

সমাধান : (গ)

$$\frac{1}{1-a+a^2} - \frac{1}{1+a+a^2} - \frac{2a}{1+a^2+a^4}$$

$$\begin{aligned}\text{এখানে, } 1+a^2+a^4 &= 1+2a^2+(a^2)^2-a^2 \\ &= (1+a^2)^2-a^2 \\ &= (1+a^2-a)(1+a^2+a)\end{aligned}$$

$$\text{হব } (1-a+a^2), (1+a+a^2), (1+a^2+a^4) \text{ এর L.S.O. } = (1+a^2-a)(1+a^2+a)$$

$$\begin{aligned}\therefore \frac{1}{1-a+a^2} - \frac{1}{1+a+a^2} - \frac{2a}{(1-a+a^2)(1+a+a^2)} \\ &= \frac{1+a+a^2-1-a-a^2-2a}{(1-a+a^2)(1+a+a^2)} \\ &= \frac{0}{(1+a^2+a^4)} \\ &= 0\end{aligned}$$

অনুশীলনী ৫.১

১। লিখিত আকারে প্রকাশ কর :

$$(ক) \frac{4x^2y^3z^5}{9x^5y^2z^3} \quad (খ) \frac{16(2x)^4(3y)^5}{(3x)^3.(2y)^6}$$

$$(গ) \frac{x^3y+xy^3}{x^2y^3+x^3y^2} \quad (ঘ) \frac{(a-b)(a+b)}{a^3-b^3}$$

$$(ঙ) \frac{x^2-6x+5}{x^2-25} \quad (চ) \frac{x^2-7x+12}{x^2-9x+20}$$

$$(ছ) \frac{(x^3-y^3)(x^2-xy+y^2)}{(x^2-y^2)(x^3+y^3)} \quad (ছ) \frac{(a^2-b^2-2bc-c^2)}{(a^2+2ab+b^2-c^2)}$$

$$(ক) \frac{4x^2y^3z^5}{9x^5y^2z^3}$$

সমাধান :

$$\text{প্রদত্ত ভগ্নাংশ } \frac{4x^2y^3z^5}{9x^5y^2z^3}$$

এখানে, 4 ও 9 এর গ.সা.গু. হলো 1

x^2 ও x^5 এর গ.সা.গু. হলো x^2

y^3 ও y^2 " " " y^2

z^5 ও z^3 " " " z^3

$\therefore 4x^2y^3z^5$ ও $9x^5y^2z^3$ এর গ.সা.গু. হলো $x^2y^2z^3$

$\frac{4x^2y^3z^5}{9x^5y^2z^3}$ এর লব ও হরকে $x^2y^2z^3$ দ্বারা ভাগ করে পাওয়া যায় $\frac{4yz^2}{9x^3}$ ।

$\frac{4x^2y^3z^5}{9x^5y^2z^3}$ এর লম্বিষ্ট আকার হলো $\frac{4yz^2}{9x^3}$.

$$(খ) \frac{16(2x)^4(3y)^5}{(3x)^3.(2y)^6}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত ভগ্নাংশ } & \frac{16(2x)^4(3y)^5}{(3x)^3.(2y)^6} \\ &= \frac{2^4 \times 2^4 \times 3^5 \times x^4 y^5}{3^3 \times 2^6 \times x^3 y^6} \\ &= \frac{2^8 3^5 x^4 y^5}{2^6 3^3 x^3 y^6} \end{aligned}$$

এখানে, 2^8 ও 2^6 এর গ.সা.গু. হলো 2^6

3^5 ও 3^3 " " " 3^3

x^4 ও x^3 " " " x^3

$$y^5 \text{ ও } y^6 \quad " \quad " \quad " \quad y^5 \\ \therefore 16(2x)^4(3y)^5 \text{ ও } (3x)^3(2y)^6 \text{ এর গ.স.গু. হলো } 2^6 3^3 x^2 y^3$$

এখন প্রদত্ত ভগ্নাংশের লব ও হরকে $2^6 3^3 x^2 y^3$ দ্বারা ভাগ করে পাওয়া যায়।

$$\frac{2^2 3^2 x}{y} = \frac{4 \times 9x}{y} = \frac{36x}{y}$$

ভগ্নাংশটির লম্বিষ্ঠ আকার হলো $\frac{36x}{y}$ ।

$$(g) \frac{x^3 y + x y^3}{x^2 y^3 + x^3 y^2}$$

সমাধান :

$$\text{প্রদত্ত ভগ্নাংশ } \frac{x^3 y + x y^3}{x^2 y^3 + x^3 y^2}$$

$$\begin{aligned} \text{এখানে, লব} &= x^3 y + x y^3 \\ &= xy(x^2 + y^2) \\ \text{হর} &= x^2 y^3 + x^3 y^2 \\ &= x^2 y^2(x + y) \end{aligned}$$

\therefore লব ও হরের গ.স.গু. xy

$$\text{প্রদত্ত ভগ্নাংশের লব ও হরকে } xy \text{ দ্বারা ভাগ করে পাই, } \frac{(x^2 + y^2)}{xy(x + y)}$$

$$\text{সুতারাং, ভগ্নাংশের লম্বিষ্ঠ আকার } \frac{(x^2 + y^2)}{xy(x + y)}$$

$$(ঘ) \frac{(a-b)(a+b)}{a^3 - b^3}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত ভগ্নাংশ} & \frac{(a-b)(a+b)}{a^3 - b^3} \\ & = \frac{(a-b)(a+b)}{(a-b)(a^2 + ab + b^2)} \end{aligned}$$

∴ লব ও হরের গ.সা.গ. $(a-b)$

$$\text{প্রদত্ত ভগ্নাংশের লব ও হরকে } (a-b) \text{ দ্বারা ভাগ করে পাই, } \frac{(a+b)}{a^2 + ab + b^2}$$

$$\text{সুতারাং ভগ্নাংশের লম্বিষ্ট আকার } \frac{(a+b)}{a^2 + ab + b^2}$$

$$(ঝ) \frac{x^2 - 6x + 5}{x^2 - 25}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত ভগ্নাংশ} & \frac{x^2 - 6x + 5}{x^2 - 25} \\ & = \frac{x^2 - 5x - x + 5}{x^2 - 5^2} \\ & = \frac{x(x-5) - 1(x-5)}{(x-5)(x+5)} \\ & = \frac{(x-5)(x-1)}{(x-5)(x+5)} \end{aligned}$$

∴ লব ও হরের গ.সা.গ. $=(x-5)$

$$\text{প্রদত্ত ভগ্নাংশের লব ও হরকে } (x-5) \text{ দ্বারা ভাগ করে পাই, } = \frac{(x-1)}{(x+5)}$$

$$\text{সুতারাং ভগ্নাংশের লম্বিষ্ট আকার } \frac{(x-1)}{(x+5)}$$

$$(চ) \frac{x^2 - 7x + 12}{x^2 - 9x + 20}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত ভগ্নাংশ} & \frac{x^2 - 7x + 12}{x^2 - 9x + 20} \\ &= \frac{x^2 - 3x - 4x + 12}{x^2 - 4x - 5x + 20} \\ &= \frac{x(x-3) - 4(x-3)}{x(x-4) - 5(x-4)} \\ &= \frac{(x-3)(x-4)}{(x-4)(x-5)} \end{aligned}$$

$$\therefore \text{লব ও হরের গ.সা.গু.} = (x-4)$$

$$\text{প্রদত্ত ভগ্নাংশের লব ও হরকে } (x-4) \text{ দ্বারা ভাগ করে পাই, } \frac{x-3}{x-5}$$

$$\text{সুতরাং, ভগ্নাংশের লম্বিষ্ঠ আকার } \frac{x-3}{x-5}$$

$$(ছ) \frac{(x^3 - y^3)(x^2 - xy + y^2)}{(x^2 - y^2)(x^3 + y^3)}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত ভগ্নাংশ} & \frac{(x^3 - y^3)(x^2 - xy + y^2)}{(x^2 - y^2)(x^3 + y^3)} \\ &= \frac{(x-y)(x^2 + xy + y^2)(x^2 - xy + y^2)}{(x+y)(x-y)(x+y)(x^2 - xy + y^2)} \end{aligned}$$

$$\therefore \text{লব ও হরের গ.সা.গু.} = (x-y)(x^2 - xy + y^2)$$

$$\text{প্রদত্ত ভগ্নাংশের লব ও হরকে } (x-y)(x^2 - xy + y^2) \text{ দ্বারা ভাগ করে পাই, } \frac{x^2 - xy + y^2}{(x+y)^2}$$

সুতরাং ভগ্নাংশের লম্বিষ্ট আকার $\frac{(x-1)}{(x+5)}$

(জ) $\frac{(a^2 - b^2 - 2bc - c^2)}{(a^2 + 2ab + b^2 - c^2)}$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত ভগ্নাংশ } & \frac{(a^2 - b^2 - 2bc - c^2)}{(a^2 + 2ab + b^2 - c^2)} \\ &= \frac{(a)^2 - (b^2 + 2bc + c^2)}{(a^2 + 2ab + b^2) - c^2} \\ &= \frac{(a)^2 - (b+c)^2}{(a+b)^2 - (c)^2} \\ &= \frac{(a+b+c)(a-b-c)}{(a+b+c)(a+b-c)} \end{aligned}$$

\therefore লব ও হরের গ.স.গ. = $(a+b+c)$

প্রদত্ত ভগ্নাংশের লব ও হরকে $(a+b+c)$ দ্বারা ভাগ করে পাই, $= \frac{(a-b-c)}{(a+b-c)}$

সুতরাং ভগ্নাংশের লম্বিষ্ট আকার $\frac{(a-b-c)}{(a+b-c)}$

২। সাধারণ হরবিশিষ্ট ভগ্নাংশে প্রকাশ কর :

(ক) $\frac{x^2}{xy}, \frac{y^2}{yz}, \frac{z^2}{zx}$ (খ) $\frac{x-y}{xy}, \frac{y-z}{yz}, \frac{z-x}{zx}$

(গ) $\frac{x}{x-y}, \frac{y}{x+y}, \frac{z}{x(x+y)}$ (ঘ) $\frac{x+y}{(x-y)^2}, \frac{x-y}{x^3+y^3}, \frac{y-z}{x^2-y^2}$

(ঙ) $\frac{a}{a^3+b^3}, \frac{b}{a^2+ab+b^2}, \frac{c}{a^3-b^3}$

$$(ট) \frac{1}{x^2 - 5x + 6}, \frac{1}{x^2 - 7x + 12}, \frac{1}{x^2 - 9x + 20}$$

$$(ই) \frac{a-b}{a^2 b^2}, \frac{b-c}{b^2 c^2}, \frac{c-a}{c^2 a^2} \quad (জ) \frac{x-y}{x+y}, \frac{y-z}{y+z}, \frac{z-x}{z+x}$$

$$(ক) \frac{x^2}{xy}, \frac{y^2}{yz}, \frac{z^2}{zx}$$

সমাধান :

$$\text{এখানে, প্রদত্ত ভগ্নাংশগুলো } \frac{x^2}{xy}, \frac{y^2}{yz}, \frac{z^2}{zx}$$

প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু. = xyz

$$xyz \div xy = z \quad \therefore \frac{x^2}{xy} = \frac{x^2 \times z}{xy \times z} = \frac{x^2 z}{xyz}$$

$$xyz \div yz = x \quad \therefore \frac{y^2}{yz} = \frac{y^2 \times x}{yz \times x} = \frac{xy^2}{xyz}$$

$$xyz \div zx = y \quad \therefore \frac{z^2}{zx} = \frac{z^2 \times y}{zx \times y} = \frac{yz^2}{xyz}$$

$$\therefore \text{সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো } \frac{x^2 z}{xyz}, \frac{xy^2}{xyz}, \frac{yz^2}{xyz}$$

$$(খ) \frac{x-y}{xy}, \frac{y-z}{yz}, \frac{z-x}{zx}$$

সমাধান :

$$\text{এখানে প্রদত্ত ভগ্নাংশগুলো } \frac{x-y}{xy}, \frac{y-z}{yz}, \frac{z-x}{zx}$$

প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু. = xyz

$$\frac{x-y}{xy} = \frac{(x-y)z}{xyz} = \frac{xz - yz}{xyz}$$

$$\frac{y-z}{yz} = \frac{(y-z)x}{xyz} = \frac{xy-xz}{xyz}$$

$$\frac{z-x}{zx} = \frac{(z-x)y}{xyz} = \frac{yz-xy}{xyz}$$

$$\therefore \text{সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো } \frac{xz-yz}{xyz}, \frac{xy-xz}{xyz}, \frac{yz-xy}{xyz}$$

$$(গ) \frac{x}{x-y}, \frac{y}{x+y}, \frac{z}{x(x+y)}$$

সমাধান :

$$\text{এখানে প্রদত্ত ভগ্নাংশগুলো } \frac{x}{x-y}, \frac{y}{x+y}, \frac{z}{x(x+y)}$$

$$\text{প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু. } = x(x+y)(x-y)$$

$$\therefore \frac{x}{x-y} = \frac{x \cdot x(x+y)}{(x-y)x(x+y)} = \frac{x^2(x+y)}{x(x^2-y^2)}$$

$$\therefore \frac{y}{x+y} = \frac{y \cdot x(x-y)}{(x+y)x(x-y)} = \frac{xy(x-y)}{x(x^2-y^2)}$$

$$\therefore \frac{z}{x(x+y)} = \frac{z \cdot (x-y)}{x(x+y)(x-y)} = \frac{z(x-y)}{x(x^2-y^2)}$$

$$\therefore \text{সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো } \frac{x^2(x+y)}{x(x^2-y^2)}, \frac{xy(x-y)}{x(x^2-y^2)}, \frac{z(x-y)}{x(x^2-y^2)}$$

$$(ঘ) \frac{x+y}{(x-y)^2}, \frac{x-y}{x^3+y^3}, \frac{y-z}{x^2-y^2}$$

সমাধান :

$$\text{এখানে প্রদত্ত ভগ্নাংশগুলো } \frac{x+y}{(x-y)^2}, \frac{x-y}{x^3+y^3}, \frac{y-z}{x^2-y^2}$$

$$1\text{ম ভগ্নাংশের হর } = (x-y)^2$$

$$= (x-y)(x-y)$$

২য় ভগ্নাংশের হর $= x^3 + y^3$

$$= (x+y)(x^2 - xy + y^2)$$

৩য় ভগ্নাংশের হর $= x^2 - y^2$

$$= (x-y)(x+y)$$

প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু. $= (x-y)(x-y)(x+y)(x^2 - xy + y^2)$

$$= (x-y)^2(x^3 + y^3)$$

$$\therefore \frac{x+y}{(x-y)^2} = \frac{(x+y)(x^3 + y^3)}{(x-y)^2(x^3 + y^3)} = \frac{(x+y)(x^3 + y^3)}{(x-y)^2(x^3 + y^3)}$$

$$\therefore \frac{x-y}{(x^3 + y^3)} = \frac{(x-y)(x-y)^2}{(x-y)^2(x^3 + y^3)} = \frac{(x-y)^3}{(x-y)^2(x^3 + y^3)}$$

$$\therefore \frac{y-z}{(x^2 - y^2)} = \frac{(y-z)(x-y)(x^2 - xy + y^2)}{(x-y)^2(x^3 + y^3)} = \frac{(y-z)(x-y)(x^2 - xy + y^2)}{(x-y)^2(x^3 + y^3)}$$

\therefore সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো

$$\frac{(x+y)(x^3 + y^3)}{(x-y)^2(x^3 + y^3)}, \frac{(x-y)^3}{(x-y)^2(x^3 + y^3)}, \frac{(y-z)(x-y)(x^2 - xy + y^2)}{(x-y)^2(x^3 + y^3)}$$

(৫) $\frac{a}{a^3 + b^3}, \frac{b}{a^2 + ab + b^2}, \frac{c}{a^3 - b^3}$

সমাধান :

এখানে প্রদত্ত ভগ্নাংশগুলো $\frac{a}{a^3 + b^3}, \frac{b}{a^2 + ab + b^2}, \frac{c}{a^3 - b^3}$

১ম ভগ্নাংশের হর $= a^3 + b^3$

$$= (a+b)(a^2 - ab + b^2)$$

২য় ভগ্নাংশের হর $= (a^2 + ab + b^2)$

৩য় ভগ্নাংশের হর $= a^3 - b^3$

$$= (a-b)(a^2 + ab + b^2)$$

$$\begin{aligned} \text{প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু.} &= (a-b)(a+b)(a^2-ab+b^2)(a^2+ab+b^2) \\ &= (a^3-b^3)(a^3+b^3) \end{aligned}$$

$$\therefore \frac{a}{a^3+b^3} = \frac{a(a^3-b^3)}{(a^3-b^3)(a^3+b^3)} = \frac{a(a^3-b^3)}{(a^6-b^6)}$$

$$\therefore \frac{b}{a^2+ab+b^2} = \frac{b(a-b)(a^3+b^3)}{(a^3-b^3)(a^3+b^3)} = \frac{b(a-b)(a^3+b^3)}{(a^6-b^6)}$$

$$\therefore \frac{c}{a^3-b^3} = \frac{c(a^3+b^3)}{(a^3-b^3)(a^3+b^3)} = \frac{c(a^3+b^3)}{(a^6-b^6)}$$

$$\therefore \text{সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো } \frac{a(a^3-b^3)}{(a^6-b^6)}, \frac{b(a-b)(a^3+b^3)}{(a^6-b^6)}, \frac{c(a^3+b^3)}{(a^6-b^6)}$$

$$(চ) \frac{1}{x^2-5x+6}, \frac{1}{x^2-7x+12}, \frac{1}{x^2-9x+20}$$

সমাধান :

$$\text{এখানে প্রদত্ত ভগ্নাংশগুলো } \frac{1}{x^2-5x+6}, \frac{1}{x^2-7x+12}, \frac{1}{x^2-9x+20}$$

$$\begin{aligned} \text{১ম ভগ্নাংশের হর} &= x^2-5x+6 \\ &= x^2-2x-3x+6 \\ &= (x-2)(x-3) \end{aligned}$$

$$\begin{aligned} \text{২য় ভগ্নাংশের হর} &= x^2-7x+12 \\ &= x^2-3x-4x+12 \\ &= (x-3)(x-4) \end{aligned}$$

$$\begin{aligned} \text{৩য় ভগ্নাংশের হর} &= x^2-9x+20 \\ &= x^2-4x-5x+20 \\ &= (x-4)(x-5) \end{aligned}$$

$$\text{প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু.} = (x-2)(x-3)(x-4)(x-5)$$

এখন,

$$\begin{aligned} \therefore \frac{1}{(x-2)(x-3)} &= \frac{1 \cdot (x-4)(x-5)}{(x-2)(x-3)(x-4)(x-5)} = \frac{(x-4)(x-5)}{(x-2)(x-3)(x-4)(x-5)} \\ \therefore \frac{1}{(x-3)(x-4)} &= \frac{1 \cdot (x-2)(x-5)}{(x-2)(x-3)(x-4)(x-5)} = \frac{(x-2)(x-5)}{(x-2)(x-3)(x-4)(x-5)} \\ \therefore \frac{1}{(x-4)(x-5)} &= \frac{1 \cdot (x-2)(x-3)}{(x-2)(x-3)(x-4)(x-5)} = \frac{(x-2)(x-3)}{(x-2)(x-3)(x-4)(x-5)} \end{aligned}$$

∴ সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো

$$\frac{(x-4)(x-5)}{(x-2)(x-3)(x-4)(x-5)}, \frac{(x-2)(x-5)}{(x-2)(x-3)(x-4)(x-5)}, \frac{(x-2)(x-3)}{(x-2)(x-3)(x-4)(x-5)}$$

(৩) $\frac{a-b}{a^2b^2}, \frac{b-c}{b^2c^2}, \frac{c-a}{c^2a^2}$

সমাধান :

$$\text{এখানে প্রদত্ত ভগ্নাংশগুলো } \frac{a-b}{a^2b^2}, \frac{b-c}{b^2c^2}, \frac{c-a}{c^2a^2}$$

$$\text{প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু. } = a^2b^2c^2$$

$$\frac{a-b}{a^2b^2} = \frac{c^2(a-b)}{c^2a^2b^2} = \frac{c^2(a-b)}{a^2b^2c^2}$$

$$\frac{b-c}{b^2c^2} = \frac{a^2(b-c)}{a^2b^2c^2} = \frac{a^2(b-c)}{a^2b^2c^2}$$

$$\frac{c-a}{c^2a^2} = \frac{b^2(c-a)}{b^2a^2c^2} = \frac{b^2(c-a)}{a^2b^2c^2}$$

$$\therefore \text{সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো } \frac{c^2(a-b)}{a^2b^2c^2}, \frac{a^2(b-c)}{a^2b^2c^2}, \frac{b^2(c-a)}{a^2b^2c^2}$$



$$(জ) \frac{x-y}{x+y}, \frac{y-z}{y+z}, \frac{z-x}{z+x}$$

সমাধান :

$$\text{এখানে প্রদত্ত ভগ্নাংশগুলো } \frac{x-y}{x+y}, \frac{y-z}{y+z}, \frac{z-x}{z+x}$$

$$\text{প্রদত্ত ভগ্নাংশের হরগুলোর ল.সা.গু. } = (x+y)(y+z)(z+x)$$

$$\frac{x-y}{x+y} = \frac{(x-y).(y+z)(z+x)}{(x+y)(y+z)(z+x)} = \frac{(x-y)(y+z)(z+x)}{(x+y)(y+z)(z+x)}$$

$$\frac{y-z}{y+z} = \frac{(y-z).(x+y)(z+x)}{(y+z).(x+y)(z+x)} = \frac{(y-z)(x+y)(z+x)}{(x+y)(y+z)(z+x)}$$

$$\frac{z-x}{z+x} = \frac{(z-x).(x+y)(y+z)}{(z+x).(x+y)(y+z)} = \frac{(z-x)(x+y)(y+z)}{(x+y)(y+z)(z+x)}$$

∴ সাধারণ হরবিশিষ্ট ভগ্নাংশগুলো

$$\frac{(x-y)(y+z)(z+x)}{(x+y)(y+z)(z+x)}, \frac{(y-z)(x+y)(z+x)}{(x+y)(y+z)(z+x)}, \frac{(z-x)(x+y)(y+z)}{(x+y)(y+z)(z+x)}$$

৩। যোগ কর :

$$(ক) \frac{a-b}{a} + \frac{a+b}{b}$$

$$(খ) \frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab}$$

$$(গ) \frac{x-y}{x} + \frac{y-z}{y} + \frac{z-x}{z}$$

$$(ঘ) \frac{x+y}{x-y} + \frac{x-y}{x+y}$$

$$(ঙ) \frac{1}{x^2 - 3x + 2} + \frac{1}{x^2 - 4x + 3} + \frac{1}{x^2 - 5x + 4}$$

$$(চ) \frac{1}{a^2 - b^2} + \frac{1}{a^2 + ab + b^2} + \frac{1}{a^2 - ab + b^2}$$

$$(ছ) \frac{1}{x-2} - \frac{1}{x+2} + \frac{4}{x^2 - 4}$$

$$(জ) \frac{1}{x^2 - 1} + \frac{1}{x^4 - 1} + \frac{4}{x^8 - 1}$$

$$(ক) \frac{a-b}{a} + \frac{a+b}{b}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত রাশি, } & \frac{a-b}{a} + \frac{a+b}{b} \\ &= \frac{b(a-b) + a(a+b)}{ab} \\ &= \frac{ab - b^2 + a^2 + ab}{ab} \\ &= \frac{a^2 + 2ab - b^2}{ab} \\ \text{সুতারাং, নির্ণয় যোগফল } & \frac{a^2 + 2ab - b^2}{ab} \end{aligned}$$

$$(খ) \frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত রাশি, } & \frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} \\ &= \frac{a^2 + b^2 + c^2}{abc} \\ \text{সুতারাং, নির্ণয় যোগফল } & \frac{a^2 + b^2 + c^2}{abc} \end{aligned}$$

$$(গ) \frac{x-y}{x} + \frac{y-z}{y} + \frac{z-x}{z}$$

সমাধান :

$$\text{প্রদত্ত রাশি, } \frac{x-y}{x} + \frac{y-z}{y} + \frac{z-x}{z}$$

$$\begin{aligned}
 &= \frac{yz(x-y) + zx(y-z) + xy(z-x)}{xyz} \\
 &= \frac{xyz - zy^2 + xyz - xz^2 + xyz - x^2y}{xyz} \\
 &= \frac{3xyz - x^2y - y^2z + xyz - z^2x}{xyz} \\
 &\text{সুতারাং, নির্ণয় যোগফল } \frac{3xyz - x^2y - y^2z + xyz - z^2x}{xyz}
 \end{aligned}$$

(ঘ) $\frac{x+y}{x-y} + \frac{x-y}{x+y}$

সমাধান :

$$\begin{aligned}
 &\text{প্রদত্ত রাশি, } \frac{x+y}{x-y} + \frac{x-y}{x+y} \\
 &= \frac{(x+y)^2 + (x-y)^2}{(x-y)(x+y)} \\
 &= \frac{x^2 + 2xy + y^2 + x^2 - 2xy + y^2}{(x-y)(x+y)} \\
 &= \frac{2x^2 + 2y^2}{(x^2 - y^2)} \\
 &= \frac{2(x^2 + y^2)}{(x^2 - y^2)} \\
 &\text{সুতারাং, নির্ণয় যোগফল } \frac{2(x^2 + y^2)}{(x^2 - y^2)}
 \end{aligned}$$

$$(4) \frac{1}{x^2 - 3x + 2} + \frac{1}{x^2 - 4x + 3} + \frac{1}{x^2 - 5x + 4}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত রাশি, } & \frac{1}{x^2 - 3x + 2} + \frac{1}{x^2 - 4x + 3} + \frac{1}{x^2 - 5x + 4} \\ &= \frac{1}{x^2 - 2x - x + 2} + \frac{1}{x^2 - 3x - x + 3} + \frac{1}{x^2 - 4x - x + 4} \\ &= \frac{1}{(x-2)(x-1)} + \frac{1}{(x-3)(x-1)} + \frac{1}{(x-4)(x-1)} \\ &= \frac{1.(x-3)(x-4) + 1.(x-2)(x-4) + 1.(x-3)(x-2)}{(x-1)(x-2)(x-3)(x-4)} \\ &= \frac{x^2 - 7x + 12 + x^2 - 6x + 8 + x^2 - 5x + 6}{(x-1)(x-2)(x-3)(x-4)} \\ &= \frac{3x^2 - 18x + 26}{(x-1)(x-2)(x-3)(x-4)} \end{aligned}$$

সুতারাং, নির্ণয় যোগফল $\frac{3x^2 - 18x + 26}{(x-1)(x-2)(x-3)(x-4)}$

$$(5) \frac{1}{a^2 - b^2} + \frac{1}{a^2 + ab + b^2} + \frac{1}{a^2 - ab + b^2}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত রাশি, } & \frac{1}{a^2 - b^2} + \frac{1}{a^2 + ab + b^2} + \frac{1}{a^2 - ab + b^2} \\ &= \frac{1}{(a-b)(a+b)} + \frac{1}{(a^2 + ab + b^2)} + \frac{1}{(a^2 - ab + b^2)} \end{aligned}$$

$$\begin{aligned}
 &= \frac{(a^2 - ab + b^2)(a^2 + ab + b^2) + (a-b)(a+b)(a^2 - ab + b^2) + (a-b)(a+b)(a^2 + ab + b^2)}{(a-b)(a+b)(a^2 + ab + b^2)(a^2 - ab + b^2)} \\
 &= \frac{a^4 - a^3b + a^2b^2 + a^3b - a^2b^2 + ab^3 + a^2b^2 - ab^3 + b^4 + (a-b)(a^3 + b^3) + (a+b)(a^3 - b^3)}{(a-b)(a^2 + ab + b^2)(a+b)(a^2 - ab + b^2)} \\
 &= \frac{a^4 + a^2b^2 + b^4 + a^4 + ab^3 - a^3b - b^4 + a^4 - ab^3 + a^3b - b^4}{(a^3 - b^3)(a^3 + b^3)} \\
 &= \frac{3a^4 + a^2b^2 - b^4}{(a^6 - b^6)}
 \end{aligned}$$

সূতরাং, নির্ণেয় যোগফল $\frac{3a^4 + a^2b^2 - b^4}{(a^6 - b^6)}$

(৩) $\frac{1}{x-2} - \frac{1}{x+2} + \frac{4}{x^2 - 4}$

সমাধান :

$$\begin{aligned}
 \text{প্রদত্ত রাশি, } & \frac{1}{x-2} - \frac{1}{x+2} + \frac{4}{x^2 - 4} \\
 &= \frac{1}{x-2} - \frac{1}{x+2} + \frac{4}{x^2 - 2^2} \\
 &= \frac{1}{x-2} - \frac{1}{x+2} + \frac{4}{(x-2)(x+2)} \\
 &= \frac{1.(x+2) - 1.(x-2) + 4}{(x-2)(x+2)} \\
 &= \frac{x+2 - x+2 + 4}{(x^2 - 4)} \\
 &= \frac{8}{(x^2 - 4)} \quad \text{সূতরাং, নির্ণেয় যোগফল } \frac{8}{(x^2 - 4)}
 \end{aligned}$$

$$(জ) \frac{1}{x^2-1} + \frac{1}{x^4-1} + \frac{4}{x^8-1}$$

সমাধান :

$$\begin{aligned} \text{প্রদত্ত রাশি, } & \frac{1}{x^2-1} + \frac{1}{x^4-1} + \frac{4}{x^8-1} \\ &= \frac{1}{x^2-1} + \frac{1}{(x^2)^2-1^2} + \frac{4}{(x^4)^2-1^2} \\ &= \frac{1}{x^2-1} + \frac{1}{(x^2-1)(x^2+1)} + \frac{4}{(x^4-1)(x^4+1)} \\ &= \frac{1}{x^2-1} + \frac{1}{(x^2-1)(x^2+1)} + \frac{4}{(x^2-1)(x^2+1)(x^4+1)} \\ &= \frac{1.(x^2+1)(x^4+1)+1.(x^4+1)+4}{(x^2-1)(x^2+1)(x^4+1)} \\ &= \frac{x^6+x^4+x^2+1+x^4+1+4}{(x^4-1)(x^4+1)} \\ &= \frac{x^6+2x^4+x^2+6}{(x^8-1)} \end{aligned}$$

$$\text{সুতারাং, নির্ণয় যোগফল } \frac{x^6+2x^4+x^2+6}{(x^8-1)}$$

৪। বিয়োগ কর :

$$(ক) \frac{a}{x-3} - \frac{a^2}{x^2-9}$$

$$(খ) \frac{1}{y(x-y)} - \frac{1}{x(x+y)}$$

$$(গ) \frac{x+1}{1+x+x^2} - \frac{x-1}{1-x+x^2}$$

$$(ঘ) \frac{a^2+16b^2}{a^2-16b^2} - \frac{a-4b}{a+4b}$$

$$(ঙ) \frac{1}{x-y} - \frac{x^2-xy+y^2}{x^3+y^3}$$

$$(ক) \frac{a}{x-3} - \frac{a^2}{x^2-9}$$

সমাধান :

এখানে, হর $x-3$ ও x^2-9 এর ল.স.গ. = x^2-9

$$\begin{aligned} & \therefore \frac{a}{x-3} - \frac{a^2}{x^2-9} \\ &= \frac{a}{x-3} - \frac{a^2}{(x-3)(x+3)} \\ &= \frac{a(x+3)-a^2}{(x-3)(x+3)} \\ &= \frac{ax+3a-a^2}{(x-3)(x+3)} \end{aligned}$$

সুতারাং, নির্ণয় বিয়োগফল $\frac{ax+3a-a^2}{(x-3)(x+3)}$

$$(খ) \frac{1}{y(x-y)} - \frac{1}{x(x+y)}$$

সমাধান :

এখানে, হর $y(x-y)$ ও $x(x+y)$ এর ল.স.গ. = $xy(x-y)(x+y)$

$$\begin{aligned} & \therefore \frac{1}{y(x-y)} - \frac{1}{x(x+y)} \\ &= \frac{1.x(x+y)-1.y(x-y)}{xy(x-y)(x+y)} \\ &= \frac{x^2+xy-xy+y^2}{xy(x^2-y^2)} \\ &= \frac{(x^2+y^2)}{xy(x^2-y^2)} \quad \text{সুতারাং, নির্ণয় বিয়োগফল } \frac{(x^2+y^2)}{xy(x^2-y^2)} \end{aligned}$$

$$(গ) \frac{x+1}{1+x+x^2} - \frac{x-1}{1-x+x^2}$$

সমাধান :

এখানে, হর $(1+x+x^2)$ ও $(1-x+x^2)$ এর ল.স.গ. = $(1+x+x^2)(1-x+x^2)$

$$\begin{aligned} & \therefore \frac{x+1}{1+x+x^2} - \frac{x-1}{1-x+x^2} \\ &= \frac{(x+1)(1-x+x^2) - (x-1)(1+x+x^2)}{(1+x+x^2)(1-x+x^2)} \\ &= \frac{(x+1)(1-x+x^2) - (x-1)(1+x+x^2)}{\{(1+x^2)+(x)\}\{(1+x^2)-(-x)\}} \\ &= \frac{(x^3+1)-(x^3-1)}{(1+x^2)^2-x^2} \\ &= \frac{x^3+1-x^3+1}{(1+2x^2+x^4-x^2)} \\ &= \frac{2}{1+x^2+x^4} \\ &\text{সুতারাং, নির্ণেয় বিয়োগফল } \frac{2}{1+x^2+x^4} \end{aligned}$$

$$(ঘ) \frac{a^2+16b^2}{a^2-16b^2} - \frac{a-4b}{a+4b}$$

সমাধান :

$$\begin{aligned} & \therefore \frac{a^2+16b^2}{a^2-16b^2} - \frac{a-4b}{a+4b} \\ &= \frac{a^2+16b^2}{(a-4b)(a+4b)} - \frac{a-4b}{a+4b} \end{aligned}$$

এখানে, হর $(a-4b)(a+4b)$ ও $(a+4b)$ এর ল.স.গ. = $(a-4b)(a+4b)$

$$\begin{aligned}
 &= \frac{a^2 + 16b^2 - (a-4b)(a+4b)}{(a-4b)(a+4b)} \\
 &= \frac{a^2 + 16b^2 - (a-4b)^2}{(a)^2 - (4b)^2} \\
 &= \frac{a^2 + 16b^2 - \{(a)^2 - 2.a.4b + (4b)^2\}}{(a)^2 - (4b)^2} \\
 &= \frac{a^2 + 16b^2 - a^2 + 8ab - 16b^2}{(a-4b)(a+4b)} \\
 &= \frac{8ab}{(a-4b)(a+4b)}
 \end{aligned}$$

সুতরাং, নির্ণয় বিয়োগফল $\frac{8ab}{(a-4b)(a+4b)}$

$$(গ) \frac{1}{x-y} - \frac{x^2 - xy + y^2}{x^3 + y^3}$$

সমাধান :

$$\begin{aligned}
 &\frac{1}{x-y} - \frac{x^2 - xy + y^2}{x^3 + y^3} \\
 &= \frac{1}{x-y} - \frac{x^2 - xy + y^2}{(x+y)(x^2 - xy + y^2)}
 \end{aligned}$$

এখানে, হ্র

$$\begin{aligned}
 (x-y) \text{ ও } (x+y)(x^2 - xy + y^2) \text{ এর ল.সা.গ. } (x-y)(x+y)(x^2 - xy + y^2) \\
 \therefore \frac{1}{x-y} - \frac{x^2 - xy + y^2}{(x+y)(x^2 - xy + y^2)} \\
 = \frac{1.(x+y)(x^2 - xy + y^2) - (x-y)(x^2 - xy + y^2)}{(x-y)(x+y)(x^2 - xy + y^2)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{x^3 + y^3 - (x^3 - x^2y + xy^2 - x^2y + xy^2 - y^3)}{(x-y)(x+y)(x^2 - xy + y^2)} \\
 &= \frac{x^3 + y^3 - x^3 + 2x^2y - 2xy^2 + y^3}{(x-y)(x+y)(x^2 - xy + y^2)} \\
 &= \frac{2x^2y - 2xy^2 + 2y^3}{(x-y)(x+y)(x^2 - xy + y^2)} \\
 &= \frac{2y(x^2 - xy + y^2)}{(x-y)(x+y)(x^2 - xy + y^2)} \\
 &= \frac{2y}{(x-y)(x+y)} \\
 &= \frac{2y}{(x^2 - y^2)}
 \end{aligned}$$

সুতারাং, নির্ণয় বিয়োগফল $\frac{2y}{(x^2 - y^2)}$

৫। সরল কর :

(ক) $\frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx}$

(খ) $\frac{x-y}{(x+y)(y+z)} + \frac{y-z}{(y+z)(z+x)} + \frac{z-x}{(z+x)(x+y)}$

(গ) $\frac{y}{(x-y)(y-z)} + \frac{x}{(z-x)(x-y)} + \frac{z}{(y-z)(x-z)}$

(ঘ) $\frac{1}{x+3y} + \frac{1}{x-3y} + \frac{1}{x^2 - 9y^2}$

(ঙ) $\frac{1}{x-y} - \frac{2}{2x+y} + \frac{1}{x+y} - \frac{2}{2x-y}$

$$(ট) \frac{1}{x-2} - \frac{x-2}{x^2+2x+4} + \frac{6x}{x^3+8}$$

$$(ই) \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} + \frac{4}{x^4+1}$$

$$(জ) \frac{x-y}{(y-z)(z-x)} + \frac{y-z}{(z-x)(x-y)} + \frac{z-x}{(x-y)(x-z)}$$

$$(ঝ) \frac{1}{a-b-c} + \frac{1}{a-b+c} + \frac{a}{a^2+b^2-c^2-2ab}$$

$$(ঞ) \frac{1}{a^2+b^2-c^2+2ab} + \frac{1}{b^2+c^2-a^2+2bc} + \frac{1}{c^2+a^2-b^2+2ca}$$

$$(ক) \frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx}$$

সমাধান :

$$\frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx}$$

এখানে, হর, xy , yz ও zx এর ল.স.গু. $= xyz$

$$\begin{aligned} & \therefore \frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx} \\ &= \frac{z(x-y) + x(y-z) + y(z-x)}{xyz} \end{aligned}$$

$$= \frac{xz - yz + xy - zx + yz - xy}{xyz}$$

$$= \frac{0}{xyz}$$

$$= 0$$

$$(x) \frac{x-y}{(x+y)(y+z)} + \frac{y-z}{(y+z)(z+x)} + \frac{z-x}{(z+x)(x+y)}$$

সমাধান :

$$\frac{x-y}{(x+y)(y+z)} + \frac{y-z}{(y+z)(z+x)} + \frac{z-x}{(z+x)(x+y)}$$

এখানে, হর, $(x+y)$, $(y+z)$ ও $(z+x)$ এর ল.স.গু. $=(x+y)(y+z)(z+x)$

$$\therefore \frac{x-y}{(x+y)(y+z)} + \frac{y-z}{(y+z)(z+x)} + \frac{z-x}{(z+x)(x+y)}$$

$$= \frac{(x-y)(z+x) + (y-z)(x+y) + (z-x)(y+z)}{(x+y)(y+z)(z+x)}$$

$$= \frac{xz + x^2 - yz - xy + xy - xz + y^2 - yz + yz - xy + z^2 - xz}{(x+y)(y+z)(z+x)}$$

$$= \frac{x^2 + y^2 + z^2 - xy - yz - xz}{(x+y)(y+z)(z+x)}$$

$$(g) \frac{y}{(x-y)(y-z)} + \frac{x}{(z-x)(x-y)} + \frac{z}{(y-z)(x-z)}$$

সমাধান :

$$\frac{y}{(x-y)(y-z)} + \frac{x}{(z-x)(x-y)} + \frac{z}{(y-z)(x-z)}$$

এখানে, হর, $(x-y)$, $(y-z)$ ও $(z-x)$ এর ল.স.গু. $=(x-y)(y-z)(z-x)$

$$\therefore \frac{y}{(x-y)(y-z)} + \frac{x}{(z-x)(x-y)} - \frac{z}{(y-z)(z-x)}$$

$$= \frac{y(z-x) + x(y-z) - z(x-y)}{(x-y)(y-z)(z-x)}$$

$$\begin{aligned}
 &= \frac{yz - xy + xy - zx - zx + yz}{(x-y)(y-z)(z-x)} \\
 &= \frac{-2zx + 2yz}{(x-y)(y-z)(z-x)} \\
 &= \frac{-2z(x-y)}{(x-y)(y-z)(z-x)} \\
 &= \frac{-2z}{(y-z)(z-x)}
 \end{aligned}$$

(ঘ) $\frac{1}{x+3y} + \frac{1}{x-3y} + \frac{2x}{x^2-9y^2}$

সমাধান :

$$\frac{1}{x+3y} + \frac{1}{x-3y} + \frac{2x}{x^2-9y^2}$$

এখানে, $x^2 - 9y^2$

$$= x^2 - (3y)^2$$

$$= (x-3y)(x+3y)$$

হব, $(x+3y)$, $(x-3y)$ ও $(x-3y)(x+3y)$ এর ল.সা.গ. = $(x-3y)(x+3y)$

$$\therefore \frac{1}{x+3y} + \frac{1}{x-3y} + \frac{2x}{(x-3y)(x+3y)}$$

$$= \frac{1.(x-3y) + 1.(x+3y) + 2x}{(x-3y)(x+3y)}$$

$$= \frac{x-3y+x+3y-2x}{(x-3y)(x+3y)}$$

$$= \frac{0}{(x-y)(y-z)(z-x)}$$

$$= 0$$

$$(৫) \frac{1}{x-y} - \frac{2}{2x+y} + \frac{1}{x+y} - \frac{2}{2x-y}$$

সমাধান :

$$\frac{1}{x-y} - \frac{2}{2x+y} + \frac{1}{x+y} - \frac{2}{2x-y}$$

এখানে, হর, $x-y$, $(2x+y)$, $x+y$ ও $(2x-y)$ এর ল.সা.গু = $(x^2 - y^2)(4x^2 - y^2)$

$$\therefore \frac{1}{x-y} + \frac{1}{x+y} - \frac{2}{2x+y} - \frac{2}{2x-y}$$

$$= \left\{ \frac{1.(x+y) + 1.(x-y)}{(x-y)(x+y)} \right\} - 2 \left(\frac{1}{2x+y} + \frac{1}{2x-y} \right)$$

$$= \left\{ \frac{x+y+x-y}{(x-y)(x+y)} \right\} - 2 \left\{ \frac{2x-y+2x+y}{(2x+y)(2x-y)} \right\}$$

$$= \frac{2x}{(x^2 - y^2)} - \frac{2 \times 4x}{(4x^2 - y^2)}$$

$$= \frac{2x(4x^2 - y^2) - 8x(x^2 - y^2)}{(x^2 - y^2)(4x^2 - y^2)}$$

$$= \frac{8x^3 - 2xy^2 - 8x^3 + 8xy^2}{(x^2 - y^2)(4x^2 - y^2)}$$

$$= \frac{6xy^2}{(x^2 - y^2)(4x^2 - y^2)}$$

$$(v) \frac{1}{x-2} - \frac{x-2}{x^2+2x+4} + \frac{6x}{x^3+8}$$

সমাধান :

$$\frac{1}{x-2} - \frac{x-2}{x^2+2x+4} + \frac{6x}{x^3+8}$$

এখানে, হর, $(x-2)$, (x^2+2x+4) ও (x^3+8) এর ল.সা.গু

$$= (x-2)(x^2+2x+4)(x^3+8)$$

$$= (x^3-2^3)(x^3+8)$$

$$= (x^3-8)(x^3+8)$$

$$= (x^3)^2 - (8)^2$$

$$= x^6 - 64$$

$$\therefore \frac{1}{x-2} - \frac{x-2}{x^2+2x+4} + \frac{6x}{x^3+8}$$

$$= \frac{1.(x^2+2x+4) - (x-2)^2}{(x-2)(x^2+2x+4)} + \frac{6x}{x^3+8}$$

$$= \frac{x^2+2x+4 - (x^2-4x+4)}{(x-2)(x^2+2x+4)} + \frac{6x}{x^3+8}$$

$$= \frac{x^2+2x+4-x^2+4x-4}{(x^3-2^3)} + \frac{6x}{x^3+8}$$

$$= \frac{6x}{x^3-8} + \frac{6x}{x^3+8}$$

$$= \frac{6x(x^3+8) + 6x(x^3-8)}{(x^3-8)(x^3+8)}$$

$$= \frac{6x^4+48x+6x^4-48x}{(x^3)^2-8^2} = \frac{12x^4}{x^6-64}$$

$$(7) \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} + \frac{4}{x^4+1}$$

সমাধান :

$$\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} + \frac{4}{x^4+1}$$

এখানে, হর, $x-1, x+1, x^2+1$ ও x^4+1 এর ল.স.গু = x^8-1

$$\therefore \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} + \frac{4}{x^4+1}$$

$$= \frac{1.(x+1)-1.(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} + \frac{4}{x^4+1}$$

$$= \frac{x+1-x+1}{x^2-1} - \frac{2}{x^2+1} + \frac{4}{x^4+1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} + \frac{2}{x^4+1}$$

$$= \frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)} + \frac{4}{x^4+1}$$

$$= \frac{2x^2+2-2x^2+2}{x^4-1} + \frac{4}{x^4+1}$$

$$= \frac{4}{x^4-1} + \frac{4}{x^4+1}$$

$$= \frac{4(x^4+1)+4(x^4-1)}{(x^4-1)(x^4+1)}$$

$$= \frac{4x^4+4+4x^4-4}{(x^4)^2-(1)^2}$$

$$= \frac{8x^4}{x^8-1}$$

$$(জ) \frac{x-y}{(y-z)(z-x)} + \frac{y-z}{(z-x)(x-y)} + \frac{z-x}{(x-y)(x-z)}$$

সমাধান :

$$\begin{aligned} & \frac{x-y}{(y-z)(z-x)} + \frac{y-z}{(z-x)(x-y)} + \frac{z-x}{(x-y)(x-z)} \\ &= \frac{x-y}{(y-z)(z-x)} + \frac{y-z}{(z-x)(x-y)} - \frac{z-x}{(x-y)(z-x)} \\ \text{এখানে, } & \text{হয়, } y-z, z-x \text{ ও } x-y \text{ এর ল.স.গু} = (x-y)(y-z)(z-x) \\ \therefore & \frac{x-y}{(y-z)(z-x)} + \frac{y-z}{(z-x)(x-y)} - \frac{z-x}{(x-y)(z-x)} \\ &= \frac{(x-y)^2 + (y-z)^2 - (z-x)(y-z)}{(x-y)(y-z)(z-x)} \\ &= \frac{x^2 - 2xy + y^2 + y^2 - 2yz + z^2 - (zy - z^2 - xy + zx)}{(x-y)(y-z)(z-x)} \\ &= \frac{x^2 - 2xy + y^2 + y^2 - 2yz + z^2 - yz + z^2 + xy - zx}{(x-y)(y-z)(z-x)} \\ &= \frac{x^2 + 2y^2 + 2z^2 - xy - 3yz - zx}{(x-y)(y-z)(z-x)} \end{aligned}$$

$$(ঝ) \frac{1}{a-b-c} + \frac{1}{a-b+c} + \frac{a}{a^2 + b^2 - c^2 - 2ab}$$

সমাধান :

$$\begin{aligned} & \frac{1}{a-b-c} + \frac{1}{a-b+c} + \frac{a}{a^2 + b^2 - c^2 - 2ab} \\ \text{এখানে, } & a^2 + b^2 - c^2 - 2ab \\ &= (a-b)^2 - c^2 \\ &= (a-b-c)(a-b+c) \end{aligned}$$

হর, $a-b-c$, $a-b+c$ ও $a^2+b^2-c^2-2ab$ এর ল.সা.গু $a^2+b^2-c^2-2ab$

$$\begin{aligned} & \therefore \frac{1}{a-b-c} + \frac{1}{a-b+c} + \frac{a}{(a-b-c)(a-b+c)} \\ & = \frac{a-b+c+a-b-c}{(a-b-c)(a-b+c)} + \frac{a}{(a-b-c)(a-b+c)} \\ & = \frac{2a-2b}{(a-b-c)(a-b+c)} + \frac{a}{(a-b-c)(a-b+c)} \\ & = \frac{2a-2b+a}{(a-b-c)(a-b+c)} \\ & = \frac{3a-2b}{a^2+b^2-c^2-2ab} \end{aligned}$$

(৫৩) $\frac{1}{a^2+b^2-c^2+2ab} + \frac{1}{b^2+c^2-a^2+2bc} + \frac{1}{c^2+a^2-b^2+2ca}$

সমাধান :

$$\frac{1}{a^2+b^2-c^2+2ab} + \frac{1}{b^2+c^2-a^2+2bc} + \frac{1}{c^2+a^2-b^2+2ca}$$

এখানে,

$$\begin{aligned} (a^2+b^2-c^2+2ab) &= (a+b)^2-c^2 \\ &= (a+b-c)(a+b+c) \end{aligned}$$

$$\begin{aligned} (b^2+c^2-a^2+2bc) &= (b+c)^2-a^2 \\ &= (b+c-a)(b+c+a) \end{aligned}$$

$$\begin{aligned} (c^2+a^2-b^2+2ca) &= (c+a)^2-b^2 \\ &= (c+a-b)(c+a+b) \end{aligned}$$

হর, $(a+b-c)(a+b+c)$, $(b+c-a)(a+b+c)$ ও $(a-b+c)(a+b+c)$ এর
 ল.সা.গু $(a+b+c)(a-b+c)(a+b-c)(b+c-a)$

$$\therefore \frac{1}{(a+b+c)(a+b-c)} + \frac{1}{(a+b+c)(b+c-a)} + \frac{1}{(a+b+c)(c+a-b)}$$

$$\begin{aligned} &= \frac{(a-b+c)(b+c-a) + (a-b+c)(a+b-c) + (a+b-c)(b+c-a)}{(a+b+c)(a-b+c)(a+b-c)(b+c-a)} \\ &\quad ab - ca - a^2 - b^2 - bc + ab + bc + c^2 - ca + a^2 + ab + ca - ab - b^2 + bc \\ &= \frac{ca + bc - c^2 + bc + ca - a^2 + b^2 + bc - ab - bc - c^2 + ca}{(a+b+c)(a-b+c)(a+b-c)(b+c-a)} \\ &= \frac{2ab + 2bc + 2ca - a^2 - b^2 - c^2}{(a+b+c)(a-b+c)(a+b-c)(b+c-a)} \end{aligned}$$

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