2(a) $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$

$= \int \frac{2\cos^2 x - 1 - (2\cos^2\theta - 1)}{\cos x - \cos\theta} dx$

$= 2\int \frac{\cos^2 x - \cos^2\theta}{\cos x - \cos\theta} dx$

$= 2\int \frac{(\cos x + \cos\theta)(\cos x - \cos\theta)}{\cos x - \cos\theta} dx$

$= 2\int (\cos x + \cos\theta) dx$

$= 2(\int \cos x dx + \cos\theta \int dx)$

$= 2(\sin x + \cos\theta . x) + c$

$= 2(\sin x + x\cos\theta) + c$

2(b) $\int (\sec x + \tan x)^2 dx$

$= \int (\sec^2 x + \tan^2 x + 2\sec x \tan x) dx$

$= \int (\sec^2 x + \sec^2 x - 1 + 2\sec x \tan x) dx$

$= \int (2\sec^2 x - 1 + 2\sec x \tan x) dx$

$= 2\tan x - x + 2\sec x + c$

3(a) $\int \sqrt{1 \pm \sin x}\, dx$

$= \int \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} \pm 2\sin\frac{x}{2}\cos\frac{x}{2}}\, dx$

$= \int \sqrt{(\sin\frac{x}{2} \pm \cos\frac{x}{2})^2}\, dx$

$= \int (\sin\frac{x}{2} \pm \cos\frac{x}{2}) dx$ ev $\int (\cos\frac{x}{2} \pm \sin\frac{x}{2}) dx$

$= 2(-\cos\frac{x}{2} \pm \sin\frac{x}{2}) + c$

বা $2(\sin\frac{x}{2} \mp \cos\frac{x}{2}) + c$

3(b) $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$

$= \int \frac{\sin x + \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}} dx$

$= \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} dx$

$= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int dx = x + c$

3(c) $\int \frac{\cos x + \sin x}{\cos x - \sin x}(1 - \sin 2x)\, dx$

$= \int \frac{\cos x + \sin x}{\cos x - \sin x}(\cos x - \sin x)^2\, dx$

$= \int (\cos x + \sin x)(\cos x - \sin x)\, dx$

$= \int (\cos^2 x - \sin^2 x)\, dx = \int \cos 2x\, dx$

$= \frac{1}{2}\sin 2x + c$

3(d) $\int (\sin\frac{x}{2} + \cos\frac{x}{2})^2 dx$

$= \int (\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}) dx$

$= \int (1 + \sin x) dx = x - \cos x + c$

4 $\int \cos^3 x\, dx = \int \frac{1}{4}(3\cos x + \cos 3x)\, dx$

$= \frac{1}{4}(3\sin x + \frac{1}{3}\sin 3x) + c$

## প্রশ্নমালা X B

নিচের যোগজগুলি নির্ণয় কর ঃ

1.(a) $\int \frac{1}{\sqrt[3]{(1-4x)}} dx = \int \frac{1}{(1-4x)^{1/3}} dx$

$= \int (1-4x)^{-\frac{1}{3}} dx = \frac{(1-4x)^{-\frac{1}{3}+1}}{(-\frac{1}{3}+1)(-4)} + c$

$= \frac{(1-4x)^{\frac{2}{3}}}{\frac{2}{3}(-4)} + c = -\frac{3}{8}(1-4x)^{\frac{2}{3}} + c$

1(b) $\int \frac{e^{5x} + e^{3x}}{e^x + e^{-x}} dx$ [প্র.ভ.প. '১২]

$= \int \frac{e^{4x}(e^x + e^{-x})}{e^x + e^{-x}} dx = \int e^{4x} dx = \frac{e^{4x}}{4} + c$

1(c) ধরি , $I = \int \sin x^0 dx$ [চ.'০৪]

এবং $x^\circ = \frac{\pi x}{180} = z$

তাহলে $\frac{\pi}{180}dx = dz \Rightarrow dx = \frac{180}{\pi}dz$ এবং

$I = \frac{180}{\pi}\int \sin z\, dz = \frac{180}{\pi}(-\cos z) + c$

$\int \sin x^\circ dx = -\frac{180}{\pi}\cos x^\circ + c$

**2(a)** ধরি, $I = \int \sin 5x\, dx$ [সি.'০৫]

এবং $5x = z$ তাহলে $5dx = dz \Rightarrow dx = \frac{1}{5}dz$

এবং $I = \frac{1}{5}\int \sin z\, dz = -\frac{1}{5}\cos z + c$

$\therefore \int \sin 5x\, dx = -\frac{1}{5}\cos 5x + c$

**2(b)** ধরি, $I = \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$ [কু.'০০]

এবং $\sqrt{x} = z$. তাহলে $\frac{dx}{2\sqrt{x}} = dz \Rightarrow \frac{dx}{\sqrt{x}} = 2dz$

এবং $I = 2\int \cos z\, dz = 2\sin z + c$

$\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\sin\sqrt{x} + c$

**2(c)** $\int \frac{1}{x^2}\sin\frac{1}{x}dx$ [ঢা.'০৪; য.'০৭]

ধরি, $\frac{1}{x} = z$ $\quad -x^{-2}dx = dz \Rightarrow \frac{1}{x^2}dx = -dz$

$\therefore \int \frac{1}{x^2}\sin\frac{1}{x}dx = \int \frac{\sin(1/x)}{x^2}dx$

$= -\int \sin z\, dz = -(-\cos z) + c = \cos\frac{1}{x} + c$

**3. (a)** ধরি, $I = \int x e^{x^2}dx$ [ব.'০৩]

এবং $x^2 = z$. তাহলে, $2xdx = dz \Rightarrow xdx = \frac{dz}{2}$

এবং $I = \frac{1}{2}\int e^z dz = \frac{1}{2}e^z + c = e^{x^2} + c$

**3(b)** ধরি, $I = \int x^2 a^{x^3}dx$ [মা.'০৯]

এবং $x^3 = z$. তাহলে, $3xdx = dz \Rightarrow xdx = \frac{dz}{3}$

এবং $I = \frac{1}{3}\int a^z dz = \frac{a^z}{3\ln a} + c = \frac{a^{x^3}}{3\ln a} + c$

**3.(c)** $\int e^x \tan e^x \sec e^x dx$

$= \int \sec e^x \tan e^x d(e^x)$ $\quad [\quad d(e^x) = e^x dx\,]$

$= \sec e^x + c$

**3(d)** ধরি, $I = \int e^{2x}\tan e^{2x}\sec e^{2x}dx$ [চ.'০৭]

এবং $e^{2x} = z$. তাহলে, $2e^{2x}dx = dz$ এবং

$I = \frac{1}{2}\int \sec z \tan z\, dz = \frac{1}{2}\sec z + c$

$\therefore \int e^{2x}\tan e^{2x}\sec e^{2x}dx = \frac{1}{2}\sec e^{2x} + c$

**4. (a)** ধরি, $I = \int \sin^2 x\cos x\, dx$ [ঢা.'০২]

এবং $\sin x = z$. তাহলে, $\cos x\, dx = dz$ এবং

$I = \int z^2\, dz = \frac{1}{3}z^3 + c = \frac{1}{3}\sin^3 x + c$

**4(b)** ধরি, $I = \int (1+\cos x)^3 \sin x\, dx$ [কু.'০৩]

এবং $1 + \cos x = z$. তাহলে, $-\sin x\, dx = dz$ এবং

$I = -\int z^3 dz = -\frac{z^4}{4} + c = -\frac{(1+\cos x)^4}{4} + c$

**4(c)** ধরি, $I = \int \sin^2\frac{x}{2}\cos\frac{x}{2}dx$ [চ.'০৩]

এবং $\sin\frac{x}{2} = z$. তাহলে, $\frac{1}{2}\cos\frac{x}{2}dx = dz$ এবং

$I = 2\int z^2 dz = 2.\frac{1}{3}z^3 + c = \frac{2}{3}\sin^3\frac{x}{2} + c$

**4(d)** ধরি, $I = \int \sqrt{1-\sin x}\cos x\, dx$ [সি.'০১]

এবং $1 - \sin x = z$. তাহলে, $-\cos dx = dz$ এবং

$$I = -\int z^{\frac{1}{2}}\, dz = -\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = -\frac{2}{3}z^{\frac{3}{2}} + c$$

$$\therefore \int \sqrt{1-\sin x}\cos x\, dx = -\frac{2}{3}(1-\sin x)^{\frac{3}{2}} + c$$

**4(e)** $\int \frac{\cos x dx}{(1-\sin x)^2}$ [ রা.'০৪,কু.'০৬;ব.'১১ ]

ধরি, $1-\sin x = z$. তাহলে, $-\cos dx = dz$ এবং

$$\int \frac{\cos x dx}{(1-\sin x)^2} = -\int \frac{dz}{z^2} = -\int z^{-2} dz$$

$$= -\frac{z^{-2+1}}{-2+1} + c = z^{-1} + c = \frac{1}{1-\sin x} + c$$

**4(f)** $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx$ [ চ. '০৭; কু. '০৮; রা.'১১]

ধরি, $\tan^{-1} x^3 = z$

$$\frac{1}{1+(x^3)^2}.3x^2\, dx = dz$$

$$\Rightarrow \frac{x^2}{1+x^6} dx = \frac{1}{3} dz$$

$$\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx = \frac{1}{3}\int z dz$$

$$= \frac{1}{3}\frac{z^2}{2} + c = \frac{1}{6}(\tan^{-1} x^3)^2 + c \text{ (Ans.)}$$

**5(a)** ধরি, $I = \int \frac{1}{x(1+\ln x)^3} dx$ [ঢা.'০১]

এবং $1+\ln x = z$. তাহলে, $\frac{1}{x}dx = dz$ এবং

$$I = \int \frac{1}{z^3} dz = \int z^{-3} dz = \frac{z^{-3+1}}{-3+1} + c = -\frac{1}{2z^2} + c$$

$$\therefore \int \frac{1}{x(\ln x)^2} dx = -\frac{1}{2(1+\ln x)^2} + c$$

**5(b)** ধরি, $I = \int \frac{(\log_{10} x)^2}{x} dx$ [প্র.ভ.প.'৮৩]

এবং $\log_{10} x = z$. তাহলে, $\frac{1}{x\ln 10}dx = dz$ এবং

$$I = \ln 10 \int z^2 dz = \ln 10.\frac{1}{3}z^3 + c$$

$$\therefore \int \frac{(\log_{10} x)^2}{x} dx = \frac{\ln 10}{3}(\log_{10} x)^3$$

**6.(a)** ধরি, $I = \int e^{\tan^{-1} x}.\frac{1}{1+x^2} dx$

[ঢা.'০৯; মা.'১২,'১৪]

এবং $\tan^{-1} x = z$. তাহলে, $\frac{1}{1+x^2}dx = dz$ এবং

$$I = \int e^z\, dz = e^z + c = e^{\tan^{-1} x} + c$$

**6(b)** $\int e^{\sin^{-1} x}.\frac{dx}{\sqrt{1-x^2}}$ [চ.'০১; প্র.ভ.প.'০৬]

ধরি, $\sin^{-1} x = z$. তাহলে, $\frac{1}{1-x^2}dx = dz$ এবং

$$\int e^{\sin^{-1} x}.\frac{dx}{\sqrt{1-x^2}} = \int e^z dz = e^z + c$$

$$= e^{\sin^{-1} x} + c$$

**6(c)** ধরি, $I = \int \frac{x}{\sqrt{1-x^2}} dx$ [য.'০৬; দি.'১১; ঢা.'১৪]

এবং $1-x^2 = z$. তাহলে, $-2xdx = dz$ এবং

$$I = -\frac{1}{2}\int \frac{dz}{\sqrt{z}} = -\frac{1}{2}\cdot 2\sqrt{z} + c$$

$$\therefore \int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} + c$$

**6(d)** ধরি, $I = \int \frac{\tan(\sin^{-1} x)}{\sqrt{1-x^2}} dx$ এবং

$\sin^{-1} x = z$ [কু. '০৭; ব. '১১,'১৪; য.'০৯,'১৩; ঢা.'১৩]

তাহলে, $\frac{1}{\sqrt{1-x^2}} dx = dz$ এবং

$$\therefore\ I = \int \tan z\, dz = \ln|\sec z| + c$$

$$= \ln|\sec(\sin^{-1} x)| + c$$

7(a) ধরি, $I=\int\frac{\sin x}{3+4\cos x}dx$ [ ঢা.'০৭, ব.'১৩ ]

এবং $3+4\cos x=z$. তাহলে, $-4\sin x dx = dz$

এবং $I=-\frac{1}{4}\int\frac{dz}{z}=-\frac{1}{4}\ln|3+4\cos x|+c$

7(b) ধরি, $I=\int\frac{\sin x}{1+2\cos x}dx$ [ রা.'০৩ ]

এবং $1+2\cos x=z$. তাহলে, $-2\sin x dx = dz$

এবং $I=-\frac{1}{2}\int\frac{dz}{z}=-\frac{1}{2}\ln|1+2\cos x|+c$

7(c) $\int\frac{\sec^2 x}{3-4\tan x}dx=-\frac{1}{4}\int\frac{-4\sec^2 x dx}{3-4\tan x}$

$=-\frac{1}{4}\ln|3-4\tan x|+c$

7(d) ধরি, $I=\int\frac{dx}{(1+x^2)\tan^{-1}x}$

[ ব.'০৪; ঢা.'১০; সি.'১১; কু.'১৩ ]

এবং $\tan^{-1}x=z$. তাহলে, $\frac{1}{1+x^2}dx=dz$ এবং

$I=\int\frac{dz}{z}=\ln|z|+c=\ln|\tan^{-1}x|+c$

8 $\int\frac{1}{x(1+\ln x)}dx$ [ব.'০৯; কু.'১২]

ধরি, $1+\ln x=z$. তাহলে, $\frac{1}{x}dx=dz$ এবং

$\int\frac{1}{x(1+\ln x)}dx=\int\frac{dz}{z}=\ln|z|+c$

$=\ln(1+\ln x)+c$

9.(a) $\int\frac{e^{3x}}{e^{3x}-1}dx=\frac{1}{3}\int\frac{(e^{3x}-0)dx}{e^{3x}-1}$

$=\frac{1}{3}\ln|e^{3x}-1|+c$

9(b) $\int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx=\int\frac{d(e^x-e^{-x})}{e^x+e^{-x}}$ [দি.'১০]

$=\ln|e^x+e^{-x}|+c$

9(c) $\int\frac{1}{e^x+1}dx=\int\frac{e^{-x}}{e^{-x}(e^x+1)}dx$ [য.'১০]

$=\int\frac{e^{-x}}{1+e^{-x}}dx=-\int\frac{(0-e^{-x})dx}{1+e^{-x}}$

$=-\ln|1+e^{-x}|+c$

10. (a) ধরি, $I=\int\frac{1}{\sqrt[3]{1-6x}}dx$ [প্র.ভ.প.'০৫]

এবং $1-6x=z$. তাহলে, $-6dx=dz$

$I=-\frac{1}{6}\int\frac{1}{\sqrt[3]{z}}dz=-\frac{1}{6}\int\frac{dz}{z^{1/3}}=-\frac{1}{6}\int z^{-\frac{1}{3}}dz$

$=-\frac{1}{6}\frac{z^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}+c=-\frac{1}{6}\frac{z^{2/3}}{\frac{2}{3}}+c$

$=-\frac{1}{4}(1-6x)^{2/3}+c$

10(b) ধরি, $I=\int\frac{x^3dx}{\sqrt{(1-2x^4)}}$ [চ.'০১]

এবং $1-2x^4=z$. তাহলে, $-8x^3dx=dz$ এবং

$I=-\frac{1}{8}\int\frac{dz}{\sqrt{z}}=-\frac{1}{8}.2\sqrt{z}+c=-\frac{1}{4}\sqrt{z}+c$

$\therefore\int\frac{x^3dx}{\sqrt{(1-2x^4)}}=-\frac{1}{4}\sqrt{1-2x^4}+c$

10(c) $\int\frac{dx}{\cos^2 x\sqrt{\tan x-1}}$ [সি.'০২]

$=\int\frac{\sec^2 x\,dx}{\sqrt{\tan x-1}}=\int\frac{(\sec^2 x-0)dx}{\sqrt{\tan x-1}}$

$=2\sqrt{\tan x-1}+c$ $[\because\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}]$

10 (d) ধরি, $I=\int\frac{\cos x}{\sqrt{\sin x}}dx$ [কু.'০৫; রা.'১০]

এবং $\sin x=z$. তাহলে, $\cos x\,dx=dz$ এবং

$I=\int\frac{dz}{\sqrt{z}}=2\sqrt{z}+c=2\sqrt{\sin x}+c$

**10(e)** ধরি, $I=\int\frac{dx}{x\sqrt{1+\ln x}}$ [কু.'০৩]

এবং $1+\ln x = z$. তাহলে, $\frac{1}{x}dx$ এবং

$I=\int\frac{dz}{\sqrt{z}}= 2\sqrt{z}+c=2\sqrt{1+\ln x}+c$

**11(a)** $\int\frac{dx}{4x^2+9}=\frac{1}{2}\int\frac{2xdx}{(2x)^2+3^2}$

$=\frac{1}{2}\frac{1}{3}\tan^{-1}\frac{2x}{3}+c=\frac{1}{6}\tan^{-1}\frac{2x}{3}+c$

**11(b)** $\int\frac{xdx}{x^4+1}dx$ [রা.'০৮; ব.'১১]

$=\frac{1}{2}\int\frac{2xdx}{1+(x^2)^2}=\frac{1}{2}.\tan^{-1}(x^2)+c$

**11(c)** ধরি, $I=\int\frac{3x^2}{1+x^6}dx$ [রা.'০১, চ.'০৮]

এবং $x^3=z$. তাহলে, $3x^2dx= dz$ এবং

$I=\int\frac{dz}{1+z^2}= \tan^{-1}z+c$

$\int\frac{3x^2}{1+x^6}dx= \tan^{-1}(x^3)+c$

**11(d)** ধরি, $I=\int\frac{e^x}{1+e^{2x}}dx$ [সি.'০৪]

এবং $e^x=z$. তাহলে, $e^x dx = dz$ এবং

$I=\int\frac{dz}{1+z^2}= \tan^{-1}z+c=\tan^{-1}(e^x)+c.$

**11(e)** $\int\frac{5e^{2x}}{1+e^{4x}}dx=\frac{5}{2}\int\frac{2e^{2x}dx}{1+(e^{2x})^2}$ [চ.'০১,'১১]

$=\frac{5}{2}\tan^{-1}(e^{2x})+c$

**11(f)** $\int\frac{1}{e^x+e^{-x}}dx$ [ঢা. '০৬; য. '০৫,'১২; রা. '০৭, '১৪; ব. '০৫,'০৭,'০৯; চ. '০৮;কু.'১২,'১৪; দি.'১৩; মা.'১৪]

$=\int\frac{e^x}{e^x(e^x+e^{-x})}dx = \int\frac{e^x}{(e^x)^2+1}dx$

ধরি, $e^x=z$. তাহলে, $e^x dx = dz$ এবং

$\int\frac{1}{e^x+e^{-x}}dx=\int\frac{dz}{1+z^2}= \tan^{-1}z+c$

$= \tan^{-1}(e^x)+c$

**12. (a)** $\int\frac{dx}{x^2-x+1}$ [চ.'০৩]

$=\int\frac{dx}{(x-\frac{1}{2})^2+1-\frac{1}{4}}\int\frac{dx}{(x-\frac{1}{2})^2+\frac{3}{4}}$

$=\int\frac{d(x-\frac{1}{2})}{(\frac{\sqrt{3}}{2})^2+(x-\frac{1}{2})^2}$ $[\because d(x-\frac{1}{2})=dx]$

$=\frac{1}{\sqrt{3}/2}\tan^{-1}\frac{x-\frac{1}{2}}{\sqrt{3}/2}+c$

$=\frac{2}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}}+c$

**12(b)** $\int\frac{dx}{\sqrt{x^2+4x+13}}$ [রা.'০২]

$=\int\frac{dx}{\sqrt{(x+2)^2+13-4}}$

$=\int\frac{d(x+2)}{\sqrt{(x+2)^2+3^2}}$

$=\ln|\sqrt{(x+2)^2+3^2}+x+2|+c$

$=\ln|\sqrt{x^2+4x+13}+x+2|+c$

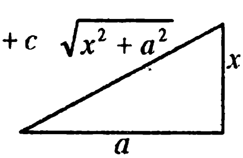
**12. (c)** $\int\frac{dx}{(a^2+x^2)^{3/2}}$ [য.'০২; প্র.ভ.প.'০৬]

ধরি, $x=a\tan\theta$. তাহলে $dx=a\sec^2\theta\, d\theta$

$\therefore\int\frac{dx}{(a^2+x^2)^{3/2}}=\int\frac{a\sec^2\theta\, d\theta}{(a^2+a^2\tan^2\theta)^{3/2}}$

$=\int\frac{a\sec^2\theta\, d\theta}{a^3(1+\tan^2\theta)^{3/2}}=\int\frac{\sec^2\theta\, d\theta}{a^2\sec^3\theta}$

$=\frac{1}{a^2}\int\cos\theta\, d\theta=\frac{1}{a^2}\sin\theta+c$



$= \dfrac{x}{a^2\sqrt{x^2+a^2}} + c$

[ চিত্র হতে $\tan\theta = \dfrac{x}{a}$ এবং $\sin\theta = \dfrac{x}{\sqrt{x^2+a^2}}$ ]

**12(d)** $\int x^2\sqrt{1-x^2}\,dx$

ধরি, $x = \sin\theta$. তাহলে $dx = \cos\theta\, d\theta$

$\int x^2\sqrt{1-x^2}\,dx$

$= \int \sin^2\theta\sqrt{1-\sin^2\theta}\cos\theta\, d\theta$

$= \int \sin^2\theta\cos^2\theta\, d\theta = \int \dfrac{1}{4}(2\sin\theta\cos\theta)^2\, d\theta$

$= \int \dfrac{1}{4}\sin^2 2\theta\, d\theta = \int \dfrac{1}{8}(1-\cos 4\theta)\, d\theta$

$= \dfrac{1}{8}(\theta - \dfrac{\sin 4\theta}{4}) + c = \dfrac{1}{8}(\theta - \dfrac{2\sin 2\theta\cos 2\theta}{4}) + c$

$= \dfrac{1}{8}(\theta - \dfrac{2\sin\theta\cos\theta\cos 2\theta}{2}) + c$

$= \dfrac{1}{8}\{\theta - \dfrac{2\sin\theta\sqrt{1-\sin^2\theta}(1-2\sin^2\theta)}{2}\} + c$

$= \dfrac{1}{8}\{\sin^{-1}x - x\sqrt{1-x^2}(1-2x^2)\} + c$

**13.(a)** $\int \dfrac{dx}{1-x^2} = \int \dfrac{dx}{1^2-x^2}$ [ব.'০৩]

$= \dfrac{1}{2.1}\ln|\dfrac{1+x}{1-x}| + c$ [সূত্র প্রয়োগ করে।]

$= \dfrac{1}{2}\ln|\dfrac{1+x}{1-x}| + c$

**13(b)** $\int \dfrac{dx}{9-4x^2} = \int \dfrac{dx}{3^2-(2x)^2}$ [সি.'১১]

$= \dfrac{1}{2}\int \dfrac{2dx}{3^2-(2x)^2} = \dfrac{1}{2}.\dfrac{1}{2.3}\ln|\dfrac{3+2x}{3-2x}| + c$

$= \dfrac{1}{12}\ln|\dfrac{3+2x}{3-2x}| + c$

**13(c)** ধরি, $I = \int \dfrac{dx}{9x^2-16}$ [ঢা.'০৩]

$= \int \dfrac{dx}{(3x)^2-4^2}$ এবং $3x = z$ .তাহলে, $3dx = dz$ এবং

$I = \dfrac{1}{3}\int \dfrac{dz}{z^2-4^2} = \dfrac{1}{3}.\dfrac{1}{2.4}\ln|\dfrac{z-4}{z+4}| + c$

$\therefore \int \dfrac{dx}{9x^2-16} = \dfrac{1}{24}\ln|\dfrac{3x-4}{3x+4}| + c$

**13(d)** $\int \dfrac{dx}{16-4x^2}$ [ক.'০০; সি.'০১]

$= \dfrac{1}{4}\int \dfrac{dx}{4-x^2} = \dfrac{1}{4}\int \dfrac{dx}{2^2-x^2}$

$= \dfrac{1}{4}.\dfrac{1}{2.2}\ln|\dfrac{2+x}{2-x}| + c = \dfrac{1}{16}\ln|\dfrac{2+x}{2-x}| + c$

**13(e)** $\int \dfrac{\cos x\, dx}{3+\cos^2 x}$ [প্র.ভ.প.'০৫]

$= \int \dfrac{\cos x\, dx}{3+1-\sin^2 x} = \int \dfrac{d(\sin x)}{2^2-(\sin x)^2}$

$= \dfrac{1}{2.2}\ln|\dfrac{2+\sin x}{2-\sin x}| + c = \dfrac{1}{4}\ln|\dfrac{2+\sin x}{2-\sin x}| + c$

**13.(f)** $\int \dfrac{1}{e^x - e^{-x}}dx$ [রা.'০১; য.'০২]

$= \int \dfrac{1}{e^x-e^{-x}}dx = \int \dfrac{e^x}{e^x(e^x-e^{-x})}dx$

$= \int \dfrac{e^x}{(e^x)^2-1}dx = \int \dfrac{d(e^x)}{(e^x)^2-1^2}$

$= \dfrac{1}{2.1}\, ln\left|\dfrac{e^x-1}{e^x+1}\right| + c = \dfrac{1}{2}\, ln\left|\dfrac{e^x-1}{e^x+1}\right| + c$

**14.(a)** $\int \dfrac{dx}{\sqrt{25-x^2}} = \int \dfrac{dx}{\sqrt{5^2-x^2}}$ [দি.'১০; চ.'১৩]

$= \sin^{-1}\dfrac{x}{5} + c$

**14(b)** $\int \dfrac{dx}{\sqrt{2-3x^2}}$

[ য.'০৫; কু.'০৭,'১০,'১৪; ঢা.,ব.'১২; সি.'১৩]

$= \dfrac{1}{\sqrt{3}}\int \dfrac{\sqrt{3}\, dx}{\sqrt{(\sqrt{2})^2-(\sqrt{3}x)^2}} = \dfrac{1}{\sqrt{3}}\sin^{-1}\dfrac{\sqrt{3}x}{\sqrt{2}} + c$

**14(c)** $\int\frac{dx}{\sqrt{5-4x^2}}$

[ব.'০৬,'০৯; রা.'০৮;ঢা.'০৯; চ.'য.'১১]

$$=\int\frac{dx}{\sqrt{(\sqrt{5})^2-(2x)^2}}$$

ধরি, $2x=z$. তাহলে $2dx=dz$

$$\int\frac{dx}{\sqrt{5-4x^2}}=\frac{1}{2}\int\frac{dz}{\sqrt{(\sqrt{5})^2-z^2}}$$

$$=\frac{1}{2}\sin^{-1}\frac{z}{\sqrt{5}}+c=\frac{1}{2}\sin^{-1}\frac{2x}{\sqrt{5}}+c$$

**14(d)** $\int\frac{dx}{\sqrt{25-16x^2}}$ [সি.'০৪]

$$=\frac{1}{4}\int\frac{d(4x)}{\sqrt{5^2-(4x)^2}}\qquad[\because d(4x)=4dx]$$

$$=\frac{1}{4}\sin^{-1}\frac{4x}{5}+c.$$

**14(e)** $\int\frac{\sin x}{\sqrt{5-\cos^2 x}}dx$ [কু.'০৪]

$$=-\int\frac{-\sin x dx}{\sqrt{(\sqrt{5})^2-(\cos x)^2}}=-\cos^{-1}(\frac{\cos x}{\sqrt{5}})+c$$

**14(f)** ধরি, $I=\int\frac{x^2}{\sqrt{1-x^6}}dx$

[ব.'০৮; য.'১১;দি.'১২]

এবং $x^3=z$. তাহলে, $3x^2dx=dz$

$$I=\int\frac{x^2dx}{\sqrt{1-(x^3)^2}}=\frac{1}{3}\int\frac{dz}{\sqrt{1-z^2}}=\frac{1}{3}\sin^{-1}z+c$$

$$=\frac{1}{3}\sin^{-1}x^3+c$$

**14.(g)** $\int\frac{dx}{\sqrt{2ax-x^2}}$ [য.'০৯]

$$=\int\frac{dx}{\sqrt{a^2-(x^2-2ax+a^2)}}$$

$$=\int\frac{(1-0)dx}{\sqrt{a^2-(x-a)^2}}=\sin^{-1}(\frac{x-a}{a})+c$$

**14(h)** ধরি, $I=\int\sqrt{1-9x^2}\,dx$ [ব.'০১]

এবং $3x=z$ তাহলে, $3dx=dz$ এবং

$$I=\int\sqrt{1-(3x)^2}\,dx=\frac{1}{3}\int\sqrt{1-z^2}\,dz$$

$$=\frac{1}{3}[\frac{z\sqrt{1-z^2}}{2}+\frac{1}{2}\sin^{-1}z]+c$$

$$=\frac{1}{3}[\frac{3x\sqrt{1-(3x)^2}}{2}+\frac{1}{2}\sin^{-1}(3x)]+c$$

$$=\frac{1}{6}[3x\sqrt{1-9x^2}+\sin^{-1}(3x)]+c$$

**15.** $\int\frac{3x-2}{\sqrt{3+2x-4x^2}}dx$

$$=\int\frac{-\frac{3}{8}(-8x+2)+\frac{3}{4}-2}{\sqrt{3+2x-4x^2}}dx$$

$$=-\frac{3}{8}\int\frac{(-8x+2)dx}{\sqrt{3+2x-4x^2}}$$

$$-\frac{5}{4}\int\frac{dx}{\sqrt{-\{(2x)^2-2.2x.\frac{1}{2}+(\frac{1}{2})^2\}+3+\frac{1}{4}}}$$

$$=-\frac{3}{8}\int\frac{d(3+2x-4x^2)}{\sqrt{3+2x-4x^2}}$$

$$-\frac{5}{4}\int\frac{dx}{\sqrt{(\frac{\sqrt{13}}{2})^2-(2x-\frac{1}{2})^2}}$$

$$=-\frac{3}{8}\cdot 2\sqrt{3+2x-4x^2}$$

$$-\frac{5}{4}\int\frac{\frac{1}{2}d(2x-\frac{1}{2})}{\sqrt{(\frac{\sqrt{13}}{2})^2+(2x-\frac{1}{2})^2}}$$

$$=-\frac{3}{4}\sqrt{3+2x-4x^2}-\frac{5}{8}\sin^{-1}\frac{2x-\frac{1}{2}}{\frac{\sqrt{13}}{2}}+c$$

$= -\frac{3}{4}\sqrt{3+2x-4x^2} - \frac{5}{8}\sin^{-1}\frac{4x-1}{\sqrt{13}} + c$

**16.(a)** $\int \frac{x+25}{x-25}dx$ [সি.'০৭]

$= \int \frac{x-25+50}{x-25}dx = \int (\frac{x-25}{x-25} + \frac{50}{x-25})dx$

$= \int (1+\frac{50}{x-25})dx = \int dx + 50\int \frac{1}{x-25}dx$

$= x + 50\ln|x-25| + c$

**16(b)** $\int \frac{x^2 dx}{x^2-4}$ [সি.'০৮; ব.'০৪; রা.'০৪,'১১]

$= \int \frac{x^2-4+4}{x^2-4}dx = \int (\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4})dx$

$= \int (1+\frac{4}{x^2-2^2})dx$

$= x + \frac{4}{2.2}\ln|\frac{x-2}{x+2}| + c = x + \ln|\frac{x-2}{x+2}| + c$

**16(c)** $\int \frac{x^2-1}{x^2-4}dx$

[কু. '০৯; সি.'০৫,'১২; য.'০৯; ঢা.'১১; ব.'১৩]

$= \int \frac{x^2-4+3}{x^2-4}dx = \int (\frac{x^2-4}{x^2-4} + \frac{3}{x^2-4})dx$

$= \int (1+\frac{3}{x^2-2^2})dx = x + \frac{3}{2.2}\ln|\frac{x-2}{x+2}| + c$

$= x + \frac{3}{4}\ln|\frac{x-2}{x+2}| + c$

**16(d)** $\int \frac{xdx}{(1-x)^2} = -\int \frac{1-x-1}{(1-x)^2}dx$

$= -\int \{\frac{1-x}{(1-x)^2} - \frac{1}{(1-x)^2}\}dx$

$= -\int \frac{1}{1-x}dx + \int \frac{1}{(1-x)^2}dx$

$= -\int \frac{-d(1-x)}{1-x} - \int \frac{d(1-x)}{(1-x)^2}$

$= \ln|1-x| - (-\frac{1}{1-x}) + c$

$= \ln|1-x| + \frac{1}{1-x} + c$

**17(a)** $\int \sqrt{\frac{5-x}{5+x}}dx = \int \frac{5-x}{\sqrt{5^2-x^2}}dx$

$= \int \frac{5}{\sqrt{5^2-x^2}}dx - \int \frac{x}{\sqrt{25-x^2}}dx$

$= \int \frac{5}{\sqrt{5^2-x^2}}dx + \frac{1}{2}\int \frac{d(25-x^2)}{\sqrt{25-x^2}}$

$= 5\sin^{-1}\frac{x}{5} + \frac{1}{2}.2\sqrt{25-x^2} + c$

$= 5\sin^{-1}\frac{x}{5} + \sqrt{25-x^2} + c$

**17(b)** $\int x\sqrt{\frac{1-x}{1+x}}dx = \int x\frac{\sqrt{1-x}\times\sqrt{1-x}}{\sqrt{1+x}\times\sqrt{1-x}}dx$

$= \int x\frac{1-x}{\sqrt{1-x^2}}dx = \int \frac{x-x^2}{\sqrt{1-x^2}}dx$

$= \int \frac{(1-x^2) - \frac{1}{2}(-2x) - 1}{\sqrt{1-x^2}}dx$

$= \int \frac{1-x^2}{\sqrt{1-x^2}}dx - \frac{1}{2}\int \frac{(-2x)}{\sqrt{1-x^2}}dx - \int \frac{1}{\sqrt{1-x^2}}dx$

$= \int \sqrt{1-x^2}\,dx - \frac{1}{2}\cdot 2\sqrt{1-x^2} - \sin^{-1}x$

$= \frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x - \sqrt{1-x^2} - \sin^{-1}x + c$

$= \frac{x\sqrt{1-x^2}}{2} - \frac{1}{2}\sin^{-1}x - \sqrt{1-x^2} + c$ (Ans.)

**নিয়ম ঃ** $\int \frac{1}{g(x)\sqrt{\varphi(x)}}dx$ **আকারের জন্য,**

**(a)** $g(x)$ ও $\varphi(x)$ উভয়ে একঘাত হলে, $\varphi(x) = z^2$ ধরতে হয়।

**(b)** $g(x)$ একঘাত ও $\varphi(x)$ দ্বিঘাত হলে, $g(x) = \frac{1}{z}$ ধরতে হয়।

**(c)** $g(x)$ দ্বিঘাত ও $\varphi(x)$ একঘাত হলে, $\varphi(x) = z^2$ ধরতে হয়।

(d) $g(x)$ ও $\varphi(x)$ উভয়ে দ্বিঘাত হলে, $x = \dfrac{1}{z}$ ধরতে হয়।

(e) $\int \dfrac{x}{g(x)\sqrt{\varphi(x)}}dx$ এবং $g(x)$ ও $\varphi(x)$ উভয়ে দ্বিঘাত হলে, $\varphi(x) = z^2$ ধরতে হয়।

18.(a) ধরি, $I=\int \dfrac{dx}{(x-3)\sqrt{x+1}}$ এবং [ঢা.'১০; ব.'১৩]

$x+1=z^2$. তাহলে $dx = 2zdz$ এবং

$$I=\int \frac{2zdz}{(z^2-1-3)\sqrt{z^2}}$$

$$\Rightarrow I=\int \frac{2zdz}{(z^2-4)z} = 2\int \frac{dz}{z^2-2^2}$$

$$=2.\frac{1}{2}\ln\left|\frac{z-2}{z+2}\right|+c = \ln\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right|+c$$

18(b) $\int \dfrac{dx}{(x-1)\sqrt{x^2-2x}} = \int \dfrac{d(x-1)}{(x-1)\sqrt{(x-1)^2-1}}$

$= \sec^{-1}(x-1)+c$

নিয়ম ঃ (a) যদি কোন যোগজ $\int \dfrac{a+bx^l}{p+qx^m}dx$ আকারে থাকে, যেখানে $l$ ও $m$ উভয়ে ভগ্নাংশ এবং তাদের হরের ল.সা.গু $n$ হয়, তবে $x = z^n$ ধরতে হয়।

(b) $\int \dfrac{dx}{x(a+bx^n)}$ আকারের যোগজের জন্য, $x^n = \dfrac{1}{z}$ ধরতে হয়।

(c) $\int \dfrac{dx}{x\sqrt{a+bx^n}}$ আকারের যোগজের জন্য, $x^n = \dfrac{1}{z^2}$ ধরতে হয়।

(d) $\int \dfrac{dx}{x^m(a+bx)^n}$ আকারের যোগজের জন্য, $a+bx = zx$ ধরতে হয়।

(e) $\int \dfrac{dx}{(x-a)^m(x-b)^n}$ আকারের যোগজের জন্য, $z = \dfrac{x-b}{x-a}$ ধরতে হয়।

19.(a) $\int \dfrac{\sqrt{x}}{1+\sqrt[3]{x}}dx = \int \dfrac{x^{1/2}}{1+x^{1/3}}dx$ [চ.'০০]

ধরি, $x = z^6$. তাহলে, $dx = 6z^5dz$

$$\therefore \int \frac{\sqrt{x}}{1+\sqrt[3]{x}}dx = \int \frac{\sqrt{z^6}\,6z^5dz}{1+\sqrt[3]{z^6}}$$

$$=\int \frac{z^3.6z^5dz}{1+z^2} = 6\int \frac{z^8\,dz}{1+z^2}$$

$$=6\int \frac{1}{z^2+1}\{z^6(z^2+1)-z^4(z^2+1)+ z^2(z^2+1)-(z^2+1)+1\}dz$$

$$=6\int (z^6-z^4+z^2-1+\frac{1}{1+z^2})dz$$

$$=6(\frac{z^7}{7}-\frac{z^5}{5}+\frac{z^3}{3}-z+\tan^{-1}z)+c$$

$$=\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{6}{3}x^{\frac{3}{6}}-6x^{\frac{1}{6}}+\tan^{-1}x^{\frac{1}{6}}+c$$

19(b) ধরি, $I = \int \dfrac{dx}{x(4+5x^{20})}$ এবং $x^{20} = \dfrac{1}{z}$

তাহলে, $20x^{19}dx = -\dfrac{dz}{z^2} \Rightarrow x^{19}dx = -\dfrac{dz}{20z^2}$



এবং $I = \int \dfrac{x^{19}dx}{x^{20}(4+5x^{20})} = \int \dfrac{-\dfrac{dz}{20z^2}}{\dfrac{1}{z}(4+5\dfrac{1}{z})}$

$$=-\frac{1}{20}\int \frac{dz}{4z+5} = -\frac{1}{20}.\frac{1}{4}\int \frac{d(4z+5)}{4z+5}$$

$$=-\frac{1}{80}\ln|4z+5|+c = -\frac{1}{80}\ln\left|\frac{4}{x^{20}}+5\right|+c$$

19. (c) ধরি, $I=\int \dfrac{dx}{x\sqrt{x^4-1}}$ [ক.'০১; রা.'১১]

এবং $x^4 = \dfrac{1}{z^2}$. তাহলে, $4x^3dx = -\dfrac{2dz}{z^3}$ এবং

$$I=\int \frac{x^3dx}{x^4\sqrt{x^4-1}} = \int \frac{-\dfrac{dz}{2z^3}}{\dfrac{1}{z^2}\sqrt{\dfrac{1}{z^2}-1}}$$

$$= -\frac{1}{2}\int \frac{dz}{\sqrt{1-z^2}} = \frac{1}{2}\cos^{-1} z + c$$

$$= \frac{1}{2}\cos^{-1}(\frac{1}{x^2}) + c = \frac{1}{2}\sec^{-1}(x^2) + c$$

**(d)** ধরি, $I = \int \frac{dx}{(x-1)^2(x-2)^3}$ এবং $z = \frac{x-1}{x-2}$

$$\Rightarrow zx - 2z = x - 1 \Rightarrow x(1-z) = 1-2z$$

$$\Rightarrow x = \frac{1-2z}{1-z} \Rightarrow x - 2 = \frac{1-2z}{1-z} - 2$$

$$\Rightarrow x - 2 = \frac{1-2z-2+2z}{1-z} = -\frac{1}{1-z}$$

$$\Rightarrow dx = -\frac{dz}{(1-z)^2}$$

$$\therefore I = \int \frac{dx}{(\frac{x-1}{x-2})^2(x-2)^5} = \int \frac{-\frac{dz}{(1-z)^2}}{z^2 \cdot \frac{-1}{(1-z)^5}}$$

$$= \int \frac{(1-z)^3 dz}{z^2} = \int \frac{(1-3z+3z^2-z^3)dz}{z^2}$$

$$= \int (\frac{1}{z^2} - 3\frac{1}{z} + 3 - z)dz$$

$$= -\frac{1}{z} - 3\ln|z| + 3z - \frac{z^2}{2} + c$$

$$= -\frac{x-2}{x-1} - 3\ln|\frac{x-1}{x-2}| + 3(\frac{x-1}{x-2}) - \frac{1}{2}(\frac{x-1}{x-2})^2$$

**20. (a)** $\int \frac{x^2+1}{x^4+1}dx = \int \frac{x^2(1+\frac{1}{x^2})}{x^2(x^2+\frac{1}{x^2})}dx$

$$= \int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2}dx = \int \frac{d(x-\frac{1}{x})}{(x-\frac{1}{x})^2+(\sqrt{2})^2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x-\frac{1}{x}}{\sqrt{2}} + c = \frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x} + c$$

**20(b)** $\int \frac{x^2-1}{x^4+1}dx = \int \frac{x^2(1-\frac{1}{x^2})}{x^2(x^2+\frac{1}{x^2})}dx$

$$= \int \frac{1-\frac{1}{x^2}}{(x+\frac{1}{x})^2-2}dx = \int \frac{d(x+\frac{1}{x})}{(x+\frac{1}{x})^2-(\sqrt{2})^2}$$

$$= \frac{1}{2\sqrt{2}}\ln|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}| + c$$

$$= \frac{1}{2\sqrt{2}}\ln|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}| + c$$

**(c)** $\int \frac{x^2dx}{x^4+a^4} = \frac{1}{2}\int \frac{(x^2+a^2)+(x^2-a^2)}{x^4+a^4}dx$

$$= \frac{1}{2}[\int \frac{x^2+a^2}{x^4+a^4}dx + \int \frac{x^2-a^2}{x^4+a^4}dx]$$

$$= \frac{1}{2}[\int \frac{x^2(1+\frac{a^2}{x^2})}{x^2(x^2+\frac{a^4}{x^2})}dx + \int \frac{x^2(1-\frac{a^2}{x^2})}{x^2(x^2+\frac{a^4}{x^2})}dx]$$

$$= \frac{1}{2}[\int \frac{d(x-\frac{a^2}{x})}{(x-\frac{a^2}{x})^2+(\sqrt{2}.a)^2} + \int \frac{d(x+\frac{a^2}{x})}{(x+\frac{a^2}{x})^2-(\sqrt{2}.a)^2}]$$

$$= \frac{1}{2}[\frac{1}{\sqrt{2}\,a}\tan^{-1}\frac{x-\frac{a^2}{x}}{\sqrt{2}\,a} + \frac{1}{2.\sqrt{2}\,a}\ln|\frac{x+\frac{a^2}{x}-\sqrt{2}\,a}{x+\frac{a^2}{x}+\sqrt{2}\,a}| + c$$

$$= \frac{1}{2\sqrt{2}\,a}[\tan^{-1}\frac{x^2-a^2}{\sqrt{2}\,ax} +$$

$$\frac{1}{2}\ln\left|\frac{x^2+a^2-\sqrt{2}\,ax}{x^2+a^2+\sqrt{2}\,ax}\right|+c$$

**21(a)** $\int \sin^2 x\cos^2 x dx$ [ য. '০৮; রা.,ঢা.'১৩]

$= \int \frac{1}{4}(2\sin x\cos x)dx = \frac{1}{4}\int \sin^2 2x dx$

$= \frac{1}{8}\int (1-\cos 4x)dx = \frac{1}{8}(x-\frac{1}{4}\sin 4x)+c$

**21(b)** ধরি, $I = \int \sin^3 x\cos^3 x\, dx$ [য.'০৬]

$= \int \sin^3 x(1-\sin^2 x)\cos x\, dx$ এবং sinx = z .

তাহলে, cosx dx = dz এবং

$I = \int z^3(1-z^2)\, dz = \int (z^3 - z^5)\, dz$

$= \frac{1}{4}z^4 - \frac{1}{6}z^6 + c = \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c$

**21(c)** ধরি, $I = \int \sin^3 x\cos^4 x\, dx$ [রা.'০১]

$= \int (1-\cos^2 x)\cos^4 x\sin x\, dx$ এবং cosx = z

তাহলে, – sinx dx = dz এবং

$I = -\int (1-z^2)z^4\, dz = \int (z^6 - z^4)\, dz$

$= \frac{1}{7}z^7 - \frac{1}{5}z^5 + c = \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + c$

**21(d)** ধরি, $I = \int \sin^4 x\cos^4 x\, dx$

$\sin^4 x\cos^4 x = \frac{1}{16}(2\sin x\cos x)^4$

$= \frac{1}{16}\sin^4 2x = \frac{1}{16}.\{\frac{1}{2}(1-\cos 4x)\}^2$

$= \frac{1}{64}(1-2\cos 4x+\cos^2 4x)$

$= \frac{1}{64}\{1-2\cos 4x+\frac{1}{2}(1+\cos 8x)\}$

$= \frac{1}{128}(3-4\cos 4x+\cos 8x)$

$\therefore I = \int \frac{1}{128}(3-4\cos 4x+\cos 8x)dx$

$= \frac{1}{128}(3x-4.\frac{1}{4}\sin 4x+\frac{1}{8}\sin 8x)+c$

$= \frac{1}{128}(3x-\sin 4x+\frac{1}{8}\sin 8x)+c$

**21(e)** $\int \sin^2 x\cos 2x\, dx$

[চ.'০৯; য.'০৫; কু.'০৭; সি.'১১]

$= \int \frac{1}{2}(1-\cos 2x)\cos 2x\, dx$

$= \frac{1}{2}\int (\cos 2x - \cos^2 2x)\, dx$

$= \frac{1}{2}\int \{\cos 2x - \frac{1}{2}(1+\cos 4x)\}\, dx$

$= \frac{1}{2}\{\frac{1}{2}\sin 2x - \frac{1}{2}(x+\frac{1}{4}\sin 4x)\}+c$

$= \frac{1}{4}(\sin 2x - x - \frac{1}{4}\sin 4x)\} + c$

**21(f)** $\int \sin^2 x\cos 2x dx$ [চ.'০২; য.'০৫; কু.'১১]

$= \int \frac{1}{2}(1-\cos 2x)\cos 2x dx$

$= \frac{1}{2}\int (\cos 2x - \cos^2 2x)dx$

$= \frac{1}{2}\int \{\cos 2x - \frac{1}{2}(1+\cos 4x)\}dx$

$= \frac{1}{2}\{\frac{1}{2}\sin 2x - \frac{1}{2}(x+\frac{1}{4}\sin 4x)\}+c$

$= \frac{1}{4}(\sin 2x - x - \frac{1}{4}\sin 4x)+c$

**22. (a)** $\int \tan^2 x dx$ [ ঢা.'০৫,'০৭ ]

$= \int (\sec^2 x - 1)dx = \tan x - x + c$

**22(b)** ধরি, $I = \int \frac{\tan^2(\ln x)}{x}dx$ [ ব.'০২ ]

এবং lnx = z তাহলে, $\frac{1}{x}dx = dz$ এবং

$I = \int \tan^2 z dz = \int (\sec^2 z - 1)dz$

$= \tan z - z + c = \tan(\ln x) - \ln x + c$

**22(c)** $\int \frac{dx}{\sin x\cos^3 x} = \int \frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}dx$

$= \int (\tan x\sec^2 x + \frac{2}{2\sin x\cos x})dx$

$= \int \tan x\sec^2 x dx + 2\int \frac{dx}{\sin 2x}$

$= \int \tan x\, d(\tan x) + \int \sec 2x\, d(2x)$

$= \frac{1}{2}\tan^2 x + \ln|\tan\frac{2x}{2}| + c$

$= \frac{1}{2}\tan^2 x + \ln|\tan x| + c$

**23.** $\int \frac{1-\tan x}{1+\tan x}dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}dx$

$= \int \frac{d(\sin x + \cos x)}{\sin x + \cos x} = \ln|\sin x + \cos x| + c$

**24. (a)** ধরি, $I = \int \frac{\sin 4x}{\sin^4 x + \cos^4 x}dx$ এবং

$z = \sin^4 x + \cos^4 x$ তাহলে,

$dz = (4\sin^3 x\cos x - 4\cos^3 x\sin x)dx$

$= 4\sin x\cos x(\sin^2 - \cos^2 x)\,dx$

$= -2\sin 2x\cos 2x\,dx = -\sin 4x\,dx$ এবং

$I = \int \frac{-dz}{z} = -\ln|z| + c$

$= -\ln|\sin^4 x + \cos^4 x| + c$

**24(b)** ধরি, $I = \int \frac{dx}{1+\cos^2 x}$ [রা.'০৬]

$= \int \frac{\sec^2 x dx}{\sec^2 x(1+\cos^2 x)} = \int \frac{\sec^2 x dx}{\sec^2 x + 1}$

$= \int \frac{\sec^2 x dx}{1+\tan^2 x + 1}$ এবং $z = \tan x \Rightarrow dz = \sec^2 x dx$

$\therefore I = \int \frac{dz}{(\sqrt{2})^2 + z^2} = \frac{1}{\sqrt{2}}\tan^{-1}(\frac{z}{\sqrt{2}}) + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}(\frac{\tan x}{\sqrt{2}}) + c$

**24(c)** $\int \frac{1-\cos 2x}{1+\cos 2x}dx$ [য.'০৩]

$= \int \frac{2\sin^2 x}{2\cos^2 x}dx = \int \tan^2 x dx$

$= \int (\sec^2 x - 1)dx = \tan x - x + c$

**24(d)** $\int \frac{1-\cos 5x}{1+\cos 5x}$ [য.'০১; সি.'০২]

$= \int \frac{2\sin^2 \frac{5x}{2}}{2\cos^2 \frac{5x}{2}}dx = \int \tan^2 \frac{5x}{2}dx$

$= \int (\sec^2 \frac{5x}{2} - 1)dx = \frac{2}{5}\tan\frac{5x}{2} - x + c$

**25(a)** ধরি, $I = \int \frac{dx}{(e^x - 1)^2} = \int \frac{dx}{\{e^x(1-e^{-x})\}^2}$

$= \int \frac{dx}{e^{2x}(1-e^{-x})^2} = \int \frac{e^{-x}.e^{-x}dx}{(1-e^{-x})^2}$ এবং

$e^{-x} = z$. তাহলে $-e^{-x}dx = dz$ এবং

$I = -\int \frac{zdz}{(1-z)^2} = \int \frac{(1-z)-1}{(1-z)^2}dz$

$= \int \{\frac{1}{1-z} - \frac{1}{(1-z)^2}\}dz$

$= -\int \{\frac{1}{1-z} - \frac{1}{(1-z)^2}\}d(1-z)$

$= -\{\ln|1-z| + \frac{1}{1-z}\} + c$

$= -\ln|1-e^{-x}| - \frac{1}{1-e^{-x}} + c$

**25(b)** $\int \frac{\sin x\,dx}{\sin(x+a)} = \int \frac{\sin x\,dx}{\sin x\cos a + \cos x\sin a}$

ধরি, $\sin x = l(\sin x\cos a + \cos x\sin a) + m(\cos x\cos a - \sin x\sin a) + n$

$\Rightarrow \sin x = (l\cos a - m\sin a)\sin x + (l\sin a\ m\cos a)\cos x + n$

উভয়পক্ষে sinx , cosx ও ধ্রুবপদ সমীকৃত করে পাই,

$n = 0$, $l\sin a + m\cos a = 0 \Rightarrow m = -\frac{l\sin a}{\cos a}$

এবং $l\cos a - m\sin a = 1$

$\Rightarrow l\cos a + \frac{l\sin a}{\cos a}\sin a = 1$

$\Rightarrow l(\sin^2 a + \cos^2 a) = \cos a \Rightarrow l = \cos a$

$$m = -\frac{\cos a \sin a}{\cos a} = -\sin a$$

$$\int \frac{\sin x\, dx}{\sin(x+a)} = \int \frac{\cos a \sin(x+a)\, dx}{\sin(x+a)} -$$

$$\int \frac{\sin a\,(\cos x \cos a - \sin x \sin a)}{\sin x \cos a + \sin a \cos x} dx$$

$$= \cos a \int dx - \sin a \ln|\sin(x+a)|$$

$$= x\cos a - \sin a \ln|\sin(x+a)| + c$$

**25(c)** $\int (\sqrt{\tan x} + \sqrt{\cot x})dx$

$$= \int \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$

$$= \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx = \sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}} dx$$

$$= \sqrt{2}\int \frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}} dx$$

$$= \sqrt{2}\int \frac{d(\sin x - \cos x)}{\sqrt{1-(\sin x - \cos x)^2}}$$

$$= \sqrt{2}\sin^{-1}(\sin x - \cos x) + c$$

## অতিরিক্ত প্রশ্ন (সমাধানসহ)

**নিচের যোগজগুলি নির্ণয় কর:**

**1(a)** $\int (e^{\frac{x}{2}} + e^{-\frac{x}{2}})dx = \frac{e^{\frac{x}{2}}}{\frac{1}{2}} + \frac{e^{-\frac{x}{2}}}{-\frac{1}{2}} + c$

$$= 2(e^{\frac{x}{2}} - e^{-\frac{x}{2}}) + c$$

**1(b)** $\int a^{4x}dx = \frac{a^{4x}}{\ln a}\frac{1}{4} + c = \frac{a^{4x}}{4\ln a} + c$

**2.(a)** ধরি, $I = \int (2x+3)\sqrt{x^2+3x}\, dx$ এবং

$x^2 + 3x = z$. তাহলে $2x+3)dx = dz$

$$\therefore I = \int z^{\frac{1}{2}}\, dz = \frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{2}{3}z^{3/2} + c$$

$$= \frac{2}{3}(x^2+3x)^{3/2} + c$$

**2(b)** $\int x^2 \cos x^3 dx = \frac{1}{3}\int \cos(x^3)(3x^2 dx)$

$$= \frac{1}{3}\sin x^3 + c$$

**2(c)** $\int \frac{(1+\tan\frac{3x}{2})^2 dx}{1+\sin 3x}$ [প্র.ভ.প. ৮৬]

$$= \int \frac{(1+\tan\frac{3x}{2})^2 dx}{1+\frac{2\tan(3x/2)}{1+\tan^2(3x\backslash 2)}}$$

$$= \int \frac{\{1+\tan(3x/2)\}^2\{1+\tan^2(3x\backslash 2)\}dx}{1+\tan^2(3x\backslash 2)+2\tan(3x/2)}$$

$$= \int \frac{\{1+\tan(3x/2)\}^2\{1+\tan^2(3x\backslash 2)\}dx}{\{1+\tan(3x/2)\}^2}$$

$$= \int \{1+\tan^2(3x\backslash 2)\}dx = \int \sec^2(3x\backslash 2)dx$$

$$= \frac{2}{3}\tan\frac{3x}{2} + c$$

**3.** $\int \frac{2x\sin^{-1}x^2}{\sqrt{1-x^4}}dx$

ধরি, $\sin^{-1}x^2 = z$

$$\frac{1}{\sqrt{1-(x^2)^2}}.2x\, dx = dz$$

$$\Rightarrow \frac{2x\, dx}{\sqrt{1-x^4}} = dz$$

$$\int \frac{2x\sin^{-1}x^2}{\sqrt{1-x^4}}dx = \int z\, dz$$

$$= \frac{z^2}{2} + c = \frac{1}{2}(\sin^{-1}x^2)^2 + c \text{ (Ans.)}$$

**4.** $\int \frac{1}{x(\ln x)^2}dx = \int (\ln x)^{-2} d(\ln x)$

$$= \frac{(\ln x)^{-2+1}}{-2+1} + c = -\frac{1}{\ln x} + c$$

5(a) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int \sin^{-1} x \, d(\sin^{-1} x)$

$= \frac{(\sin^{-1} x)^2}{2} + c$

5(b) $\int \frac{1+\tan^2 x}{(1+\tan x)^2} dx$ [প্র.ভ.প.'৯৩]

$= \int \frac{\sec^2 x}{(1+\tan x)^2} dx$

$= \int (1+\tan x)^{-2} \, d(1+\tan x)$

$= \frac{(1+\tan x)^{-2+1}}{-2+1} + c = -\frac{1}{1+\tan x} + c$

5.(c) ধরি, $I = \int \frac{\cos 2x}{(\sqrt{\sin 2x+3})^3} dx$ [প্র.ভ.প.'৯৫]

এবং $\sin 2x + 3 = z$. তাহলে, $2\cos 2x \, dx = dz$ এবং

$I = \frac{1}{2}\int \frac{dz}{z^{3/2}} = \frac{1}{2}\int z^{-\frac{3}{2}} dz$

$= \frac{1}{2} \frac{z^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c = \frac{1}{2} \frac{z^{-\frac{1}{2}}}{-\frac{1}{2}}. + c = -\frac{1}{\sqrt{z}} + c$

$= -\frac{1}{\sqrt{\sin 2x+3}} + c$

6. (a) $\int \cos ec \frac{x}{2} dx = \frac{1}{1/2} \ln|\tan(\frac{x/2}{2})| + c$

$= 2\ln|\tan\frac{x}{4}| + c$

6(b) $\int \sec\sqrt{x} \frac{dx}{\sqrt{x}} = 2\int \sec(\sqrt{x})(\frac{1}{2\sqrt{x}} dx)$

$= 2\ln|\sec\sqrt{x} + \tan\sqrt{x}| + c$

6(c) $\int (\frac{3}{x-1} - \frac{4}{x-2}) dx$

$= 3 \, ln|x-1| - 4 \, ln|x-2| + c$

6(d) $\int \frac{\sin x}{1+\cos x} dx = -\int \frac{(-\sin x dx)}{1+\cos x}$

$= -ln|1+\cos x| + c$

7. $\int \frac{1}{x\ln x} dx$

$= \int \frac{d(\ln x)}{\ln x}$ [ $d(\ln x) = \frac{1}{x} dx$ ]

$= \ln(\ln x) + c$

8. (a) $\int \frac{dx}{16+x^2} = \int \frac{dx}{4^2+x^2} = \frac{1}{4}\tan^{-1}\frac{x}{4} + c$

8(b) $\int \frac{4}{16a^2+x^2} dx = 4\int \frac{dx}{(4a)^2+x^2}$

$= 4.\frac{1}{4a}\tan^{-1}\frac{x}{4a} + c = \frac{1}{a}\tan^{-1}\frac{x}{4a} + c$

8 (c) $\int \frac{x^2 \, dx}{e^{x^3} + e^{-x^3}}$ [প্র.ভ.প. '৮৯,'০১]

$= \int \frac{x^2 e^{x^3} dx}{e^{x^3}(e^{x^3} + e^{-x^3})} = \int \frac{x^2 e^{x^3} dx}{(e^{x^3})^2 + 1}$

$= \int \frac{d(e^{x^3})}{1+(e^{x^3})^2} . \frac{1}{3}$ [$\because d(e^{x^3}) = e^{x^3} 3x^2 dx$]

$= \frac{1}{3}\tan^{-1}(e^{x^3}) + c$

9(a) $\int \frac{dx}{x^2+6x+25} = \int \frac{dx}{(x+3)^2+25-9}$

$= \int \frac{d(x+3)}{(x+3)^2+4^2} = \frac{1}{4}\tan^{-1}\frac{x+3}{4} + c$

9(b) $\int \frac{dx}{(x^2+9)^2}$ [প্র.ভ.প.'০০]

$= \frac{1}{18}\int \frac{(x^2+9)-(x^2-9)}{(x^2+9)^2} dx$

$= \frac{1}{18}\{\int \frac{x^2+9}{(x^2+9)^2} dx - \int \frac{x^2-9}{(x^2+9)^2} dx\}$

$= \frac{1}{18}\{\int \frac{dx}{x^2+9} - \int \frac{x^2(1-\frac{9}{x^2})}{x^2(x+\frac{9}{x})^2} dx$

$$= \frac{1}{18}\{\int\frac{dx}{x^2+3^2} - \int\frac{d(x+\frac{9}{x})}{(x+\frac{9}{x})^2}\}$$

$$= \frac{1}{18}\{\frac{1}{3}\tan^{-1}\frac{x}{3} - (-\frac{1}{x+\frac{9}{x}})\} + c$$

$$= \frac{1}{18}(\frac{1}{3}\tan^{-1}\frac{x}{3} + \frac{x}{x^2+9}) + c$$

**বিকল্প পদ্ধতি ঃ** ধরি, $x = 3\tan\theta$. তাহলে

$\theta = \tan^{-1}\frac{x}{3}$ এবং $dx = 3\sec^2\theta\, d\theta$

$$\int\frac{dx}{(x^2+9)^2} = \int\frac{3\sec^2\theta\, d\theta}{(9\tan^2\theta+9)^2}$$

$$= \int\frac{3\sec^2\theta\, d\theta}{81(\tan^2\theta+1)^2} = \int\frac{\sec^2\theta\, d\theta}{27\sec^4\theta}$$

$$= \frac{1}{27}\int\cos^2\theta\, d\theta = \frac{1}{27}\int\frac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{54}(\theta + \frac{1}{2}\sin 2\theta) + c$$

$$= \frac{1}{54}(\theta + \frac{1}{2}\frac{2\tan\theta}{1+\tan^2\theta}) + c$$

$$= \frac{1}{54}(\tan^{-1}\frac{x}{3} + \frac{x/3}{1+x^2/9}) + c$$

$$= \frac{1}{54}(\tan^{-1}\frac{x}{3} + \frac{3x}{9+x^2}) + c$$

**10.** $\int\frac{dx}{x^2-3x+2}$ [প্র.ভ.প.'০৪]

$$= \int\frac{dx}{(x-\frac{3}{2})^2+2-\frac{9}{4}} = \int\frac{dx}{(x-\frac{3}{2})^2-(\frac{1}{2})^2}$$

$$= \frac{1}{2.\frac{1}{2}}\ln\left|\frac{x-\frac{3}{2}-\frac{1}{2}}{x-\frac{3}{2}+\frac{1}{2}}\right| + c = \ln|\frac{x-2}{x-1}| + c$$

**11(a)** $\int\frac{dx}{\sqrt{x+4}\sqrt{x+3}} = \int\frac{dx}{\sqrt{x^2+7x+12}}$

$$= \int\frac{dx}{\sqrt{(x+\frac{7}{2})^2+12-\frac{49}{4}}} = \int\frac{dx}{\sqrt{(x+\frac{7}{2})^2-(\frac{1}{2})^2}}$$

$$= \ln|\sqrt{(x+\frac{7}{2})^2-(\frac{1}{2})} + x + \frac{7}{2}| + c$$

$$= \ln|\sqrt{x^2+7x+12} + x + \frac{7}{2}| + c$$

**11(b)** $\int\sqrt{16-9x^2}\, dx = \frac{1}{3}\sqrt{(4)^2-(3x)^2}\, d(3x)$

$$= \frac{1}{3}[\frac{3x\sqrt{4^2-(3x)^2}}{2} + \frac{4^2}{2}\sin^{-1}\frac{3x}{4}] + c$$

$$= \frac{x\sqrt{16-9x^2}}{2} + \frac{8}{3}\sin^{-1}\frac{3x}{4} + c \text{ (Ans.)}$$

**12 (a)** $\int\frac{xdx}{\sqrt{4+x}} = \int\frac{4+x-4}{\sqrt{4+x}}dx$

$$= \int(\frac{4+x}{\sqrt{4+x}} - \frac{4}{\sqrt{4+x}})dx$$

$$= \int\sqrt{4+x}dx - 4\int\frac{1}{\sqrt{4+x}}dx$$

$$= \frac{(4+x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 4.2\sqrt{4+x} + c$$

$$= \frac{2}{3}(4+x)^{3/2} - 8\sqrt{4+x} + c$$

**12(b)** $\int\frac{6x-10}{(2x+1)^2}dx = \int\frac{3(2x+1)-13}{(2x+1)^2}dx$

$$= \int\frac{3}{2x+1}dx - \int\frac{13}{(2x+1)^2}dx$$

$$= \frac{3}{2}\int\frac{d(2x+1)}{2x+1} - \frac{13}{2}\int(2x+1)^{-2}d(2x+1)$$

$$= \frac{3}{2}\ln|2x+1| - \frac{13}{2}\frac{(2x+1)^{-2+1}}{-2+1} + c$$

$$= \frac{3}{2}\ln|2x+1| + \frac{13}{2(2x+1)} + c \text{ (Ans.)}$$

**12(c)** $\int\frac{x\,dx}{4-x} = \int\frac{-(4-x-4)}{4-x}dx$ [প্র.ভ.প'৮৮]

$$= -\int \frac{4-x}{4-x}dx + 4\int \frac{dx}{4-x}$$

$$= -\int dx - 4\int \frac{d(4-x)}{4-x} = -x - 4\ln|4-x| + c$$

**13(a)** $\int \sqrt{\frac{a+x}{x}}dx = \int \frac{(\sqrt{a+x})^2}{\sqrt{x(a+x)}}dx$

$$= \int \frac{(a+x)dx}{\sqrt{x^2+ax}} = \int \frac{\frac{1}{2}(2x+a)+\frac{a}{2}}{\sqrt{x^2+ax}}dx$$

$$= \frac{1}{2}\int \frac{(2x+a)}{\sqrt{x^2+ax}}dx + \frac{a}{2}\int \frac{dx}{\sqrt{(x+\frac{a}{2})^2 - (\frac{a}{2})^2}}$$

$$= \frac{1}{2}.2\sqrt{x^2+ax}$$

$$+\frac{a}{2}\ln\left|\sqrt{(x+\frac{a}{2})^2-(\frac{a}{2})^2} + x + \frac{a}{2}\right| + c$$

$$= \sqrt{x^2+ax} + \frac{a}{2}\ln\left|\sqrt{x^2+ax} + x + \frac{a}{2}\right| + c$$

**13.(b)** ধরি, $I = \int \frac{\sqrt{x+3}}{x+2}dx$ এবং $x+3 = z^2$

তাহলে, $dx = 2zdz$ এবং $I = \int \frac{\sqrt{z^2}\,2zdz}{z^2-3+2}$

$$\Rightarrow I = \int \frac{2z^2dz}{z^2-1} = 2\int \frac{z^2-1+1}{z^2-1}dz$$

$$= 2\int dz + 2\int \frac{1}{z^2-1}dz$$

$$= 2z + 2.\frac{1}{2.1}\ln\left|\frac{z-1}{z+1}\right| + c$$

$$= 2\sqrt{x+3} + \ln\left|\frac{\sqrt{x+3}-1}{\sqrt{x+3}+1}\right| + c$$

**14(a)** ধরি, $I = \int \frac{dx}{(1-x)\sqrt{1-x^2}}$ এবং $1-x = \frac{1}{z}$

তাহলে $z = \frac{1}{1-x}$ এবং $-dx = -\frac{1}{z^2}dz$

$$I = \int \frac{dz}{z^2.\frac{1}{z}\sqrt{1-(1-\frac{1}{z})^2}}$$

$$= \int \frac{dz}{z\sqrt{1-1+2\frac{1}{z}-\frac{1}{z^2}}}$$

$$= \int \frac{dz}{\sqrt{2z-1}} = \frac{1}{2}\int \frac{d(2z-1)}{\sqrt{2z-1}}$$

$$= \frac{1}{2}.2\sqrt{2z-1} + c$$

$$= \sqrt{2.\frac{1}{1-x}-1} + c = \sqrt{\frac{2-1+x}{1-x}} + c$$

$$\therefore \int \frac{dx}{(1-x)\sqrt{1-x^2}} = \sqrt{\frac{1+x}{1-x}} + c \text{ (Ans.)}$$

**14 (b)** ধরি, $I = \int \frac{dx}{(2x+3)\sqrt{x^2+3x+2}}$ এবং

$2x+3 = \frac{1}{z}$ তাহলে $z = \frac{1}{2x+3}$ এবং

$2dx = -\frac{1}{z^2}dz \Rightarrow dx = -\frac{dz}{2z^2}$

$$\therefore I = \int \frac{-dz/2z^2}{\frac{1}{z}\sqrt{(\frac{1-3z}{2z})^2 + 3.\frac{1-3z}{2z} + 2}}$$

$$= -\int \frac{dz}{2z\sqrt{\frac{1-6z+9z^2}{4z^2} + \frac{3-9z}{2z} + 2}}$$

$$= -\int \frac{dz}{2z\sqrt{\frac{1-6z+9z^2+6z-18z^2+8z^2}{4z^2}}}$$

$$= -\int \frac{dz}{\sqrt{1-z^2}} = \cos^{-1}z + c$$

$$= \cos^{-1}(\frac{1}{2x+3}) + c = \sec^{-1}(2x+3) + c$$

বিকল্প পদ্ধতি : $\int \frac{dx}{(2x+3)\sqrt{x^2+3x+2}}$

$$= \int \frac{dx}{(2x+3)\sqrt{\frac{1}{4}(4x^2+12x+8)}}$$

$$= \int \frac{dx}{(2x+3)\frac{1}{2}\sqrt{(2x+3)^2-1}}$$

$$= \int \frac{d(2x+3)}{(2x+3)\sqrt{(2x+3)^2-1}}$$

$$= \sec^{-1}(2x+3)+c$$

**15 (a)** $\int \frac{x^{-3/4}}{1+\sqrt{x}}dx$

ধরি, $x=z^4$ . তাহলে, $dx=4z^3dz$ এবং

$$\int \frac{x^{-3/4}}{1+\sqrt{x}}dx = \int \frac{(z^4)^{-3/4}}{1+\sqrt{z^4}}4z^3dz$$

$$= \int \frac{z^{-3}}{1+z^2}4z^3dz = 4\int \frac{dz}{1+z^2}$$

$$= 4\tan^{-1}z+c = 4\tan^{-1}(x^{1/4})+c \text{ (Ans.)}$$

**15(b)** ধরি, $I = \int \frac{1+x^{1/4}}{1+x^{1/2}}dx$ এবং $x=z^4$ .

তাহলে, $dx=4z^3dz$ এবং

$$I = \int \frac{(1+z)4z^3dz}{1+z^2} = 4\int \frac{z^4+z^3}{1+z^2}dz$$

$$= 4\int \frac{z^2(z^2+1)-(z^2+1)+z(z^2+1)-z-1}{1+z^2}dz$$

$$= 4\{\int (z^2-1+z)dz - \int \frac{zdz}{z^2+1} - \int \frac{dz}{z^2+1}\}$$

$$= 4\{\frac{z^3}{3}-z+\frac{z^2}{2}-\frac{1}{2}\ln(z^2+1)-\tan^{-1}z\}+c$$

$$= 4\{\frac{x^{3/4}}{3}-x^{1/4}+\frac{x^{1/2}}{2}-\frac{1}{2}\ln(x^{1/2}+1) - \tan^{-1}x^{1/4}\}+c$$

**15(c)** ধরি, $I = \int \frac{dx}{x(x^3+2)}$ এবং $x^3 = \frac{1}{z}$

তাহলে, $3x^2dx = -\frac{1}{z^2}dz \Rightarrow x^2dx = -\frac{dz}{3z^2}$

এবং $I = \int \frac{x^2dx}{x^3(x^3+2)} = \int \frac{-\frac{dz}{3z^2}}{\frac{1}{z}(\frac{1}{z}+2)}$

$$= -\frac{1}{3}\int \frac{dz}{1+2z} = -\frac{1}{3}\cdot\frac{1}{2}\int \frac{d(1+2z)}{1+2z}$$

$$= -\frac{1}{6}\ln|1+2z|+c = -\frac{1}{6}\ln|1+\frac{2}{x^3}|+c$$

**15(d)** ধরি, $I = \int \frac{dx}{x\sqrt{2+3\sqrt{x}}}$ এবং $\sqrt{x} = \frac{1}{z^2}$

তাহলে, $\frac{1}{2\sqrt{x}}dx = -\frac{2}{z^3}dz \Rightarrow \frac{z^2}{2}dx = -\frac{2}{z^3}dz$

$\Rightarrow dx = -\frac{4dz}{z^5}$ এবং $I = \int \frac{-\frac{4dz}{z^5}}{\frac{1}{z^4}\sqrt{2+\frac{3}{z^2}}}$

$$= -4\int \frac{dz}{\sqrt{2z^2+3}} = -4\int \frac{dz}{\sqrt{2}\sqrt{z^2+(\sqrt{3/2})^2}}$$

$$= -2\sqrt{2}\ln|z+\sqrt{z^2+\frac{3}{2}}|+c$$

$$= -2\sqrt{2}\ln|\frac{1}{x^{1/4}}+\sqrt{\frac{1}{x^{1/2}}+\frac{3}{2}}|+c$$

**15(e)** ধরি, $I = \int \frac{dx}{x+x^n}, n \neq 1$ এবং $x^{n-1} = \frac{1}{z}$

তাহলে, $(n-1)x^{n-2}dx = -\frac{dz}{z^2}$

$$\Rightarrow x^{n-2}dx = \frac{-dz}{(n-1)z^2}$$

এবং $I = \int \frac{dx}{x(1+x^{n-1})} = \int \frac{x^{n-2}dx}{x^{n-1}(1+x^{n-1})}$

$$= \int \frac{-\frac{dz}{(n-1)z^2}}{\frac{1}{z}(1+\frac{1}{z})} = -\frac{1}{n-1}\int \frac{dz}{1+z}$$

$$= -\frac{1}{n-1}\ln|1+z|+c$$

$$= -\frac{1}{n-1}\ln|1+\frac{1}{x^{n-1}}|+c$$

**16(a)** ধরি, $I=\int\frac{dx}{x\sqrt{x^3+4}}$ এবং $x^3=\frac{1}{z^2}$.

তাহলে, $3x^2dx=-\frac{2dz}{z^3}\Rightarrow x^2dx=-\frac{2dz}{3z^3}$ এবং

$$I=\int\frac{x^2dx}{x^3\sqrt{x^3+4}}=\int\frac{-\frac{2dz}{3z^3}}{\frac{1}{z^2}\sqrt{\frac{1}{z^2}+4}}$$

$$=-\frac{2}{3}\int\frac{dz}{\sqrt{1+4z^2}}=-\frac{2}{3}.\frac{1}{2}\int\frac{dz}{\sqrt{(\frac{1}{2})^2+z^2}}$$

$$=-\frac{1}{3}\ln|z+\sqrt{\frac{1}{4}+z^2}|+c$$

$$=-\frac{1}{3}\ln|\frac{1}{x^{3/2}}+\sqrt{\frac{1}{4}+\frac{1}{x^3}}|+c$$

**16(b)** $\int\frac{dx}{x^3(3+5x)^2}$

ধরি, $3+5x=zx\Rightarrow(z-5)x=3$

$\Rightarrow x=\frac{3}{z-5}$. তাহলে, $dx=-\frac{3dz}{(z-5)^2}$ এবং

$$\int\frac{dx}{x^3(3+5x)^2}=\int\frac{\frac{-3dz}{(z-5)^2}}{\frac{27}{(z-5)^3}(3+5\frac{3}{z-5})^2}$$

$$=\int\frac{-3(z-5)^3dz}{27(3z-15+15)^2}$$

$$=-\frac{1}{81}\int\frac{z^3-15z^2+75z-125)dz}{z^2}$$

$$=-\frac{1}{81}\int(z-15+\frac{75}{z}-125\frac{1}{z^2})dz$$

$$=-\frac{1}{81}\{\frac{z^2}{2}-15z+75\ln|z|-125(-\frac{1}{z})\}+c$$

$$=-\frac{1}{81}\{\frac{1}{2}(\frac{3+5x}{x})^2-15(\frac{3+5x}{x})+$$

$$75\ln|\frac{3+5x}{x}|+125(\frac{x}{3+5x})\}+c$$

**17(a)** $\int\frac{a^2+x^2}{(x^2-a^2)^2}dx=\int\frac{x^2(1+\frac{a^2}{x^2})}{x^2(x-\frac{a^2}{x})^2}dx$

$$=\int\frac{d(x-\frac{a^2}{x})}{(x-\frac{a^2}{x})^2}=-\frac{1}{x-\frac{a^2}{x}}+c=-\frac{x}{x^2-a^2}+c$$

**17(b)** $\int\frac{(x^2-1)dx}{x^4+6x^3+7x^2+6x+1}$

$$=\int\frac{(1-\frac{1}{x^2})dx}{x^2+\frac{1}{x^2}+6(x+\frac{1}{x})+7}$$

$$=\int\frac{(1-\frac{1}{x^2})dx}{(x+\frac{1}{x})^2+6(x+\frac{1}{x})+5}$$

$$=\int\frac{(1-\frac{1}{x^2})dx}{(x+\frac{1}{x}+3)^2+5-9}=\int\frac{d(x+\frac{1}{x}+3)}{(x+\frac{1}{x}+3)^2-2^2}$$

$$=\frac{1}{2.2}\ln|\frac{x+\frac{1}{x}+3-2}{x+\frac{1}{x}+3-2}|+c$$

$$=\frac{1}{4}\ln|\frac{x^2+1+x}{x^2+1+5x}|+c$$

**18(a)** $\int\cot^2 xdx=\int(\cos ec^2x-1)dx$

$$=-\cot x-x+c$$

**18(b)** $\int\tan^2\frac{x}{2}dx=\int(\sec^2\frac{x}{2}-1)dx$

$$=2\int\sec^2\frac{x}{2}d(\frac{x}{2})-\int dx=2\tan\frac{x}{2}-x+c$$

**18 (c)** $\int\frac{dx}{\sin x\cos^2 x}=\int\frac{\sin^2 x+\cos^2 x}{\sin x\cos^2 x}dx$

$= \int \tan x \sec x dx + \int \cos ec x\, dx$

$= \sec x + \ln(\cos ec x - \cot x) + c$

**19(a)** $\int \frac{dx}{4-5\sin^2 x} = \int \frac{\sec^2 x dx}{\sec^2 x(4-5\sin^2 x)}$

$= \int \frac{\sec^2 dx}{4\sec^2 x - 5\tan^2 x}$

$= \int \frac{\sec^2 dx}{4(1+\tan^2 x) - 5\tan^2 x} = \int \frac{\sec^2 dx}{4-\tan^2 x}$

$= \int \frac{d(\tan x)}{2^2 - (\tan x)^2} = \frac{1}{2.2} \ln|\frac{2+\tan x}{2-\tan x}| + c$

$= \frac{1}{4} \ln|\frac{2+\tan x}{2-\tan x}| + c$

**19(b)** $\int \frac{\sin 2x}{\sin x + \cos x} dx$

$= \int \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x - 1}{\sin x + \cos x} dx$

$= \int \frac{(\sin x + \cos x)^2 - 1}{\sin x + \cos x} dx$

$= \int (\sin x + \cos x - \frac{1}{\sin x + \cos x}) dx$

$= \cos x - \sin x - \int \frac{dx}{\sqrt{2}(\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4})}$

$= \cos x - \sin x - \frac{1}{\sqrt{2}} \int \frac{dx}{\sin(x+\frac{\pi}{4})}$

$= \cos x - \sin x - \frac{1}{\sqrt{2}} \int \cos ec(x+\frac{\pi}{4}) dx$

$= \cos x - \sin x - \frac{1}{\sqrt{2}} \ln|\tan\frac{1}{2}(x+\frac{\pi}{4})| + c$

$= \cos x - \sin x - \frac{1}{\sqrt{2}} \ln|\tan(\frac{x}{2}+\frac{\pi}{8})| + c$

**20** ধরি, $I = \int \frac{dx}{\sqrt{x+\sqrt{1-x}}}$ এবং

$x = \sin^2\theta$. তাহলে $dx = 2\sin\theta\cos\theta\, d\theta$,

$\sin\theta = \sqrt{x} \Rightarrow \theta = \sin^{-1}\sqrt{x}$ এবং

$I = \int \frac{2\sin\theta\cos\theta\, d\theta}{\sqrt{\sin^2\theta + \sqrt{1-\sin^2\theta}}}$

$= \int \frac{2\sin\theta\cos\theta\, d\theta}{\sin\theta + \cos\theta}$

$= \int \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta + \cos\theta} d\theta$

$= \int \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta + \cos\theta} d\theta$

$= \int (\sin\theta + \cos\theta - \frac{1}{\sin x + \cos x}) d\theta$

$= \cos\theta - \sin\theta - \int \frac{d\theta}{\sqrt{2}(\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4})}$

$= \cos\theta - \sin\theta - \frac{1}{\sqrt{2}} \int \frac{d\theta}{\sin(\theta+\frac{\pi}{4})}$

$= \cos\theta - \sin\theta - \frac{1}{\sqrt{2}} \int \cos ec(\theta+\frac{\pi}{4}) d\theta$

$= \sqrt{1-\sin^2\theta} - \sin\theta - \frac{1}{\sqrt{2}} \ln|\tan\frac{1}{2}(\theta+\frac{\pi}{4})| + c$

$= \sqrt{1-x} - \sqrt{x} - \frac{1}{\sqrt{2}} \ln|\tan(\frac{1}{2}\sin^{-1}\sqrt{x} + \frac{\pi}{8})| + c$

## প্রশ্নমালা X C

[ সূত্র (MCQ এর ক্ষেত্রে) ঃ $\int x^m e^{nx} dx$

$\{\frac{1}{n}x^m - \frac{1}{n^2}\frac{d}{dx}(x^m) + \frac{1}{n^3}\frac{d^2}{dx^2}(x^m) - \frac{1}{n^4}\frac{d^3}{dx^3}(x^m) + \cdots\cdots\} e^{nx}$ ]

**1.(a)** $\int xe^x dx$

$= x\int e^x dx - \int\{\frac{d}{dx}(x)\int e^x dx\}dx$

$= xe^x - \int 1.e^x dx = xe^x - e^x + c$

**(b)** $\int x^2 e^x dx$ [কু.'০৪; সি.'০৯]

$= x^2\int e^x dx - \int\{\frac{d}{dx}(x^2)\int e^x dx\}dx$

$= x^2e^x - \int (2x)e^x dx$

$= x^2e^x - 2[x\int e^x - \int \{\frac{d}{dx}(x)\int e^x dx\}dx]$

$= x^2e^x - 2[xe^x - \int 1.e^x dx]$

$= x^2e^x - 2xe^x + 2e^x + c$

$= (x^2 - 2x + 2)e^x + c$

**(c)** $\int x^2e^{-3x}dx$

$= x^2\int e^{-3x}dx - \int \{\frac{d}{dx}(x^2)\int e^{-3x}dx\}dx$

$= x^2(-\frac{1}{3})e^{-3x} - \int (2x)(-\frac{1}{3})e^{-3x}dx$

$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}[x\int e^{-3x} - \int \{\frac{d}{dx}(x)\int e^{-3x}dx\}dx]$

$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}[x(-\frac{e^{-3x}}{3}) - \int (-\frac{e^{-3x}}{3})dx]$

$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}[\frac{xe^{-3x}}{3} + \frac{1}{3}(-\frac{e^{-3x}}{3})] + c$

$= -\frac{1}{3}(x^2 + \frac{2}{3}x + \frac{2}{9})e^{-3x} + c$

**(d)** ধরি, $I = \int x^3e^{x^2}dx$ এবং $x^2 = z$. তাহলে

$2xdx = dz \Rightarrow xdx = \frac{1}{2}dz$ এবং

$I = \int x^2e^{x^2}(xdx) = \frac{1}{2}\int ze^z dz$

$= \frac{1}{2}[z\int e^z dz - \int \{\frac{d}{dz}(z)\int e^z dz\}dz]$

$= \frac{1}{2}[ze^z - \int 1.e^z dz] = \frac{1}{2}(ze^z - e^z) + c$

$= \frac{1}{2}(x^2 - 1)e^{x^2} + c$

**2. সূত্র (MCQ এর জন্য):** $\int x^n \sin x dx$

$= x^n(-\cos x) - (nx^{n-1})(-\sin x) + \cdots$

**(a)** $\int x\sin 3x dx$

$= x\int \sin 3x dx - \int \{\frac{d}{dx}(x)\int \sin 3x\, dx\}\, dx$

$= x(-\frac{1}{3}\cos 3x) - \int 1.(-\frac{1}{3}\cos 3x)dx$

$= -\frac{1}{3}x\cos 3x + \frac{1}{3}(\frac{1}{3}\sin 3x) + c$

$= \frac{1}{9}\sin 3x - \frac{1}{3}x\cos 3x + c$

**(b)** $\int x^3 \sin x\, dx$

$= x^3\int \sin x dx - \int \{\frac{d}{dx}(x^3)\int \sin x\, dx\}\, dx$

$= x^3(-\cos x) - \int 3x^2(-\cos x)dx$

$= -x^3\cos x + 3[x^2\int \cos x - \int \{\frac{d}{dx}(x^2)\int \cos x\, dx\}\, dx]$

$= -x^3\cos x + 3[x^2\sin x - \int 2x\sin x dx]$

$= -x^3\cos x + 3[x^2\sin x - 2\{x(-\cos x) - \int 1(-\cos x)dx\}]$

$= -x^3\cos x + 3[x^2\sin x - 2(-x\cos x + \sin x)] + c$

$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + c$

[MCQ এর ক্ষেত্রে, $\int x^3\sin x\, dx = x^3(-\cos x) - (3x^2)(-\sin x) + (6x)(\cos x) - 6\sin x$

$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + c$]

**(c)** ধরি, $I = \int e^{2x}\cos e^x dx$ এবং $e^x = z$.

তাহলে $e^x dx = dz$ এবং

$I = \int e^x\cos e^x (e^x dx) = \int z\cos z\, dz$

$= z\int \cos z\, dz - \int \{\frac{d}{dz}(z)\int \cos z\, dz\}dz$

$= z\sin z - \int 1.\sin z\, dz$

$= z\sin z - (-\cos z) + c$