$$= \sqrt{3}\frac{\frac{1}{2}\{\cos(20^0+10^0)+\cos(20^0-10^0)\}-\frac{1}{2}\cos 10^0}{\frac{1}{2}\sin 10^0+\frac{1}{2}\{\sin(20^0+10^0)-\sin(20^0-10^0)\}}$$

$$= \sqrt{3}.\frac{\frac{1}{2}\cos 30^0+\frac{1}{2}\cos 10^0-\frac{1}{2}\cos 10^0}{\frac{1}{2}\sin 10^0+\frac{1}{2}\sin 30^0-\frac{1}{2}\sin 10^0}$$

$$= \sqrt{3}.\frac{\frac{1}{2}.\frac{\sqrt{3}}{2}}{\frac{1}{2}.\frac{1}{2}} = \sqrt{3}.\frac{\sqrt{3}}{4}\times 4$$

$$= \sqrt{3}.\sqrt{3} = 3 = \text{R.H.S.}$$

2.(a) $\cos\theta\ \cos(60°-\theta)\cos(60°+\theta) = \frac{1}{4}\cos 3\theta$

L.H.S. $= \cos\theta\cos(60°-\theta)\cos(60°+\theta)$

$= \cos\theta.\frac{1}{2}\{\cos(60°+\theta+60°-\theta)$

$+\cos(60°+\theta-60°+\theta)\}$

$= \frac{1}{2}\cos\theta(\cos 120°+\cos 2\theta)$

$= \frac{1}{2}\cos\theta(-\frac{1}{2})+\frac{1}{2}\cos\theta\cos 2\theta$

$= -\frac{1}{4}\cos\theta+\frac{1}{2}.\frac{1}{2}\{\cos(2\theta+\theta)+\cos(2\theta-\theta)\}$

$= -\frac{1}{4}\cos\theta+\frac{1}{4}\cos 3\theta+\frac{1}{4}\cos\theta$

$= \frac{1}{4}\cos 3\theta = \text{R.H.S.}$ (Proved)

2(b) $\cos(36°-\theta)\cos(36°+\theta)+\cos(54°+\theta)\cos(54°-\theta) = \cos 2\theta$

L.H.S. $= \cos(36°-\theta)\cos(36°+\theta)+\cos(54°+\theta)\cos(54°-\theta)$

$= \frac{1}{2}(\cos 72°+\cos 2\theta).+\frac{1}{2}(\cos 108°+\cos 2\theta)$

$= \frac{1}{2}\{\cos(90°-18°)+\cos 2\theta\}+$

$\frac{1}{2}\{\cos(90°+18°)+\cos 2\theta\}$

$= \frac{1}{2}(\cos 2\theta+\cos 18°)+\frac{1}{2}(\cos 2\theta-\cos 18°)$

$= \frac{1}{2}(\cos 2\theta+\cos 18°+\cos 2\theta-\cos 18°)$

$= \frac{1}{2}.2\cos 2\theta = \cos 2\theta = \text{R.H.S.}$ (Proved)

3. প্রমাণ কর যে,

(a) $\cos(60°-\theta)+\cos(60°+\theta)-\cos\theta = 0$

L.H.S. $= \cos(60°-\theta)+\cos(60°+\theta)-\cos\theta$

$= 2\cos 60°\cos\theta-\cos\theta$

$= 2.\ \frac{1}{2}\cos\theta-\cos\theta$

$= \cos\theta-\cos\theta = 0 = \text{R.H.S.}$ (Proved)

(b) $\sin\theta+\sin(120°+\theta)+\sin(240°+\theta) = 0$ [ঢা.'১২]

L.H.S. $= \sin\theta+\sin(120°+\theta)+\sin(240°+\theta)$

$= \sin\theta+\sin\{180°-(60°-\theta)\}+$
$\sin\{180°+(60°+\theta)\}$

$= \sin\theta+\sin(60°-\theta)-\sin(60°+\theta)$

$= \sin\theta-\{\sin(60°+\theta)-\sin(60°-\theta)\}$

$= \sin\theta-2\cos 60°\sin\theta = \sin\theta-2.\ \frac{1}{2}\sin\theta$

$= \sin\theta-\sin\theta = 0 = \text{R.H.S.}$ (Proved)

3(c) $\cos 70°-\cos 10°+\sin 40° = 0$

L.H.S. $= \cos 70°-\cos 10°+\sin 40°$

$= 2\sin\frac{1}{2}(70°+10°)\sin\frac{1}{2}(10°-70°)+\sin 40°$

$= 2\sin 40°\sin(-30°)+\sin 40°$

$= -2\sin 40°.(\frac{1}{2})+\sin 40°$

$= -\sin 40°+\sin 40° = 0 = \text{R.H.S.}$

4(a) $\sin 18°+\cos 18° = \sqrt{2}\cos 27°$ [ব'১১]

L.H.S. $= \sin 18°+\cos 18°$

$= \sin(90°-72°)+\cos 18°$

$= \cos 72°+\cos 18°$

$= 2\cos\frac{1}{2}(72°+18°)\cos\frac{1}{2}(72°-18°)$

$= 2\cos 45° \cos 27° = 2.\dfrac{1}{\sqrt{2}}\cos 27°$

$= \sqrt{2}\cos 27°$

4.(b) $\dfrac{\cos 10^0 - \sin 10^0}{\cos 10^0 + \sin 10^0} = \tan 35°$

L.H.S.$= \dfrac{\cos 10^0 - \sin 10^0}{\cos 10^0 + \sin 10^0}$

$= \dfrac{\cos 10^0(1-\tan 10^0)}{\cos 10^0(1+\tan 10^0)} = \dfrac{\tan 45^0 - \tan 10^0}{1+\tan 45^0 \tan 10^0}$

$= \tan(45°-10°) = \tan 35° =$ R.H.S. (Proved)

5.(a) $\cot(A+15°) - \tan(A-15°) = \dfrac{4\cos 2A}{2\sin 2A+1}$

L.H.S. $= \cot(A+15°) - \tan(A-15°)$

$= \dfrac{\cos(A+15^0)}{\sin(A+15^0)} - \dfrac{\sin(A-15^0)}{\cos(A-15^0)}$

$= \dfrac{\cos(A+15^0)\cos(A-15^0) - \sin(A+15^0)\sin(A-15^0)}{\sin(A+15^0)\cos(A-15^0)}$

$= \dfrac{\cos(A+15^0+A-15^0)}{\frac{1}{2}(\sin 2A + \sin 30^0)} = \dfrac{2\cos 2A}{\sin 2A + \frac{1}{2}}$

$= \dfrac{4\cos 2A}{2\sin 2A+1} =$ R.H.S. (Proved)

5(b) $(\cos\alpha + \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\cos^2\frac{1}{2}(\alpha+\beta)$ [য.'১২]

L.H.S. $= (\cos\alpha + \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

$= \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta - 2\sin^2\alpha\sin\beta$

$= 1 + 1 + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$

$= 2\{1 + \cos(\alpha+\beta)\}$

$= 2.\ 2\cos^2\frac{1}{2}(\alpha+\beta)$

$= 4\cos^2\frac{1}{2}(\alpha+\beta) =$ R.H.S. (Prived)

5(c) $2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13} = 0$

L.H.S. $= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + 2\cos\dfrac{1}{2}(\dfrac{5\pi}{13} + \dfrac{3\pi}{13})\cos\dfrac{1}{2}(\dfrac{5\pi}{13} - \dfrac{3\pi}{13})$

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + 2\cos\dfrac{4\pi}{13}\cos\dfrac{\pi}{13}$

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} + 2\cos(\pi - \dfrac{9\pi}{13})\cos\dfrac{\pi}{13}$

$= 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13} - 2\cos\dfrac{\pi}{13}\cos\dfrac{9\pi}{13}$

$= 0 =$ R.H.S. (Proved)

6. $\left(\dfrac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\dfrac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$= 2\cot^n\frac{1}{2}(A-B)$ **অথবা 0 যখন n যথাক্রমে জোড় অথবা বিজোড় সংখ্যা।**

$\left(\dfrac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\dfrac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$= \left(\dfrac{2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}{2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)}\right)^n + \left(\dfrac{2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)}{2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(B-A)}\right)^n$

$= \left(\cot\frac{1}{2}(A-B)\right)^n + \left(\dfrac{\cos\frac{1}{2}(A-B)}{-\sin\frac{1}{2}(A-B)}\right)^n$

$= \cot^n\frac{1}{2}(A-B) + \left(-\cot\frac{1}{2}(A-B)\right)^n$

$= \cot^n\frac{1}{2}(A-B) + (-1)^n\cot^n\frac{1}{2}(A-B)$

যখন n বিজোড় সংখ্যা,

$\cot^n\frac{1}{2}(A-B) + (-1)^n\cot^n\frac{1}{2}(A-B)$

$$= \cot^n \frac{1}{2}(A-B) - \cot^n \frac{1}{2}(A-B) = 0,$$

যখন n জোড় সংখ্যা,

$$\cot^n \frac{1}{2}(A-B) + (-1)^n \cot^n \frac{1}{2}(A-B)$$

$$= \cot^n \frac{1}{2}(A-B) + \cot^n \frac{1}{2}(A-B)$$

$$= 2\cot^n \frac{1}{2}(A-B)$$

$$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n =$$

$2\cot^n \frac{1}{2}(A-B)$ অথবা o যখন যথাক্রমে জোড় অথবা বিজোড় সংখ্যা।

7. (a) $a\cos\alpha + b\sin\alpha = a\cos\beta + b\sin\beta$ হলে দেখাও যে, $\cos^2\frac{\alpha+\beta}{2} - \sin^2\frac{\alpha+\beta}{2} = \frac{a^2-b^2}{a^2+b^2}$

দেওয়া আছে,

$$a\cos\alpha + b\sin\alpha = a\cos\beta + b\sin\beta$$

$$\Rightarrow a(\cos\alpha - \cos\beta) = b(\sin\beta - \sin\alpha)$$

$$\Rightarrow a.2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$$

$$= b.2\sin\frac{\beta-\alpha}{2}\cos\frac{\alpha+\beta}{2}$$

$$\Rightarrow \frac{\cos\frac{\alpha+\beta}{2}}{\sin\frac{\alpha+\beta}{2}} = \frac{a}{b} \Rightarrow \frac{\cos^2\frac{\alpha+\beta}{2}}{\sin^2\frac{\alpha+\beta}{2}} = \frac{a^2}{b^2}$$

$$\Rightarrow \frac{\cos^2\frac{\alpha+\beta}{2} + \sin^2\frac{\alpha+\beta}{2}}{\cos^2\frac{\alpha+\beta}{2} - \sin^2\frac{\alpha+\beta}{2}} = \frac{a^2+b^2}{a^2-b^2}$$

[যোজন - বিয়োজন করে ।]

$$\Rightarrow \frac{1}{\cos^2\frac{\alpha+\beta}{2} - \sin^2\frac{\alpha+\beta}{2}} = \frac{a^2+b^2}{a^2-b^2}$$

$$\cos^2\left(\frac{\alpha+\beta}{2}\right) - \sin^2\left(\frac{\alpha+\beta}{2}\right) = \frac{a^2-b^2}{a^2+b^2}$$

7.(b) $\cos x = k\cos y$ হলে দেখাও যে, $\tan\frac{x+y}{2} = \frac{k-1}{k+1}\cot\frac{y-x}{2}$

প্রমাণ ঃ দেওয়া আছে, $\cos x = k\cos y$

$$\Rightarrow \frac{\cos x}{\cos y} = \frac{k}{1} \Rightarrow \frac{\cos x + \cos y}{\cos x - \cos y} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{2\cos\frac{x+y}{2}\cos\frac{y-x}{2}}{2\sin\frac{y-x}{2}\sin\frac{x+y}{2}} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\cot\frac{y-x}{2}}{\tan\frac{x+y}{2}} = \frac{k+1}{k-1}$$

$$\tan\frac{x+y}{2} = \frac{k-1}{k+1}\cot\frac{x+y}{2}$$

7(c) $\sin\theta = k\sin(\alpha - \theta)$ হলে দেখাও যে, $\tan(\theta - \frac{\alpha}{2}) = \frac{k-1}{k+1}\tan\frac{\alpha}{2}$ [সি.বো.'০৩; কু.বো.'০৮]

প্রমাণ ঃ দেওয়া আছে, $\sin\theta = k\sin(\alpha - \theta)$

$$\Rightarrow \frac{\sin\theta}{\sin(\alpha-\theta)} = \frac{k}{1}$$

$$\Rightarrow \frac{\sin\theta + \sin(\alpha-\theta)}{\sin\theta - \sin(\alpha-\theta)} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{2\sin\frac{\theta+\alpha-\theta}{2}\cos\frac{\theta-\alpha+\theta}{2}}{2\cos\frac{\theta+\alpha-\theta}{2}\sin\frac{\theta-\alpha+\theta}{2}} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\tan\frac{\alpha}{2}}{\tan(\theta - \frac{\alpha}{2})} = \frac{k+1}{k-1}$$

$$\tan(\theta - \frac{\alpha}{2}) = \frac{k-1}{k+1}\tan\frac{\alpha}{2} \quad \text{(Showed).}$$

7(d) $\frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)} = \frac{a}{b}$ হলে দেখাও যে, $\frac{a+b}{a-b}\sin^2(\alpha - \beta) = \sin^2(\theta+\alpha) - \sin^2(\theta+\beta)$

প্রমাণ ঃ দেওয়া আছে, $\frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)} = \frac{a}{b}$

$$\Rightarrow \frac{\tan(\theta+\alpha)+\tan(\theta+\beta)}{\tan(\theta+\alpha)-\tan(\theta+\beta)} = \frac{a+b}{a-b}$$

[যোজন – বিয়োজন করে ।]

$$\Rightarrow \frac{\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)}+\frac{\sin(\theta+\beta)}{\cos(\theta+\beta)}}{\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)}-\frac{\sin(\theta+\beta)}{\cos(\theta+\beta)}} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cos(\theta+\beta)+\sin(\theta+\beta)\cos(\theta+\alpha)}{\sin(\theta+\alpha)\cos(\theta+\beta)-\sin(\theta+\beta)\cos(\theta+\alpha)} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin\{(\theta+\alpha)+(\theta+\beta)\}}{\sin\{(\theta+\alpha)-(\theta+\beta)\}} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{a+b}{a-b}\sin(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}$$

$$\Rightarrow \frac{a+b}{a-b}\sin^2(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}\sin\{(\theta+\alpha)-(\theta+\beta)\}$$

$$\therefore \frac{a+b}{a-b}\sin^2(\alpha-\beta) = \sin^2(\theta+\alpha)-\sin^2(\theta+\beta)$$

[$\because \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$]

8. $\dfrac{x}{\tan(\theta+\alpha)} = \dfrac{y}{\tan(\theta+\beta)} = \dfrac{z}{\tan(\theta+\gamma)}$ **হলে**

দেখাও যে, $\dfrac{x+y}{x-y}\sin^2(\alpha-\beta) + \dfrac{y+z}{y-z}\sin^2(\beta-\gamma) + \dfrac{z+x}{z-x}\sin^2(\gamma-\alpha) = 0$

প্রমাণ ঃ দেওয়া আছে ,

$$\frac{x}{\tan(\theta+\alpha)} = \frac{y}{\tan(\theta+\beta)} = \frac{z}{\tan(\theta+\gamma)}$$

১ম ও ২য় অনুপাত হতে পাই,

$$\frac{}{\tan(\theta+\alpha)} = \frac{}{\tan(\theta+\beta)}$$

$$\Rightarrow \frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)} = \frac{x}{y}$$

$$\Rightarrow \frac{\tan(\theta+\alpha)+\tan(\theta+\beta)}{\tan(\theta+\alpha)-\tan(\theta+\beta)} = \frac{x+y}{x-y}$$

[যোজন – বিয়োজন করে ।]

$$\Rightarrow \frac{\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)}+\frac{\sin(\theta+\beta)}{\cos(\theta+\beta)}}{\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)}-\frac{\sin(\theta+\beta)}{\cos(\theta+\beta)}} = \frac{x+y}{x-y}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cos(\theta+\beta)+\sin(\theta+\beta)\cos(\theta+\alpha)}{\sin(\theta+\alpha)\cos(\theta+\beta)-\sin(\theta+\beta)\cos(\theta+\alpha)} = \frac{x+y}{x-y}$$

$$\Rightarrow \frac{\sin\{(\theta+\alpha)+(\theta+\beta)\}}{\sin\{(\theta+\alpha)-(\theta+\beta)\}} = \frac{x+y}{x-y}$$

$$\Rightarrow \frac{x+y}{x-y}\sin(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}$$

$$\Rightarrow \frac{x+y}{x-y}\sin^2(\alpha-\beta) = \sin\{(\theta+\alpha)+(\theta+\beta)\}\sin\{(\theta+\alpha)-(\theta+\beta)\}$$

$$\therefore \frac{x+y}{x-y}\sin^2(\alpha-\beta) = \sin^2(\theta+\alpha)-\sin^2(\theta+\beta)$$

অনুরূপভাবে, $\dfrac{y}{\tan(\theta+\beta)} = \dfrac{z}{\tan(\theta+\gamma)}$

$$\Rightarrow \frac{y+z}{y-z}\sin^2(\beta-\gamma) = \sin^2(\theta+\beta)-\sin^2(\theta+\gamma)$$

এবং $\dfrac{z}{\tan(\theta+\gamma)} = \dfrac{x}{\tan(\theta+\alpha)}$

$$\Rightarrow \frac{z+x}{z-x}\sin^2(\gamma-\alpha) = \sin^2(\theta+\gamma)-\sin^2(\theta+\alpha)$$

$$\frac{x+y}{x-y}\sin^2(\alpha-\beta) + \frac{y+z}{y-z}\sin^2(\beta-\gamma) + \frac{z+x}{z-x}\sin^2(\gamma-\alpha) = \sin^2(\theta+\alpha)-\sin^2(\theta+\beta)+\sin^2(\theta+\beta)-\sin^2(\theta+\gamma)+\sin^2(\theta+\gamma)-\sin^2(\theta+\alpha) = 0$$

9 (a) $\sin A + \cos A = \sin B + \cos B$ **হলে দেখাও যে,** $A + B = \dfrac{\pi}{2}$ [সি.'০৯; চ.,দি.'১০; কু.'১২]

প্রমাণঃ দেওয়া আছে, $\sin A + \cos A = \sin B + \cos B$

$$\Rightarrow \sin A - \sin B = \cos B - \cos A$$

$$\Rightarrow 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$= 2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\Rightarrow \frac{\sin\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A+B)} = 1$$

$$\Rightarrow \tan\frac{1}{2}(A+B) = \tan\frac{\pi}{4} \Rightarrow \frac{1}{2}(A+B) = \frac{\pi}{4}$$

$$\therefore A+B = \frac{\pi}{2}$$

9(b) $\sin\theta + \sin\varphi = a$ এবং $\cos\theta + \cos\varphi = b$ হলে দেখাও যে, $\tan\frac{\theta-\varphi}{2} = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

প্রমাণ ঃ দেওয়া আছে , $\sin\theta + \sin\varphi = a$

$$\Rightarrow 2\sin\frac{1}{2}(\theta+\varphi)\cos\frac{1}{2}(\theta-\varphi) = a$$

উভয় পক্ষকে বর্গ করে আমরা পাই ,

$$4\sin^2\frac{1}{2}(\theta+\varphi)\cos^2\frac{1}{2}(\theta-\varphi) = a^2 \cdots (1)$$

এবং $\cos\theta + \cos\varphi = b$

$$\Rightarrow 2\cos\frac{1}{2}(\theta+\varphi)\cos\frac{1}{2}(\theta-\varphi) = b$$

উভয় পক্ষকে বর্গ করে আমরা পাই ,

$$4\cos^2\frac{1}{2}(\theta+\varphi)\cos^2\frac{1}{2}(\theta-\varphi) = b^2 \cdots (2)$$

(1) ও (2) যোগ করে আমরা পাই ,

$$4\cos^2\frac{1}{2}(\theta-\varphi)\{\sin^2\frac{1}{2}(\theta+\varphi) + \cos^2\frac{1}{2}(\theta+\varphi)\} = a^2+b^2$$

$$\Rightarrow \cos^2\frac{1}{2}(\theta-\varphi) = \frac{a^2+b^2}{4}$$

$$\Rightarrow \sec^2\frac{1}{2}(\theta-\varphi) = \frac{4}{a^2+b^2}$$

$$\Rightarrow 1 + \tan^2\frac{1}{2}(\theta-\varphi) = \frac{4}{a^2+b^2}$$

$$\Rightarrow \tan^2\frac{1}{2}(\theta-\varphi) = \frac{4}{a^2+b^2} - 1$$

$$= \frac{4-a^2-b^2}{a^2+b^2}$$

$$\therefore \tan\frac{1}{2}(\theta-\varphi) = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

9.(c) cosec A + sec A = cosec B + sec B হলে দেখাও যে, $\tan A\tan B = \cot\frac{A+B}{2}$

প্রমাণ ঃ দেওয়া আছে ,

$$\text{cosec } A + \sec A = \text{cosec } B + \sec B$$

$$\Rightarrow \text{cosec } A - \text{cosec } B = \sec B - \sec A$$

$$\Rightarrow \frac{1}{\sin A} - \frac{1}{\sin B} = \frac{1}{\cos B} - \frac{1}{\cos A}$$

$$\Rightarrow \frac{\sin B - \sin A}{\sin A\sin B} = \frac{\cos A - \cos B}{\cos A\cos B}$$

$$\Rightarrow \frac{\sin B - \sin A}{\cos A - \cos B} = \frac{\sin A\sin B}{\cos A\cos B}$$

$$\Rightarrow \frac{2\cos\frac{A+B}{2}\sin\frac{B-A}{2}}{2\sin\frac{A+B}{2}\sin\frac{B-A}{2}} = \tan A\tan B$$

$$\tan A\tan B = \cot\left(\frac{A+B}{2}\right)$$

10. $x\cos\alpha + y\sin\alpha = k = x\cos\beta + y\sin\beta$ হলে দেখাও যে,

$$\frac{x}{\cos\frac{1}{2}(\alpha+\beta)} = \frac{y}{\sin\frac{1}{2}(\alpha+\beta)} = \frac{k}{\cos\frac{1}{2}(\alpha-\beta)}$$

প্রমাণ ঃ দেওয়া আছে ,

$$x\cos\alpha + y\sin\alpha - k = 0 \cdots\cdots\cdots (1)$$

$$x\cos\beta + y\sin\beta - k = 0 \cdots\cdots\cdots (2)$$

বজ্রগুণন প্রক্রিয়ায় সাহায্যে (1) ও (2) হতে আমরা পাই

$$\frac{x}{\sin\alpha + \sin\beta} = \frac{y}{-\cos\beta + \cos\alpha} = \frac{k}{\cos\alpha\sin\beta - \sin\alpha\cos\beta}$$

$$\Rightarrow \frac{x}{2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)} = \frac{y}{2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)} = \frac{k}{\sin(\beta-\alpha)}$$

$$\Rightarrow \frac{x}{2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)} = \frac{y}{2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\beta-\alpha)} = \frac{k}{2\sin\frac{1}{2}(\beta-\alpha)\cos\frac{1}{2}(\beta-\alpha)}$$

$$\therefore \frac{x}{\cos\frac{1}{2}(\alpha+\beta)} = \frac{y}{\sin\frac{1}{2}(\alpha+\beta)} = \frac{k}{\cos\frac{1}{2}(\alpha-\beta)}$$

11. $\sin\frac{\pi}{16}.\sin\frac{3\pi}{16}.\sin\frac{5\pi}{16}.\sin\frac{7\pi}{16}$ এর মান নির্ণয় কর।

সমাধান: $\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$

$= \frac{1}{4}(2\sin\frac{7\pi}{16}\sin\frac{\pi}{16})(2\sin\frac{5\pi}{16}\sin\frac{3\pi}{16})$

$= \frac{1}{4}\{\cos(\frac{7\pi}{16}-\frac{\pi}{16})-\cos(\frac{7\pi}{16}+\frac{\pi}{16})\}$
$\{\cos(\frac{5\pi}{16}-\frac{3\pi}{16})-\cos(\frac{5\pi}{16}+\frac{3\pi}{16})\}$

$= \frac{1}{4}(\cos\frac{3\pi}{8}-\cos\frac{\pi}{2})(\cos\frac{\pi}{8}-\cos\frac{\pi}{2})$

$= \frac{1}{4}\{\cos(\frac{\pi}{2}-\frac{\pi}{8})-0\}(\cos\frac{\pi}{8}-0)$

$= \frac{1}{4}\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \frac{1}{8}\sin 2.\frac{\pi}{8}$

$= \frac{1}{8}\sin\frac{\pi}{4} = \frac{1}{8}.\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{16}$ (Ans.)

অতিরিক্ত প্রশ্ন (সমাধানসহ)

প্রমাণ কর যে,

1(a) $\cos 10° \cos 50° \cos 70° = \frac{\sqrt{3}}{8}$ [প্র.ভ.প.'৯৩]

L.H.S $= \cos10° \cos50° \cos70°$

$= \frac{1}{2}\{\cos(50°+10°)+\cos(50°-10°)\}$
$\cos(90°-20°)$

$= \frac{1}{2}(\cos60° + \cos 40°)\sin 20°$

$= \frac{1}{2}.\frac{1}{2}\sin20° + \frac{1}{2}\cos40°\sin20°$

$= \frac{1}{4}\sin20° + \frac{1}{2}.\frac{1}{2}\{\sin(40° + 20°) -$
$\sin(40° - 20°)\}$

$= \frac{1}{4}\sin20° + \frac{1}{4}\sin60° - \frac{1}{4}\sin20°$

$= \frac{1}{4}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8} =$ R.H.S. (Proved)

1.(b) $\sin 20° \sin 40° \sin 60° \sin 80° = \frac{3}{16}$

L.H.S $= \sin 20° \sin40°\sin60°\sin80°$

$= \frac{1}{2}\{\cos(40° - 20°) -$
$\cos(40° + 20°)\}.\frac{\sqrt{3}}{2}.\sin80°$

$= \frac{\sqrt{3}}{4}(\cos20° - \cos60°)\sin(90° - 10°)$

$= \frac{\sqrt{3}}{4}(\cos20° - \frac{1}{2})\cos10°$

$= \frac{\sqrt{3}}{4}\cos20°\cos10° - \frac{\sqrt{3}}{8}\cos10°$

$= \frac{\sqrt{3}}{4}\frac{1}{2}\{\cos(20° - 10°)+\cos(20° + 10°)\}$

$- \frac{\sqrt{3}}{8}\cos10°$

$= \frac{\sqrt{3}}{8}\cos10° + \frac{\sqrt{3}}{8}\cos30° - \frac{\sqrt{3}}{8}\cos10°$

$= \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2} = \frac{3}{16} =$ R.H.S. (Proved)

1(c) $\cos 10° \cos 30° \cos 50° \cos 70° = \frac{3}{16}$

L.H.S. $= \cos10°\cos30°\cos50°\cos70°$

$= \cos10°.\frac{\sqrt{3}}{2}\frac{1}{2}\{\cos(70° + 50°) +$

$\cos(70° - 50°)\}$

$= \frac{\sqrt{3}}{4}.\cos10°\cos120° + \frac{\sqrt{3}}{4}\cos20°\cos10°$

$= \frac{\sqrt{3}}{4}\cos10°.(-\frac{1}{2}) + \frac{\sqrt{3}}{4}.\frac{1}{2}\{\cos(20°+10°) + \cos(20°-10°)\}$

$= -\frac{\sqrt{3}}{8}\cos10° + \frac{\sqrt{3}}{8}\cos30° + \frac{\sqrt{3}}{8}\cos10°$

$= \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2} = \frac{3}{16}$ = R.H.S. (Proved)

2(a) $4\cos\theta\cos(\frac{2\pi}{3}+\theta)\cos(\frac{4\pi}{3}+\theta) = \cos 3\theta$

LH.S. $= 4\cos\theta\cos(\frac{2\pi}{3}+\theta)\cos(\frac{4\pi}{3}+\theta)$

$= 4\cos\theta.\frac{1}{2}\{\cos(\frac{4\pi}{3}+\frac{2\pi}{3}+2\theta) + \cos(\frac{4\pi}{3}-\frac{2\pi}{3})\}$

$= 2\cos\theta\{\cos(2\pi+2\theta) + \cos\frac{2\pi}{3}\}$

$= 2\cos\theta\cos2\theta + 2\cos\theta(-\frac{1}{2})$

$= \cos(2\theta+\theta) + \cos(2\theta-\theta) - \cos\theta$

$= \cos3\theta + \cos\theta - \cos\theta$

$= \cos3\theta =$ R.H.S. (Proved)

2(b) $\sin(45°+A)\sin(45°-A) = \frac{1}{2}\cos2A$

L.H.S. $= \sin(45°+A)\sin(45°-A)$

$= \frac{1}{2}\{\cos(45°+A-45°+A) - \cos(45°+A+45°-A)\}$

$= \frac{1}{2}(\cos2A - \cos90°) = \frac{1}{2}(\cos2A - 0)$

$= \frac{1}{2}\cos2A$ = R.H.S. (Proved)

2(c) $4\cos\frac{B+C}{2}\cos\frac{C+A}{2}\cos\frac{A+B}{2}$
$= \cos A + \cos B + \cos C + \cos(A+B+C)$

L.H.S. $= 4\cos\frac{B+C}{2}\cos\frac{C+A}{2}\cos\frac{A+B}{2}$

$= 2\{\cos\frac{1}{2}(B+C+C+A) + \cos\frac{1}{2}(B+C-C-A)\}\cos\frac{A+B}{2}$

$= 2\cos\frac{1}{2}(B+2C+A)\cos\frac{A+B}{2} + 2\cos\frac{1}{2}(B-A)\cos\frac{A+B}{2}$

$= \cos\frac{1}{2}(A+B+2C+A+B) + \cos\frac{1}{2}(A+B+2C-A-B) + \cos\frac{1}{2}(B-A+A+B) + \cos\frac{1}{2}(B-A-A-B)$

$= \cos(A+B+C) + \cos C + \cos B + \cos(-A)$

$= \cos A + \cos B + \cos C + \cos(A+B+C)$

= R.H.S. (Proved)

3(a) $\sin\theta + \sin(60°-\theta) - \sin(60°+\theta) = 0$

L.H.S. $= \sin\theta + \sin(60°-\theta) - \sin(60°+\theta)$

$= \sin\theta - \{\sin(60°+\theta) - \sin(60°-\theta)\}$

$= \sin\theta - 2\sin\theta\cos60° = \sin\theta - 2(\frac{1}{2})\sin\theta$

$= \sin\theta - \sin\theta = 0 =$ R.H.S. (Proved)

(b) $\cos40° + \cos80° + \cos160° = 0$

L.H.S. $= \cos40° + \cos80° + \cos160°$

$= \cos40° + 2\cos\frac{1}{2}(160°+80°)\cos\frac{1}{2}(160°-80°)$

$= \cos40° + 2\cos120°\cos40°$

$= \cos40° + 2(-\frac{1}{2})\cos40°$

$= \cos40° - \cos40° = 0 =$ R.H.S.

4. $\sin65° + \cos65° = \sqrt{2}\cos20°$

L.H.S. $= \sin 65° + \cos 65°$
$= \sin 65° + \cos (90° - 25°)$
$= \sin 65° + \sin 25°$
$= 2 \sin \frac{1}{2}(65° + 25°) \cos (65° - 25°)$
$= 2 \sin 45° \cos 20° = 2.\frac{1}{\sqrt{2}} \cos 20°$
$= \sqrt{2} \cos 20° =$ R.H.S. (Proved)

5.(a) $\tan(\frac{\pi}{6} + \theta) \tan(\frac{\pi}{6} - \theta) = \frac{2\cos 2\theta - 1}{2\cos 2\theta + 1}$

L.H.S.$= \tan(\frac{\pi}{6} + \theta) \tan(\frac{\pi}{6} - \theta)$

$= \frac{\sin(\frac{\pi}{6}+\theta)\sin(\frac{\pi}{6}-\theta)}{\cos(\frac{\pi}{6}+\theta)\cos(\frac{\pi}{6}-\theta)}$

$= \frac{2\sin(\frac{\pi}{6}+\theta)\sin(\frac{\pi}{6}-\theta)}{2\cos(\frac{\pi}{6}+\theta)\cos(\frac{\pi}{6}-\theta)}$

$= \frac{\cos(\frac{\pi}{6}+\theta-\frac{\pi}{6}+\theta)-\cos(\frac{\pi}{6}+\theta+\frac{\pi}{6}-\theta)}{\cos(\frac{\pi}{6}+\theta-\frac{\pi}{6}+\theta)+\cos(\frac{\pi}{6}+\theta+\frac{\pi}{6}-\theta)}$

$= \frac{\cos 2\theta - \cos\frac{\pi}{3}}{\cos 2\theta + \cos\frac{\pi}{3}} = \frac{\cos 2\theta - \frac{1}{2}}{\cos 2\theta + \frac{1}{2}}$

$= \frac{2\cos 2\theta - 1}{2\cos 2\theta + 1} =$ R.H.S. (Proved)

5.(b) $\sin(\alpha + \beta + \gamma) + \sin(\alpha - \beta - \gamma) + \sin(\alpha + \beta - \gamma) + \sin(\alpha - \beta + \gamma) = 4 \sin\alpha \cos\beta \cos\gamma$

L.H.S.$= \sin(\alpha + \beta + \gamma) + \sin(\alpha - \beta - \gamma) + \sin(\alpha + \beta - \gamma) + \sin(\alpha - \beta + \gamma)$
$= \sin\{\alpha + (\beta + \gamma)\} + \sin\{\alpha - (\beta + \gamma)\} + \sin\{\alpha + (\beta - \gamma)\} + \sin\{\alpha - (\beta - \gamma)\}$
$= 2 \sin\alpha \cos(\beta + \gamma) + 2 \sin\alpha \cos(\beta - \gamma)$
$= 2 \sin\alpha\{\cos(\beta + \gamma) + \cos(\beta - \gamma)\}$
$= 2 \sin\alpha .2 \cos\beta \cos\gamma$
$= 4 \sin\alpha \cos\beta \cos\gamma =$ R.H.S. (Prived)

6 $\sin x = k \sin y$ **হলে দেখাও যে,**
$\tan\frac{x-y}{2} = \frac{k-1}{k+1} \tan\frac{x+y}{2}$ [প্র.ভ.প.'১৭]

প্রমাণ ঃ দেওয়া আছে , $\sin x = k \sin y$

$\Rightarrow \frac{\sin x}{\sin y} = \frac{k}{1} \Rightarrow \frac{\sin x + \sin y}{\sin x - \sin y} = \frac{k+1}{k-1}$

$\Rightarrow \frac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\sin\frac{x-y}{2}\cos\frac{x+y}{2}} = \frac{k+1}{k-1}$

$\Rightarrow \frac{\tan\frac{x+y}{2}}{\tan\frac{x-y}{2}} = \frac{k+1}{k-1}$

$\therefore \tan\frac{x-y}{2} = \frac{k-1}{k+1} \tan\frac{x+y}{2}$

7. $x \sin\varphi = y \sin(2\theta + \varphi)$ **হলে দেখাও যে,**
$\cot(\theta + \varphi) = \frac{x-y}{x+y} \cot\theta$

প্রমাণ ঃ দেওয়া আছে , $x \sin\varphi = y \sin(2\theta + \varphi)$

$\Rightarrow \frac{\sin(2\theta+\varphi)}{\sin\varphi} = \frac{x}{y}$

$\Rightarrow \frac{\sin(2\theta+\varphi) - \sin\varphi}{\sin(2\theta+\varphi) + \sin\varphi} = \frac{x-y}{x+y}$

$\Rightarrow \frac{2\cos\frac{2\theta+\varphi+\varphi}{2}\sin\frac{2\theta+\varphi-\varphi}{2}}{2\sin\frac{2\theta+\varphi+\varphi}{2}\cos\frac{2\theta+\varphi-\varphi}{2}} = \frac{x-y}{x+y}$

$\Rightarrow \frac{\cot(\theta+\varphi)}{\cot\theta} = \frac{x-y}{x+y}$

$\therefore \cot(\theta + \varphi) = \frac{x-y}{x+y} \cot\theta$ (Showed)

প্রশ্নমালা –VII D

প্রমাণ কর যে,

1. (a) $\frac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$

L.H.S.$=\frac{1+\cos 2\theta}{\sin 2\theta}=\frac{2\cos^2\theta}{2\sin\theta\cos\theta}=\frac{\cos\theta}{\sin\theta}$
$=\cot\theta=$ R.H.S. (proved)

1(b) $\sin 2x \tan 2x = \frac{4\tan^2 x}{1-\tan^4 x}$

L.H.S. $=\sin 2x \tan 2x$
$=\frac{2\tan x}{1+\tan^2 x}\times\frac{2\tan x}{1-\tan^2 x}$
$=\frac{4\tan^2 x}{1-\tan^4 x}=$ R.H.S. (proved)

1(c) $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta = \cot\theta$ [য.'০২; সি.'০৮]

প্রমাণ ঃ $4\tan 4\theta + 8\cot 8\theta$
$=4(\frac{\sin 4\theta}{\cos 4\theta}+2\frac{\cos 8\theta}{\sin 8\theta})$
$=4(\frac{\sin 4\theta}{\cos 4\theta}+\frac{2\cos 8\theta}{2\sin 4\theta\cos 4\theta})$
$=4(\frac{\sin^2 4\theta+1-2\sin^2 4\theta}{\sin 4\theta\cos 4\theta})$
$=4\ \frac{1-\sin^2 4\theta}{\sin 4\theta\cos 4\theta})=4(\frac{\cos^2 4\theta}{\sin 4\theta\cos 4\theta})$
$=4\cot 4\theta$
অনুরূপভাবে প্রমাণ করা যায় ,
$2\tan 2\theta+4\cot 4\theta=2\cot 2\theta$ এবং
$\tan\theta+2\cot 2\theta=\cot\theta$
L.H.S.$=\tan\theta+2\tan 2\theta+4\tan 4\theta+8\cot 8\theta$
$=\tan\theta+2\tan 2\theta+4\cot 4\theta$
$=\tan\theta+2\cot 2\theta=\cot\theta=$ R.H.S. (Proved)

2.(a) $4(\sin^3 10° + \cos^3 20°) = 3(\sin 10° + \cos 20°)$

L.H.S. $=4(\sin^3 10°+\cos^3 20°)$
$=4\sin^3 10°+4\cos^3 20°$
$=3\sin 10°-\sin(3.10°)+\cos(3.20°)+3\cos 20°$
$=3(\sin 10°+\cos 20°)-\sin 30°+\cos 60°$
$=3(\sin 10°+\sin 20°)-\frac{1}{2}+\frac{1}{2}$
$=3(\sin 10°+\cos 20°)=$ R.H.S. (Proved)

(b) $\sin^2(60°+A)+\sin^2 A+\sin^2(60°-A)=\frac{3}{2}$

L.H.S. $=\sin^2(60°+A)+\sin^2 A+\sin^2(60-A)$
$=\frac{1}{2}\{1-\cos 2(60°+A)+1-\cos 2A+1-\cos 2(60°-A)\}$
$=\frac{1}{2}\{3-\cos(120°+2A)-\cos(120°-2A)-\cos 2A\}$
$=\frac{1}{2}[3-\{\cos(120°+2A)+\cos(120°-2A)\}-\cos 2A]$
$=\frac{1}{2}\{3-2.\cos 120°\cos 2A-\cos 2A\}$
$=\frac{1}{2}\{3-2(-\frac{1}{2})\cos 2A-\cos 2A\}$
$=\frac{1}{2}\{3+\cos 2A-\cos 2A\}=\frac{3}{2}=$ R.H.S.

2(c) $\sin^2(\frac{\pi}{8}+\frac{\theta}{2})-\sin^2(\frac{\pi}{8}-\frac{\theta}{2})=\frac{1}{\sqrt{2}}\sin\theta$ [রা.'১১]

L.H.S. $=\sin^2(\frac{\pi}{8}+\frac{\theta}{2})-\sin^2(\frac{\pi}{8}-\frac{\theta}{2})$
$=\frac{1}{2}\{1-\cos 2(\frac{\pi}{8}+\frac{\theta}{2})\}-\frac{1}{2}\{1-\cos 2(\frac{\pi}{8}-\frac{\theta}{2})\}$
$=\frac{1}{2}\{1-\cos(\frac{\pi}{4}+\theta)-1+\cos(\frac{\pi}{4}-\theta)\}$
$=\frac{1}{2}\{\cos(\frac{\pi}{4}-\theta)-\cos(\frac{\pi}{4}+\theta)\}$
$=\frac{1}{2}.2\sin\frac{\pi}{4}\sin\theta=\frac{1}{\sqrt{2}}\sin\theta=$ R.H.S.

2. (d) $\cos^2(A-120°)+\cos^2 A+\cos^2(A+120°)=3/2$ [ঢা.'০৩; কু.'০৭; য.'০৮]

L.H.S. $=\cos^2(A-120°)+\cos^2 A+\cos^2(A+120°)$
$=\frac{1}{2}\{1+\cos 2(A-120°)+1+\cos 2A+1+\cos 2(A+120°)\}$