

$= \int \{1 - \frac{4x+3}{(x+2)^2}\}dx$ এবং

$\frac{4x+3}{(x+2)^2} \equiv \frac{A}{x+2} + \frac{B}{(x+2)^2}$

$\Rightarrow 4x + 3 = A(x+2) + B \cdots(1)$

(1) এ $x = -2$ বসিয়ে পাই, $B = -8 + 3 = -5$

(1) এর উভয়পক্ষ থেকে x এর সহগ সমীকৃত করে পাই,

$4 = A \Rightarrow A = 4$

$\therefore I = \int \{1 - \frac{4}{x+2} + \frac{5}{(x+2)^2}\}dx$

$= x - 4\ln|x+2| - \frac{5}{x+2} + c$

ভর্তি পরীক্ষার MCQ