নির্দিষ্ট যোগজ এর প্রয়োগ
প্রশ্নমালা X D

মান নির্ণয় কর :

1(a) $\int_0^3 (3-2x+x^2)dx$ [কু.'০৬,'০৭]

$= \left[3x - 2.\frac{x^2}{2} + \frac{x^3}{3}\right]_0^3 = \{(3.3 - 3^2 + \frac{3^3}{3}) - 0\}$

$= (9-9+9) = 9$

(b) $\int_0^{\pi/2} (\sin\theta + \cos\theta)\, dx$ [চ.'০৪]

$= [-\cos\theta + \sin\theta]_0^{\pi/2} = [-\cos\theta + \sin\theta]_0^{\pi/2}$

$= (\sin\frac{\pi}{2} - \cos\frac{\pi}{2}) - (\sin 0 - \cos 0)$

$= (1-0) - (0-1) = 2$

(c) $\int_0^{\pi} \frac{1-\cos 2x}{2} dx = \left[\frac{1}{2}(x - \frac{1}{2}\sin 2x)\right]_0^{\pi}$

$= \frac{1}{2}\{(\pi - \frac{1}{2}\sin 2\pi) - (0 - \frac{1}{2}\sin 2.0)\} = \frac{\pi}{2}$

(d) $\int_{-\pi/2}^{\pi/2} \frac{\sec x + 1}{\sec x} dx$ [য.'০৬; কু.,'০৯]

$= \int_{-\pi/2}^{\pi/2} (1 + \frac{1}{\sec x}) dx = \int_{-\pi/2}^{\pi/2} (1+\cos x) dx$

$= x[1 + \sin x)]_{-\pi/2}^{\pi/2}$

$= \frac{\pi}{2} + \sin\frac{\pi}{2} - \{-\frac{\pi}{2} + \sin(-\frac{\pi}{2})\}$

$= \frac{\pi}{2} + 1 - (-\frac{\pi}{2} - 1) = \frac{\pi}{2} + \frac{\pi}{2} + 2 = \pi + 2$

(e) $\int_{-1}^{1} |x|\, dx$ [প্র.ভ.প.'০৬]

$= \int_{-1}^{0} |x|\, dx + \int_{0}^{1} |x|\, dx = \int_{-1}^{0} (-x) dx + \int_0^1 x dx$

$[\because |x| = x, x \geq 0; |x| = -x, x \leq 0]$

$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = -0 + \frac{1}{2} + \frac{1}{2} - 0 = 1$

2.(a) $\int_0^{\pi/3} \frac{1}{1-\sin x} dx$

[ঢা. '০৯,'১৩; য.'০৯; সি.'১০; রা.'১৩]

$= \int_0^{\pi/3} \frac{1+\sin x}{(1-\sin x)(1+\sin x)} dx$

$= \int_0^{\pi/3} \frac{1+\sin x}{1-\sin^2 x} dx = \int_0^{\pi/3} \frac{1+\sin x}{\cos^2 x} dx$

$= \int_0^{\pi/3} \{\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\} dx$

$= \int_0^{\pi/3} \{\sec^2 x + \sec x \tan x\} dx$

$= [\tan x + \sec x]_0^{\pi/3}$

$= \tan\frac{\pi}{3} + \sec\frac{\pi}{3} - (\tan 0 + \sec 0)$

$= \sqrt{3} + 2 - 0 - 1 = \sqrt{3} + 1$

2(b) $\int_0^{\pi/2} \frac{1}{1+\cos x} dx$ [ব.'০৮; ঢা.,সি.'১১]

$= \int_0^{\pi/2} \frac{1}{2\cos^2\frac{x}{2}} dx = \frac{1}{2}\int_0^{\pi/2} \sec^2\frac{x}{2} dx$

$= \frac{1}{2}\left[2\tan\frac{x}{2}\right]_0^{\pi/2} = \tan\frac{\pi}{4} - \tan 0 = 1$

3. $\int_0^{\pi/4} \frac{\cos 2\theta}{\cos^2\theta} d\theta$ [ব.'১১]

$= \int_0^{\pi/4} \frac{2\cos^2\theta - 1}{\cos^2\theta} d\theta$

$= \int_0^{\pi/4} (2 - \sec^2\theta) dx = [2\theta - \tan\theta]_0^{\pi/4}$

$= 2.\frac{\pi}{4} - \tan\frac{\pi}{4} - (2.0 - \tan 0) = \frac{\pi}{2} - 1$

4(a) $\int_0^{\pi/2} \cos^2 x dx$ [চ. '০৪; রা.'০৫ ,'০৯; সি.'১১]

$= \int_0^{\pi/2} \frac{1}{2}(1+\cos 2x) dx = \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/2}$

$= \frac{1}{2}\{(\frac{\pi}{2} + \frac{1}{2}\sin\pi) - (0 + \frac{1}{2}\sin 0)\} = \frac{\pi}{4}$

4 (b) $\int_0^{\pi/2} \cos^3 x dx$ [সি.'০৬,'০৭; য. '০৭,'০৯, '১৩; ব. '০৮; মা.'০৬; দি.'১৩]

$$= \int_0^{\pi/2} \frac{1}{4}(3\cos x + \cos 3x)dx$$

$$= \frac{1}{4}\left[3\sin x + \frac{1}{3}\sin 3x\right]_0^{\pi/2}$$

$$= \frac{1}{4}(3\sin\frac{\pi}{2} + \frac{1}{3}\sin\frac{3\pi}{2} - 3\sin 0 - \frac{1}{3}\sin 0)$$

$$= \frac{1}{4}(3.1 + \frac{1}{3}(-1) - 0 - 0) = \frac{1}{4}\times\frac{8}{3} = \frac{2}{3}$$

4(c) $\int_0^{\pi/2} \cos^4 x\, dx$ [য.'০৪]

$$\cos^4 x = \frac{1}{4}(2\cos^2 x)^2 = \frac{1}{4}(1 + \cos 2x)^2$$

$$= \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$$

$$= \frac{1}{4}\{1 + 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\}$$

$$= \frac{1}{4}(\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x)$$

$$\int_0^{\pi/2} \cos^4 x\, dx$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{2}{2}\sin 2x + \frac{1}{2}.\frac{1}{4}\sin 4x\right]_0^{\pi/2}$$

$$= \frac{1}{4}(\frac{3}{2}.\frac{\pi}{2} + \sin\pi + \frac{1}{8}\sin 2\pi - 0)$$

$$= \frac{1}{4}(\frac{3\pi}{4} + 0) = \frac{3\pi}{16}$$

4(d) $\int_0^{\pi/4} \tan^2 x\, dx = \int_0^{\pi/4} (\sec^2 x - 1)\, dx$

$$= [\tan x - x]_0^{\pi/4} = \tan\frac{\pi}{4} - \frac{\pi}{4} - (\tan 0 - 0)$$

$$= 1 - \frac{\pi}{4}$$

4(e) $\int_0^{\pi/2} \sin^2 2\theta\, d\theta$ [মা.বো.'০৯]

$$= \int_0^{\pi/2} \frac{1}{2}(1 - \cos 4\theta)\, d\theta = \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\{\frac{\pi}{2} - \frac{\sin 2\pi}{4} - (0 - \frac{\sin 0}{4})\}$$

$$= \frac{1}{2}\{\frac{\pi}{2} - 0 - (0 - 0)\} = \frac{\pi}{4}$$

5 (a) $\int_0^{\pi/2} \cos^5 x \sin x dx$ [ঢা.'০৩; দি.'১০; য.'১১]

$$= -\int_0^{\pi/2} (\cos x)^5 (-\sin x)dx$$

$$= -\left[\frac{1}{6}(\cos x)^6\right]_0^{\pi/2}$$

$$= -\frac{1}{6}\{(\cos\frac{\pi}{2})^6 - (\cos 0)^6\}$$

$$= -\frac{1}{6}\{0 - 1\} = \frac{1}{6}$$

5(b) ধরি $I = \int_0^{\pi/4} \sin^4 x \cos^4 x\, dx$ [প্র.ভ.প'৮৯]

$$\sin^4 x\cos^4 x = \frac{1}{16}(2\sin x\cos x) = \frac{1}{16}\sin^4 2x$$

$$= \frac{1}{16}.\{\frac{1}{2}(1 - \cos 4x)\}^2$$

$$= \frac{1}{64}(1 - 2\cos 4x + \cos^2 4x)$$

$$= \frac{1}{64}\{1 - 2\cos 4x + \frac{1}{2}(1 + \cos 8x)\}$$

$$= \frac{1}{128}(3 - 4\cos 4x + \cos 8x)$$

$$\therefore I = \frac{1}{128}\left[3x - 4.\frac{1}{4}\sin 4x + \frac{1}{8}\sin 8x\right]_0^{\pi/4}$$

$$= \frac{1}{128}(\frac{3\pi}{4} - \sin\pi + \frac{1}{8}\sin 2\pi - 0)$$

$$= \frac{1}{128}\times\frac{3\pi}{4} = \frac{3\pi}{512}$$

5(c) $\int_0^{\pi/2} \sin^2 x \sin 3x\, dx$

[ব.'০৫; মা.'০৪; য.'১৪]

$$= \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x)\sin 3x\, dx$$

$$= \int_0^{\pi/2} (\frac{1}{2}\sin 3x - \frac{1}{2}\cos 2x\sin 3x)\, dx$$

$$= \int_0^{\pi/2} \{\frac{1}{2}\sin 3x - \frac{1}{4}(\sin 5x + \sin x)\}\, dx$$

$$\left[-\frac{1}{2}.\frac{1}{3}\cos 3x-\frac{1}{4}(-\frac{1}{5}\cos 5x-\cos x)\right]_0^{\pi/2}$$

$$=-\frac{1}{6}(\cos\frac{3\pi}{2}-\cos 0)+\frac{1}{20}(\cos\frac{5\pi}{2}-\cos 0)$$

$$+\frac{1}{4}(\cos\frac{\pi}{2}-\cos 0)$$

$$-\frac{1}{6}(0-1)+\frac{1}{20}(0-1)+\frac{1}{4}(0-1)$$

$$\frac{1}{6}-\frac{1}{20}-\frac{1}{4} \quad \frac{10-3-15}{60} \quad \frac{-8}{60}=\frac{-2}{15}$$

5(d) ধরি $\int_0^{\pi} 3\sqrt{1-\cos x}\sin x\,dx$ [চ.'০৪]

$z=\cos x \qquad dz=-\sin x dx$

$x=0$ হলে $z=1$ $\quad x=\pi$ হলে $z=-1$

$$-3\int_1^{-1}\sqrt{1-z}dz=-3\left[-\frac{2}{3}(1-z)^{\frac{3}{2}}\right]_1^{-1}$$

$$2\{(1+1)^{\frac{3}{2}}-(1-1)^{\frac{3}{2}}\} \quad 2\times 2\sqrt{2}=4\sqrt{2}$$

১০) $\int_0^{\pi/2}(1+\cos\theta)^2\sin\theta\,d\theta$

[বুয়েট, ০৮-০৯;ঢ.'১১]

$z=1+\cos x \qquad dz=-\sin x dx$

$x=0$ হলে $z=2 \qquad x=\frac{\pi}{2} \qquad z=1$

$$\int_0^{\pi/2}(1+\cos\theta)^2\sin\theta\,d\theta=-\int_2^1 z^2dz$$

$$\left[-\frac{z^3}{3}\right]_2^1=-(\frac{1^3}{3}-\frac{2^3}{3})=-(\frac{1}{3}-\frac{8}{3})=\frac{7}{3}$$

$$\int_0^{\pi/2}\sin x\sin 2x\,dx$$

$$\int_0^{\pi/2}\frac{1}{2}(\cos x-\cos 3x)\,dx$$

$$\frac{1}{2}\left[\sin x-\frac{1}{3}\sin 3x\right]_0^{\pi/2}$$

$$\frac{1}{2}(\sin\frac{\pi}{2}-\frac{1}{3}\sin\frac{3\pi}{2}-\sin 0+\frac{1}{3}\sin 0)$$

$$\frac{1}{2}\{1-\frac{1}{3}(-1)-0+0\}=\frac{1}{2}\times\frac{4}{3}=\frac{2}{3}$$

6(b) $\int_0^{\pi/2}\cos 2x\cos 3x\,dx$ [কু.

$$\int_0^{\pi/2}\frac{1}{2}(\cos 5x+\cos x)\,dx$$

$$\frac{1}{2}\left[\frac{1}{5}\sin 5x+\sin x\right]_0^{\pi/2}$$

$$\frac{1}{2}(\frac{1}{5}\sin\frac{5\pi}{2}+\sin\frac{\pi}{2}-\frac{1}{5}\sin 0-\sin 0)$$

$$\frac{1}{2}(\frac{1}{5}.1+1)=\frac{1}{2}\times\frac{6}{5}=\frac{3}{5}$$

6(c) $\int_0^{\pi/2}\sin 2x\cos x\,dx$ [দি.'০৫; রা

$$\int_0^{\pi/2}\frac{1}{2}(\sin 3x+\sin x)\,dx$$

$$=\frac{1}{2}\left[-\frac{1}{3}\cos 3x-\cos x\right]_0^{\pi/2}$$

$$\frac{1}{2}(-\frac{1}{3}\cos 3\frac{\pi}{2}-\cos\frac{\pi}{2}+\frac{1}{3}\cos 0+\cos 0)$$

$$\frac{1}{2}[-\frac{1}{3}(\cos 3\frac{\pi}{2}-\cos 0)-(\cos\frac{\pi}{2}-\cos 0)]$$

$$\frac{1}{2}[-\frac{1}{3}(0-1)-(0-1)]=\frac{1}{2}(\frac{1}{3}+1)=\frac{2}{3}$$

7.(a) ধরি, $I=\int_0^{\pi/2}\sqrt{\cos x}\sin^3 x dx$

$$\int_0^{\pi/2}\sqrt{\cos x}\sin^2 x\sin x dx$$

$$\int_0^{\pi/2}\sqrt{\cos x}(1-\cos^2 x)\sin x dx$$

$z=\cos x \qquad dz=-\sin x dx$

$x=0$ হলে $z=1 \qquad x=\frac{\pi}{2}$ হলে $z=0$

$$=-\int_1^0\sqrt{z}(1-z^2)\,dz$$

$$-\int_1^0(\sqrt{z}-z^{5/2})\,dz=-\left[\frac{z^{3/2}}{3/2}-\frac{z^{7/2}}{7/2}\right]_1^0$$

$$-\{\frac{2}{3}(0-1)-\frac{2}{7}(0-1)\}=-(-\frac{2}{3}+\frac{2}{7})$$

$-\frac{-14+6}{21}=\frac{8}{21}$

7(b) ধরি $\int_0^{\pi/2}\frac{\cos^3 xdx}{\sqrt{\sin x}}$ [ব.,য.'১০;

$\int_0^{\pi/2}\frac{\cos^2 x\cos xdx}{\sqrt{\sin x}}$

$\int_0^{\pi/2}\frac{(1-\sin^2 x)\cos xdx}{\sqrt{\sin x}}$

$z=\sin x$ $\quad dz=\cos x\,dx$

$x=0$ হলে $z=0$ $\quad x=\frac{\pi}{2}$ $\quad z=1$

I $\int_0^1(\frac{1-z^2}{\sqrt{z}})\,dz$ $\quad \int_0^1(\frac{1}{\sqrt{z}}-z^{3/2})\,dz$

$=\left[2\sqrt{z}-\frac{z^{5/2}}{5/2}\right]_0^1$ $\quad 2(1-0)-\frac{2}{5}(1-0)$

$=2-\frac{2}{5}=\frac{8}{5}$

8.(a) ধরি $\int_0^1\frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}dx$ '১১]

$z=\sin^{-1}x$ $\quad dz=\frac{1}{\sqrt{1-x^2}}dx$

$x=0$ হলে $z=0$ এবং $x=1$ $\quad z=\frac{\pi}{2}$

I $\int_0^{\pi/2}z^2dz=\left[\frac{z^3}{3}\right]_0^{\pi/2}=\frac{1}{3}\{(\frac{\pi}{2})^3-0\}$

$=\frac{\pi^3}{24}$

8(b) $\int_0^1\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ [ম.'০৪;সি.০৭;ব.'০৮;

ধরি, $z=\sin^{-1}x$ $\quad dz=\frac{1}{\sqrt{1-x^2}}dx$

$x=0$ হলে $z=0$ এবং $x=1$ $\quad z=\frac{\pi}{2}$

$\int_0^1\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ $\quad \int_0^{\pi/2}zdz=\left[\frac{z^2}{2}\right]_0^{\pi/2}$

$=\frac{1}{2}\{(\frac{\pi}{2})^2-0\}=\frac{\pi^2}{8}$

8(c) ধরি, $\int_0^1\frac{\tan^{-1}x}{1+x^2}dx$ [বুয়েট'০৯; ১]

$z=\tan^{-1}x$ $\quad dz=\frac{1}{1+x^2}dx$

সীমা: $x=0$ হলে $z=0$ $\quad x=1$ হলে $z=\frac{\pi}{4}$

$\int_0^{\pi/4}zdz=\left[\frac{z^2}{2}\right]_0^{\pi/4}$ $\quad \frac{1}{2}\cdot\frac{\pi^2}{16}=\frac{\pi^2}{32}$

9(a) $\int_0^1\frac{xdx}{\sqrt{1-x^2}}$ [চ.'০৭; ব.'০৭]

$-\frac{1}{2}\int_0^1\frac{(-2x)dx}{\sqrt{1-x^2}}=-\frac{1}{2}\left[2\sqrt{1-x^2}\right]_0^1$

$-(\sqrt{1-1^2}-\sqrt{1-0^2})=-(0-1)$

9(b) $\int_4^8\frac{xdx}{\sqrt{x^2-15}}=\frac{1}{2}\int_4^8\frac{d(x^2-15)}{\sqrt{x^2-15}}$

$\frac{1}{2}\left[2\sqrt{x^2-15}\right]_4^8$ $\quad \sqrt{64-15}-\sqrt{16-15}$

$\sqrt{64-15}-\sqrt{16-15}$ $\quad \sqrt{49}-\sqrt{1}$ $\quad 6$

9(c) $\int_0^2\frac{xdx}{\sqrt{9-2x^2}}$

[বুয়েট'০৯; ব.'১০; রা.'১২; সি.'১৩; দি.,রু.'১৪]

$=-\frac{1}{4}\int_0^2\frac{d(9-2x^2)}{\sqrt{9-2x^2}}$ $\quad -\frac{1}{4}\left[2\sqrt{9-2x^2}\right]_0^2$

$-\frac{1}{2}(\sqrt{9-8}-\sqrt{9-0})=-\frac{1}{2}(1-3)=$

9(d) ধরি, I $=\int_0^1\frac{xdx}{\sqrt{4-x^2}}$

রা., কু.'১০; দি.'১৩]

এবং $z=4-x^2$ $\quad dz=-2xdx$

$x=0$ হলে $z=4$ $\quad x=1$ $\quad z=3$

$\therefore \; -\frac{1}{2}\int_4^3 \frac{dz}{\sqrt{z}} = -\frac{1}{2}\left[2\sqrt{z}\right]_4^3$

$-(\sqrt{3}-\sqrt{4}) = 2-\sqrt{3}$

9(e) $\int_{-2}^{5} \frac{7x}{\sqrt{x^2+3}}dx$ [সা.বো.'০৪]

$z = x^2+3 \quad dz = 2xdx$

$x=-2$ হলে $z=7 \qquad x=5 \qquad z=28$

$\therefore \; \frac{7}{2}\int_7^{28} \frac{dz}{\sqrt{z}} = \frac{7}{2}\left[2\sqrt{z}\right]_7^{28}$

$7(\sqrt{28}-\sqrt{7}) = 7(2\sqrt{7}-\sqrt{7}) = 7\sqrt{7}$

9(f) ধরি, $I = \int_0^1 x^3\sqrt{1+3x^4}\,dx$ '০৯;

'০৮; সা.'০৫, '০৯]

এবং $z = 1+3x^4 \quad dz = 12x^3dx$

$x=0$ হলে $z=1 \qquad x=1 \qquad z=4$

$\therefore \; \frac{1}{12}\int_1^4 \sqrt{z}dz = \frac{1}{12}\left[\frac{z^{3/2}}{3/2}\right]_1^4$

$\frac{1}{12}\times\frac{2}{3}(4^{3/2}-1) = \frac{1}{18}(8-1) = \frac{7}{18}$

10. $\int_1^2 x^2e^{x^3}dx$ সা.'০৪,'০৬; ব.'১০]

$z = x^3 \quad dz = 3x^2dx \Rightarrow x^2dx = \frac{1}{3}dz$

$x=1$ হলে $z=1$ এবং $x=2$ হলে $z=8$

$\int_1^2 x^2e^{x^3}dx = \frac{1}{3}\int_1^8 e^z dz = \frac{1}{3}\left[e^z\right]_1^8$

$= \frac{1}{3}(e^8-e^1) = \frac{1}{3}(e^8-e)$

10(b) $\int_0^1 xe^{x^2}dx$ [

কু.দি.'১২]

$z = x^2 \quad dz = 2xdx \qquad xdx = \frac{1}{2}dz$

$x=0$ হলে $z=0 \qquad x=1 \qquad z=1$

$\int_0^1 xe^{x^2}dx \quad \frac{1}{2}\int_0^1 e^z dz = \frac{1}{2}\left[e^z\right]_0^1$

$\frac{1}{2}(e^1-e^0) = \frac{1}{2}(e-1)$

10(c) $\int_0^{\ln 2} \frac{e^x}{1+e^x}dx$ য.'১০;

কু.'১৩;

ধরি $z = 1+e^x \quad dz = e^x dx$

$x=0 \qquad z = 1+e^0 = 1+1 = 2$

$x=\ln 2$ হলে $z = 1+e^{\ln 2} = 1+2 = 3$

$\int_0^{\ln 2}\frac{e^x}{1+e^x}dx = \int_2^3 \frac{dz}{z} = \left[\ln z\right]_2^3$

$\ln 3-\ln 2 = \ln\frac{3}{2}$

10(d) $\int_1^3 \frac{1}{x}\cos(\ln x)\,dx$ '০৮; ঢা.'০৮,'১৪; ব]

ধরি, $z = \ln x \quad dz = \frac{dx}{x}$

সীমা: $x=1$ হলে $z = \ln 1 = 0$

$x=3 \qquad z = \ln 3$

$\int_1^3 \frac{1}{x}\cos(\ln x)\,dx \quad \int_0^{\ln 3}\cos z\,dz$

$\left[\sin z\right]_0^{\ln 3} = \sin(\ln 3)-\sin 0 = \sin(\ln 3)$

11. (a) $\int_{\pi/3}^{\pi/2} \frac{\cos^5 x}{\sin^7 x}dx$

[ব. '০৭; য.'০৮; দি.'১১; ঢ.'১২; কুয়েট'০৯]

$\int_{\pi/3}^{\pi/2} \cot^5 x\,\mathrm{cosec}^2 x\,dx$

ধরি, $\cot x = z \qquad -\mathrm{cosec}^2 x\,dx = dz$

সীমা: $x = \frac{\pi}{3} \qquad z = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{2}$ হলে $z = \cot\frac{\pi}{2} = 0$

$\int_{\pi/3}^{\pi/2}\frac{\cos^5 x}{\sin^7 x}dx \quad \int_{1/\sqrt{3}}^{0} z^5(-dz)$

$-\left[\frac{1}{6}z^6\right]_{1/\sqrt{3}}^{0} \quad -\frac{1}{6}\{0-(\frac{1}{\sqrt{3}})^6\} = \frac{1}{162}$

11.(b) ধরি, $I=\int_0^{\pi/4}\tan^3 x\sec^2 x\,dx$ [য.'০৬; মা.'০৬,'০৮; কু., সি, দি.'০৯; ঢা.,ব.'১১; সি.'১৩]

এবং $\tan x=z \therefore \sec^2 x\,dx=dz$

সীমা: $x=0$ হলে $z=\tan 0=0$ এবং

$x=\frac{\pi}{4}$ হলে $z=\tan\frac{\pi}{4}=1$

$$\therefore I=\int_0^1 z^3dz=\left[\frac{1}{4}z^4\right]_0^1=\frac{1}{4}(1^4-0^4)=\frac{1}{4}$$

11(c) $\int_0^{\pi/4}(\tan^3 x+\tan x)dx$ [কু.'০৮]

$$=\int_0^{\pi/4}(\tan^2 x+1)\tan x dx$$

$$=\int_0^{\pi/4}\sec^2 x\tan x dx$$

$$=\int_0^{\pi/4}(\tan x)d(\tan x)=\left[\frac{1}{2}(\tan x)^2\right]_0^{\pi/4}$$

$$=\frac{1}{2}\{(\tan\frac{\pi}{4})^2-(\tan 0)^2\}=\frac{1}{2}\{(1)^2-0\}=\frac{1}{2}$$

11(d) $\int_0^{\pi/4}\tan^2 x\sec^2 x\,dx$ [ঢা.'০৩,'১৩; কু.'০৪,'০৬; য.'০৪; ঢা.'০৫; রা.'০৫; চ.'১১]

ধরি, $\tan x=z \therefore \sec^2 x\,dx=dz$

সীমা: $x=0$ হলে $z=\tan 0=0$ এবং

$x=\frac{\pi}{4}$ হলে $z=\tan\frac{\pi}{4}=1$

$$\int_0^{\pi/4}\tan^2 x\sec^2 x\,dx=\int_0^1 z^2\,dz$$

$$=\left[\frac{1}{3}z^3\right]_0^1=\frac{1}{3}(1^3-0^3)=\frac{1}{3}$$

12. (a) $\int xe^{-3x}dx$ [দি.'১০]

$$=x\int e^{-3x}dx-\int\{\frac{d}{dx}(x)\int e^{-3x}dx\}dx$$

$$=x(-\frac{1}{3}e^{-3x})-\int 1.(-\frac{1}{3}e^{-3x})dx$$

$$=-x\frac{1}{3}e^{-3x}+\frac{1}{3}(-\frac{1}{3}e^{-3x})$$

$$=-\frac{1}{3}xe^{-3x}-\frac{1}{9}e^{-3x}=-\frac{1}{9}(3x+1)e^{-3x}$$

$$\therefore\int_0^1 xe^{-3x}dx=\left[-\frac{1}{9}(3x+1)e^{-3x}\right]_0^1$$

$$=-\frac{1}{9}\{(3+1)e^{-3}-(0+1)e^{-0}\}$$

$$=-\frac{1}{9}(4e^{-3}-1)=\frac{1}{9}(1-4e^{-3})$$

12(b) $\int\ln(2x)dx$ [য. '০১;ব.'০৯]

$$=\ln(2x)\int dx-\int[\frac{d}{dx}\{\ln(2x)\}\int dx]dx$$

$$=x\ln(2x)-\int\frac{2}{2x}.xdx$$

$$=x\ln(2x)-\int dx=x\ln(2x)-x+c$$

$$\therefore\int_2^4\ln(2x)\,dx=[x\ln(2x)-x]_2^4$$

$$=4\ln 8-4-(2\ln 4-2)$$

$$=4\ln 2^3-4-2\ln 2^2+2$$

$$=12\ln 2-2-4\ln 2=8\ln 2-2$$

12(c) $\int\frac{\ln x}{\sqrt{x}}dx$ [প্র.ভ.প'১৬]

$$=\ln x\int\frac{1}{\sqrt{x}}dx-\int[\frac{d}{dx}(\ln x)\int\frac{1}{\sqrt{x}}dx]dx$$

$$=2\sqrt{x}\ln x-\int\frac{1}{x}.2\sqrt{x}dx$$

$$=2\sqrt{x}\ln x-2\int\frac{1}{\sqrt{x}}dx$$

$$=2\sqrt{x}\ln x-2.2\sqrt{x}+c$$

$$=2\sqrt{x}(\ln x-2)+c$$

$$\int_1^4\frac{\ln x}{\sqrt{x}}dx=\left[2\sqrt{x}(\ln x-2)\right]_1^4$$

$$=2\sqrt{4}(\ln 4-2)-2\sqrt{1}(\ln 1-2)$$

$$=4\ln 2^2-8-2(0-2)$$

$$=8\ln 2-8+4=8\ln 2-4$$

12(d) $\int x^2\cos x\,dx$ [কু.'০৪]

$= x^2\int\cos x\,dx-\int\{\frac{d}{dx}(x^2)\int\cos x\,dx\}dx$

$= x^2\sin x-\int 2x\sin x\,dx$

$= x^2\sin x-2[x\int\sin x\,dx-\int 1.(-\cos x)dx]$

$= x^2\sin x-2[x(-\cos x)+\sin x]+c$

$= x^2\sin x+2x\cos x-2\sin x+c$

$\int_0^{\pi/2}x^2\cos x\,dx$

$=\left[x^2\sin x+2x\cos x-2\sin x\right]_0^{\pi/2}$

$=(\frac{\pi}{2})^2\sin\frac{\pi}{2}+2.\frac{\pi}{2}\cos\frac{\pi}{2}-2\sin\frac{\pi}{2}-0$

$=\frac{\pi^2}{4}.1+2.\frac{\pi}{2}.0-2.1=\frac{\pi^2}{4}-2$

12(e) $\int x\tan^{-1}x\,dx$

[রা. '০৮,'১২; চ. '০৮,'১২; য.'১১; সি.'১২; কু.'১৪]

$= \tan^{-1}x\int xdx-\int\{\frac{d}{dx}(\tan^{-1}x)\int xdx\}dx$

$=\frac{x^2}{2}\tan^{-1}x-\int\frac{1}{1+x^2}.\frac{x^2}{2}dx$

$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$

$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int(1-\frac{1}{1+x^2})dx$

$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}(x-\tan^{-1}x)+c$

$=\frac{1}{2}\{(x^2+1)\tan^{-1}x-x\}+c$

$\int_1^{\sqrt{3}}x\tan^{-1}x\,dx=\left[\frac{(x^2+1)\tan^{-1}x-x}{2}\right]_1^{\sqrt{3}}$

$=\frac{(3+1)\tan^{-1}\sqrt{3}-\sqrt{3}-(1+1)\tan^{-1}1+1}{2}$

$=\frac{1}{2}(4.\frac{\pi}{3}-\sqrt{3}-2.\frac{\pi}{4}+1)$

$=\frac{1}{2}(\frac{4\pi}{3}-\frac{\pi}{2}-\sqrt{3}+1)$

$=\frac{1}{2}(\frac{8\pi-3\pi}{6}-\sqrt{3}+1)=\frac{1}{12}(5\pi-6\sqrt{3}+6)$

12(f) ধরি, $I=\int_0^{\pi/2}e^x(\sin x+\cos x)\,dx$

[কু.'০৫,'১১; রা.'১০]

এবং $f(x)=\sin x\ \therefore\ f'(x)=\cos x$

$\therefore\ I=\int_0^{\pi/2}e^x\{f(x)+f'(x)\}dx$

$=\left[e^xf(x)\right]_0^{\pi/2}=\left[e^x\sin x\right]_0^{\pi/2}$

$=e^{\pi/2}\sin\frac{\pi}{2}-e^0\sin 0=e^{\pi/2}-0=e^{\pi/2}$

12(g) $\int\ln x\,dx$ **[প্র.ভ.প.'০৫]**

$=\ln x\int dx-\int\{\frac{d}{dx}(\ln x)\int dx\}dx$

$=x\ln x-\int\frac{1}{x}.xdx=x\ln x-\int dx$

$=x\ln x-x+c=x(\ln x-1)+c$

$\therefore\ \int_1^0\ln x\,dx=\left[x(\ln x-1)\right]_1^0$

$=0-1(\ln 1-1)=-1(0-1)=1$

12(h) $\int x\sin^2x\,dx$ **[প্র.ভ.প.'০৫]**

$=\int\frac{x}{2}(1-\cos 2x)\,dx=\frac{1}{2}\int(x-x\cos 2x)\,dx$

$=\frac{1}{2}.\frac{x^2}{2}-\frac{1}{2}[x\int\cos 2xdx-\int\{1.\frac{1}{2}\sin 2x\,dx\}]$

$=\frac{1}{4}x^2-\frac{1}{2}[x.\frac{1}{2}\sin 2x-\frac{1}{2}\int\sin 2x\,dx]$

$=\frac{1}{4}x^2-\frac{1}{4}[x\sin 2x-(-\frac{1}{2}\cos 2x)]+c$

$=\frac{1}{4}(x^2-x\sin 2x-\frac{1}{2}\cos 2x)+c$

$\therefore\ \int_0^{\pi}x\sin^2x\,dx=\frac{1}{4}\left[x^2-x\sin 2x-\frac{1}{2}\cos 2x\right]_0^{\pi}$

$=\frac{1}{4}\{(\pi^2-\pi\sin 2\pi-\frac{1}{2}\cos 2\pi)+\frac{1}{2}\cos 0\}$

$=\frac{1}{4}\{\pi^2-0-\frac{1}{2}.1)+\frac{1}{2}.1\}=\frac{1}{4}\pi^2$

12(i) $\int x\cot^{-1}x\,dx$ [বুয়েট'০৯]

$= \cot^{-1}x\int xdx - \int\{\frac{d}{dx}(\cot^{-1}x)\int xdx\}dx$

$= \frac{x^2}{2}\cot^{-1}x + \int\frac{1}{1+x^2}.\frac{x^2}{2}dx$

$= \frac{x^2}{2}\cot^{-1}x + \frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$

$= \frac{x^2}{2}\cot^{-1}x + \frac{1}{2}\int(1-\frac{1}{1+x^2})dx$

$= \frac{x^2}{2}\cot^{-1}x + \frac{1}{2}(x+\cot^{-1}x)+c$

$= \frac{1}{2}\{(x^2+1)\cot^{-1}x + x\}+c$

$\int_1^{\sqrt{3}} x\cot^{-1}x\,dx = \left[\frac{(x^2+1)\cot^{-1}x + x}{2}\right]_1^{\sqrt{3}}$

$= \frac{(3+1)\cot^{-1}\sqrt{3}+\sqrt{3}-(1+1)\cot^{-1}1-1}{2}$

$= \frac{1}{2}(4.\frac{\pi}{6}+\sqrt{3}-2.\frac{\pi}{4}-1)$

$= \frac{1}{2}(\frac{2\pi}{3}-\frac{\pi}{2}+\sqrt{3}-1)$

$= \frac{1}{2}(\frac{4\pi-3\pi}{6}+\sqrt{3}-1) = \frac{1}{12}(\pi+6\sqrt{3}-6)$

(j) $\int x\ln x\,dx$ [য.'০৫; রা.'১৪]

$= \ln x\int x\,dx - \int\{\frac{d}{dx}(\ln x)\int xdx\}dx$

$= \ln x.\frac{x^2}{2} - \int(\frac{1}{x}\times\frac{x^2}{2})dx$

$= \ln x.\frac{x^2}{2} - \frac{1}{2}\int xdx = \frac{x^2}{2}\ln x - \frac{1}{2}\times\frac{x^2}{2}+c$

$\int_1^{\sqrt{e}} x\ln x\,dx = \left[\frac{x^2}{2}\ln x - \frac{1}{4}x^2\right]_1^{\sqrt{e}}$

$= \frac{(\sqrt{e})^2}{2}\ln\sqrt{e} - \frac{1}{4}(\sqrt{e})^2 - \frac{1}{2}\ln 1 + \frac{1}{4}$

$= \frac{e}{2}.\frac{1}{2}\ln e - \frac{1}{4}e - \frac{1}{2}\times 0 + \frac{1}{4}$

$= \frac{e}{4}.1 - \frac{1}{4}e - \frac{1}{2}\times 0 + \frac{1}{4} = \frac{1}{4}$

13(a) $\int_0^1\frac{x\,dx}{1+x^4}$ [প্র.ভ.প.'০৬]

$= \frac{1}{2}\int_0^1\frac{2x\,dx}{1+(x^2)^2} = \left[\frac{1}{2}\tan^{-1}(x^2)\right]_0^1$

$= \frac{1}{2}(\tan^{-1}1 - \tan^{-1}0) = \frac{1}{2}(\frac{\pi}{4}-0) = \frac{\pi}{8}$

13(b) $\int_0^1\frac{1+x}{1+x^2}dx$

[রা. '০৬, '০৯; ব. '০৭ ; ঢা. '০৯; কু.,সি.'১২,'১৪]

$= \int_0^1(\frac{1}{1+x^2}+\frac{x}{1+x^2})dx$

$= \int_0^1(\frac{1}{1+x^2}+\frac{1}{2}\frac{2x}{1+x^2})dx$

$= \left[\tan^{-1}x + \frac{1}{2}\ln(1+x^2)\right]_0^1$

$= \tan^{-1}1 + \frac{1}{2}\ln 2 - \tan^{-1}0 - \frac{1}{2}\ln 1$

$= \frac{\pi}{4} + \frac{1}{2}\ln 2 - 0 + 0 = \frac{\pi}{4} + \frac{1}{2}\ln 2$

13(c) $\int_0^\pi\frac{\sin x}{1+\cos^2 x}dx$ [ঢা.'০৭]

$= -\int_0^\pi\frac{(-\sin x)}{1^2+\cos^2 x}dx = -\left[\tan^{-1}(\cos x)\right]_0^\pi$

$= -\{\tan^{-1}(\cos\pi) - \tan^{-1}(\cos 0)\}$

$= -\{\tan^{-1}(-1) - \tan^{-1}(1)\}$

$= -(-\frac{\pi}{4}-\frac{\pi}{4}) = \frac{\pi}{2}$

13(d) ধরি, $I = \int_0^{\pi/4}\frac{\sin 2x}{\cos^4 x + \sin^4 x}dx$ [প্র.ভ.প.'০৭]

$\cos^4 x + \sin^4 x = (\cos^2 x)^2 + (\sin^2 x)^2$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$$= 1-\frac{1}{2}(2\sin x\cos x)^2 = 1-\frac{1}{2}\sin^2 2x$$

$$= 1-\frac{1}{2}(1-\cos^2 2x) = \frac{1}{2}(1+\cos^2 2x)$$

$$I = 2\int_0^{\pi/4}\frac{\sin 2x}{1+\cos^2 2x}dx$$

$$= 2(-\frac{1}{2})\int_0^{\pi/4}\frac{(-2\sin 2x)}{1^2+(\cos 2x)^2}dx$$

$$= -\left[\tan^{-1}(\cos 2x)\right]_0^{\pi/4}$$

$$= -\{\tan^{-1}(\cos\frac{\pi}{2}) - \tan^{-1}(\cos 0)\}$$

$$= -\{\tan^{-1}0 - \tan^{-1}1\} = -\{0-\frac{\pi}{4}\} = \frac{\pi}{4}$$

13(e) $\int_0^1 \frac{dx}{e^x+e^{-x}}$

[রা.'১২; সি. '০৭; কু.'০৮; ব.'১৩; ঢা.'১৪]

$$= \int_0^1 \frac{e^x dx}{e^x(e^x+e^{-x})} = \int_0^1 \frac{e^x dx}{(e^x)^2+1}$$

ধরি, $e^x = z \quad \therefore e^x dx = dz$

সীমা : $x=0$ হলে, $z=e^0=1$

$x=1$ হলে, $z=e^1=e$

$$\int_0^1 \frac{dx}{e^x+e^{-x}} = \int_1^e \frac{dz}{z^2+1} = \left[\tan^{-1}z\right]_1^e$$

$$= \tan^{-1}e - \tan^{-1}(1) = \tan^{-1}e - \frac{\pi}{4}$$

14(a) $\int_3^4 \frac{dx}{25-x^2}$ [ব.'১৩]

$$= \int_3^4 \frac{dx}{5^2-x^2} = \left[\frac{1}{2.5}\ln\left|\frac{5+x}{5-x}\right|\right]_3^4$$

$$= \frac{1}{10}(\ln\left|\frac{5+4}{5-4}\right| - \ln\left|\frac{5+3}{5-3}\right|)$$

$$= \frac{1}{10}(\ln 9 - \ln 4) = \frac{1}{10}\ln\frac{9}{4} = \frac{1}{10}\ln(\frac{3}{2})^2$$

$$= \frac{1}{10}\times 2\ln(\frac{3}{2}) = \frac{1}{5}\ln(\frac{3}{2})$$

(b) $\int_0^{\pi/2}\frac{\cos x dx}{9-\sin^2 x}dx$ [ঢা.'০৫; যা.'০৮; চ., সি.'০৯]

ধরি, $\sin x = z$. $\cos x dx = dz$

সীমা : $x=0$ হলে $z=0$ এবং $x=\frac{\pi}{2}$ হলে $z=1$

$$\therefore \int_0^{\pi/2}\frac{\cos x dx}{9-\sin^2 x}dx = \int_0^1 \frac{dz}{3^2-z^2}$$

$$= \left[\frac{1}{2.3}\ln\left|\frac{3+z}{3-z}\right|\right]_0^1 = \frac{1}{6}(\ln\left|\frac{3+1}{3-1}\right| - \ln\left|\frac{3+0}{3-0}\right|)$$

$$= \frac{1}{6}(\ln 2 - \ln 1) = \frac{1}{6}\ln 2$$

15 (a) $\int_0^1 \frac{dx}{\sqrt{2x-x^2}} = \int_0^1 \frac{dx}{\sqrt{1-(x^2-2x+1)}}$

$$= \int_0^1 \frac{d(x-1)}{\sqrt{1-(x-1)^2}} = \left[\sin^{-1}(x-1)\right]_0^1$$

$$= \sin^{-1}(1-1) - \sin^{-1}(0-1) = \sin^{-1}0 + \sin^{-1}1$$

$$= \frac{\pi}{2}$$

15 (b) $\int_{1/2}^1 \frac{dx}{x\sqrt{4x^2-1}}$ [প্র.ভ.প. '০৪]

$$= \int_{1/2}^1 \frac{2dx}{2x\sqrt{(2x)^2-1}} = \left[\sec^{-1}(2x)\right]_{1/2}^1$$

$$= \sec^{-1}2 - \sec^{-1}1 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$$

15(c) ধরি $I = \int_1^2 \frac{dx}{x^2\sqrt{4-x^2}}$ [প্র.ভ.প.'০৪]

এবং $x = 2\cos\theta$. তাহলে $dx = -2\sin\theta\, d\theta$

সীমা : $x=1$ হলে $\theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$ এবং

$x=2$ হলে $\theta = \cos^{-1}1 = 0$

$$\therefore I = \int_{\pi/3}^0 \frac{-2\sin\theta\, d\theta}{4\cos^2\theta\sqrt{4(1-\cos^2\theta)}}$$

$$= \int_{\pi/3}^0 \frac{-2\sin\theta\, d\theta}{4\cos^2\theta . 2\sin\theta} = -\frac{1}{4}\int_{\pi/3}^0 \sec^2\theta\, d\theta$$

$$= -\frac{1}{4}\left[\tan\theta\right]_{\pi/3}^0 = -\frac{1}{4}(\tan 0 - \tan\frac{\pi}{3})$$

$$= -\frac{1}{4}(0-\sqrt{3}) = \frac{\sqrt{3}}{4}$$

15 (d) $\int_0^{\pi/6} \frac{dx}{1-\tan^2 x}$ [বুয়েট-০৭-০৮]

$= \int_0^{\pi/6} \frac{\cos^2 x dx}{\cos^2 x - \sin^2 x}$

$= \int_0^{\pi/6} \frac{\frac{1}{2}(1+\cos 2x)dx}{\cos 2x} = \frac{1}{2}\int_0^{\pi/6}(\sec 2x + 1)dx$

$= \frac{1}{2}\left[\frac{1}{2}\ln|\tan 2x + \sec 2x| + x\right]_0^{\pi/6}$

$= \frac{1}{2}\{\frac{1}{2}\ln|\tan\frac{\pi}{3} + \sec\frac{\pi}{3}| + \frac{\pi}{6} - 0\}$

$= \frac{1}{4}\ln|\sqrt{3}+2| + \frac{\pi}{12} = \frac{1}{4}\ln(\sqrt{3}+2) + \frac{\pi}{12}$

16. (a) ধরি $I = \int_0^a \sqrt{a^2 - x^2}\,dx$ [সি. '০৭; রা. '০৫; কু.'০৯,'১৩; চ. '০৯; য.,ব. '১২, দি.'১২,'১৪]

এবং $x = a\sin\theta$. তাহলে $dx = a\cos\theta\, d\theta$

সীমা : $x = 0$ হলে $\theta = \sin^{-1} 0 = 0$ এবং

$x = a$ হলে $\theta = \sin^{-1} 1 = \frac{\pi}{2}$

$\therefore I = \int_0^{\pi/2} \sqrt{a^2(1-\sin^2\theta)}\, a\cos\theta\, d\theta$

$= a^2\int_0^{\pi/2}\cos^2\theta\, d\theta = \frac{a^2}{2}\int_0^{\pi/2}(1+\cos 2\theta)\, d\theta$

$= \frac{a^2}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$= \frac{a^2}{2}\{(\frac{\pi}{2} + \frac{1}{2}\sin\pi) - (0 + \frac{1}{2}\sin 0)\}$

$= \frac{a^2}{2}.\frac{\pi}{2} = \frac{1}{4}\pi a^2$

16(b) ধরি $I = \int_0^{\sqrt{2}} \frac{x^2}{(4-x^2)^{3/2}}dx$ [প্র.ভ.প.'৮৫]

এবং $x = 2\sin\theta$. তাহলে $dx = 2\cos\theta\, d\theta$

সীমা : $x = 0$ হলে $\theta = \sin^{-1} 0 = 0$ এবং

$x = \sqrt{2}$ হলে $\theta = \sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

$\therefore I = \int_0^{\pi/4} \frac{4\sin^2\theta . 2\cos\theta\, d\theta}{\{4(1-\sin^2\theta)\}^{3/2}}$

$= \int_0^{\pi/4}\frac{8\sin^2\theta\cos\theta\, d\theta}{8\cos^3\theta} = \int_0^{\pi/4}\tan^2\theta\, d\theta$

$= \int_0^{\pi/4}(\sec^2\theta - 1)\, d\theta = [\tan\theta - \theta]_0^{\pi/4}$

$= \tan\frac{\pi}{4} - \frac{\pi}{4} - (\tan 0 - 0) = 1 - \frac{\pi}{4}$

17. ধরি, $I = \int_0^4 y\sqrt{4-y}\, dy$

[ব.'০৫; রা.'০৭;ঢা.'০৯,'১২; রা.'১৩; চ.'১০,'১৪]

এবং $4 - y = z^2$. $\therefore -dy = 2z\, dz$

সীমা : $y = 0$ হলে $z = 2$ এবং $y = 4$ হলে $z = 0$

$\therefore I = \int_2^0 (4-z^2)\sqrt{z^2}.(-2z\, dz)$

$= 2\int_2^0 (z^4 - 4z^2)\, dz = 2\left[\frac{1}{5}z^5 - \frac{4}{3}z^3\right]_2^0$

$= 2(-\frac{1}{5}\times 2^5 + \frac{4}{3}\times 2^3) = 2^6(-\frac{1}{5}+\frac{1}{3}) = \frac{128}{15}$

18. $\int_1^{15}\frac{x+2}{(x+1)(x+3)}dx$ [প্র.ভ.প.'৯৫]

$= \int_1^{15}\{\frac{-1+2}{(x+1)(-1+3)} + \frac{-3+2}{(-3+1)(x+3)}\}dx$

$= \int_1^{15}\{\frac{1}{2(x+1)} + \frac{1}{2(x+3)}\}dx$

$= \frac{1}{2}[\ln|x+1| + \ln|x+3|]_1^{15}$

$= \frac{1}{2}[\ln|(x+1)(x+3)|]_1^{15}$

$= \frac{1}{2}\{\ln|(15+1)(15+3)| - \ln|(1+1)(1+3)|\}$

$= \frac{1}{2}\{\ln(16\times 18) - \ln(2\times 4)\}$

$= \frac{1}{2}\ln\frac{16\times 18}{2\times 4} = \frac{1}{2}\ln 6^2 = \frac{2}{2}\ln 6 = \ln 6$

অতিরিক্ত প্রশ্ন (সমাধানসহ)

1. $\int_0^{\pi/2} \sqrt{1+\sin\theta}\, d\theta$

$= \int_0^{\pi/2} \sqrt{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\, d\theta$

$= \int_0^{\pi/2} \sqrt{(\sin\frac{\theta}{2} + \cos\frac{\theta}{2})^2}\, d\theta$

$= \int_0^{\pi/2} (\sin\frac{\theta}{2} + \cos\frac{\theta}{2})\, d\theta$

$= \left[-2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2}\right]_0^{\pi/2}$

$= 2\{-\cos\frac{\pi}{4} + \sin\frac{\pi}{4} - (-\cos 0 + \sin 0)\}$

$= 2\{-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (-1+0)\} = 2$

2. $\int_{\pi/2}^{\pi/4} \frac{dx}{\sin x} = \int_{\pi/2}^{\pi/4} \operatorname{cosec} x dx$

$= \left[\ln|\tan\frac{x}{2}|\right]_{\pi/2}^{\pi/4}$

$= \ln|\tan\frac{\pi}{8}| - \ln|\tan\frac{\pi}{4}| = \ln(\tan\frac{\pi}{8}) - \ln 1$

$= \ln(\tan\frac{\pi}{8}) - 0 = \ln(\tan\frac{\pi}{8})$

3. $\int_0^{\pi/2} \sin^3 x dx = \int_0^{\pi/2} \frac{1}{4}(3\sin x - \sin 3x) dx$

$= \frac{1}{4}\left[-3\cos x + \frac{1}{3}\cos 3x\right]_0^{\pi/2}$

$= \frac{1}{4}\{-3\cos\frac{\pi}{2} + \frac{1}{3}\cos\frac{3\pi}{2} - (-3\cos 0 + \frac{1}{3}\cos 0)\}$

$= \frac{1}{4}\{(-0+0) - (-3.1 + \frac{1}{3})\} = \frac{1}{4}\times\frac{8}{3} = \frac{2}{3}$

4(a) $\int_0^{\pi/2} \sin^5 x \cos x dx$

$= \int_0^{\pi/2} (\sin x)^5 d(\sin x)$

$= \left[\frac{1}{6}(\sin x)^6\right]_0^{\pi/2} = \frac{1}{6}\{(\sin\frac{\pi}{2})^6 - (\sin 0)^6\}$

$= \frac{1}{6}\{1-0\} = \frac{1}{6}$

4(b) $\int_0^{\pi/4} \cos x \sin^3 x\, dx$

$= \int_0^{\pi/4} (\sin x)^3 d(\sin x)$

$= \left[\frac{1}{4}(\sin x)^4\right]_0^{\pi/4} = \frac{1}{4}\{(\sin\frac{\pi}{4})^4 - (\sin 0)^4\}$

$= \frac{1}{4}\{(\frac{1}{\sqrt{2}})^4 - 0\} = \frac{1}{4}.\frac{1}{4} = \frac{1}{16}$

5. $\int_0^{\pi/6} \sin 3x \cos 3x\, dx$

$= \int_0^{\pi/6} \frac{1}{2}\sin 6x\, dx = \frac{1}{2}\left[-\frac{\cos 6x}{6}\right]_0^{\pi/6}$

$= -\frac{1}{12}(\cos\pi - \cos 0) = -\frac{1}{12}(-1-1) = \frac{1}{6}$

6(a) $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \frac{1}{2}\int_0^1 e^{\sqrt{x}} d(\sqrt{x})$

$= 2\left[e^{\sqrt{x}}\right]_0^1 = 2(e^{\sqrt{1}} - e^{\sqrt{0}}) = 2(e-1)$

6(b) $\int_0^2 2x\cos(1+x^2)\, dx$

$= \int_0^2 \cos(1+x^2)\, d(1+x^2)$

$= \left[\sin(1+x^2)\right]_0^2 = \sin(1+2^2) - \sin(1+0^2)$

$= \sin(5) - \sin(1)$

7(a) ধরি, $I = \int 2x^3 e^{-x^2} dx$ এবং $x^2 = z$.

তাহলে $2x\,dx = dz$ এবং

$I = \int x^2 e^{-x^2}(2xdx) = \int z e^{-z} dz$

$= z\int e^{-z} dz - \int\{\frac{d}{dz}(z)\int e^{-z} dz\} dz$

$= z(-e^{-z}) - \int 1.(-e^{-z}) dz$

$= -ze^{-z} + (-e^{-z}) = -(x^2+1)e^{-x^2}$

$$\int_0^1 2x^3 e^{-x^2}dx = \left[-(x^2+1)e^{-x^2}\right]_0^1$$
$$= -(1+1)e^{-1} + (0+1)e^0 = 1 - 2e^{-1}$$

7(b) $\int \ln(1+x)dx$

$$= \ln(1+x)\int dx - \int[\frac{d}{dx}\{\ln(1+x)\}\int dx]dx$$
$$= x\ln(1+x) - \int \frac{1}{1+x}.xdx$$
$$= x\ln(1+x) - \int \frac{1+x-1}{1+x}dx$$
$$= x\ln(1+x) - \int (1-\frac{1}{1+x})dx$$
$$= x\ln(1+x) - \{x - \ln(1+x)\} + c$$
$$= (x+1)\ln(1+x) - x + c$$
$$\int_0^1 \ln(1+x)dx = [(x+1)\ln(1+x) - x]_0^1$$
$$= 2\ln 2 - 1 - \ln 1 = 2\ln 2 - 1 - 0 = 2\ln 2 - 1$$

8(a) $\int_1^{\sqrt{3}} \frac{3dx}{1+x^2} = 3[\tan^{-1}x]_1^{\sqrt{3}}$

$$= 3(\tan^{-1}\sqrt{3} - \tan^{-1}1) = 3(\frac{\pi}{3} - \frac{\pi}{4})$$
$$= 3\times\frac{\pi}{12} = \frac{\pi}{4}$$

8(b) $\int_{-2}^{2} \frac{dx}{x^2+4} = \int_{-2}^{2}\frac{dx}{x^2+2^2} = \left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_{-2}^{2}$

$$= \frac{1}{2}\{\tan^{-1}1 - \tan^{-1}(-1)\} = \frac{1}{2}\{\frac{\pi}{4} + \frac{\pi}{4}\} = \frac{\pi}{4}$$

8(c) $\int_0^a \frac{dx}{a^2+x^2} = \left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_0^a$

$$= \frac{1}{a}(\tan^{-1}1 - \tan^{-1}0) = \frac{1}{a}(\frac{\pi}{4} - 0) = \frac{\pi}{4a}$$

9. $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]_0^1$

$$= \sin^{-1}1 - \sin^{-1}0 = \frac{\pi}{2}$$

10(a) $\int_0^1 x(1-\sqrt{x})^2 dx = \int_0^1 x(1 - 2\sqrt{x} + x)dx$

$$= \int_0^1 (x - 2x^{\frac{3}{2}} + x^2)dx = \left[\frac{x^2}{2} - 2\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^3}{3}\right]_0^1$$
$$= (\frac{1}{2} - 2\times\frac{2}{5} + \frac{1}{3}) - 0 = \frac{15-24+10}{30} = \frac{1}{30}$$

(b) $\int_1^2 \frac{(x^2-1)^2}{x^2}dx = \int_1^2 \frac{x^4 - 2x^2 + 1}{x^2}dx.$

$$= \int_1^2 (x^2 - 2 + \frac{1}{x^2})dx = \left[\frac{x^3}{3} - 2x - \frac{1}{x}\right]_1^2$$
$$= (\frac{8}{3} - 4 - \frac{1}{2}) - (\frac{1}{3} - 2 - 1)$$
$$= \frac{8}{3} - 1 - \frac{1}{2} - \frac{1}{3} = \frac{16-6-3-2}{6} = \frac{5}{6}$$

(e) $\int_{\pi/2}^{\pi} (1 + \sin 2\theta)d\theta = \left[\theta - \frac{1}{2}\cos 2\theta\right]_{\pi/2}^{\pi}$

$$= (\pi - \frac{1}{2}\cos 2\pi) - (\frac{\pi}{2} - \frac{1}{2}\cos 2.\frac{\pi}{2})$$
$$= \pi - \frac{1}{2}.1 - \frac{\pi}{2} + \frac{1}{2}(-1) = \frac{\pi}{2} - 1$$

11. $\int_{-\pi/4}^{0} \tan(\frac{\pi}{4} + x)dx$

$$= \left[-\ln|\cos(\frac{\pi}{4} + x)|\right]_{-\pi/4}^{0}$$
$$= -\ln|\cos\frac{\pi}{4}| + \ln|\cos(\frac{\pi}{4} - \frac{\pi}{4})|$$
$$= -\ln|\frac{1}{\sqrt{2}}| + \ln|\cos 0| = -\ln 2^{-\frac{1}{2}} + \ln 1$$
$$= \frac{1}{2}\ln 2 + 0 = \frac{1}{2}\ln 2$$

12(a) $\int_0^{\pi/2} \sin^2 x dx$ [য.'০১; কু.'০২]

$$= \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x)dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2}$$

$=\frac{1}{2}\{(\frac{\pi}{2}-\frac{1}{2}\sin\pi)-(0-\frac{1}{2}\sin 0)\}=\frac{\pi}{4}$

12(b) $\int_0^{\pi/2}\sin^5 x\cos^4 x dx$

$=\int_0^{\pi/2}\sin^4 x\cos^4 x\sin x dx$

$=\int_0^{\pi/2}(1-\cos^2 x)^2\cos^4 x\sin x dx$

মনে করি, $\cos x = z \quad \therefore -\sin x\, dx = dz$.

$x=0$ হলে, $z=\cos 0=1$;

$x=\frac{\pi}{2}$ হলে, $z=\cos\frac{\pi}{2}=0$

$\therefore \int_0^{\pi/2}\sin^5 x\cos^4 x dx = -\int_1^0(1-z^2)^2 z^4 dz$

$=-\int_1^0(1-2z^2+z^4)z^4 dz$

$=-\int_1^0(z^4-2z^6+z^8)dz$

$=-\left[\frac{1}{5}z^5-2.\frac{1}{7}z^7+\frac{1}{9}z^9\right]_1^0$

$=-\{0-(\frac{1}{5}-\frac{2}{7}+\frac{1}{9})\}=\frac{63-90+35}{315}$

$=\frac{98-90}{315}=\frac{8}{315}$

12(c) ধরি, $I=\int_0^{\pi/2}\frac{\cos x}{(1+\sin x)^3}dx$

এবং $z=1+\sin x \qquad dz=\cos x dx$

সীমা: $x=0$ হলে $z=1$ এবং $x=\frac{\pi}{2}$ হলে $z=2$

$\therefore I=\int_1^2\frac{dz}{z^3}=\int_1^2 z^{-3}dz=\left[\frac{z^{-2}}{-2}\right]_1^2=\left[-\frac{1}{2z^2}\right]_1^2$

$=-\frac{1}{2}(\frac{1}{2^2}-\frac{1}{1^2})=-\frac{1}{2}(\frac{1}{4}-1)=\frac{3}{8}$

13. ধরি, $I=\int_0^1\frac{\cos^{-1}x}{\sqrt{1-x^2}}dx$ **[প্র.ভ.প.'০৪]**

এবং $z=\cos^{-1}x \qquad dz=-\frac{1}{\sqrt{1-x^2}}dx$

সীমা: $x=0$ হলে $z=\frac{\pi}{2}$ এবং $x=1$ হলে $z=0$

$\therefore I=-\int_{\pi/2}^0 zdz=-\left[\frac{z^2}{2}\right]_{\pi/2}^0$

$=-\frac{1}{2}\{0-(\frac{\pi}{2})^2\}=\frac{\pi^2}{8}$

14(a) $\int_1^3\frac{2xdx}{1+x^2}=\int_1^3\frac{d(1+x^2)}{1+x^2}$

$=\left[\ln(1+x^2)\right]_1^3=\ln(1+9)-\ln(1+1)$

$=\ln\frac{10}{2}=\ln 5$

14(b) $\int_0^4\frac{dx}{\sqrt{(2x+1)}}=\frac{1}{2}\int_0^4\frac{d(2x+1)}{\sqrt{(2x+1)}}$

$=\frac{1}{2}\left[2\sqrt{2x+1}\right]_0^4=\sqrt{8+1}-\sqrt{0+1}=3-1=2$

15(a) $\int\ln(x^2+1)dx$

$=\ln(x^2+1)\int dx-\int[\frac{d}{dx}\{\ln(x^2+1)\}\int dx]dx$

$=\ln(x^2+1)-\int\frac{2x}{x^2+1}.xdx$

$=x\ln(x^2+1)-2\int\frac{x^2+1-1}{x^2+1}dx$

$=x\ln(x^2+1)-2\int(1-\frac{1}{x^2+1})dx$

$=x\ln(x^2+1)-2(x-\tan^{-1}x)+c$

$=x\ln(x^2+1)-2x+2\tan^{-1}x)+c$

$\int_0^1\ln(x^2+1)dx=\left[x\ln(x^2+1)-2x+2\tan^{-1}x\right]_0^1$

$=\ln 2-2+2\tan^{-1}1-0$

$=\ln 2-2+2.\frac{\pi}{4}=\ln 2-2+\frac{\pi}{2}$

15(b) ধরি, $I=\int_2^e\{\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\}dx$ [প্র.ভ.প'১৪,'০২]

এবং $\ln x=y\Rightarrow x=e^y \qquad dx=e^y dy$

$\therefore \int\{\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\}dx=\int\{\frac{1}{y}-\frac{1}{y^2}\}e^y dy$

$$=\int e^y\{\frac{1}{y}+D(\frac{1}{y})\}dy=\frac{e^y}{y}+c=\frac{x}{\ln x}$$

$$\therefore I=\left[\frac{x}{\ln x}\right]_2^e=\frac{e}{\ln e}-\frac{2}{\ln 2}=e-\frac{2}{\ln 2}$$

16(a) $\int_0^1\frac{3\,dx}{1+x^2}=3[\tan^{-1}x]_0^1$

$$=3(\tan^{-1}1-\tan^{-1}0)=\frac{3\pi}{4}$$

16(b) $\int_0^{\pi/2}\frac{\cos x}{1+\sin^2 x}dx=\int_0^{\pi/2}\frac{d(\sin x)}{1^2+(\sin x)^2}$

$$=[\tan^{-1}(\sin x)]_0^{\pi/2}=\tan^{-1}(\sin\frac{\pi}{2})-\tan^{-1}(\sin 0)$$

$$=\tan^{-1}1-\tan^{-1}0=\frac{\pi}{4}-0=\frac{\pi}{4}$$

17(a) $\int_{-1}^{2}\frac{dx}{x^2-9}=\int_{-1}^{2}\frac{dx}{x^2-3^2}$

$$=\left[\frac{1}{2.3}\ln\left|\frac{x-3}{x+3}\right|\right]_{-1}^{2}$$

$$=\frac{1}{6}\{\ln\left|\frac{2-3}{2+3}\right|-\ln\left|\frac{-1-3}{-1+3}\right|\}$$

$$=\frac{1}{6}(\ln\frac{1}{5}-\ln 2)=\frac{1}{6}\ln\frac{1}{5\times 2}=\frac{1}{6}\ln(0\cdot 1)$$

17(b) $\int_0^{a/2}\frac{1}{a^2-x^2}dx=\left[\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|\right]_0^{a/2}$

$$=\frac{1}{2a}\ln\left|\frac{a+\frac{a}{2}}{a-\frac{a}{2}}\right|=\frac{1}{2a}\ln\left|\frac{3a}{a}\right|=\frac{1}{2a}\ln 3$$

18(a) $\int_0^a\frac{dx}{\sqrt{a^2-x^2}}=\left[\sin^{-1}\frac{x}{a}\right]_0^a$

$$=\sin^{-1}\frac{a}{a}-\sin^{-1}\frac{0}{a}=\sin^{-1}1-\sin^{-1}0=\frac{\pi}{2}$$

18(b) $\int_0^1\frac{dx}{\sqrt{4-3x^2}}$ [কু.বো.'০১; প্র.ভ.প.'৮৩]

$$=\frac{1}{\sqrt{3}}\int_0^1\frac{\sqrt{3}dx}{\sqrt{2^2-(\sqrt{3}x)^2}}=\left[\frac{1}{\sqrt{3}}\sin^{-1}\frac{\sqrt{3}x}{2}\right]_0^1$$

$$=\frac{1}{\sqrt{3}}(\sin^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}0)=\frac{1}{\sqrt{3}}(\frac{\pi}{3}-0)=\frac{\pi}{3\sqrt{3}}$$

18 (c) ধরি, $I=\int_0^{\pi/2}\frac{\cos x dx}{\sqrt{4-\sin^2 x}}$ এবং

$\sin x=z$. তাহলে $\cos x dx=dz$

সীমা : $x=0$ হলে $z=0$ এবং $x=\frac{\pi}{2}$ হলে $z=1$

$$\therefore I=\int_0^1\frac{dz}{\sqrt{2^2-z^2}}=\left[\sin^{-1}\frac{x}{2}\right]_0^1$$

$$=\sin^{-1}\frac{1}{2}-\sin^{-1}0=\frac{\pi}{2}-0=\frac{\pi}{2}$$

18 (d) $\int_2^3\frac{dx}{(x-1)\sqrt{x^2-2x}}$ [প্র.ভ.প.'০১,'০৩]

$$=\int_2^3\frac{dx}{(x-1)\sqrt{(x^2-2x+1)-1}}$$

$$=\int_2^3\frac{d(x-1)}{(x-1)\sqrt{(x-1)^2-1}}$$

$$=[\sec^{-1}(x-1)]_2^3=\sec^{-1}(3-1)-\sec^{-1}(2-1)$$

$$=\sec^{-1}2-\sec^{-1}1=\frac{\pi}{3}-0=\frac{\pi}{3}$$

19. $\int_0^a\frac{a^2-x^2}{(a^2+x^2)^2}dx$ [প্র.ভ.প. '০০]

$$=\int_0^a\frac{x^2(\frac{a^2}{x^2}-1)}{\{x(\frac{a^2}{x}+x)\}^2}dx=\int_0^a\frac{(\frac{a^2}{x^2}-1)}{(\frac{a^2}{x}+x)^2}dx$$

$$=\int_0^a\frac{-(-\frac{a^2}{x^2}+1)}{(\frac{a^2}{x}+x)^2}dx=-\left[-\frac{1}{\frac{a^2}{x}+x}\right]_0^a$$

$$=\left[\frac{x}{a^2+x^2}\right]_0^a=\frac{a}{a^2+a^2}-0=\frac{1}{2a}$$

20. $\int_8^{27} \frac{dx}{x-x^{1/3}} = \int_8^{27} \frac{dx}{x(1-x^{-2/3})}$

ধরি $x^{-\frac{2}{3}} = z$. তাহলে $-\frac{2}{3}x^{-\frac{5}{3}}dx = dz$

$\Rightarrow -\frac{2}{3}x^{-\frac{2}{3}}\frac{dx}{x} = dz \Rightarrow -\frac{2}{3}z\frac{dx}{x} = dz$

$\Rightarrow \frac{dx}{x} = -\frac{3}{2}\frac{dz}{z}$

সীমা : $x = 8$ হলে $z = 2^{-2} = \frac{1}{4}$ এবং

$x = 27$ হলে $z = 3^{-2} = \frac{1}{9}$

$\therefore \int_8^{27} \frac{dx}{x-x^{1/3}} = -\frac{3}{2}\int_{1/4}^{1/9} \frac{dz}{z(1-z)}$

$= \frac{3}{2}\int_{1/4}^{1/9}\{\frac{1}{z-1} - \frac{1}{z}\}dz$

$= \frac{3}{2}[\ln|z-1| - \ln|z|]_{1/4}^{1/9} = \frac{3}{2}\left[\ln\left|\frac{z-1}{z}\right|\right]_{1/4}^{1/9}$

$= \frac{3}{2}\{\ln\left|\frac{\frac{1}{9}-1}{\frac{1}{9}}\right| - \ln\left|\frac{\frac{1}{4}-1}{\frac{1}{4}}\right|\}$

$= \frac{3}{2}\{\ln|-8| - \ln|-3|\} = \frac{3}{2}(\ln 8 - \ln 3)$

$= \frac{3}{2}\ln\frac{8}{3}$

21. $\int_{-1}^{1} \frac{1-x}{1+x}dx$ **[প্র.ভ.প.'৮৪]**

$= \int_{-1}^{1} \frac{-(1+x)+2}{1+x}dx = \int_{-1}^{1}(-1+\frac{2}{1+x})dx$

$= [-x + 2\ln|1+x|]_{-1}^{1}$

$= -1 + 2\ln|1+1| - (1 + 2\ln|1-1|)$

$= -1 + 2\ln 2 - 1 - 2\ln 0$

$= 2(\ln 2 - 1)$

প্রশ্নমালা X E

1(a) Sol^n : $\int \sin ax\, dx = -\frac{1}{a}\cos ax + c$

Ans. **A**

(b) Sol^n : $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$\therefore$ Ans. **B**

(c) Sol^n : : ক্যালকুলেটরের সাহায্যে

$\int_0^{\pi/2} \cos^5 x dx = 0.533$,যা 8/15 এর সমান।

Ans. D.

(d) Sol^n : ন্যূনতম হতে হলে, $\frac{d}{dx}\{F(x)\} = 0$ হতে হবে।

এখানে, $\frac{d}{dx}\{F(x)\} = \frac{t-3}{t^2+7} = 0 \Rightarrow t = 3$

$\therefore$ Ans. D.

(e) $y = \frac{1}{2}x^2 + 1$ পরাবৃত্ত ও তার উপকেন্দ্রিক লম্ব দ্বারা বেষ্টিত ক্ষেত্রের ক্ষেত্রফল কত?

$Sol^n.: x^2 = 2y - 2 = 2(y-1) = 4 \times \frac{1}{2}(y-1)$

পরাবৃত্তের শীর্ষ $(0,1)$, উপকেন্দ্রিক লম্ব, $y - 1 = \frac{1}{2}$

$\Rightarrow y = \frac{3}{2}$ নির্ণেয় ক্ষেত্রফল $= \int_1^{3/2} xdy$

$= \int_1^{3/2}\sqrt{2(y-1)}dy = 0.666 = \frac{2}{3}$ Ans. C

(f) Sol^n : সবগুলি তথ্য সত্য। $\therefore$ Ans. D

(g) Sol^n : $\int \frac{dx}{ay-bx} = -\frac{1}{b}\int \frac{d(ay-bx)}{ay-bx}$

$= -\frac{1}{b}\ln(ay-bx) + c$ $\therefore$ Ans. **A**

(h) Sol^n : $\int \frac{dx}{\sqrt{9-16x^2}} = \frac{1}{4}\int \frac{d(4x)}{\sqrt{3^2-(4x)^2}}$

$= \frac{1}{4}\sin^{-1}\frac{4x}{3} + c$ $\therefore$ Ans. **B**

(i) Sol^n : $\int_0^{1/a} d(\tan^{-1}ax) = [\tan^{-1}ax]_0^{1/a}$

(j) Sol^n : কৌশল : $\int_a^b f(x) = \int_{a+c}^{b+c} f(x-c)$