$$[x^2 \frac{d}{dx}\{\ln(1+x^2)\} + \ln(1+x^2)\frac{d}{dx}(x^2)]$$

$$= (1+x^2)^{x^2}[\frac{x^2}{1+x^2}(2x) + \ln(1+x^2).(2x)]$$

$$= 2x(1+x^2)^{x^2}[\frac{x^2}{1+x^2} + \ln(1+x^2)]$$

2(e) $(\sqrt{x})^{\sqrt{x}}$ [ব.'১২;চ.'১০; কু.'১১; প্র.ভ.প.'০৫]

$$\frac{d}{dx}\{(\sqrt{x})^{\sqrt{x}}\}$$

$$= (\sqrt{x})^{\sqrt{x}}[\sqrt{x}\frac{d}{dx}(\ln\sqrt{x}) + \ln\sqrt{x}\frac{d}{dx}(\sqrt{x})]$$

$$= (\sqrt{x})^{\sqrt{x}}[\sqrt{x}.\frac{1}{\sqrt{x}}.\frac{1}{2\sqrt{x}} + \ln\sqrt{x}.\frac{1}{2\sqrt{x}}]$$

$$= (\sqrt{x})^{\sqrt{x}}[\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}\ln\sqrt{x}]$$

$$= (\sqrt{x})^{\sqrt{x}}[\frac{1+\ln\sqrt{x}}{2\sqrt{x}}] \quad \text{(Ans.)}$$

2(f) ধরি, $y = x^{\ln x}$ [রা.'০২; কু.'০৮; সি.'১১]

$$\frac{dy}{dx} = x^{\ln x}[\ln x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\ln x)]$$

$$[\quad \frac{d}{dx}(u^v) = u^v[v\frac{d}{dx}(\ln u) + \ln u\frac{dv}{dx}]$$

$$= x^{\ln x}[2\ln x.\frac{1}{x}] = \frac{2\ln x}{x}x^{\ln x}$$

অর্থাৎ, $\frac{d}{dx}(x^{\ln x}) = \frac{2\ln x}{x}x^{\ln x}$

2(g) $\frac{d}{dx}(\sin^{-1}x)^x = (\sin^{-1}x)^x$

$$[x\frac{d}{dx}\{\ln(\sin^{-1}x)\} + \ln(\sin^{-1}x)\frac{d}{dx}(x)]$$

$$= (\sin^{-1}x)^x[x\frac{1}{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}} + \ln(\sin^{-1}x).1]$$

$$= (\sin^{-1}x)^x[\frac{x}{\sqrt{1-x^2}\sin^{-1}x} + \ln(\sin^{-1}x)]$$

2(h) $\frac{d}{dx}(\sin x)^x$ [য.'০৭]

$$= (\sin x)^x[x\frac{d}{dx}\{\ln(\sin x)\} + \ln(\sin x)\frac{d}{dx}(x)]$$

$$= (\sin x)^x[x\frac{1}{\sin x}.\cos x + \ln(\sin x).1]$$

$$= (\sin x)^x[x\cot x + \ln(\sin x)]$$

2(i) $\frac{d}{dx}(lnx)^x$

$$= (lnx)^x[x\frac{d}{dx}\{\ln(\ln x)\} + \ln(\ln x)\frac{d}{dx}(x)]$$

$$= (lnx)^x[x\frac{1}{\ln x}.\frac{1}{x} + \ln(\ln x).1]$$

$$= (lnx)^x[\frac{1}{\ln x} + \ln(\ln x)]$$

2 (j) $\frac{d}{dx}(\log x)^x = (\log x)^x$

$$[x\frac{d}{dx}\{\ln(\log x)\} + \ln(\log x)\frac{d}{dx}(x)]$$

$$= (\log x)^x[x\frac{1}{\log x}.\frac{1}{x\ln 10} + \ln(\log x).1]$$

$$= (\log x)^x[\frac{1}{\ln 10\log x} + \ln(\log x)]$$

2(k) $x^{\cos^{-1}x}$ [কু.'১০,'১৩; সি. ' ০৬,'০৮; ঢা. '১০, '১৩; রা. '০৫, '০৭; ব.'০৬,'১০; দি.'০৯; য.'১০]

$$\frac{d}{dx}(x^{\cos^{-1}x})$$

$$= x^{\cos^{-1}x}[\cos^{-1}x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\cos^{-1}x)]$$

$$= x^{\cos^{-1}x}[\cos^{-1}x.\frac{1}{x} + \ln x\frac{-1}{\sqrt{1-x^2}}]$$

$$= x^{\cos^{-1}x}[\frac{\cos^{-1}x}{x} - \frac{\ln x}{\sqrt{1-x^2}}]$$

2(*l*) $\frac{d}{dx}(x^{-1/x})$ [বুয়েট''০৭]

$$= x^{-1/x}[-\frac{1}{x}\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(-\frac{1}{x})]$$

$$= x^{-1/x}[-\frac{1}{x}.\frac{1}{x} + \ln x\{-(-\frac{1}{x^2})\}]$$

$= x^{-1/x} \times \frac{1}{x^2} (\ln x - 1) = \frac{1}{x^{2+1/x}} (\ln x - 1)$

3(a) $\frac{d}{dx}(e^{x^x}) = e^{x^x} \frac{d}{dx}(x^x)$

$= e^{x^x} x^x [x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x)]$

$= e^{x^x} . x^x \{x . \frac{1}{x} + \ln x . 1\}$

$= e^{x^x} . x^x (1 + \ln x)$

3(b) $\frac{d}{dx}(x^{e^x})$

$= x^{e^x} [\, e^x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(e^x)]$

$= x^{e^x} [\, e^x \frac{1}{x} + \ln x . e^x]$

$= x^{e^x} \; e^x (\frac{1}{x} + \ln x)$

(c) $\frac{d}{dx}(a^{a^x})$ [দি.'১২]

$= a^{a^x} . lna . \frac{d}{dx}(a^x)$

$= a^{a^x} lna . a^x . lna = a^{a^x} \; a^x (lna)^2$

3(d) $(\cot x)^{\tan x}$ [চ.'০৫; ব., দি.'০৯; য.'১২]

$\frac{d}{dx}(\cot x)^{\tan x} = (\cot x)^{\tan x}$

$[\tan x \frac{d}{dx}\{\ln(\cot x)\} + \ln(\cot x) \frac{d}{dx}(\tan x)]$

$= (\cot x)^{\tan x} [\frac{\tan x}{\cot x}(-\operatorname{cosec}^2 x) + \ln(\cot x).(\sec^2 x)]$

$= (\cot x)^{\tan x} [-\frac{\sin^2 x}{\cos^2 x} . \frac{1}{\sin^2 x} + \ln(\cot x).(\sec^2 x)]$

$= (\cot x)^{\tan x} [-\sec^2 x + \ln(\cot x).(\sec^2 x)]$

$= (\cot x)^{\tan x} . \sec^2 x [\ln(\cot x) - 1]$

4. (a) x^{x^x} [রা.'০৬,'০৮; য.'১১; প্র.ভ.প.'০৫]

$\frac{d}{dx}(x^{x^x}) = x^{x^x} [x^x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x^x)]$

$= x^{x^x} [x^x . \frac{1}{x} + \ln x . x^x \{x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x)\}]$

$= x^{x^x} \quad x^x \; [\frac{1}{x} + \ln x . \{x . \frac{1}{x} + \ln x . 1\}]$

$= x^{x^x} . x^x \; [\frac{1}{x} + \ln x . (1 + \ln x\}]$

4(b) $(x^x)^x$ [য.,মা.'০৯; কু.'০৫; ঢা.,ব.,দি.'১১; রা.'১২]

$(x^x)^x = x^{x^2}$

$\frac{d}{dx}(x^x)^x = x^{x^2} [x^2 \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x^2)]$

$= x^{x^2} [x^2 . \frac{1}{x} + \ln x . (2x)]$

$= x^{x^2} [x + 2x \ln x] = (x^x)^x . x [1 + 2 \ln x]$

4(c) $\frac{d}{dx}(\sec x)^{x^x} = (\sec x)^{x^x}$

$[x^x \frac{d}{dx}\{\ln(\sec x)\} + \ln(\sec x) \frac{d}{dx}(x^x)]$

$= (\sec x)^{x^x} [x^x \frac{1}{\sec x} . \sec x \tan x + \ln(\sec x) . x^x \{x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x)\}]$

$= (\sec x)^{x^x} . x^x [\tan x + \ln(\sec x) \{x . \frac{1}{x} + \ln x . 1\}]$

$= (\sec x)^{x^x} . \; x^x [\tan x + (1 + \ln x) \ln(\sec x)]$

5.(a) $\frac{d}{dx}(x^x \, ln\, x)$ [কু. '০৪; দি.'১০; ব.'১২]

$= x^x \frac{d}{dx}(lnx) + lnx \frac{d}{dx}(x^x)$

$= x^x . \frac{1}{x} + lnx . \; x^x \{ x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(x)\}$

$= x^x [\frac{1}{x} + lnx . \{ x . \frac{1}{x} + \ln x . 1\}]$

$= x^x \{ \frac{1}{x} + lnx . (1 + \ln x)\}$

$=x^{-1/x}\times\frac{1}{x^2}(\ln x-1)=\frac{1}{x^{2+1/x}}(\ln x-1)$

3(a) $\frac{d}{dx}(e^{x^x})=e^{x^x}\frac{d}{dx}(x^x)$

$=e^{x^x}x^x[x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x)]$

$=e^{x^x}.x^x\{x.\frac{1}{x}+\ln x.1\}$

$=e^{x^x}.x^x(1+\ln x)$

3(b) $\frac{d}{dx}(x^{e^x})$

$=x^{e^x}[e^x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(e^x)]$

$=x^{e^x}[e^x\frac{1}{x}+\ln x.e^x]$

$=x^{e^x}e^x(\frac{1}{x}+\ln x)$

(c) $\frac{d}{dx}(a^{a^x})$ [দি.'১২]

$=a^{a^x}.lna.\frac{d}{dx}(a^x)$

$=a^{a^x}lna.a^x.lna=a^{a^x}a^x(lna)^2$

3(d) $(\cot x)^{\tan x}$ [চ.'০৫; ব., দি.'০৯; য.'১২]

$\frac{d}{dx}(\cot x)^{\tan x}=(\cot x)^{\tan x}$

$[\tan x\frac{d}{dx}\{\ln(\cot x)\}+\ln(\cot x)\frac{d}{dx}(\tan x)]$

$=(\cot x)^{\tan x}[\frac{\tan x}{\cot x}(-\cos ec^2x)+\ln(\cot x).(\sec^2x)]$

$=(\cot x)^{\tan x}[-\frac{\sin^2 x}{\cos^2 x}.\frac{1}{\sin^2 x}+\ln(\cot x).(\sec^2x)]$

$=(\cot x)^{\tan x}[-\sec^2x+\ln(\cot x).(\sec^2x)]$

$=(\cot x)^{\tan x}.\sec^2x[\ln(\cot x)-1]$

4. (a) x^{x^x} [রা.'০৬,'০৮; য.'১১; প্র.ভ.প.'০৫]

$\frac{d}{dx}(x^{x^x})=x^{x^x}[x^x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x^x)]$

$=x^{x^x}[x^x.\frac{1}{x}+\ln x.x^x\{x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x)\}]$

$=x^{x^x}\ x^x\ [\frac{1}{x}+\ln x.\{x.\frac{1}{x}+\ln x.1\}]$

$=x^{x^x}.x^x\ [\frac{1}{x}+\ln x.(1+\ln x\}]$

4(b) $(x^x)^x$ [য.,মা.'০৯; কু.'০৫; ঢা.,ব.,দি.'১১; রা.'১২]

$(x^x)^x=x^{x^2}$

$\frac{d}{dx}(x^x)^x=x^{x^2}[x^2\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x^2)]$

$=x^{x^2}[x^2.\frac{1}{x}+\ln x.(2x)]$

$=x^{x^2}[x+2x\ln x]=(x^x)^x.x[1+2\ln x]$

4(c) $\frac{d}{dx}(\sec x)^{x^x}=(\sec x)^{x^x}$

$[x^x\frac{d}{dx}\{\ln(\sec x)\}+\ln(\sec x)\frac{d}{dx}(x^x)]$

$=(\sec x)^{x^x}[x^x\frac{1}{\sec x}.\sec x\tan x+\ln(\sec x).x^x\{x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x)\}]$

$=(\sec x)^{x^x}.x^x[\tan x+\ln(\sec x)\{x.\frac{1}{x}+\ln x.1\}]$

$=(\sec x)^{x^x}.x^x[\tan x+(1+\ln x)\ln(\sec x)]$

5.(a) $\frac{d}{dx}(x^x\,ln\,x)$ [কু. '০৪; দি.'১০; ব.'১২]

$=x^x\frac{d}{dx}(lnx)+lnx\frac{d}{dx}(x^x)$

$=x^x.\frac{1}{x}+lnx.\ x^x\{x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x)\}$

$=x^x[\frac{1}{x}+lnx.\ \{x.\frac{1}{x}+\ln x.1\}]$

$=x^x\{\frac{1}{x}+lnx.\ (1+\ln x)\}$

5(b) $\frac{d}{dx}(ax)^{bx}$

$= (ax)^{bx}[\,bx\frac{d}{dx}\{\ln(ax)\} + \ln(ax)\frac{d}{dx}(bx)]$

$= (ax)^{bx}[\,bx.\frac{1}{ax}.a + \ln(ax).b]$

$= (ax)^{bx}.b\,[\,1 + ln\,(ax)\,]$

5(c) ধরি, $y = (xe^x)^{\sin x}$

$\ln y = \ln(xe^x)^{\sin x} = \sin x\,(\ln x + \ln e^x)$

$= \sin x\,(\ln x + x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{1}{y}\frac{dy}{dx} = \sin x\,(\frac{1}{x}+1) + (\ln x + x)\cos x$

$\Rightarrow \frac{dy}{dx} = y[(\frac{1}{x}+1)\,\sin x + (\ln x + x)\cos x]$

$= (xe^x)^{\sin x}[\sin x.(\frac{1}{x}+1) + (\ln x + x)\cos x]$

5(d) $\frac{d}{dx}(e^{x^2} + x^{x^2})$ [ঢা.'০৬,'১২]

$= \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(x^{x^2})$

$= e^{x^2}(2x) + x^{x^2}[x^2\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x^2)]$

$= 2x\,e^{x^2} + x^{x^2}[\frac{x^2}{x} + \ln x.(2x)]$

$= 2x\,e^{x^2} + x^{x^2}[x + 2x\ln x]$

5(e) $\frac{d}{dx}\{(\tan x)^x + x^{\tan x}\}$

$= \frac{d}{dx}(\tan x)^x + \frac{d}{dx}(x^{\tan x})$

$= (\tan x)^x[x\frac{d}{dx}\{\ln(\tan x)\} + \ln(\tan x)\frac{d}{dx}(x)]$

$+ x^{\tan x}[\tan x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\tan x)]$

$= (\tan x)^x[x\frac{1}{\tan x}.\sec^2 x + \ln(\tan x).1]$

$+ x^{\tan x}[\tan x.\frac{1}{x} + \ln x.\sec^2 x]$

$= (\tan x)^x[x\frac{\cos x}{\sin x}.\frac{1}{\cos^2 x} + \ln(\tan x)]$

$+ x^{\tan x}[\frac{1}{x}\tan x + \sec^2 x\ln x]$

$= (\tan x)^x[2x\cos ec2x + \ln(\tan x)]$

$+ x^{\tan x}[\frac{1}{x}\tan x + \sec^2 x\ln x]$

5.(f) $\frac{d}{dx}(x^{\ln x} + x^{\log x})$

$= \frac{d}{dx}(x^{\ln x}) + \frac{d}{dx}(x^{\ln x})$

$= x^{\ln x}[\ln x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\ln x)]$

$+ x^{\log x}[\log x\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(\log x)]$

$= x^{\ln x}\,2\ln x.\frac{1}{x} + x^{\log x}[\log x.\frac{1}{x} + \ln x.\frac{1}{x\ln 10}]$

$= \frac{2\ln x}{x}.x^{\ln x} + x^{\log x}[\frac{\log x}{x} + \frac{\ln x}{x\ln 10}]$

5(g) $\frac{d}{dx}\{(\ln x)^x + (\log x)^x\}$

$= \frac{d}{dx}(\ln x)^x + \frac{d}{dx}(\log x)^x$

$= (\ln x)^x[x\frac{d}{dx}\{\ln(\ln x)\} + \ln(\ln x)\frac{d}{dx}(x)] +$

$(\log x)^x[x\frac{d}{dx}\{\ln(\log x)\} + \ln(\log x)\frac{d}{dx}(x)]$

$= (\ln x)^x[x\frac{1}{\ln x}.\frac{1}{x} + \ln(\ln x).1] +$

$(\log x)^x[x\frac{1}{\log x}.\frac{1}{x\ln 10} + \ln(\log x).1]$

$= (\ln x)^x[\frac{1}{\ln x} + \ln(\ln x)] +$

$(\log x)^x[\frac{1}{\ln 10\log x} + \ln(\log x)]$

5(h) $\frac{d}{dx}\{(\tan x)^{\cot x} + (\cot x)^{\tan x}\}$

$= \frac{d}{dx}(\tan x)^{\cot x} + \frac{d}{dx}(\cot x)^{\tan x}$

$= (\tan x)^{\cot x}[\cot x \frac{d}{dx}\{\ln(\tan x)\} + \ln(\tan x)$

$\frac{d}{dx}(\cot x)] + (\cot x)^{\tan x}\ [\tan x \frac{d}{dx}\{\ln(\cot x)\}$

$+ \ln(\cot x)\frac{d}{dx}(\tan x)]$

$= (\tan x)^{\cot x}[\frac{\cot x}{\tan x}\sec^2 x + \ln(\tan x).$

$(-\operatorname{cosec}^2 x)] + (\cot x)^{\tan x}\ [\frac{\tan x}{\cot x}$

$(-\operatorname{cosec}^2 x) + \ln(\cot x).(\sec^2 x)]$

$= (\tan x)^{\cot x}[\frac{\cos^2 x}{\sin^2 x}\frac{1}{\cos^2 x} - \ln(\tan x).$

$\operatorname{cosec}^2 x] + (\cot x)^{\tan x}[-\frac{\sin^2 x}{\cos^2 x}.\frac{1}{\sin^2 x}$

$+ \ln(\cot x).(\sec^2 x)]$

$= (\tan x)^{\cot x}.\operatorname{cosec}^2 x[1 - \ln(\tan x)]$

$+(\cot x)^{\tan x}.\sec^2 x\ [\ln(\cot x) - 1]$

5(i) $\frac{d}{dx}(x^x \log x)$ **[চ.'১২]**

$= x^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^x)$

$= x^x \frac{1}{x\ln 10} + \log x\ [x^x\{x \frac{d}{dx}(\ln x)$

$+ \ln x \frac{d}{dx}(x)\}]$

$= \frac{x^x}{x\ln 10} + x^x \log x\ \{x\frac{1}{x} + \ln x\}$

$= \frac{x^x}{x\ln 10} + x^x \log x\ \{1 + \ln x\}$

প্রশ্নমালা IX H

1. $\frac{dy}{dx}$ **নির্ণয় কর ঃ**

(a) $x^a y^b = (x-y)^{a+b}$ **[প্র.ভ.প. '০৬]**

$\ln(x^a y^b) = \ln(x-y)^{a+b}$

$\Rightarrow \ln(x^a) + \ln(y^b) = (a+b)\ln(x-y)$

$\Rightarrow a\ln x + b\ln y = (a+b)\ln(x-y)$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$a.\frac{1}{x} + b.\frac{1}{y}\frac{dy}{dx} = (a+b)\frac{1}{x-y}(1 - \frac{dy}{dx})$

or, $(\frac{b}{y} + \frac{a+b}{x-y})\frac{dy}{dx} = \frac{a+b}{x-y} - \frac{a}{x}$

or, $\frac{bx - by + ay + by}{y(x-y)}.\frac{dy}{dx} = \frac{ax + bx - ax + ay}{x(x-y)}$

or, $\frac{bx + ay}{y(x-y)}.\frac{dy}{dx} = \frac{bx + ay}{x(x-y)}$

$\frac{dy}{dx} = \frac{y}{x}$

1(b) $y = \sin(x+y)^2$ **[রা.'০৪; কু.'০৭; য.'১১]**

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{dy}{dx} = \cos(x+y)^2 \frac{d}{dx}(x+y)^2$

$\Rightarrow \frac{dy}{dx} = \cos(x+y)^2.2(x+y)(1 + \frac{dy}{dx})$

$\Rightarrow \{1 - 2(x+y)\cos(x+y)^2\}\frac{dy}{dx}$

$= 2(x+y)\cos(x+y)^2$

$\frac{dy}{dx} = \frac{2(x+y)\cos(x+y)^2}{1 - 2(x+y)\cos(x+y)^2}$

1(c) $x + y = \sin^{-1}(y/x)$

$\Rightarrow \sin(x+y) = \frac{y}{x} \Rightarrow y = x\sin(x+y)$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{dy}{dx} = x\cos(x+y)(1 + \frac{dy}{dx}) + \sin(x+y)$

$\Rightarrow \{1 - x\cos(x+y)\}\frac{dy}{dx} = x\cos(x+y) +$

$\sin(x+y)$

$\frac{dy}{dx} = \frac{x\cos(x+y) + \sin(x+y)}{1 - x\cos(x+y)}$