**1(a)** $\sin(-1230°) - \cos\{(2n+1)\pi + \frac{\pi}{3}\}$

$= -\sin 1230° - \cos\{2n\pi + (\pi + \frac{\pi}{3})\}$

$= -\sin(3.360° + 150°) - \cos(\pi + \frac{\pi}{3})$

$= -\sin 150° - (-\cos\frac{\pi}{3})$

$= -\sin(180° - 30°) + \cos\frac{\pi}{3}$

$= -\sin 30° + \cos\frac{\pi}{3} = -\frac{1}{2} + \frac{1}{2} = 0$ (Ans.)

**1(b)** $\sin 780° \cos 390° + \sin(-330°)\cos(-300°)$ [চ.'০১]

$= \sin 780° \cos 390° - \sin 330° \cos 300°$

$= \sin(2.360° + 60°)\cos(360° + 30°) - \sin(360° - 30°)\cos(360° - 60°)$

$= \sin 60° \cos 30° - (-\sin 30°)\cos 60°$

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$ (Ans.)

**2.** **মান নির্ণয় কর ঃ**

**(a)** $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$

[ঢা. '০২; সি. '০৯; মা.বো.'০৯; ব.'১০; য.'১১]

$= \sin^2\frac{\pi}{7} + \sin^2(\frac{\pi}{2} - \frac{\pi}{7}) + \sin^2(\pi + \frac{\pi}{7}) + \sin^2(\frac{\pi}{2} + \frac{\pi}{7})$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}$

$= 2(\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}) = 2.1 = 2$ (Ans.)

**2(b)** $\sin^2\frac{\pi}{12} + \sin^2\frac{3\pi}{12} + \sin^2\frac{5\pi}{12} + \sin^2\frac{7\pi}{12} + \sin^2\frac{9\pi}{12} + \sin^2\frac{11\pi}{12}$

$= \sin^2\frac{\pi}{12} + \sin^2\frac{3\pi}{12} + \sin^2\frac{5\pi}{12} + \sin^2(\frac{\pi}{2} + \frac{\pi}{12}) + \sin^2(\frac{\pi}{2} + \frac{3\pi}{12}) + \sin^2(\frac{\pi}{2} + \frac{5\pi}{12})$

$= \sin^2\frac{\pi}{12} + \sin^2\frac{3\pi}{12} + \sin^2\frac{5\pi}{12} + \cos^2\frac{\pi}{12} + \cos^2\frac{3\pi}{12} + \cos^2\frac{5\pi}{12}$

$= (\sin^2\frac{\pi}{12} + \cos^2\frac{\pi}{12}) + (\sin^2\frac{3\pi}{12} + \cos^2\frac{3\pi}{12}) + (\sin^2\frac{5\pi}{12} + \cos^2\frac{5\pi}{12})$

$= 1 + 1 + 1 = 3$ (Ans.)

**2.(c)** $\sin^2\frac{17\pi}{18} + \sin^2\frac{5\pi}{8} + \cos^2\frac{37\pi}{18} + \cos^2\frac{3\pi}{8}$

$= \sin^2(\pi - \frac{\pi}{18}) + \sin^2(\pi - \frac{3\pi}{8}) + \cos^2(2\pi + \frac{\pi}{18}) + \cos^2\frac{3\pi}{8}$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{3\pi}{8} + \cos^2\frac{\pi}{18} + \cos^2\frac{3\pi}{8}$

$= (\sin^2\frac{\pi}{18} + \cos^2\frac{\pi}{18}) + (\sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8})$

$= 1 + 1 = 2$ (Ans.)

**3.(a)** $\sec^2\frac{14\pi}{17} - \sec^2\frac{39\pi}{17} + \cot^2\frac{41\pi}{34} - \cot^2\frac{23\pi}{34}$

$= \sec^2(\pi - \frac{3\pi}{17}) - \sec^2(2\pi + \frac{5\pi}{17}) + \cot^2(\pi + \frac{7\pi}{34}) - \cot^2(\pi - \frac{11\pi}{34})$

$= \sec^2\frac{3\pi}{17} - \sec^2\frac{5\pi}{17} + \cot^2\frac{7\pi}{34} - \cot^2\frac{11\pi}{34}$

$= \sec^2\frac{3\pi}{17} - \sec^2\frac{5\pi}{17} + \cot^2(\frac{\pi}{2} - \frac{5\pi}{17}) - \cot^2(\frac{\pi}{2} - \frac{3\pi}{17})$

$= \sec^2\frac{3\pi}{17} - \sec^2\frac{5\pi}{17} + \tan^2\frac{5\pi}{17} - \tan^2\frac{3\pi}{17}$

$= (\sec^2\frac{3\pi}{17} - \tan^2\frac{3\pi}{17}) - (\sec^2\frac{5\pi}{17} - \tan^2\frac{5\pi}{17})$

$= 1 - 1 = 0$ (Ans.)

**3(b)** $\tan15°+\tan45°+\tan75°+\cdots+\tan165°$

$= \tan15°+\tan45°+\tan75°+\tan105°+\tan135°+\tan165°$

$= \tan15° + \tan45° + \tan(90°-15°) + \tan(90°+15°) + \tan(180-45°) + \tan(180°-15°)$

$= \tan15° + \tan45° + \cot15° - \cot15° - \tan45° - \tan15° = 0$ (Ans.)

**3(c)** $\cos^2 15° + \cos^2 25° + \cos^2 35° + \cdots\cdots + \cos^2 75°$

$= \cos^2 15° + \cos^2 25° + \cos^2 35° + \cos^2 45° + \cos^2 55° + \cos^2 65° + \cos^2 75°$

$= \cos^2 15° + \cos^2 25° + \cos^2 35° + (\frac{1}{\sqrt{2}})^2 + \cos^2(90°-35°) + \cos^2(90°-25°) + \cos^2(90°-15°)$

$= \cos^2 15° + \cos^2 25° + \cos^2 35° + \frac{1}{2} + \sin^2 35° + \sin^2 25° + \sin^2 15°$

$= \sin^2 5° + \cos^2 5°) + (\sin^2 25° + \cos^2 25°) + (\sin^2 35° + \cos^2 35°) + \frac{1}{2}$

$= 1 + 1 + 1 + \frac{1}{2} = 3 + \frac{1}{2} = \frac{7}{2}$ (Ans.)

**4a)** **প্রমাণ ঃ** দেওয়া আছে , **[দি.'১৪;' য.'১২; চ.'০৯]**

$\sin\theta = \frac{5}{13}$ এবং $\frac{\pi}{2} < \theta < \pi$

$\text{cosec}\theta = \frac{13}{5}$ , $\cos\theta = -\sqrt{1-\sin^2\theta}$

$= -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$

$\sec\theta = -\frac{13}{12}$ এবং

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{13} \times (-\frac{13}{12}) = -\frac{5}{12}$

$\Rightarrow \cot\theta = -\frac{12}{5}$

এখন , $\frac{\tan\theta + \sec(-\theta)}{\cot\theta + \text{cos}ec(-\theta)} = \frac{\tan\theta + \sec\theta}{\cot\theta - \text{cos}ec\theta}$

$= \dfrac{\frac{-5}{12} + \frac{-13}{12}}{\frac{-12}{5} - \frac{13}{5}} = \dfrac{\frac{-5-13}{12}}{\frac{-12-13}{5}}$

$= (-\frac{18}{12}) \times (-\frac{5}{25}) = \frac{3}{2} \times \frac{1}{5} = \frac{3}{10}$

$\therefore \frac{\tan\theta + \sec(-\theta)}{\cot\theta + \text{cos}ec(-\theta)} = \frac{3}{10}$

**4.(b)** যেহেতু $\cot\theta = \frac{3}{4} \Rightarrow \tan\theta = \frac{4}{3}$ এবং $\cos\theta$ ঋণাত্মক

$\therefore \sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\frac{16}{9}}$

$= -\sqrt{\frac{25}{9}} = -\frac{5}{3}$

$\therefore \cos\theta = -\frac{3}{5}$ এবং

$\sin\theta = \tan\theta \cos\theta = \frac{4}{3} \times (-\frac{3}{5}) = -\frac{4}{5}$

$\therefore \text{cosec}\theta = -\frac{5}{4}$

এখন , $\frac{\cot(-\theta) + \text{cos}ec\theta}{\cos\theta + \sin(-\theta)} = \frac{-\cot\theta + \text{cos}ec\theta}{\cos\theta - \sin\theta}$

$= \dfrac{-\frac{3}{4} + (-\frac{5}{4})}{-\frac{3}{5} - \frac{-4}{5}} = \frac{-3-5}{4} \times \frac{5}{-3+4}$

$= -\frac{40}{4} = -10$ (Ans.)

**5. সমাধান ঃ**

**(a)** $\sin x + \sin(\pi + x) + \sin(2\pi + x) + \cdots\cdots$ (n +1)তম পদ পর্যন্ত

$= \sin x - \sin x + \sin x - \sin x + \cdots\cdots$ (n +1) তম পদ পর্যন্ত

$n = 1$ হলে, $(1+1)$ বা ২য় পদ পর্যন্ত যোগফল
$= \sin x - \sin x = 0$
$n = 3$ হলে, $(3+1)$ বা ৪র্থ পদ পর্যন্ত
যোগফল $= \sin x - \sin x + \sin x - \sin x = 0$
তদ্রূপ, n যেকোন বিজোড় সংখ্যা হলে নির্ণেয় যোগফল $= 0$
আবার, $n = 2$ হলে $(2+1)$ বা ৩য় পদ পর্যন্ত যোগফল
$= \sin x - \sin x + \sin x = \sin x$
$n = 4$ হলে, $(4+1)$ বা ৫ম পদ পর্যন্ত যোগফল
$= \sin x - \sin x + \sin x - \sin x + \sin x$
$= \sin x$
তদ্রূপ, n যেকোন জোড় সংখ্যা হলে নির্ণেয় যোগফল $= \sin x$

**5(b)** $\tan\theta + \tan(\pi+\theta) + \tan(2\pi+\theta) + \cdots + \tan(n\pi+\theta)$
$= \tan\theta + \tan\theta + \tan\theta + \cdots$ n তম পদ পর্যন্ত
$= (n+1)\tan\theta$ (Ans.)

**6(a)** দেওয়া আছে, $\theta = \frac{\pi}{20} \Rightarrow \frac{\pi}{2} = 10\theta$

L.H.S.$= \cot\theta \cot 3\theta \cot 5\theta \cot 7\theta \cot 9\theta \cot 11\theta \cot 13\theta \cot 15\theta \cot 17\theta \cot 19\theta$
$= \cot\theta \cot 3\theta \cot 5\theta \cot 7\theta \cot 9\theta \cot(10\theta+\theta) \cot(10\theta+3\theta) \cot(10\theta+5\theta) \cot(10\theta+7\theta) \cot(10\theta+9\theta)$
$= \cot\theta \cot 3\theta \cot 5\theta \cot 7\theta \cot 9\theta \cot(\frac{\pi}{2}+\theta) \cot(\frac{\pi}{2}+3\theta) \cot(\frac{\pi}{2}+5\theta) \cot(\frac{\pi}{2}+7\theta) \cot(\frac{\pi}{2}+9\theta)$
$= \frac{1}{\tan\theta \tan 3\theta \tan 5\theta \tan 7\theta \tan 9\theta}(-\tan\theta)(-\tan 3\theta)(-\tan 5\theta)(-\tan 7\theta)(-\tan 9\theta)$
$= -1 =$ R.H.S.

**6. (b)** দেওয়া আছে, $\theta = \frac{\pi}{28} \Rightarrow \frac{\pi}{2} = 14\theta$

L.H.S $= \tan\theta \tan 3\theta \tan 5\theta \tan 7\theta \tan 9\theta \tan 11\theta \tan 13\theta$
$= \tan\theta \tan 3\theta \tan 5\theta \tan 7\theta \tan(14\theta-5\theta) \tan(14\theta-3\theta) \tan(14\theta-\theta)$
$= \frac{1}{\tan\theta \tan 3\theta \tan 5\theta} \tan\frac{\pi}{4} \tan(\frac{\pi}{2}-5\theta) \tan(\frac{\pi}{2}-3\theta) \tan(\frac{\pi}{2}-\theta)$
$= \frac{1}{\tan\theta \tan 3\theta \tan 5\theta}.1.\tan 5\theta.\tan 3\theta.\tan\theta$
$= 1 =$ R.H.S.

**6(c)** $\tan\theta.\tan 2\theta.\tan 3\theta. \quad \tan(2n-1)\theta$
এখানে, পদসংখ্যা $= 2n-1$, যা বিজোড় সংখ্যা।
$\frac{2n-1+1}{2}$ অর্থাৎ n তম পদ মধ্যপদ।
$\therefore$ মধ্যপদ $= \tan n\theta = \tan\frac{\pi}{4} = 1$ [$\because 4n\theta = \pi$]
$\tan\theta.\tan(2n-1)\theta = \tan\theta.\tan(2n\theta-\theta)$
$= \tan\theta.\tan(\frac{\pi}{2}-\theta)$ [$\because 4n\theta = \pi$]
$= \tan\theta.\cot\theta = 1$
$\tan 2\theta.\tan(2n-2)\theta = \tan 2\theta.\tan(2n\theta-2\theta)$
$= \tan 2\theta.\tan(\frac{\pi}{2}-2\theta)$
$= \tan 2\theta.\cot 2\theta = 1$
অনুরূপভাবে, $\tan 3\theta.\tan(2n-3)\theta = 1$
$\tan 4\theta.\tan(2n-4)\theta = 1, \cdots$ ইত্যাদি।
অর্থাৎ, মধ্যপদ হতে সমদূরবর্তী পদ দুইটির গুণফল $= 1$
$\therefore \tan\theta.\tan 2\theta.\tan 3\theta.\cdots\cdots\tan(2n-1)\theta = 1$

## অতিরিক্ত প্রশ্ন (সমাধানসহ)

**1.** মান নির্ণয় কর :

**(a)** $\tan(-1590°) = -\tan(1590°)$
$= -\tan(4.360° + 150°) = -\tan 150°$
$= -\tan(180° - 30°) = +\tan 30° = \frac{1}{\sqrt{3}}$

**(b)** $\cos 420° \sin(-300°) - \sin 870° \cos 570°$
$= \cos 420°(-\sin 300°) - \sin 870° \cos 570°$
$= -\cos(360° + 60°) \sin(360° - 60°)$

$-\sin(2.360° + 150°)\cos(2.360° - 150°)$

$= -\cos 60°(-\sin 60°) - \sin 150° \cos 150°$

$= \cos 60° \sin 60° - \sin(180° - 30°) \cos(180° - 30°)$

$= \cos 60° \sin 60° - \sin 30° (-\cos 30°)$

$= \frac{1}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{\sqrt{3}}{2} = 2.\frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (Ans.)

2. $\cos^2\frac{\pi}{24} + \cos^2\frac{19\pi}{24} + \cos^2\frac{31\pi}{24} + \cos^2\frac{37\pi}{24}$

$= \cos^2\frac{\pi}{24} + \cos^2\frac{19\pi}{24} + \cos^2(\frac{\pi}{2} + \frac{19\pi}{24}) + \cos^2(3.\frac{\pi}{2} + \frac{\pi}{24})$

$= \cos^2\frac{\pi}{24} + \cos^2\frac{19\pi}{24} + \sin^2\frac{\pi}{24} + \sin^2\frac{19\pi}{24}$

$= (\sin^2\frac{\pi}{24} + \cos^2\frac{\pi}{24}) + (\sin^2\frac{19\pi}{24} + \cos^2\frac{19\pi}{24})$

$= 1 + 1 = 2$ (Ans.)

3(a) $\cos^2 25° + \cos^2 35° + \cos^2 45° + \cos^2 55° + \cos^2 65°$

$= \cos^2 25° + \cos^2 35° + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2(90° - 35°) + \cos^2(90° - 25°)$

$= \cos^2 25° + \cos^2 35° + \frac{1}{2} + \sin^2 35° + \sin^2 25°$

$= (\sin^2 25° + \cos^2 25°) + \frac{1}{2} + (\sin^2 25° + \cos^2 25°)$

$= 1 + \frac{1}{2} + 1 = \frac{5}{2}$ (Ans.)

3(b) $\sin^2 10° + \sin^2 20° + \sin^2 30° + \cdots + \sin^2 80°$

$= \sin^2 10° + \sin^2 20° + \sin^2 30° + \sin^2 40° + \sin^2 50° + \sin^2 60° + \sin^2 70° + \sin^2 80°$

$= \sin^2 10° + \sin^2 20° + \sin^2 30° + \sin^2 40° + \sin^2(90° - 40°) + \sin^2(90° - 30°) + \sin^2(90° - 20°) + \sin^2(90° - 10°)$

$= \sin^2 10° + \sin^2 20° + \sin^2 30° + \sin^2 40° + \cos^2 40° + \cos^2 30° + \cos^2 20° + \cos^2 10°$

$= (\sin^2 10° + \cos^2 10°) + (\sin^2 20° + \cos^2 20°) + (\sin^2 30° + \cos^2 30°) + (\sin^2 40° + \cos^2 40°)$

$= 1 + 1 + 1 + 1 = 4$ (Ans.)

4. $\tan\theta = \frac{3}{4}$ এবং $\cos\theta$ ঋণাত্মক হলে, $\frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta}$ এর মান নির্ণয় কর ।

সমাধান ঃ দেওয়া আছে ,

$\tan\theta = \frac{3}{4}$ এবং $\cos\theta$ ঋণাত্মক

$\therefore \sec\theta = -\sqrt{1 + \tan^2\theta} = -\sqrt{1 + \frac{9}{16}}$

$= -\sqrt{\frac{25}{16}} = -\frac{5}{4}$ $\quad \therefore \cos\theta = -\frac{4}{5}$ এবং

$\sin\theta = \tan\theta \cos\theta = \frac{3}{4}(-\frac{4}{5}) = -\frac{3}{5}$

এখন , $\frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} = \frac{-\frac{3}{5} - \frac{4}{5}}{-\frac{5}{4} + \frac{3}{4}}$

$= -\frac{3+4}{5} \times \frac{4}{-5+3} = -\frac{7}{5} \times \frac{4}{-2} = \frac{14}{5}$ (Ans.)

5. $\sin\theta = \frac{12}{13}$ এবং $90° < \theta < 180°$ হলে দেখাও যে, $\frac{\tan\theta + \sec(-\theta)}{\cot\theta + \operatorname{cosec}(-\theta)} = \frac{10}{3}$

প্রমাণ ঃ যেহেতু $\sin\theta = \frac{12}{13} \Rightarrow \operatorname{cosec}\theta = \frac{13}{12}$ এবং $90° < \theta < 180°$,

$\therefore \cos\theta = -\sqrt{1 - \sin^2\theta}$

$$= -\sqrt{1-\frac{144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$$

$$\sec\theta = -\frac{13}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12}{13}\times\left(-\frac{13}{5}\right) = -\frac{12}{5}$$

$$\Rightarrow \cot\theta = -\frac{5}{12}$$

এখন , $\dfrac{\tan\theta+\sec(-\theta)}{\cot\theta+\operatorname{cosec}(-\theta)} = \dfrac{\tan\theta+\sec\theta}{\cot\theta-\operatorname{cosec}\theta}$

$$= \frac{-\frac{12}{5}-\frac{13}{5}}{-\frac{5}{12}-\frac{13}{12}} = \frac{-25}{5}\times\frac{12}{-5-13}$$

$$= 5\times\frac{12}{18} = \frac{10}{3}$$

**6. যোগফল নির্ণয় কর :** $\cos\theta + \cos(\pi+\theta) + \cos(2\pi+\theta) + \cdots + \cos(n\pi+\theta)$

**সমাধান:** $\cos\theta + \cos(\pi+\theta) + \cos(2\pi+\theta) + \cdots + \cos(n\pi+\theta)$

$= \cos\theta + \{-\cos\theta + \cos\theta - \cos\theta + \cdots + (-1)^n\cos\theta\}$

$n = 2$ হলে যোগফল $= \cos\theta + \{-\cos\theta + \cos\theta\}$
$= \cos\theta$

$n = 4$ হলে যোগফল $= \cos\theta + \{-\cos\theta + \cos\theta - \cos\theta + \cos\theta\} = \cos\theta$

তদ্রূপ, n যেকোন জোড় হলে নির্ণেয় যোগফল $= \cos x$

$n = 1$ হলে যোগফল $= \cos\theta + (-\cos\theta) = 0$

$n = 3$ হলে যোগফল $= \cos\theta + \{-\cos\theta + \cos\theta - \cos\theta\} = 0$

তদ্রূপ, n যেকোন বিজোড় হলে নির্ণেয় যোগফল = 0

**7.** $n \in \mathbb{Z}$ হলে , $\sin\{n\pi + (-1)^n\frac{\pi}{4}\}$ এর মান নির্ণয় কর ।

**সমাধান : (a)** $\sin\{n\pi + (-1)^n\frac{\pi}{4}\}$

n জোড় সংখ্যা হলে মনে করি, $n = 2m$ , যেখানে $m\in\mathbb{N}$.

$\therefore \sin\{n\pi + (-1)^n\frac{\pi}{4}\}$

$= \sin\{2m\pi + (-1)^{2m}\frac{\pi}{4}\}$

$= \sin(2m\pi + \frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

n বিজোড় সংখ্যা হলে মনে করি , $n = 2m+1$; $m\in\mathbb{N}$.

$\therefore \sin\{n\pi + (-1)^n\frac{\pi}{4}\}$

$= \sin\{(2m+1)\pi + (-1)^{2m+1}\frac{\pi}{4}\}$

$= \sin\{2m\pi + (\pi - \frac{\pi}{4})\}$

$= \sin(\pi - \frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ (Ans.)

**8.** দেখাও যে ,$\tan\frac{\pi}{12}\tan\frac{5\pi}{12}\tan\frac{7\pi}{12}\tan\frac{11\pi}{12} = 1$

**প্রমাণ:** $\tan\frac{\pi}{12}\tan\frac{5\pi}{12}\tan\frac{7\pi}{12}\tan\frac{11\pi}{12}$

$= \tan\frac{\pi}{12}\tan\frac{5\pi}{12}\tan(\frac{\pi}{2}-\frac{\pi}{12})\tan(\frac{\pi}{2}-\frac{5\pi}{12})$

$= \tan\frac{\pi}{12}\tan\frac{5\pi}{12}\cot\frac{\pi}{12}\cot\frac{5\pi}{12}$

$= (\tan\frac{\pi}{12}.\cot\frac{\pi}{12})(\tan\frac{5\pi}{12}.\cot\frac{5\pi}{12})$

$= 1.1 = 1$ $[\because \tan\theta.\cot\theta = 1]$

## প্রশ্নমালা VII B

**1. মান নির্ণয় কর : (a)** $\tan 105°$ **(b)** $\cot 165°$ **(c)** $\operatorname{cosec} 165°$

**(a)** $\tan 105° = \tan(60°+45°)$

$$= \frac{\tan 60^0 + \tan 45^0}{1-\tan 60^0\tan 45^0} = \frac{\sqrt{3}+1}{1-\sqrt{3}.1}$$

$$= \frac{(1+\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{1+2\sqrt{3}+3}{1-3}$$

$$= \frac{2(\sqrt{3}+2)}{-2} = -(\sqrt{3}+2)$$

1(b) $\cot 165° = \cot(90° + 75°) = -\tan 75°$

$= -\tan(30° + 45°) = -\dfrac{\tan 30^0 + \tan 45^0}{1 - \tan 30^0 \tan 45^0}$

$= -\dfrac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}.1} = -\dfrac{1+\sqrt{3}}{\sqrt{3}-1} = -\dfrac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= -\dfrac{3+2\sqrt{3}+1}{3-1} = -\dfrac{2(\sqrt{3}+2)}{2} = -(\sqrt{3}+2)$

1(c) $\operatorname{cosec} 165° = \operatorname{cosec}(90° + 75°)$

$= \sec 75° = \dfrac{1}{\cos 75^0} = \dfrac{1}{\cos(45^0 + 30^0)}$

$= \dfrac{1}{\cos 45^0 \cos 30^0 - \sin 45^0 \sin 30^0}$

$= \dfrac{1}{\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}} = \dfrac{2\sqrt{2}}{\sqrt{3}-1}$

$= \dfrac{2\sqrt{2}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \dfrac{2(\sqrt{6}+\sqrt{3})}{3-1}$

$= \dfrac{2(\sqrt{6}+\sqrt{3})}{2} = \sqrt{6}+\sqrt{3}$

2. মান নির্ণয় কর ঃ

(a) $\cos 38°15' \sin 68°15' - \cos 51°45' \sin 21°45'$

$= \cos 38°15' \sin 68°15' - \cos(90° - 38°15') \sin(90° - 68°15')$

$= \cos 38°15' \sin 68°15' - \sin 38°15' \cos 68°15'$

$= \sin(68°15' - 38°15') = \sin 30° = \dfrac{1}{2}$

2(b) $\cos 69°22' \cos 9°22' + \cos 80°38' \cos 20°38'$

$= \cos 69°22' \cos 9°22' + \cos(90° - 9°22') \cos(90° - 69°22')$

$= \cos 69°22' \cos 9°22' + \sin 9°22' \sin 69°22'$

$= \cos(69°22' - 9°22') = \cos 60° = \dfrac{1}{2}$

3. প্রমাণ কর যে ,

**(a) L.H.S.** $= \sin(25° + A)\cos(25° - A) + \cos(25° + A)\cos(115° - A)$

$= \sin(25° + A)\cos(25° - A) + \cos(25° + A)\cos\{90° + (25° - A)\}$

$= \sin(25° + A)\cos(25° - A) - \cos(25° + A)\sin(25° - A)$

$= \sin\{(25° + A) - (25° - A)\}$

$= \sin(25° + A - 25° + A)$

$= \sin 2A =$ R.H.S. (Proved)

3(b) $\cos(\frac{\pi}{3} - \alpha)\cos(\frac{\pi}{6} - \beta) - \sin(\frac{\pi}{3} - \alpha)\sin(\frac{\pi}{6} - \beta)$

$= \cos\{(\frac{\pi}{3} - \alpha) + (\frac{\pi}{6} - \beta)\}$

$= \cos\{(\frac{\pi}{3} + \frac{\pi}{6}) - (\alpha + \beta)\}$

$= \cos\{\frac{\pi}{2} - (\alpha + \beta)\}$

$= \sin(\alpha + \beta) =$ R.H.S. (Proved)

**3(c) L.H.S.**$= \sin(n+1)x \cos(n-1)x - \cos(n+1)x \sin(n-1)x$

$= \sin\{(n+1)x - (n-1)x\}$

$= \sin(nx + x - nx + x)$

$= \sin 2x =$ R.H.S. (Proved)

4. প্রমাণ কর যে,

(a) L.H.S.$= \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B)$

$= \sin A(\sin B \cos C - \sin C \cos B) + \sin B(\sin C \cos A - \sin A \cos C) + \sin C(\sin A \cos B - \sin B \cos A)$

$= \sin A \sin B \cos C - \sin A \cos B \sin C + \cos A \sin B \sin C - \sin A \sin B \cos C + \sin A \cos B \sin C - \cos A \sin B \sin C$

$= 0 =$ R.H.S. (Proved)

**4(b)** **L.H.S.** $= \sin(B+C)\sin(B-C) + \sin(C+A)\sin(C-A) + \sin(A+B)\sin(A-B)$

$= \sin^2B - \sin^2C + \sin^2C - \sin^2A + \sin^2A - \sin^2B$

$= 0 =$ R.H.S. (Proved)

**4(c)** **L.H.S.** $= \sin(135° - A) + \cos(135° + A)$

$= \sin\{180° - (45° + A)\} + \cos\{180° - (45° - A)\}$

$= \sin(45° + A) - \cos(45° - A)$

$= \sin(45° + A) - \cos\{90° - (45° + A)\}$

$= \sin(45° + A) - \sin(45° + A)$

$= 0 =$ R.H.S. (Proved)

**5. প্রমাণ কর যে,**

**(a)** **L.H.S.**$= \dfrac{\cos 15^0 + \sin 15^0}{\cos 15^0 - \sin 15^0}$

$= \dfrac{\cos 15^0 (1 + \dfrac{\sin 15^0}{\cos 15^0})}{\cos 15^0 (1 - \dfrac{\sin 15^0}{\cos 15^0})} = \dfrac{1 + \tan 15^0}{1 - \tan 15^0}$

$= \dfrac{\tan 45^0 + \tan 15^0}{1 - \tan 45^0 \tan 15^0} = \tan(45° + 15°)$

$= \tan 60° = \sqrt{3} =$ R.H.S. (Proved)

**5(b)** **L.H.S.**$= \dfrac{\cos 25^0 - \sin 25^0}{\cos 25^0 + \sin 25^0}$

$= \dfrac{\cos 25^0 (1 - \dfrac{\sin 25^0}{\cos 25^0})}{\cos 25^0 (1 + \dfrac{\sin 25^0}{\cos 25^0})} = \dfrac{1 - \tan 25^0}{1 - \tan 25^0}$

$= \dfrac{\tan 45^0 - \tan 25^0}{1 + \tan 45^0 \tan 25^0} = \tan(45° - 25°)$

$= \tan 20° =$ R.H.S. (proved)

**5(c)** **L.H.S.**$= \dfrac{\sin 75^0 + \sin 15^0}{\sin 75^0 - \sin 15^0}$

$= \dfrac{\sin(90^0 - 15^0) + \sin 15^0}{\sin(90^0 - 15^0) - \sin 15^0}$

$= \dfrac{\cos 15^0 + \sin 15^0}{\cos 15^0 - \sin 15^0} = \dfrac{\cos 15^0 (1 + \dfrac{\sin 15^0}{\cos 15^0})}{\cos 15^0 (1 - \dfrac{\sin 15^0}{\cos 15^0})}$

$= \dfrac{1 + \tan 15^0}{1 - \tan 15^0} = \dfrac{\tan 45^0 + \tan 15^0}{1 - \tan 45^0 \tan 15^0}$

$= \tan(45° + 15°) = \tan 60° = \sqrt{3}$

**6. প্রমাণ কর যে,**

**(a)** $\tan\dfrac{\pi}{4} = \tan(\dfrac{\pi}{20} + \dfrac{\pi}{5})$

$\Rightarrow 1 = \dfrac{\tan\dfrac{\pi}{20} + \tan\dfrac{\pi}{5}}{1 - \tan\dfrac{\pi}{20}\tan\dfrac{\pi}{5}}$

$\Rightarrow \tan\dfrac{\pi}{20} + \tan\dfrac{\pi}{5} = 1 - \tan\dfrac{\pi}{20}\tan\dfrac{\pi}{5}$

$\therefore \tan\dfrac{\pi}{20} + \tan\dfrac{\pi}{5} + \tan\dfrac{\pi}{20}\tan\dfrac{\pi}{5} = 1$

**6(b)** $\tan 70° = \tan(50° + 20°)$ [চ.'০৫; ঢা.'১০; প্র.ভ.প.'০৩]

$\Rightarrow \tan 70° = \dfrac{\tan 50^0 + \tan 20^0}{1 - \tan 50^0 \tan 20^0}$

$\Rightarrow \tan 70° - \tan 70° \tan 50° \tan 20° = \tan 50° + \tan 20°$

$\Rightarrow \tan 70° - \tan(90° - 20°)\tan 50° \tan 20° = \tan 50° + \tan 20°$

$\Rightarrow \tan 70° - \cot 20° \tan 50° \tan 20° = \tan 50° + \tan 20°$

$\Rightarrow \tan 70° - \tan 50° = \tan 50° + \tan 20°$

$\therefore \tan 70° = \tan 20° + 2\tan 50°$

**6(c)** $\tan(A - B) = -\tan(B - A)$

$= -\tan\{(B - C) + (C - A)\}$

$= -\dfrac{\tan(B-C) + \tan(C-A)}{1 - \tan(B-C)\tan(C-A)}$

$\Rightarrow \tan(A - B) - \tan(A - B)\tan(B - C)$

$\tan(C-A) = -\tan(B-C) - \tan(C-A)$

$\tan(B-C) + \tan(C-A) + \tan(A-B)$
$= \tan(B-C)\tan(C-A)\tan(A-B)$

**7(a) L.H.S.** $= 2\sin(\theta + \frac{\pi}{4})\sin(\theta - \frac{\pi}{4})$

$= 2(\sin\theta \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\theta)$

$(\sin\theta\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\theta)$

$= 2(\sin\theta . \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cos\theta)$

$(\sin\theta . \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\cos\theta)$

$= 2. \frac{1}{2}(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$

$= \sin^2\theta - \cos^2\theta =$ R.H.S. (Proved)

**বিকল্প পদ্ধতি:** L.H.S.$= 2\sin(\theta + \frac{\pi}{4})\sin(\theta - \frac{\pi}{4})$

$= 2(\sin^2\theta - \sin^2\frac{\pi}{4})$

$[\because \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B]$

$= 2(\sin^2\theta - \frac{1}{2}) = 2\sin^2\theta - 1$

$= 2\sin^2\theta - (\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta =$ R.H.S. (Proved)

**7(b)** L.H.S.$= \tan(A+B)\tan(A-B)$

$= \frac{\sin(A+B)\sin(A-B)}{\cos(A+B)\cos(A-B)}$

$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B} =$ R.H.S.

**7.(c) L.H.S.**$= \frac{\tan(\frac{\pi}{4}+\theta) - \tan(\frac{\pi}{4}-\theta)}{\tan(\frac{\pi}{4}+\theta) + \tan(\frac{\pi}{4}-\theta)}$

$= \{\frac{\sin(\frac{\pi}{4}+\theta)}{\cos(\frac{\pi}{4}+\theta)} - \frac{\sin(\frac{\pi}{4}-\theta)}{\cos(\frac{\pi}{4}-\theta)}\} \div$

$\{\frac{\sin(\frac{\pi}{4}+\theta)}{\cos(\frac{\pi}{4}+\theta)} + \frac{\sin(\frac{\pi}{4}-\theta)}{\cos(\frac{\pi}{4}-\theta)}\}$

$= \frac{\sin(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}-\theta) - \cos(\frac{\pi}{4}+\theta)\sin(\frac{\pi}{4}-\theta)}{\cos(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}-\theta)} \times$

$\frac{\sin(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}-\theta) + \cos(\frac{\pi}{4}+\theta)\sin(\frac{\pi}{4}-\theta)}{\cos(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}-\theta)}$

$= \frac{\sin(\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta)}{\sin(\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta)} = \frac{\sin 2\theta}{\sin\frac{\pi}{2}}$

$= \sin 2\theta =$ R.H.S. (Proved)

**8. (a)** $a\cos(x+\alpha) = b\cos(x-\alpha)$ **হলে দেখাও যে,** $(a+b)\tan x = (a-b)\cot\alpha$ **[ঢা.'০৫]**

**প্রমাণ ঃ** দেওয়া আছে, $a\cos(x+\alpha) = b\cos(x-\alpha)$

$\Rightarrow a(\cos x\cos\alpha - \sin x\sin\alpha)$

$= b(\cos x\cos\alpha + \sin x\sin\alpha)$

$\Rightarrow (a-b)\cos x\cos\alpha = (a+b)\sin x\sin\alpha$

$\Rightarrow (a+b)\frac{\sin x}{\cos x} = (a-b)\frac{\cos\alpha}{\sin\alpha}$

$\therefore (a+b)\tan x = (a-b)\cot\alpha$

**8(b)** $a\sin(x+\theta) = b\sin(x-\theta)$ **হলে দেখাও যে,** $(a+b)\tan\theta + (a-b)\tan x = 0$

**প্রমাণ ঃ** দেওয়া আছে, $a\sin(x+\theta) = b\sin(x-\theta)$

$\Rightarrow a(\sin x\cos\theta + \sin\theta\cos x)$

$= b(\sin x\cos\theta - \sin\theta\cos x)$

$\Rightarrow (a-b)\sin x\cos\theta = -(a+b)\sin\theta\cos x$

$\Rightarrow (a-b)\frac{\sin x}{\cos x} = -(a+b)\frac{\sin\theta}{\cos\theta}$

$\Rightarrow (a-b)\tan x = -(a+b)\tan\theta$

$\therefore (a+b)\tan\theta + (a-b)\tan x = 0$

**8.(c)** $\theta$ **কোণকে** $\alpha$ **এবং** $\beta$ **এই দুই অংশে এমন ভাবে বিভক্ত করা হল যেন,** $\tan\alpha : \tan\beta = x : y$ **হয়।**

**দেখাও যে, $\sin(\alpha-\beta) = \dfrac{x-y}{x+y}\sin\theta$**

**প্রমাণ ঃ** দেওয়া আছে , $\theta = \alpha+\beta$ এবং

$\tan\alpha \quad \tan\beta = x : y$

$$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{x}{y} \Rightarrow \frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta} = \frac{x+y}{x-y}$$

$$\Rightarrow \tan\alpha+\tan\beta = \frac{x+y}{x-y}(\tan\alpha-\tan\beta)$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta} = \frac{x+y}{x-y}\left(\frac{\sin\alpha}{\cos\alpha}-\frac{\sin\beta}{\cos\beta}\right)$$

$$\Rightarrow \frac{\sin\alpha\cos\beta+\sin\beta\cos\alpha}{\cos\alpha\cos\beta} = \frac{x+y}{x-y}\left(\frac{\sin\alpha\cos\beta-\sin\beta\cos\alpha}{\cos\alpha\cos\beta}\right)$$

$$\Rightarrow \sin(\alpha+\beta) = \frac{x+y}{x-y}\sin(\alpha-\beta)$$

$$\Rightarrow \sin\theta = \frac{x+y}{x-y}\sin(\alpha-\beta)$$

$$\sin(\alpha-\beta) = \frac{x-y}{x+y}\sin\theta$$

**8(d) $\tan\theta+\sec\theta = \dfrac{x}{y}$ হলে দেখাও যে,**

$$\sin\theta = \frac{x^2-y^2}{x^2+y^2}$$

**প্রমাণ :** দেওয়া আছে , $\tan\theta+\sec\theta = \dfrac{x}{y}$

$$\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta} = \frac{x}{y} \Rightarrow \frac{1+\sin\theta}{\cos\theta} = \frac{x}{y}$$

$$\Rightarrow \frac{1+2\sin\theta+\sin^2\theta}{\cos^2\theta} = \frac{x^2}{y^2}$$ [উভয় পক্ষকে বর্গ করে।]

$$\Rightarrow \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{1+2\sin\theta+\sin^2\theta-\cos^2\theta} = \frac{x^2+y^2}{x^2-y^2}$$

[ যোজন-বিয়োজন করে।]

$$\Rightarrow \frac{1+2\sin\theta+(\sin^2\theta+\cos^2\theta)}{(1-\cos^2\theta)+2\sin\theta+\sin^2\theta} = \frac{x^2+y^2}{x^2-y^2}$$

$$\Rightarrow \frac{1+2\sin\theta+1}{\sin^2\theta+2\sin\theta+\sin^2\theta} = \frac{x^2+y^2}{x^2-y^2}$$

$$\Rightarrow \frac{2(1+\sin\theta)}{2\sin\theta(1+\sin\theta)} = \frac{x^2+y^2}{x^2-y^2}$$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{x^2+y^2}{x^2-y^2}$$

$$\therefore \sin\theta = \frac{x^2-y^2}{x^2+y^2}$$ (Showed)

**8.(e) $\sin(A+B) = n\sin(A-B)$ এবং $n \neq 1$ হলে দেখাও যে, $\cot A = \dfrac{n-1}{n+1}\cot B$**

**প্রমাণ ঃ** দেওয়া আছে , $\sin(A+B) = n\sin(A-B)$

$$\Rightarrow \frac{\sin(A+B)}{\sin(A-B)} = n$$

$$\Rightarrow \frac{\sin(A+B)+\sin(A-B)}{\sin(A+B)-\sin(A-B)} = \frac{n+1}{n-1}$$

[ যোজন-বিয়োজন করে।]

$$\Rightarrow \frac{2\sin A\cos B}{2\sin B\cos A} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\cot B}{\cot A} = \frac{n+1}{n-1}$$

$$\therefore \cot A = \frac{n-1}{n+1}\cot B$$

**9. (a) $a\sin(\theta+\alpha) = b\sin(\theta+\beta)$ হলে দেখাও যে, $\cot\theta = \dfrac{a\cos\alpha-b\cos\beta}{b\sin\beta-a\sin\alpha}$** [য .'০৫]

**প্রমাণ:** দেওয়া আছে, $a\sin(\theta+\alpha) = b\sin(\theta+\beta)$

$$\Rightarrow a(\sin\theta\cos\alpha+\sin\alpha\cos\theta) = b(\sin\theta\cos\beta+\sin\beta\cos\theta)$$

$$\Rightarrow a\sin\theta\cos\alpha - b\sin\theta\cos\beta = b\sin\beta\cos\theta - a\sin\alpha\cos\theta$$

$$\Rightarrow (a\cos\alpha-b\cos\beta)\sin\theta = (b\sin\beta-a\sin\alpha)\cos\theta$$

$$\therefore \cot\theta = \frac{a\cos\alpha-b\cos\beta}{b\sin\beta-a\sin\alpha}$$ (Showed)

**9.(b) $\sin\theta = k\cos(\theta-\alpha)$ হলে দেখাও যে,**

$$\cot\theta = \frac{1+k\sin\alpha}{k\cos\alpha}$$ [কু.'১২]

প্রমাণ : দেওয়া আছে, $\sin\theta = k\cos(\theta-\alpha)$

$\Rightarrow \sin\theta = k(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

$\Rightarrow \sin\theta + k\sin\theta\sin\alpha = k\cos\theta\cos\alpha$

$\Rightarrow (1+k\sin\alpha)\sin\theta = k\cos\theta\cos\alpha$

$\Rightarrow \frac{1+k\sin\alpha}{k\cos\alpha} = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \frac{1+k\sin\alpha}{k\cos\alpha}$

**9(c) $\cot\alpha + \cot\beta = a$, $\tan\alpha + \tan\beta = b$ এবং $\alpha+\beta = \theta$ হলে দেখাও যে, $(a-b)\tan\theta = ab$**

[ঢা.'০১,'১১; য.'০১; ব.'০৮]

প্রমাণ : দেওয়া আছে,

$\cot\alpha + \cot\beta = a\cdots(1)$, $\tan\alpha + \tan\beta = b\cdots(2)$

এবং $\alpha+\beta = \theta\cdots\cdots(3)$

(1) হতে আমরা পাই, $\frac{1}{\tan\alpha} + \frac{1}{\tan\beta} = a$

$\Rightarrow \frac{\tan\beta+\tan\alpha}{\tan\alpha\tan\beta} = a$

$\Rightarrow \frac{b}{\tan\alpha\tan\beta} = a \Rightarrow \tan\alpha\tan\beta = \frac{b}{a}$

এখন, $\theta = \alpha+\beta$

$\Rightarrow \tan\theta = \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \frac{b}{1-\frac{b}{a}} = \frac{ab}{a-b}$

$\therefore (a-b)\tan\theta = ab$

**9(d) $\frac{\sin(\alpha+\theta)}{\sin\alpha} = \frac{2\sin(\beta+\theta)}{\sin\beta}$ হলে দেখাও যে, $\cot\alpha - \cot\theta = 2\cot\beta$** [কু.'১২]

প্রমাণ : দেওয়া আছে, $\frac{\sin(\alpha+\theta)}{\sin\alpha} = \frac{2\sin(\beta+\theta)}{\sin\beta}$

$\Rightarrow \sin\beta.\sin(\alpha+\theta) = 2\sin\alpha.\sin(\beta+\theta)$

$\Rightarrow (\sin\alpha\cos\theta + \cos\alpha\sin\theta)\sin\beta$

$= 2\sin\alpha(\sin\beta\cos\theta + \sin\theta\cos\beta)$

$\Rightarrow \sin\alpha\cos\theta\sin\beta + \cos\alpha\sin\theta\sin\beta$

$= 2\sin\alpha\sin\beta\cos\theta + 2\sin\alpha\sin\theta\cos\beta$

$\Rightarrow \cos\alpha\sin\theta\sin\beta - \sin\alpha\sin\beta\cos\theta$

$= 2\sin\alpha\sin\theta\cos\beta$

ধরি, $\sin\theta\sin\alpha\sin\beta \neq 0$ এবং উভয় পক্ষকে $\sin\theta\sin\alpha\sin\beta$ দ্বারা ভাগ করে আমরা পাই,

$\frac{\cos\alpha}{\sin\alpha} - \frac{\cos\theta}{\sin\theta} = 2\frac{\cos\beta}{\sin\beta}$

$\therefore \cot\alpha - \cot\theta = 2\cot\beta$

**10. $A+B = \frac{\pi}{4}$ হলে দেখাও যে, $(1+\tan A)(1+\tan B) = 2$**

প্রমাণ : দেওয়া আছে, $A+B = \frac{\pi}{4}$

$\Rightarrow \tan(A+B) = \tan\frac{\pi}{4} \Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B} = 1$

$\Rightarrow \tan A + \tan B = 1 - \tan A\tan B$

$\Rightarrow \tan A + \tan B + \tan A\tan B + 1 = 2$

$\Rightarrow 1(1+\tan A) + \tan B(1+\tan A) = 2$

$\therefore (1+\tan A)(1+\tan B) = 2$ (Showed)

**11.(a) $\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$ হলে প্রমাণ কর যে, $1+\cot\alpha\tan\beta = 0$** [য.'০৭]

প্রমাণ : দেওয়া আছে,

$\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$

$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta = 1$

$\Rightarrow \cos(\alpha+\beta) = 1 \Rightarrow \cos(\alpha+\beta) = \cos 0$

$\therefore \alpha+\beta = 0 \Rightarrow \beta = -\alpha$

এখন, L.H.S. $= 1 + \cot\alpha\tan(-\alpha)$

$= 1 + \frac{1}{\tan\alpha}(-\tan\alpha) = 1 - 1 = 0 =$ R.H.S.

**11. (b) $\tan\beta = \frac{2\sin\alpha\sin\gamma}{\sin(\alpha+\gamma)}$ হলে দেখাও যে, $\frac{1}{\tan\alpha} + \frac{1}{\tan\gamma} = \frac{2}{\tan\beta}$.**

প্রমাণ : দেওয়া আছে, $\tan\beta = \frac{2\sin\alpha\sin\gamma}{\sin(\alpha+\gamma)}$

$\Rightarrow \frac{\sin\beta}{\cos\beta} = \frac{2\sin\alpha\sin\gamma}{\sin(\alpha+\gamma)}$

$\Rightarrow \sin\beta(\sin\alpha\cos\gamma + \sin\gamma\cos\alpha)$

$= 2\sin\alpha\cos\beta\sin\gamma$

$\Rightarrow \sin\alpha\sin\beta\cos\gamma + \cos\alpha\sin\beta\sin\gamma = 2\sin\alpha\cos\beta\sin\gamma$

ধরি, $\sin\alpha\sin\beta\sin\gamma \neq 0$ এবং উভয় পক্ষকে $\sin\alpha\sin\beta\sin\gamma$ দ্বারা ভাগ করে আমরা পাই,

$$\frac{\cos\gamma}{\sin\gamma}+\frac{\cos\alpha}{\sin\alpha}=2\frac{\cos\beta}{\sin\beta}$$

$\Rightarrow \cot\gamma + \cot\alpha = 2\cot\beta$

$\therefore \frac{1}{\tan\alpha}+\frac{1}{\tan\gamma}=\frac{2}{\tan\beta}$ (Showed)

**11(c)** $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$ **হলে দেখাও যে, $\tan(\alpha-\beta)=(1-n)\tan\alpha$**

প্রমাণ ঃ $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$ .........(1)

এখন, $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$$=\frac{\frac{\sin\alpha}{\cos\alpha}-\frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}{1+\frac{\sin\alpha}{\cos\alpha}.\frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}}$$

$$=\frac{\frac{\sin\alpha}{\cos\alpha}(1-\frac{n\cos^2\alpha}{1-n\sin^2\alpha})}{1+\frac{n\sin^2\alpha}{1-n\sin^2\alpha}}$$

$$=\tan\alpha(\frac{1-n\sin^2\alpha-n\cos^2\alpha}{1-n\sin^2\alpha})\times\frac{1-n\sin^2\alpha}{1-n\sin^2\alpha+n\sin^2\alpha}$$

$$=\tan\alpha\,\frac{1-n(\sin^2\alpha+\cos^2\alpha)}{1}$$

$\therefore \tan(\alpha-\beta)=(1-n)\tan\alpha$ (Showed)

**12(a)** $\tan\alpha-\tan\beta = x$ **এবং** $\cot\beta-\cot\alpha=y$ **হলে দেখাও যে,** $\cot(\alpha-\beta)=\dfrac{1}{x}+\dfrac{1}{y}$.

প্রমাণ ঃ দেওয়া আছে, $\tan\alpha-\tan\beta=x$ এবং $\cot\beta-\cot\alpha=y$

এখন, $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{\tan\alpha-\tan\beta}+\dfrac{1}{\cot\beta-\cot\alpha}$

$$=\frac{1}{\frac{1}{\cot\alpha}-\frac{1}{\cot\beta}}+\frac{1}{\cot\beta-\cot\alpha}$$

$$=\frac{\cot\alpha\cot\beta}{\cot\beta-\cot\alpha}+\frac{1}{\cot\beta-\cot\alpha}$$

$$=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}=\cot(\alpha-\beta)$$

$\therefore \cot(\alpha-\beta)=\dfrac{1}{x}+\dfrac{1}{y}$ (Showed)

**(b)** $\tan\theta=\dfrac{x\sin\varphi}{1-x\cos\varphi}$ **এবং** $\tan\varphi=\dfrac{y\sin\theta}{1-y\cos\theta}$ **হলে দেখাও যে,** $\dfrac{\sin\theta}{\sin\varphi}=\dfrac{x}{y}$.

প্রমাণ ঃ দেওয়া আছে, $\tan\theta=\dfrac{x\sin\varphi}{1-x\cos\varphi}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta}=\dfrac{x\sin\varphi}{1-x\cos\varphi}$

$\Rightarrow x\cos\theta\sin\varphi = \sin\theta - x\sin\theta\cos\varphi$

$\Rightarrow x(\cos\theta\sin\varphi+\sin\theta\cos\varphi)=\sin\theta$

$\Rightarrow x\cos(\theta+\varphi)=\sin\theta \Rightarrow x=\dfrac{\sin\theta}{\sin(\theta+\varphi)}$

এবং $\tan\varphi=\dfrac{y\sin\theta}{1-y\cos\theta}\Rightarrow\dfrac{\sin\varphi}{\cos\varphi}=\dfrac{y\sin\theta}{1-y\cos\theta}$

$\Rightarrow y(\sin\theta\cos\varphi+\sin\varphi\cos\theta)=\sin\varphi$

$\Rightarrow y=\dfrac{\sin\varphi}{\sin(\theta+\varphi)}$

এখন, $\dfrac{x}{y}=\dfrac{\sin\theta}{\sin(\theta+\varphi)}\times\dfrac{\sin(\theta+\varphi)}{\sin\varphi}=\dfrac{\sin\theta}{\sin\varphi}$

$\therefore \dfrac{\sin\theta}{\sin\varphi}=\dfrac{x}{y}$ (Showed)

**13.(a)** $\sin x+\sin y=a$ এবং $\cos x+\cos y=b$ হলে প্রমাণ কর যে, $\sin\frac{1}{2}(x-y)=\pm\frac{1}{2}\sqrt{4-a^2-b^2}$

প্রমাণ ঃ দেওয়া আছে, $\sin x+\sin y=a$

$\Rightarrow \sin^2 x+\sin^2 y+2\sin x\sin y=a^2 \cdots (1)$

এবং $\cos x+\cos y=b$

$\Rightarrow \cos^2 x+\cos^2 y+2\cos x\cos y=b^2 \cdots (2)$

(1) ও (2) যোগ করে পাই,

$(\sin^2 x+\cos^2 x)+(\sin^2 y+\cos^2 y)+$
$2(\cos x\cos y+\sin x\sin y)=a^2+b^2$

$\Rightarrow 1+1+2\cos(x-y)=a^2+b^2$

$\Rightarrow 2\{1+\cos(x-y)\}=a^2+b^2$

$\Rightarrow 2\{2\cos^2\frac{1}{2}(x-y)\}=a^2+b^2$

$\Rightarrow 4\{1-\sin^2\frac{1}{2}(x-y)\}=a^2+b^2$

$\Rightarrow 4\sin^2\frac{1}{2}(x-y)=4-a^2+b^2$

$\Rightarrow \sin^2\frac{1}{2}(x-y)=\frac{1}{4}(4-a^2-b^2)$

$\sin\frac{1}{2}(x-y)=\pm\frac{1}{2}\sqrt{4-a^2-b^2}$

**13(b) cos (α − β) cos γ = cos ( α − γ + β ) হলে দেখাও যে, cot α , cot γ এবং cot β সমান্তর প্রগমন ভুক্ত।**

**প্রমাণ ঃ** $\cos(\alpha-\beta)\cos\gamma=\cos(\alpha-\gamma+\beta)$

$\Rightarrow \cos(\alpha-\beta)\cos\gamma-\cos\{(\alpha+\beta)-\gamma\}=0$

$\Rightarrow \cos(\alpha-\beta)\cos\gamma-\{\cos(\alpha+\beta)\cos\gamma+\sin(\alpha+\beta)\sin\gamma\}=0$

$\Rightarrow \{\cos(\alpha-\beta)-\cos(\alpha+\beta)\}\cos\gamma-\sin(\alpha+\beta)\sin\gamma\}=0$

$\Rightarrow 2\sin\alpha\sin\beta\cos\gamma-(\sin\alpha\cos\beta+\sin\beta\cos\alpha)\sin\gamma=0$

$\Rightarrow 2\sin\alpha\sin\beta\cos\gamma-\sin\alpha\cos\beta\sin\gamma-\sin\beta\cos\alpha\sin\gamma=0$

$\Rightarrow 2\cot\gamma-\cos\beta-\cot\alpha=0$

[ উভয় পক্ষকে $\sin\alpha\sin\beta\sin\gamma$ দ্বারা ভাগ করে]

$\Rightarrow \cot\gamma-\cos\beta=\cot\alpha-\cot\gamma$

$\Rightarrow \cot\alpha-\cot\gamma=\cot\gamma-\cos\beta$

$\cot\alpha$ , $\cot\gamma$ একং $\cot\beta$ সমান্তর প্রগমন ভুক্ত।

**13(c) cos (β − γ) + cos (γ − α) + cos (α − β) $=-\frac{3}{2}$ হলে দেখাও যে, Σ cosα = 0 এবং Σsinα = 0**

**প্রমাণ ঃ** দেওয়া আছে ,

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$

$\Rightarrow 2(\cos\beta\cos\gamma+\sin\beta\sin\gamma+\cos\gamma\cos\alpha+\sin\gamma\sin\alpha+\cos\alpha\cos\beta+\sin\alpha\sin\beta)=-3$

$\Rightarrow 2(\cos\alpha\cos\beta+\cos\beta\cos\gamma+\cos\gamma\cos\alpha)+2(\sin\alpha\sin\beta+\sin\beta\sin\gamma+\sin\gamma\sin\alpha)+1+1+1=0$

$\Rightarrow 2(\cos\alpha\cos\beta+\cos\beta\cos\gamma+\cos\gamma\cos\alpha)+2(\sin\alpha\sin\beta+\sin\beta\sin\gamma+\sin\gamma\sin\alpha)+(\sin^2\alpha+\cos^2\alpha)+(\sin^2\beta+\cos^2\beta)+(\sin^2\gamma+\cos^2\gamma)=0$

$\Rightarrow\{\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2(\cos\alpha\cos\beta+\cos\beta\cos\gamma+\cos\gamma\cos\alpha)\}+\{\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2(\sin\alpha\sin\beta+\sin\beta\sin\gamma+\sin\gamma\sin\alpha)\}=0$

$\Rightarrow(\cos\alpha+\cos\beta+\cos\gamma)^2+(\sin\alpha+\sin\beta+\sin\gamma)^2=0$

$\therefore \cos\alpha+\cos\beta+\cos\gamma=0$ এবং

$\sin\alpha+\sin\beta+\sin\gamma=0$

[∵ দুইটি সংখ্যার বর্গের সমষ্টি শূন্য হলে সংখ্যা দুইটি পৃথক পৃথক ভাবে শূন্য হয় । ]

$\therefore \sum\cos\alpha=0$ এবং $\sum\sin\alpha=0$

## অতিরিক্ত প্রশ্ন (সমাধানসহ)

**1. মান নির্ণয় কর ঃ**

**(a)** $\sin 76°40'\cos 16°40'-\cos 73°20'\sin 13°20'$

$=\sin76°40'\cos16°40'-\cos(90°-16°40')\sin(90°-76°40')$

$=\sin 76°40'\cos 16°40'-\sin16°40'\cos 76°40'$

$=\sin(76°40'-16°40')=\sin60°=\frac{\sqrt{3}}{2}$

**(b)** $\cos 17°40'\sin 77°40'+\cos 107°40'\sin 12°20'$

$=\cos 17°40'\sin 77°40'+$

$\cos(90° + 17°40') \sin(90° - 77°40')$

$= \cos 17°40' \sin 77°40' - \sin 17°40' \cos 77°40'$

$= \sin(77°40' - 17°40') = \sin 60° = \frac{\sqrt{3}}{2}$

**(c)** $\frac{\tan 68^0 35' - \cot 66^0 25'}{1 + \tan 68^0 35' \cot 66^0 25'}$

$= \frac{\tan 68^0 35' - \cot(90^0 - 23^0 35')}{1 + \tan 68^0 35' \cot(90^0 - 23^0 35')}$

$= \frac{\tan 68^0 35' - \tan 23^0 35'}{1 + \tan 68^0 35' \tan 23^0 35'}$

$= \tan(68°35' - 23^0 35') = \tan 45° = 1$ (Ans.)

**প্রমাণ কর যে,**

**2.** $\cos(A - B)\cos(A - C) + \sin(A - B)\sin(A - C) = \cos(B - C)$

**L.H.S.** $= \cos(A - B)\cos(A - C) + \sin(A - B)\sin(A - C)$

$= \cos\{(A - B) - (A - C)\}$

$= \cos(A - B - A + C) = \cos(-B + C)$

$= \cos(B - C) =$ R.H.S. (Proved)

**3.** $\frac{\cot(3A - B)\cot B - 1}{-\cot B - \cot(3A - B)} = -\cot 3A$

**L.H.S.** $= \frac{\cot(3A - B)\cot B - 1}{-\cot B - \cot(3A - B)}$

$= \frac{\cot(3A - B)\cot B - 1}{-\{\cot B + \cot(3A - B)\}}$

$= -\frac{\cot(3A - B)\cot B - 1}{\cot B + \cot(3A - B)}$

$= -\cot(3A - B + B) = -\cot 3A$

= R.H.S. (Proved)

**4.** $\cos A + \cos(\frac{2\pi}{3} - A) + \cos(\frac{2\pi}{3} + A) = 0$

**L.H.S.** $= \cos A + \cos(\frac{2\pi}{3} - A) + \cos(\frac{2\pi}{3} + A)$

$= \cos A + 2\cos\frac{2\pi}{3}\cos A$

$= \cos A + 2.(-\frac{1}{2})\cos A$

$= \cos A - \cos A = 0 =$ R.H.S.

(Proved)

**5.** $\frac{\sin 75^0 - \sin 15^0}{\sin 75^0 + \sin 15^0} = \frac{1}{\sqrt{3}}$

**L.H.S.** $= \frac{\sin 75^0 - \sin 15^0}{\sin 75^0 + \sin 15^0}$

$= \frac{\sin(90^0 - 15^0) - \sin 15^0}{\sin(90^0 - 15^0) + \sin 15^0}$

$= \frac{\cos 15^0 - \sin 15^0}{\cos 15^0 + \sin 15^0} = \frac{\cos 15^0 (1 - \frac{\sin 15^0}{\cos 15^0})}{\cos 15^0 (1 + \frac{\sin 15^0}{\cos 15^0})}$

$= \frac{1 - \tan 15^0}{1 + \tan 15^0} = \frac{\tan 45^0 - \tan 15^0}{1 + \tan 45^0 \tan 15^0}$

$= \tan(45° - 15°) = \tan 30°$

$= \frac{1}{\sqrt{3}} =$ R.H.S. (proved)

**6. (a)** $\tan 5A \tan 3A \tan 2A = \tan 5A - \tan 3A - \tan 2A$

**(b)** $\tan 32° + \tan 13° + \tan 32° \tan 13° = 1$

**(c)** $\tan\frac{\pi}{20} + \tan\frac{\pi}{5} + \tan\frac{\pi}{20}\tan\frac{\pi}{5} = 1$

**প্রমাণ: (a)** $\tan 5A = \tan(3A + 2A)$

$\Rightarrow \tan 5A = \frac{\tan 3A + \tan 2A}{1 - \tan 3A \tan 2A}$

$\Rightarrow \tan 3A + \tan 2A = \tan 5A - \tan 5A \tan 3A \tan 2A$

$\therefore \tan 5A \tan 3A \tan 2A = \tan 5A - \tan 3A - \tan 2A$

**(b)** $\tan 45° = \tan(32° + 13°)$

$\Rightarrow 1 = \frac{\tan 32^0 + \tan 13^0}{1 - \tan 32^0 \tan 13^0}$

$\Rightarrow \tan 32° + \tan 13° = 1 - \tan 32° \tan 13°$

$\therefore \tan 32° + \tan 13° + \tan 32° \tan 13° = 1$

**(c)** $\tan 50° = \tan 40° + 10°)$

$\Rightarrow \tan 50° = \dfrac{\tan 40^0 + \tan 10^0}{1 - \tan 40^0 \tan 10^0}$

$\Rightarrow \tan 50° - \tan 50° \tan 40° \tan 10°$
$= \tan 40° + \tan 10°$

$\Rightarrow \tan 50° - \tan (90° - 40°) \tan 40° \tan 10° = \tan 40° + \tan 10°$

$\Rightarrow \tan 50° - \cot 40° \tan 40° \tan 10°$
$= \tan 40° + \tan 10°$

$\Rightarrow \tan 50° - \tan 10° = \tan 40° + \tan 10°$

$\tan 50° = \tan 40° + 2\tan 10°$

**7.** **(a)** $\tan (45° + A) \tan (45° - A) = 1$

**(b)** $\cos^2 (A - B) - \sin^2 (A + B) = \cos 2A \cos 2B$.

**(a) L.H.S.** $= \tan (45° + A) \tan (45° - A)$
$= \tan (45° + A) \tan\{90° - (45° + A)\}$
$= \tan (45° + A) .\cot (45° + A)$
$= 1 =$ R.H.S. (Proved)

**(b) L.H.S.**$= \cos^2(A - B) - \sin^2(A + B)$
$= \cos\{(A - B) + (A + B)\}$
$\cos\{(A - B) - (A + B)\}$
$= \cos (A - B + A + B) \cos(A - B - A - B)$
$= \cos 2A \cos(-2B) = \cos 2A \cos 2B =$ R.H.S.

**11.(a) $\sin \alpha = k \sin (\alpha + \beta)$ হলে দেখাও যে,**

**$\tan (\alpha + \beta) = \dfrac{\sin\beta}{\cos\beta - k}$.**

প্রমাণ ঃ দেওয়া আছে, $\sin\alpha = k \sin (\alpha + \beta)$

$\Rightarrow \sin\alpha = k (\sin\alpha \cos\beta + \sin\beta \cos\alpha)$

$\Rightarrow \sin\alpha = k \sin\alpha \cos\beta + k \sin\beta \cos\alpha$

$\Rightarrow \sin\alpha (1 - k \cos\beta) = k \sin\beta \cos\alpha$

$\Rightarrow \tan\alpha = \dfrac{k\sin\beta}{1 - k\cos\beta}$

এখন, $\tan (\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$$= \dfrac{\dfrac{k\sin\beta}{1 - k\cos\beta} + \dfrac{\sin\beta}{\cos\beta}}{1 - \dfrac{k\sin\beta}{1 - k\cos\beta}\dfrac{\sin\beta}{\cos\beta}}$$

$$= \dfrac{\dfrac{k\sin\beta\cos\beta + \sin\beta - k\sin\beta\cos\beta}{(1 - k\cos\beta)\cos\beta}}{\dfrac{\cos\beta - k\cos^2\beta - k\sin^2\beta}{(1 - k\cos\beta)\cos\beta}}$$

$$= \dfrac{\sin\beta}{\cos\beta - k(\cos^2\beta + \sin^2\beta)}$$

$\tan (\alpha + \beta) = \dfrac{\sin\beta}{\cos\beta - k}$ (Showed)

**(b) $\tan \alpha = \dfrac{b}{a}$ হলে দেখাও যে,**

**$a \cos \theta + b \sin \theta = \sqrt{a^2 + b^2} \cos (\theta - \alpha)$.**

প্রমাণ ঃ দেওয়া আছে, $\tan\alpha = \dfrac{b}{a}$

এখন, $\sqrt{a^2 + b^2} \cos (\theta - \alpha)$

$= \sqrt{a^2(1 + \dfrac{b^2}{a^2})} \cos (\theta - \alpha)$

$= a\sqrt{1 + \tan^2\alpha} \cos (\theta - \alpha)$

$= a\sqrt{\sec^2\alpha} \cos (\theta - \alpha) = a\sec\alpha \cos (\theta - \alpha)$

$= \dfrac{a}{\cos\alpha}(\cos\alpha \cos\theta + \sin\alpha \sin\theta)$

$= a \cos\theta + a \sin\theta \tan\alpha$

$= a \cos\theta + a \sin\theta \dfrac{b}{a}$

$= a \cos\theta + b \sin\theta$

$a\cos\theta + b\sin\theta = \sqrt{a^2 + b^2} \cos (\theta - \alpha)$

বিকল্প পদ্ধতি: দেওয়া আছে, $\tan\alpha = \dfrac{b}{a} \Rightarrow \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{b}{a}$

$\Rightarrow \dfrac{\sin\alpha}{b} = \dfrac{\cos\alpha}{a} = \dfrac{\sqrt{\sin^2\alpha + \cos^2\alpha}}{\sqrt{b^2 + a^2}} = \dfrac{\sqrt{1}}{\sqrt{a^2 + b^2}}$

$b = \sqrt{a^2 + b^2} \sin\alpha$, $a = \sqrt{a^2 + b^2} \cos\alpha$

এখন, $a \cos\theta + b \sin\theta$

$= \sqrt{a^2 + b^2} (\cos\alpha \cos\theta + \sin\alpha \sin\theta)$

$\therefore a \cos\theta + b \sin\theta = \sqrt{a^2 + b^2} \cos (\theta - \alpha)$

(showed)

**12.(a) $\cos\alpha + \cos\beta = a$ এবং $\sin\alpha + \sin\beta = b$ হলে দেখাও যে, $\cos(\alpha - \beta) = \frac{1}{2}(a^2 + b^2 - 2)$**

প্রমাণ ঃ দেওয়া আছে, $\cos\alpha + \cos\beta = a$

$\Rightarrow \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = a^2 \quad \cdots(1)$

এবং $\sin\alpha + \sin\beta = b$

$\Rightarrow \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = b^2 \quad (2)$

(1) ও (2) যোগ করে পাই,

$(\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = a^2 + b^2$

$\Rightarrow 1 + 1 + 2\cos(\alpha - \beta) = a^2 + b^2$

$\Rightarrow 2\cos(\alpha - \beta) = a^2 + b^2 - 2$

$\cos(\alpha - \beta) = \frac{1}{2}(a^2 + b^2 - 2)$.(Showed)

**(b) $\tan\theta = \dfrac{a\sin x + b\sin y}{a\cos x + b\cos y}$ হলে দেখাও যে, $a\sin(\theta - x) + b\sin(\theta - y) = 0$.**

প্রমাণ ঃ দেওয়া আছে, $\tan\theta = \dfrac{a\sin x + b\sin y}{a\cos x + b\cos y}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{a\sin x + b\sin y}{a\cos x + b\cos y}$

$\Rightarrow a\sin\theta\cos x + b\sin\theta\cos y = a\sin x\cos\theta + b\cos\theta\sin y$

$\Rightarrow a(\sin\theta\cos x - \sin x\cos\theta) + b(\sin\theta\cos y - \cos\theta\sin y) = 0$

$a\sin(\theta - x) + b\sin(\theta - y) = 0$

(Showed)

**(c) $\tan\beta = \dfrac{\sin 2\alpha}{5 + \cos 2\alpha}$ হলে দেখাও যে, $3\tan(\alpha - \beta) = 2\tan\alpha$.**

প্রমাণ ঃ দেওয়া আছে, $\tan\beta = \dfrac{\sin 2\alpha}{5 + \cos 2\alpha}$

$\Rightarrow \tan\beta = \dfrac{\dfrac{2\tan\alpha}{1+\tan^2\alpha}}{5 + \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}}$

$= \dfrac{\dfrac{2\tan\alpha}{1+\tan^2\alpha}}{\dfrac{5 + 5\tan^2\alpha + 1 - \tan^2\alpha}{1+\tan^2\alpha}} = \dfrac{2\tan\alpha}{6 + 4\tan^2\alpha}$

$= \dfrac{\tan\alpha}{3 + 2\tan^2\alpha}$

এখন, $3\tan(\alpha - \beta) = 3\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= 3\dfrac{\tan\alpha - \dfrac{\tan\alpha}{3 + 2\tan^2\alpha}}{1 + \tan\alpha.\dfrac{\tan\alpha}{3 + 2\tan^2\alpha}}$

$= 3\dfrac{3\tan\alpha + 2\tan^3\alpha - \tan\alpha}{3 + 2\tan^2\alpha + \tan^2\alpha}$

$= 3\dfrac{2\tan\alpha + 2\tan^3\alpha}{3 + 3\tan^2\alpha}$

$= 3\dfrac{2\tan\alpha(1 + \tan^3\alpha)}{3(1 + \tan^2\alpha)} = 2\tan\alpha$

$\therefore 3\tan(\alpha - \beta) = 2\tan\alpha$

**13. (a) $\cos(\alpha + \beta)\sin(\gamma + \theta) = \cos(\alpha - \beta)\sin(\gamma - \theta)$ হলে দেখাও যে, $\tan\theta = \tan\alpha\tan\beta\tan\gamma$**

প্রমাণ ঃ দেওয়া আছে, $\cos(\alpha + \beta)\sin(\gamma + \theta) = \cos(\alpha - \beta)\sin(\gamma - \theta)$

$\Rightarrow \dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \dfrac{\sin(\gamma-\theta)}{\sin(\gamma+\theta)}$

$\Rightarrow \dfrac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{\cos(\alpha+\beta) - \cos(\alpha-\beta))} = \dfrac{\sin(\gamma-\theta) + \sin(\gamma+\theta)}{\sin(\gamma-\theta) - \sin(\gamma+\theta)}$

$\Rightarrow \dfrac{2\cos\alpha\cos\beta}{-2\sin\alpha\sin\beta} = \dfrac{2\sin\gamma\cos\theta}{-2\sin\theta\cos\gamma}$

$\Rightarrow \dfrac{1}{\tan\alpha\tan\beta} = \dfrac{\tan\gamma}{\tan\theta}$

$\tan\theta = \tan\alpha\tan\beta\tan\gamma$ (Showed)

**(b) $(\theta - \varphi)$ সূক্ষ্মকোণ এবং $\sin\theta + \sin\varphi = \sqrt{3}(\cos\varphi - \cos\theta)$ হলে দেখাও যে, $\sin 3\theta + \sin 3\varphi = 0$**

প্রমাণ ঃ $\sin\theta + \sin\varphi = \sqrt{3}(\cos\varphi - \sin\theta)$

$\Rightarrow\ 2\sin\frac{1}{2}(\theta+\varphi)\cos\frac{1}{2}(\theta-\varphi)=$

$\sqrt{3}\{\ 2\sin\frac{1}{2}(\theta+\varphi)\sin\frac{1}{2}(\theta-\varphi)\}$

$\Rightarrow\ \cos\frac{1}{2}(\theta-\varphi)=\sqrt{3}\ \sin\frac{1}{2}(\theta-\varphi)$

$\Rightarrow\ \cot\frac{1}{2}(\theta-\varphi)=\sqrt{3}=\cot 30^0$

$\frac{1}{2}(\theta-\varphi)=30^0$ যেহেতু $(\theta-\varphi)$ সূক্ষ্মকোণ ।

$\Rightarrow\ \theta-\varphi=60^0$

এখন, $\sin 3\theta+\sin 3\varphi$

$=2\sin\frac{3}{2}(\theta+\varphi)\cos\frac{3}{2}(\theta-\varphi)$

$=2\sin\frac{3}{2}(\theta+\varphi)\cos\frac{3}{2}(60^0)$

$=2\sin\frac{3}{2}(\theta+\varphi)\cos 90^0$

$=2\sin\frac{3}{2}(\theta+\varphi)\times 0$

$\therefore \sin 3\theta+\sin 3\varphi=0$

## প্রশ্নমালা VII C

1. প্রমাণ কর যে,

**(a) $\sin 10°\ \sin 30°\ \sin 50°\ \sin 70° = \frac{1}{16}$**

L.H.S.= $\sin 10°\sin 30°\sin 50°\sin 70°$

$=\sin 10°.\frac{1}{2}.\frac{1}{2}\{\cos(70°-50°)-$
$\cos(70°+50°)\}$

$=\frac{1}{4}\sin 10°(\cos 20°-\cos 120°)$

$=\frac{1}{4}\sin 10°\cos 20°-\frac{1}{4}\ (-\frac{1}{2})\ \sin 10°$

$=\frac{1}{4}.\frac{1}{2}\{\sin(20°+10°)-$
$\sin(20°-10°)\}+\frac{1}{8}\sin 10°$

$=\frac{1}{8}\sin 30°-\frac{1}{8}\sin 10°+\frac{1}{8}\sin 10°$

$=\frac{1}{8}.\frac{1}{2}=\frac{1}{16}$ = R.H.S. (Proved)

**1(b) $\cos 20°\ \cos 40°\ \cos 60°\cos 80°=\frac{1}{16}$**

L.H.S.= $\cos 20°\cos 40°\cos 60°\cos 80°$

$=\frac{1}{2}\{\cos(40°+20°)+$
$\cos(40°-20°)\}\frac{1}{2}.\cos 80°$

$=\frac{1}{4}\{\cos 60°+\cos 20°\}\cos(90°-10°)$

$=\frac{1}{4}\ (\ \frac{1}{2}+\cos 20°)\sin 10°$

$=\frac{1}{8}\sin 10°+\frac{1}{4}\cos 20°\sin 10°$

$=\frac{1}{8}\sin 10°+\frac{1}{8}\{\sin(20°+10°)$
$-\sin(20°-10°)\}$

$=\frac{1}{8}\sin 10°+\frac{1}{8}\sin 30°-\frac{1}{8}\sin 10°$

$=\frac{1}{8}.\frac{1}{2}=\frac{1}{16}$ = R.H.S. (Proved)

**1(c) $\tan 20°\ \tan 40°\ \tan 60°\ \tan 80°=3$**

L.H.S. = $\tan 20°\tan 40°\tan 60°\tan 80°$

$=\tan 20°\tan 40°.\sqrt{3}.\tan 80°$

$=\sqrt{3}\ \tan 20°\tan 40°\tan 60°$

$=\sqrt{3}.\dfrac{2\sin 20^0\sin 40^0\sin 80^0}{2\cos 20^0\cos 40^0\cos 80^0}$

$=\dfrac{\sqrt{3}\{\cos(40^0-20^0)-\cos(40^0+20^0)\}\sin(90^0-10^0)}{\{\cos(40^0+20^0)+\cos(40^0-20^0)\}\cos(90^0-10^0)}$

$=\sqrt{3}\dfrac{(\cos 20^0-\cos 60^0)\cos 10^0}{(\cos 60^0+\cos 20^0)\sin 10^0}$

$=\sqrt{3}\dfrac{\cos 20^0\cos 10^0-\frac{1}{2}\cos 10^0}{\frac{1}{2}\sin 10^0+\cos 20^0\sin 10^0}$