$$f'(2) = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{x^5-2^5}{x-2}$$

$$= 5\times(2)^4 \quad [\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}]$$

$$= 5\times 16 = 80$$

4(b) মূল নিয়মে $x = a$ -তে e^{mx} এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = e^{mx}$ $\quad f(a) = e^{ma}$

$$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a}\frac{e^{mx}-e^{ma}}{x-a} = \lim_{x\to a}\frac{e^{ma}(e^{mx-ma}-1)}{x-a}$$

$$= e^{ma}\lim_{x-a\to 0}\frac{e^{m(x-a)}-1}{m(x-a)}\times m$$

$$= me^{ma}.1 \qquad [\quad \lim_{x\to 0}\frac{e^x-1}{x} = 1]$$

$$= me^{ma}$$

4(c) মূল নিয়মে $x = \frac{\pi}{4}$-তে $\tan x$ এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = \tan x.$ $\quad f(\frac{\pi}{4}) = \tan\frac{\pi}{4}$

$$f'(\frac{\pi}{4}) = \lim_{x\to\frac{\pi}{4}}\frac{f(x)-f(\frac{\pi}{4})}{x-\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\tan x-\tan\frac{\pi}{4}}{x-\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}}{(x-\frac{\pi}{4})\cos x\cos\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\sin(x-\frac{\pi}{4})}{(x-\frac{\pi}{4})\cos x\cos\frac{\pi}{4}}$$

$$= \lim_{x-\frac{\pi}{4}\to 0}\frac{\sin(x-\frac{\pi}{4})}{x-\frac{\pi}{4}}\times\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x\cos\frac{\pi}{4}}$$

$$= 1.\frac{1}{\cos\frac{\pi}{4}\cos\frac{\pi}{4}} = \frac{1}{(1/\sqrt{2})^2} = 2$$

প্রশ্নমালা IX D

x এর সাপেক্ষে অন্তরক সহগ নির্ণয় কর :

1(a) $\frac{d}{dx}\{x^2\ln(x)\}$

$$= x^2\frac{d}{dx}\{ln(x)\} + ln(x)\frac{d}{dx}(x^2)$$

$$= x^2\frac{1}{x} + ln(x).(2x) = x + 2x\,ln(x)$$

1(b) $5e^x\log_a x$ [ব.'০৮;দি.'১৩]

মনে করি, $y = 5e^x\log_a x$

$$\frac{dy}{dx} = 5\{e^x\frac{d}{dx}(\log_a x)+\log_a x\frac{d}{dx}(e^x)\}$$

$$= 5\{e^x\frac{1}{x\ln a}+\log_a x.\,e^x\}$$

$$\therefore \frac{d}{dx}\{5e^x\log_a x\} = 5e^x\{\frac{1}{x\ln a}+\log_a x\}$$

1(c) $\log_{10} x$ [দি.'১১,'১৩]

মনে করি, $y = \log_{10} x = \log_{10} e\times\log_e x$

$$\Rightarrow y = \frac{1}{\log_e 10}\times\ln x = \frac{1}{\ln 10}\times\ln x$$

$$\frac{dy}{dx} = \frac{1}{\ln 10}\frac{d}{dx}(\ln x) = \frac{1}{\ln 10}\times\frac{1}{x}$$

$$\frac{d}{dx}(\log_{10} x) = \frac{1}{x\ln 10} \text{ (Ans.)}$$

1(d) $\log_a x$ [ঢা.'১৩]

মনে করি, $y = \log_a x = \log_a e\times\log_e x$

$$\Rightarrow y = \frac{1}{\log_e a}\times\ln x = \frac{1}{\ln a}\times\ln x$$

$$\frac{dy}{dx}=\frac{1}{\ln a}\frac{d}{dx}(\ln x)=\frac{1}{\ln a}\times\frac{1}{x}$$

$$\frac{d}{dx}(\log_a x)=\frac{1}{x\ln a} \text{ (Ans.)}$$

2. (a) $a^x \ln(x)+be^x \sin x$

$$\frac{d}{dx}\{a^x \ln(x)+be^x \sin x\}= a^x \frac{d}{dx}\{\ln(x)\}$$

$$+\ \ln(x)\frac{d}{dx}(a^x)\ +\ b\{e^x\frac{d}{dx}(\sin\ x)\ +$$

$$\sin x\frac{d}{dx}(e^x)\}$$

$$=\ a^x\frac{1}{x}\ +\ \ln(x)(a^x\, lna)\ +\ b\{e^x(\cos\ x)\ +$$

$$\sin x\,(e^x)\}$$

$$=a^x\{\frac{1}{x}+lna\ \ln(x)\}+ b\,e^x(\cos x+\sin x)$$

2(b) $x^2\log_a x - x^3\ln a^x + 6x\,e^x \ln x$

ধরি, $y = x^2\log_a x - x^3\ln a^x + 6x\,e^x\ln x$

$$= x^2\log_a x - x^4\ln a + 6x\,e^x\ln x$$

$$\frac{dy}{dx}= x^2\frac{d}{dx}(\log_a x)+\log_a x\frac{d}{dx}(x^2)-$$

$$\ln a\frac{d}{dx}(x^4)\ +6\{x\,e^x\frac{d}{dx}(\ln x)+$$

$$x\ \ln x\frac{d}{dx}(e^x)+e^x\ln x\frac{d}{dx}(x)\ \}$$

$$=\ x^2\frac{1}{x\ln a}+\log_a x\,.(2x)-\ln a.\,(4x^3)$$

$$+6\{x\,e^x.\frac{1}{x}+\ x\ \ln x.\,e^x+e^x\ln x.1\}$$

$$=\ x(\frac{1}{\ln a}+2\log_a x-4x^2\ln a)$$

$$+6\,e^x(1+\ x\ \ln x+\ln x)$$

3. (a) মনে করি, $y=\dfrac{x}{x^2+a^2}$

$$\frac{dy}{dx}=\frac{(x^2+a^2)\frac{d}{dx}(x)-x\frac{d}{dx}(x^2+a^2)}{(x^2+a^2)^2}$$

$$=\frac{(x^2+a^2).1-x(2x+0)}{(x^2+a^2)^2}=\frac{x^2+a^2-2x^2}{(x^2+a^2)^2}$$

$$\frac{d}{dx}(\frac{x}{x^2+a^2})=\frac{a^2-x^2}{(x^2+a^2)^2}$$

3(b) $\dfrac{d}{dx}(\dfrac{1-\tan x}{1+\tan x})$ **[দি.'১০; ব.'১৩]**

$$=\frac{(1+\tan x)\frac{d}{dx}(1-\tan x)-(1-\tan x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)(-\sec^2 x)-(1-\tan x)(\sec^2 x)}{(1+\tan x)^2}$$

$$=\frac{(-1-\tan x-1+\tan x)\sec^2 x}{(1+\tan x)^2}$$

$$=\frac{-2\sec^2 x}{(1+\tan x)^2} \text{ (Ans.)}$$

3(c) $\dfrac{d}{dx}(\dfrac{1+\sin x}{1+\cos x})=$ **[কু.'০৪]**

$$\frac{(1+\cos x)\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}=$$

$$\frac{(1+\cos x)(\cos x)-(1+\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$=\frac{\cos x+\cos^2 x+\sin x+\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{\cos x+\sin x+1}{(1+\cos x)^2} \text{ (Ans.)}$$

3(d) $\dfrac{1+\sin x}{1-\sin x}$

[ঢা.'১৩; ব. '০৭; রা.'০৯; চ.'১২; দি.'১৪]

$$\frac{d}{dx}(\frac{1+\sin x}{1-\sin x})=$$

$$\frac{(1-\sin x)\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$$

$$=\frac{(1-\sin x)(\cos x)-(1+\sin x)\frac{d}{dx}(-\cos x)}{(1-\sin x)^2}$$

$$=\frac{(1-\sin x+1+\sin x)\cos x}{(1-\sin x)^2}$$

$$=\frac{2\cos x}{(1-\sin x)^2} \text{ (Ans.)}$$

3(e) $\dfrac{\cos x-\cos 2x}{1-\cos x}$ [ব.'১০; রা., কু.'০৮; য.'১৩; ঢা.'১৪]

$$\frac{\cos x-\cos 2x}{1-\cos x}=\frac{\cos x-(2\cos^2 x-1)}{1-\cos x}$$

$$=\frac{1+\cos x-2\cos^2 x}{1-\cos x}$$

$$=\frac{(1-\cos x)(1+2\cos x)}{1-\cos x}=1+2\cos x$$

$$\frac{d}{dx}\left(\frac{\cos x-\cos 2x}{1-\cos x}\right)=-2\sin x$$

3(f) $\dfrac{\sin x+\cos x}{\sqrt{1+\sin 2x}}$ [ঢা.'০৯; ব.'০৯,'১১; য.'১৪]

$$\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}=\frac{\sin x+\cos x}{\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}}$$

$$=\frac{\sin x+\cos x}{\sqrt{(\sin x+\cos x)^2}}=\frac{\sin x+\cos x}{\sin x+\cos x}=1$$

$$\frac{d}{dx}\left(\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\right)=0 \text{ (Ans.)}$$

3(g) ধরি, $y=\dfrac{x\ln x}{\sqrt{1+x^2}}$ [প্র.ভ.প.'০৫]

$$\frac{dy}{dx}=\frac{\sqrt{1+x^2}\frac{d}{dx}(x\ln x)-x\ln x\frac{d}{dx}(\sqrt{1+x^2})}{(\sqrt{1+x^2})^2}$$

$$=\frac{1}{1+x^2}\left[\sqrt{1+x^2}\left(x.\frac{1}{x}+\ln x\right)-x\ln x\frac{2x}{2\sqrt{1+x^2}}\right]$$

$$=\frac{1}{1+x^2}\left[\frac{(1+x^2)(1+\ln x)-x^2\ln x}{\sqrt{1+x^2}}\right]$$

$$\frac{d}{dx}\left(\frac{x\ln x}{\sqrt{1+x^2}}\right)=\frac{1+x^2+\ln x}{(\sqrt{1+x^2})^3}$$

বিকল্প পদ্ধতি ঃ ধরি, $y=\dfrac{x\ln x}{\sqrt{1+x^2}}$

$$\frac{dy}{dx}=\frac{x\ln x}{\sqrt{1+x^2}}\left[\frac{1}{x}\frac{d}{dx}(x)+\frac{1}{\ln x}\frac{d}{dx}(\ln x)-\frac{1}{\sqrt{1+x^2}}\frac{d}{dx}(\sqrt{1+x^2})\right]$$

$$=\frac{x\ln x}{\sqrt{1+x^2}}\left[\frac{1}{x}+\frac{1}{\ln x}.\frac{1}{x}-\frac{1}{\sqrt{1+x^2}}.\frac{2x}{2\sqrt{1+x^2}}\right]$$

$$=\frac{x\ln x}{\sqrt{1+x^2}}\;\frac{\ln x(1+x^2)+1+x^2-x^2\ln x}{x(1+x^2)\ln x}$$

$$\frac{d}{dx}\left(\frac{x\ln x}{\sqrt{1+x^2}}\right)=\frac{1+x^2+\ln x}{(\sqrt{1+x^2})^3} \text{ (Ans.)}$$

প্রশ্নমালা IX E

1.(a) $(1+\sin 2x)^2$ [চ.'০৪]

ধরি, $y=(1+\sin 2x)^2$

$$\frac{dy}{dx}=2(1+\sin 2x)\frac{d}{dx}(1+\sin 2x)$$

$$=2(1+\sin 2x)(0+\cos 2x)\frac{d}{dx}(2x)$$

$$=2(1+\sin 2x)\cos 2x\,(2.1)$$

$$\frac{d}{dx}\{(1+\sin 2x)^2\}=4\cos 2x(1+\sin 2x)$$

1(b) a^{px+q} [চ'০১]

ধরি, $y=a^{px+q}$

$$\frac{dy}{dx}=a^{px+q}.\ln a\frac{d}{dx}(px+q)$$

$$\left[\because \frac{d}{dx}(a^x)=a^x\ln a\right]$$

$$=a^{px+q}.\ln a\,(p.1+0)$$

$$\frac{d}{dx}(a^{px+q})=p\,a^{px+q}.\ln a \text{ (Ans.)}$$

1(c) $a^{\cos x}$ [চ.'০০]

$$\frac{d}{dx}(a^{\cos x}) = a^{\cos x}.\ln a.\frac{d}{dx}(\cos x)$$
$$= a^{\cos x}.\ln a.(-\sin x)$$
$$= -a^{\cos x}\sin x.\ln a$$

1(d) $10^{\ln(\sin x)}$ [সি.'০২ '০৫; চ.'০৭]

ধরি, $y = 10^{\ln(\sin x)}$

$$\frac{dy}{dx} = 10^{\ln(\sin x)}.\ln 10\ \frac{d}{dx}\{\ln(\sin x)\}$$
$$= 10^{\ln(\sin x)}.\ln 10\ \frac{1}{\sin x}\frac{d}{dx}(\sin x)$$
$$= 10^{\ln(\sin x)}.\ln 10\ \frac{1}{\sin x}(\cos x)$$
$$\frac{d}{dx}\{10^{\ln(\sin x)}\} = 10^{\ln(\sin x)}.\ln 10.\cot x$$

1(e) $10^{\ln(\tan x)}$

ধরি, $y = 10^{\ln(\tan x)}$

$$\frac{dy}{dx} = 10^{\ln(\tan x)}.\ln 10\ \frac{d}{dx}\{\ln(\tan x)\}$$
$$= 10^{\ln(\tan x)}.\ln 10\ \frac{1}{\tan x}\frac{d}{dx}(\tan x)$$
$$= 10^{\ln(\sin x)}.\ln 10\ \frac{\cos x}{\sin x}(\sec^2 x)$$
$$= 10^{\ln(\sin x)}.\ln 10\ \frac{\cos x}{\sin x}.\frac{1}{\cos^2 x}$$
$$= 10^{\ln(\sin x)}.\ln 10\ \frac{2}{2\sin x\cos x}$$
$$= 10^{\ln(\sin x)}.\ln 10\ \frac{2}{\sin 2x}$$
$$= 2\operatorname{cosec} 2x.10^{\ln(\sin x)}.\ln 10$$

1(f) $a^{\ln(\cos x)}$ [রা.'০৫]

ধরি, $y = a^{\ln(\cos x)}$

$$\frac{dy}{dx} = a^{\ln(\cos x)}.\ln a\ \frac{d}{dx}\{\ln(\cos x)\}$$
$$= a^{\ln(\cos x)}.\ln a\frac{1}{\cos x}\frac{d}{dx}(\cos x)$$
$$= a^{\ln(\cos x)}.\ln a\ \frac{1}{\cos x}(-\sin x)$$
$$\frac{d}{dx}\{a^{\ln(\cos x)}\} = -\tan x\, a^{\ln(\cos x)}\ln a$$

1(g) $e^{2\ln(\tan 5x)}$ [ব.'০৬,'১১; কু.'০৭; সি.'১০,'১৩]

$$e^{2\ln(\tan 5x)} = e^{\ln(\tan 5x)^2} = (\tan 5x)^2$$
$$\frac{d}{dx}\{e^{2\ln(\tan 5x)}\} = 2\tan 5x\frac{d}{dx}(\tan 5x)$$
$$= 2\tan 5x(\sec^2 5x)\frac{d}{dx}(5x)$$
$$= 2\tan 5x\sec^2 5x(5)$$
$$= 10\tan 5x\sec^2 5x$$

1(h) $(\ln\sin x^2)^n$ [সি.'০৬; রা.'০৯]

ধরি, $y = (\ln\sin x^2)^n$

$$\frac{dy}{dx} = n(\ln\sin x^2)^{n-1}\frac{d}{dx}(\ln\sin x^2)$$
$$= n(\ln\sin x^2)^{n-1}.\frac{1}{\sin x^2}\frac{d}{dx}(\sin x^2)$$
$$= n(\ln\sin x^2)^{n-1}.\frac{1}{\sin x^2}(\cos x^2)(2x)$$
$$\frac{d}{dx}\{(\ln\sin x^2)^n\} = nx\cot x^2(\ln\sin x^2)^{n-1}$$

1(i) $\cos(e^{\tan^2 2x})$

$$\frac{d}{dx}\{\cos(e^{\tan^2 2x})\} = \frac{d\{\cos(e^{\tan^2 2x})\}}{d(e^{\tan^2 2x})}$$
$$\frac{d(e^{\tan^2 2x})}{d(\tan^2 2x)}\frac{d(\tan^2 2x)}{d(\tan 2x)}\frac{d(\tan 2x)}{d(2x)}\frac{d(2x)}{dx}$$
$$= -\sin(e^{\tan^2 2x}).e^{\tan^2 2x}.2\tan 2x.\sec^2 2x.2$$
$$= -4\tan 2x\sec^2 2x\sin(e^{\tan^2 2x})\ e^{\tan^2 2x}$$

1(j) $\frac{d}{dx}(\sin^3 x^2)$ [চ.'০৯]

$$= \frac{d(\sin x^2)^3}{d(\sin x^2)}\frac{d(\sin x^2)}{d(x^2)}\frac{d(x^2)}{dx}$$
$$= 3(\sin x^2)^2.\cos x^2.2x$$
$$= 6x\sin^2 x^2\cos x^2 \quad \text{(Ans.)}$$

1(k) $e^{5\ln(\tan x)}$ [চ.'১২]

$$= e^{\ln(\tan x)^5} = (\tan x)^5$$

$$\frac{d}{dx}\{e^{5\ln(\tan x)}\} = 5\tan^4 x\frac{d}{dx}(\tan x)$$
$$= 5\tan^4 x\sec^2 x$$

1(*l*) $x^n\ln(2x)$ [চ.'০৭]

মনে করি, $y = x^n\ln(2x)$

$$\frac{dy}{dx} = x^n\frac{d}{dx}\{ln(2x)\} + ln(2x)\frac{d}{dx}(x^n)$$
$$= x^n\frac{1}{2x}\frac{d}{dx}(2x) + ln(2x).nx^{n-1}$$
$$= x^{n-1}\frac{1}{2}.(2) + nx^{n-1}ln(2x)$$
$$\frac{d}{dx}\{x^n\ln(2x)\} = x^{n-1}\{1 + n\,ln(2x)\}$$

1(m) $x\sqrt{\sin x}$ [ঢা.'০৮]

মনে করি, $y = x\sqrt{\sin x} = x(\sin x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = x\frac{d}{dx}\{(\sin x)^{\frac{1}{2}}\} + (\sin x)^{\frac{1}{2}}\frac{d}{dx}(x)$$
$$= x.\frac{1}{2}(\sin x)^{-\frac{1}{2}}\frac{d}{dx}(\sin x) + \sqrt{\sin x}.1$$
$$= \frac{1}{2}x\frac{1}{\sqrt{\sin x}}(\cos x) + \sqrt{\sin x}$$
$$\frac{d}{dx}(x\sqrt{\sin x}) = \frac{x\cos x + 2\sin x}{2\sqrt{\sin x}}$$

1(n) $e^{ax}\tan^2 x$ [ঢা.'০৯]

মনে করি, $y = e^{ax}\tan^2 x$

$$\frac{dy}{dx} = e^{ax}\frac{d}{dx}(\tan^2 x) + \tan^2 x\frac{d}{dx}(e^{ax})$$
$$= e^{ax}(2\tan x)\frac{d}{dx}(\tan x) + \tan^2 x.e^{ax}(a)$$
$$= e^{ax}\tan x(2\sec^2 x + a\tan x) \text{ (Ans.)}$$

2.(a) $ln(\cos x)$ [রা.'০৩,'০৫,'১০]

$$\frac{d}{dx}\{ln(\cos x)\} = \frac{1}{\cos x}\frac{d}{dx}(\cos x)$$
$$= \frac{1}{\cos x}(-\sin x) = -\tan x \text{ (Ans.)}$$

$$\frac{d}{dx}\{ln(e^x + e^{-x})\} = \frac{1}{e^x + e^{-x}}\frac{d}{dx}(e^x + e^{-x})$$
$$= \frac{1}{e^x + e^{-x}}(e^x - e^{-x}) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

2(c) $\log_x a$ [রা.'০১; চ.'০৬;'০৮]

$$\log_x a = \log_x e\times\log_e a = ln\,a\frac{1}{\log_e x}$$
$$= ln\,a\frac{1}{\ln x} = ln\,a\,(ln\,x)^{-1}$$
$$\therefore \frac{d}{dx}(\log_x a) = ln\,a\{-1(ln\,x)^{-2}\frac{d}{dx}(ln\,x)\}$$
$$= -ln\,a\frac{1}{(\ln x)^2}.\frac{1}{x} = -\frac{\ln a}{x(\ln x)^2}$$

2(d) $\log_{10} 3x$ [য.'০৬,'১৩]

$$\log_{10} 3x = \log_{10} e\times\log_e 3x = \frac{1}{\log_e 10}\ln(3x)$$
$$\frac{d}{dx}(\log_{10} 3x) = \frac{1}{\ln 10}\frac{1}{3x}\frac{d}{dx}(3x)$$
$$= \frac{1}{\ln 10}\frac{1}{3x}(3.1) = \frac{1}{x\ln 10} \text{ (Ans.)}$$

2(e) $\log_a x + \log_x a$

$$= \log_a e\times\log_e x + \log_x e\times\log_e a$$
$$= \frac{1}{\log_e a}\times\ln x + \frac{1}{\log_e x}\times\ln a$$
$$= \frac{1}{\ln a}\times\ln x + \ln a\times(\ln x)^{-1}$$
$$\frac{d}{dx}(\log_a x + \log_x a)$$
$$= \frac{1}{\ln a}\frac{1}{x} + \ln a\times\{-1(\ln x)^{-2}\frac{1}{x}\}$$
$$= \frac{1}{x\ln a} - \frac{\ln a}{x(\ln x)^2}$$

2(f) ধরি, $y = \log_x \tan x = \log_x e\times\log_e \tan x$

$$= \frac{1}{\log_e x}\times\ln(\tan x) = \frac{\ln(\tan x)}{\ln x}$$

$$\therefore \frac{dy}{dx} = \frac{\ln x \frac{d}{dx}\{\ln(\tan x)\} - \ln(\tan x)\frac{d}{dx}(\ln x)}{(\ln x)^2}$$

$$= \frac{\ln x \frac{1}{\tan x}\sec^2 x - \ln(\tan x).\frac{1}{x}}{(\ln x)^2}$$

$$= \frac{\ln x \frac{\cos x}{\sin x}\frac{1}{\cos^2 x} - \frac{1}{x}\ln(\tan x)}{(\ln x)^2}$$

$$= \frac{\ln x \frac{2}{\sin 2x} - \frac{1}{x}\ln(\tan x)}{(\ln x)^2}$$

$$= \frac{2x\ln x \cos ec 2x - \ln(\tan x)}{x(\ln x)^2} \text{ (Ans.)}$$

2(g) $ln(\sin 2x)$ [ঢা.'১১; সি.'১৩]

$$\frac{d}{dx}\{ln(\sin 2x)\} = \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x)$$

$$= \frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x) = 2\cot 2x$$

(h) $\ln(\sin x^2)$ [রা.'১২]

$$\frac{d}{dx}\{\ln(\sin x^2)\} = \frac{1}{\sin x^2}\frac{d}{dx}(\sin x^2)$$

$$= \frac{1}{\sin x^2}(\cos x^2)\frac{d}{dx}(x^2) = 2x\cot x^2$$

3(a) $ln\ [x - \sqrt{x^2-1}]$ [রা. '০২; কু. '০৩; চ.'০৫]

$$\frac{d}{dx}\{ln\ (x - \sqrt{x^2-1})\}$$

$$= \frac{1}{x-\sqrt{x^2-1}}\frac{d}{dx}(x-\sqrt{x^2-1})$$

$$= \frac{1}{x-\sqrt{x^2-1}}\{1 - \frac{1}{2\sqrt{x^2-1}}(2x)\}$$

$$= \frac{1}{x-\sqrt{x^2-1}}\{\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}}\}$$

$$= -\frac{1}{\sqrt{x^2-1}} \text{ (Ans.)}$$

3(b) $ln\ [x - \sqrt{x^2+1}]$ [রা. '০২; কু. '০৩,'১০]

$$\frac{d}{dx}\{ln\ (x - \sqrt{x^2+1})\}$$

$$= \frac{1}{x-\sqrt{x^2+1}}\frac{d}{dx}(x-\sqrt{x^2+1})$$

$$= \frac{1}{x-\sqrt{x^2+1}}\{1 - \frac{1}{2\sqrt{x^2+1}}(2x)\}$$

$$= \frac{1}{x-\sqrt{x^2+1}}\{\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\}$$

$$= -\frac{1}{\sqrt{x^2+1}} \text{ (Ans.)}$$

3(c) $ln\ (\sqrt{x-a}+\sqrt{x-b})$ [কু.'০১]

$$\frac{d}{dx}\{ln\ (\sqrt{x-a}+\sqrt{x-b})\}$$

$$= \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\frac{d}{dx}(\sqrt{x-a}+\sqrt{x-b})$$

$$= \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\{\frac{1}{2\sqrt{x-a}} + \frac{1}{2\sqrt{x-b}}\}$$

$$= \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\{\frac{\sqrt{x-b}+\sqrt{x-a}}{2\sqrt{x-a}\sqrt{x-b}}\}$$

$$= \frac{1}{2\sqrt{(x-a)(x-b)}} \text{ (Ans.)}$$

3(d) $ln\left\{e^x\left(\frac{x-1}{x+1}\right)^{3/2}\right\}$ [চ.'০০]

ধরি, $y = ln\left\{e^x\left(\frac{x-1}{x+1}\right)^{3/2}\right\}$

$$= ln\, e^x + \frac{3}{2}\{ln\,(x-1) - ln\,(x+1)\}$$

$$= x + \frac{3}{2}\{ln\,(x-1) - ln\,(x+1)\}$$

$$\frac{dy}{dx} = 1 + \frac{3}{2}\{\frac{1}{x-1} - \frac{1}{x+1}\}$$

$$= 1 + \frac{3}{2}\{\frac{x+1-x+1}{(x-1)(x+1)}\}$$

$$= 1 + \frac{3}{2}\{\frac{2}{x^2-1}\} = \frac{x^2-1+3}{x^2-1}$$

$$= \frac{x^2+2}{x^2-1} \text{ (Ans.)}$$

4. (a) $\frac{\tan x - \cot x}{\tan x + \cot x}$ [চ.'০৭; য.'০৬]

$$\frac{\tan x - \cot x}{\tan x + \cot x} = \frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x} = \frac{-\cos 2x}{1} = -\cos 2x$$

$$\frac{d}{dx}(\frac{\tan x - \cot x}{\tan x + \cot x}) = \sin 2x.2 = 2\sin 2x$$

4(b) $\left(\frac{\sin 2x}{1+\cos 2x}\right)^2$ [কু.'০৩]

$$= \left(\frac{2\sin x \cos x}{2\cos^2 x}\right)^2 = \left(\frac{\sin x}{\cos x}\right)^2 = \tan^2 x$$

$$\frac{d}{dx}\left(\frac{\sin 2x}{1+\cos 2x}\right)^2 = 2\tan x \frac{d}{dx}(\tan x)$$

$$= 2\tan x \sec^2 x$$

4(c) $ln\sqrt{\frac{1-\cos x}{1+\cos x}}$ [ঢা.'০৭,'১৩; রা.'১১; কু.'১৪]

$$= ln\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}} = ln\sqrt{\tan^2\frac{x}{2}} = ln\tan\frac{x}{2}$$

$$\frac{d}{dx}\{ln\sqrt{\frac{1-\cos x}{1+\cos x}}\} = \frac{1}{\tan\frac{x}{2}}\sec^2\frac{x}{2}.\frac{1}{2}$$

$$= \frac{1}{2}\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\frac{1}{\cos^2\frac{x}{2}} = \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \frac{1}{\sin x} = \text{cosec } x \text{ (Ans.)}$$

4(d) $\sqrt{\frac{1+x}{1-x}}$ [প্র.ভ.প.'৮৩; রা.'১১]

ধরি, $y = \sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}}$

$$\frac{dy}{dx} = \frac{\sqrt{1-x}\frac{d}{dx}(\sqrt{1+x}) - \sqrt{1+x}\frac{d}{dx}(\sqrt{1-x})}{(\sqrt{1-x})^2}$$

$$= \frac{\sqrt{1-x}\frac{1}{2\sqrt{1+x}}.1 - \sqrt{1+x}\frac{1}{2\sqrt{1-x}}(-1)}{1-x}$$

$$= \frac{\sqrt{1-x}\frac{1}{2\sqrt{1+x}}.1 - \sqrt{1+x}\frac{1}{2\sqrt{1-x}}(-1)}{1-x}$$

$$= \frac{1-x+1+x}{2(1-x)\sqrt{(1+x)(1-x)}} = \frac{2}{2(1-x)\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\sqrt{\frac{1+x}{1-x}}) = \frac{1}{(1-x)\sqrt{1-x^2}}$$

4.(e) $\ln\sqrt[3]{\frac{1-\cos x}{1+\cos x}}$ [দি.'১২; প্র.ভ.প.'০৫]

$$= \ln\left(\frac{2\sin^2(x/2)}{2\cos^2(x/2)}\right)^{1/3} = \frac{1}{3}\ln\tan^2\frac{x}{2}$$

$$= \frac{2}{3}\ln\tan\frac{x}{2}$$

$$\frac{d}{dx}(\ln\sqrt[3]{\frac{1-\cos x}{1+\cos x}}) = \frac{2}{3}\frac{\sec^2(x/2)}{\tan(x/2)}.\frac{1}{2}$$

$$= \frac{1}{3}\frac{\cos\frac{x}{2}}{\cos^2\frac{x}{2}\sin\frac{x}{2}} = \frac{2}{3}\frac{1}{2\cos\frac{x}{2}\sin\frac{x}{2}}$$

$$= \frac{2}{3}\frac{1}{\sin x} = \frac{2}{3}\cos ecx$$

5. (a) $\sin^2[ln(\sec x)]$ [রা.'০৭,'১৩; কু.,সি., মা.বো.'০৯; চ.'১১; ঢা.'১২; য., দি.'১৩]

ধরি, $y = \sin^2[ln(\sec x)]$

$$\therefore \frac{dy}{dx} = \frac{d\{\sin[\ln(\sec x)]\}^2}{d\{\sin[\ln(\sec x)]\}}\frac{d\{\sin[\ln(\sec x)]\}}{d\{\ln(\sec x)\}}$$

$$\frac{d\{\ln(\sec x)\}}{d(\sec x)}\frac{d(\sec x)}{dx}$$

$$= 2\sin[\ln(\sec x)]\cos[\ln(\sec x)]\frac{1}{\sec x}$$
$$\sec x \tan x$$

$$= \tan x \sin[2\ln(\sec x)]$$

5(b) $\sin^2\{ln(x^2)\}$

[য.'০৭,'০৮; চ.'০৬,'১৩; ঢা.,সি,'১৪]

$$\frac{d}{dx}[\sin^2\{ln(x^2)\}] = \frac{d[\sin\{\ln(x^2)\}]^2}{d[\sin\{\ln(x^2)\}]}$$
$$\frac{d[\sin\{\ln(x^2)\}]}{d[\ln(x^2)]}\frac{d[\ln(x^2)]}{d(x^2)}\frac{d(x^2)}{dx}$$

$$= 2\sin\{\ln(x^2)\}\cos\{\ln(x^2)\}\frac{1}{x^2}.2x$$

$$= \frac{2}{x}\sin\{2\ln(x^2)\} = \frac{2}{x}\sin\{4\ln(x)\}$$

5(c) $\sqrt{\sin\sqrt{x}}$ [চ.'০১; ঢা.'০৫,'০৭]

$$\frac{d}{dx}(\sqrt{\sin\sqrt{x}})$$

$$= \frac{d(\sqrt{\sin\sqrt{x}})}{d(\sin\sqrt{x})}\frac{d(\sin\sqrt{x})}{d(\sqrt{x})}\frac{d(\sqrt{x})}{dx}$$

$$= \frac{1}{2\sqrt{\sin\sqrt{x}}}\cos\sqrt{x}.\frac{1}{2\sqrt{x}}$$

$$= \frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}} \text{ (Ans.)}$$

5(d) cos (lnx)+ ln (tan$x$)

[ব.'০৩; সি.'০৬]

$$\frac{d}{dx}\{\cos(lnx) + ln(\tan x)\}$$

$$= \frac{d}{dx}\{\cos(lnx)\} + \frac{d}{dx}\{ln(\tan x)\}$$

$$= -\sin(lnx).\frac{1}{x} + \frac{1}{\tan x}.\sec^2 x$$

$$= -\frac{1}{x}\sin(lnx) + \frac{\cos x}{\sin x}\frac{1}{\cos^2 x}$$

$$= \frac{2}{2\sin x\cos x} - \frac{1}{x}\sin(lnx)$$

$$= 2\text{ cesec } 2x - \frac{1}{x}\sin(lnx)$$

5(e) 2cosec2x cos (ln tan$x$) [রা.'০৬]

$$\frac{d}{dx}\{2\text{ cosec } 2x\cos(ln\tan x)\}$$

$$= 2[\text{cosec } 2x\frac{d}{dx}\{\cos(ln\tan x)\} +$$
$$\cos(ln\tan x)\frac{d}{dx}(\text{cosec } 2x)]$$

$$= 2[\text{cosec } 2x\{-\sin(ln\tan x)\}.\frac{1}{\tan x}.$$
$$\sec^2 x + \cos(ln\tan x)(-\text{cosec}2x\cot 2x.\,.2)]$$

$$= 2[-\text{cosec } 2x\sin(ln\tan x)\}.\frac{\cos x}{\sin x}.$$
$$\frac{1}{\cos^2 x} - 2\text{cosec}2x\cot 2x\cos(ln\tan x)]$$

$$= 2[-\text{cosec } 2x\sin(ln\tan x)\}\frac{2}{2\sin x\cos x}$$
$$- 2\text{cosec}2x\cot 2x\cos(ln\tan x)]$$
$$= -4[\text{cosec}^2 2x\sin(ln\tan x)\}$$
$$+ \text{cosec } 2x\cot 2x\cos(ln\tan x)]$$

5(f) $\frac{d}{dx}\{1+\tan(1+\sqrt{x})\}^{1/3}$

$$= \frac{1}{3}\{1+\tan(1+\sqrt{x})\}^{\frac{1}{3}-1}\{0+\sec^2(1+\sqrt{x})\}$$
$$(0+\frac{1}{2\sqrt{x}})$$

$$= \frac{1}{6\sqrt{x}}\{1+\tan(1+\sqrt{x})\}^{-\frac{2}{3}}\sec^2(1+\sqrt{x})$$

5(g) $\frac{d}{dx}(\sqrt{\tan e^{x^2}})$ [য.'০৯]

$$= \frac{d(\sqrt{\tan e^{x^2}})}{d(\tan e^{x^2})}\frac{d(\tan e^{x^2})}{d(e^{x^2})}\frac{d(e^{x^2})}{d(x^2)}\frac{d(x^2)}{dx}$$