1. (d) $x^2 = 5y^2 + \sin y$ [প্র.ভ.প.'০৬]

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$2x = 10y\frac{dy}{dx} + \cos y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x}{10y + \cos y} \text{ (Ans.)}$$

1(e) $(\cos x)^y = (\sin y)^x$ [প্র.ভ.প.'০৩]

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$(\cos x)^y [\, y\frac{d}{dx}\{\ln(\cos x)\} + \ln(\cos x)\frac{dy}{dx}]$$

$$= (\sin y)^x \;[\, x\frac{d}{dx}\{\ln(\sin y)\} + \ln(\sin y)\frac{d}{dx}(x)]$$

$$\Rightarrow \frac{y}{\cos x}(-\sin x) + \ln(\cos x)\frac{dy}{dx}$$

$$= \frac{x}{\sin y}(\cos y)\frac{dy}{dx} + \ln(\sin y).1$$

$$[\because (\cos x)^y = (\sin y)^x\,]$$

$$\Rightarrow \{\ln(\cos x) - x\cot y\}\frac{dy}{dx} = \ln(\sin y) + y\tan x$$

$$\frac{dy}{dx} = \frac{\ln(\sin y) + y\tan x}{\ln(\cos x) - x\cot y}$$

1(f) $\sqrt{x/y} + \sqrt{y/x} = 1$

$$\Rightarrow \frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} = 1 \Rightarrow x + y = \sqrt{xy}$$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$1 + \frac{dy}{dx} = \frac{1}{2\sqrt{xy}}(x\frac{dy}{dx} + y.1)$$

$$\Rightarrow (1 - \frac{\sqrt{x}}{2\sqrt{y}})\frac{dy}{dx} = \frac{\sqrt{y}}{2\sqrt{x}} - 1$$

$$\Rightarrow \frac{2\sqrt{y} - \sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} = \frac{\sqrt{y} - 2\sqrt{x}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{\sqrt{y}(\sqrt{y} - 2\sqrt{x})}{\sqrt{x}(2\sqrt{y} - \sqrt{x})} \text{ (Ans.)}$$

2. $\frac{dy}{dx}$ নির্ণয় কর ঃ

2(a) $x^y = e^{x-y}$ [য.বো.'০৫]

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$x^y[\, y\frac{d}{dx}(\ln x) + \ln x\frac{dy}{dx}] = e^{x-y}(1 - \frac{dy}{dx})$$

$$\Rightarrow \frac{y}{x} + \ln x\frac{dy}{dx} = 1 - \frac{dy}{dx} \quad [\quad x^y = e^{x-y}\,]$$

$$\Rightarrow (1 + \ln x)\frac{dy}{dx} = 1 - \frac{y}{x} = \frac{x-y}{x}$$

$$\frac{dy}{dx} = \frac{x-y}{x(1+\ln x)}$$

2(b) $y + x = x^{-y}$ [রা.'১১; য.'১৩; প্র.ভ.প. '১৫]

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx} + 1 = x^{-y}\,[-y\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(-y)]$$

$$\Rightarrow \frac{dy}{dx} + 1 = x^{-y}\,[\frac{-y}{x} - \ln x\frac{dy}{dx}]$$

$$\Rightarrow (\,1 + x^{-y}\; ln\, x\,)\frac{dy}{dx} = -1 - y.x^{-y-1}$$

$$\frac{dy}{dx} = -\frac{1 + yx^{-y-1}}{1 + x^{-y}\ln x} \text{ (Ans.)}$$

2(c) $x^y + y^x = 1$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$x^y[\, y\frac{d}{dx}(\ln x) + \ln x\frac{dy}{dx}] +$$

$$y^x[x\frac{d}{dx}(\ln y) + \ln y\frac{d}{dx}(x)] = 0$$

$$\Rightarrow x^y[\frac{y}{x} + \ln x\frac{dy}{dx}] + y^x[\frac{x}{y}\frac{dy}{dx} + \ln y.1] = 0$$

$$\Rightarrow (x^y\ln x + xy^{x-1})\frac{dy}{dx} = -\,(\,x^{y-1}y + y^x\ln y\,)$$

$$\frac{dy}{dx} = -\frac{x^{y-1}y + y^x\ln y}{x^y\ln x + xy^{x-1}}$$

2(d) $x^p y^p = (x+y)^{p+q}$

$$p\ln x + q\ln y = (p+q)\ln(x+y)$$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}(1 + \frac{dy}{dx})$$

$\Rightarrow (\frac{q}{y} - \frac{p+q}{x+y})\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$

$\Rightarrow \frac{qx+qy-py-qy}{y(x+y)}\frac{dy}{dx} = \frac{px+qx-px-py}{(x+y)x}$

$\Rightarrow \frac{qx-py}{y(x+y)}\frac{dy}{dx} = \frac{qx-py}{(x+y)x}$

$\frac{dy}{dx} = \frac{y}{x}$ (Ans.)

2(e) $y = x^{y^x} \therefore \ln y = y^x \ln x \cdots (1)$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{1}{y}\frac{dy}{dx} = y^x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(y^x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y^x \frac{1}{x} + \ln x \,.\, y^x \{\frac{x}{y}\frac{dy}{dx} + \ln y\}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\ln y}{x \ln x} + \ln y\{\frac{x}{y}\frac{dy}{dx} + \ln y\}$

[(1) দ্বারা]

$\Rightarrow (\frac{1}{y} - \frac{x}{y}\ln y)\frac{dy}{dx} = \ln y(\frac{1}{x\ln x} + \ln y)$

$\Rightarrow (\frac{1 - x\ln y}{y})\frac{dy}{dx} = \ln y(\frac{1 + x\ln x \ln y}{x \ln x})$

$\frac{dy}{dx} = \frac{y \ln y(1 + x\ln x\ln y)}{x\ln x(1 - x\ln y)}$

(f) $y = \sqrt{x\sqrt{x\sqrt{x.......\infty}}} = \sqrt{x\sqrt{x\sqrt{x\sqrt{x.......\infty}}}}$

$\Rightarrow y = \sqrt{xy} \Rightarrow y^2 = xy \Rightarrow y = x$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{dy}{dx} = 1$ (Ans.)

2.(g) $\ln (xy) = x + y$ **[রা.'০৫; কু. '০৬]**

$\Rightarrow \ln x + \ln y = x + y$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow y + x\frac{dy}{dx} = xy + xy\frac{dy}{dx}$

$\Rightarrow x(1-y)\frac{dy}{dx} = y(x-1)$

$\frac{dy}{dx} = \frac{y(x-1)}{x(1-y)}$ (Ans.)

2(h) $\log (x^n y^n) = x^n + y^n$ **[বুয়েট ০৭-০৮]**

$\Rightarrow n \log x + n \log y = x^n + y^n$

$\Rightarrow n \log_{10} e \times \log_e x + n \log_{10} e \times \log_e y = x^n + y^n$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$n\frac{\log_{10} e}{x} + n\frac{\log_{10} e}{y}\frac{dy}{dx} = nx^{n-1} + ny^{n-1}\frac{dy}{dx}$

$\Rightarrow (\frac{\log_{10} e}{y} - y^{n-1})\frac{dy}{dx} = x^{n-1} - \frac{\log_{10} e}{x}$

$\Rightarrow \frac{\log_{10} e - y^n}{y}\frac{dy}{dx} = \frac{x^n - \log_{10} e}{x}$

$\frac{dy}{dx} = \frac{y(x^n - \log_{10} e)}{x(\log_{10} e - y^n)}$

3. (a) $\tan y = \sin x$ **হলে, দেখাও যে,**

$\frac{dy}{dx} = \frac{1}{(1-x^2)^{3/2}}$ **[প্র.ভ.প.'৮৪]**

প্রমাণ : $\tan y = \sin x$

$\Rightarrow y = \tan^{-1} \sin x$

$\Rightarrow y = \tan^{-1} \tan \frac{x}{\sqrt{1-x^2}}$

$= \frac{x}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\frac{d}{dx}(x) - x\frac{d}{dx}(\sqrt{1-x^2})}{(\sqrt{1-x^2})^2}$

$= \frac{\sqrt{1-x^2}.1 - x\frac{1}{2\sqrt{1-x^2}}(-2x)}{1-x^2}$

$= \frac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}} = \frac{1}{(1-x^2)^{3/2}}$

3(b) $x\sqrt{1+y}+y\sqrt{1+x}=0$ **হলে, দেখাও যে,** $\dfrac{dy}{dx}=-\dfrac{1}{(1+x)^2}$ **[প্র.ভ.প. '০২, '০৪]**

প্রমাণ : $x\sqrt{1+y}+y\sqrt{1+x}=0$

$\Rightarrow x\sqrt{1+y}=-\ y\sqrt{1+x}$

$\Rightarrow x^2(1+y)=\ y^2(1+x)$ [বর্গ করে।]

$\Rightarrow x^2+x^2y=y^2+xy^2$

$\Rightarrow x^2-y^2+xy(x-y)=0$

$\Rightarrow (x-y)(x+y+xy)=0$

$x+y+xy=0$ হলে, $(1+x)\,y=-x$

$\Rightarrow y=\dfrac{-x}{1+x}\quad \dfrac{dy}{dx}=\dfrac{(1+x)(-1)+x(1)}{(1+x)^2}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{-1-x+x}{(1+x)^2}=-\dfrac{1}{(1+x)^2}$

3.(c) $x=a\,(t-\sin t)$ **এবং** $y=a\,(1+\cos t)$ **হলে, দেখাও যে,** $t=\dfrac{5\pi}{3}$ **যখন** $\dfrac{dy}{dx}=\sqrt{3}$.

[প্র.ভ.প. '৮৫]

প্রমাণ : $\dfrac{dx}{dt}=a(1-\cos t)$, $\dfrac{dy}{dt}=a(0-\sin t)$

$\dfrac{dy}{dx}=\dfrac{dy}{dt}\times\dfrac{dt}{dx}=\dfrac{-a\sin t}{a(1-\cos t)}$

$=\dfrac{-2\sin\frac{t}{2}\cos\frac{t}{2}}{2\sin^2\frac{t}{2}}=-\cot\dfrac{t}{2}$

এখন, $\dfrac{dy}{dx}=\sqrt{3}$ হলে , $\cot\dfrac{t}{2}=-\sqrt{3}$

$\Rightarrow \tan\dfrac{t}{2}=-\dfrac{1}{\sqrt{3}}=-\tan\dfrac{\pi}{6}=\tan(\pi-\dfrac{\pi}{6})$

$\Rightarrow \tan\dfrac{t}{2}=\tan\dfrac{5\pi}{6}\quad \dfrac{t}{2}=\dfrac{5\pi}{6}\Rightarrow t=\dfrac{5\pi}{3}$

3(d) $f(x)=\left(\dfrac{a+x}{b+x}\right)^{a+b+2x}$ **হলে, প্রমাণ কর যে,**

$f'(0)=(2\ln\dfrac{a}{b}+\dfrac{b^2-a^2}{ab})(\dfrac{a}{b})^{a+b}$

প্রমাণ: $f(x)=\left(\dfrac{a+x}{b+x}\right)^{a+b+2x}\qquad f(0)=(\dfrac{a}{b})^{a+b}$

এবং $\ln\{f(x)\}=$

$(a+b+2x)\{\ln(a+x)-\ln(b+x)\}$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\dfrac{1}{f(x)}f'(x)=(a+b+2x)\{\dfrac{1}{a+x}-\dfrac{1}{b+x}\}+\{\ln(a+x)-\ln(b+x)\}2$

$f'(0)=f(0)\,[\,(a+b)(\dfrac{1}{a}-\dfrac{1}{b})+2(\ln a-\ln b)\,]$

$\Rightarrow f'(0)=(\dfrac{a}{b})^{a+b}[\,(a+b)(\dfrac{b-a}{ab})+2\ln\dfrac{a}{b}\,]$

$f'(0)=(2\ln\dfrac{a}{b}+\dfrac{b^2-a^2}{ab})(\dfrac{a}{b})^{a+b}$

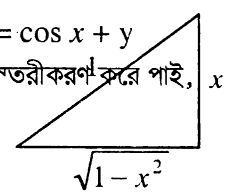
(e) $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x\ldots\ldots\infty}}}$ **হলে, প্রমাণ কর যে,** $(2y-1)\dfrac{dy}{dx}+\sin x=0.$

প্রমাণ : $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x\ldots\ldots\infty}}}$

$\Rightarrow y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x\cdots\infty}}}}$

$\Rightarrow y=\sqrt{\cos x+y}\Rightarrow y^2=\cos x+y$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,



$2y\dfrac{dy}{dx}=-\sin x+\dfrac{dy}{dx}$

$\Rightarrow (2y-1)\dfrac{dy}{dx}+\sin x=0$

3(f) $x^y=y^{x^n}$ **হলে দেখাও যে,**

$\dfrac{dy}{dx}=\dfrac{y^{n+1}(n\ln x-1)}{x^{n+1}(n\ln y-1)}$ **[বুয়েট ০৮–০৯]**

প্রমাণ : $x^y=y^{x^n}\ \therefore y^n\ln x=x^n\ln y\cdots(1)$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\dfrac{y^n}{x}+\ln x.(ny^{n-1})\dfrac{dy}{dx}=\dfrac{x^n}{y}\dfrac{dy}{dx}+\ln y.nx^{n-1}$

$\Rightarrow y^{n+1}+x\ln x.ny^n\dfrac{dy}{dx}=x^{n+1}\dfrac{dy}{dx}+y\ln y.nx^n$

$$\Rightarrow (nx\ln x.y^n - x^{n+1})\frac{dy}{dx} = y\ln y.nx^n - y^{n+1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{nyx^n \ln y - y^{n+1}}{nxy^n \ln x - x^{n+1}}$$

$$= \frac{ny.y^n \ln x - y^{n+1}}{nx.x^n \ln y - x^{n+1}} \quad [(1) \text{ দ্বারা }]$$

$$= \frac{y^{n+1}(n\ln x - 1)}{x^{n+1}(n\ln y - 1)}$$

অতিরিক্ত প্রশ্ন (সমাধানসহ)

x এর সাপেক্ষে নিম্নের ফাংশনগুলির অন্তরক সহগ নির্ণয় কর ঃ

1. $\frac{d}{dx}(5x^3 + 3x^2 - 4x - 9)$

$= 5\frac{d}{dx}(x^3) + 3\frac{d}{dx}(x^2) - 4\frac{d}{dx}(x) - \frac{d}{dx}(9)$

$= 5(3x^2) + 3(2x) - 4 - 0$

$= 15x^2 + 6x - 4$ (Ans.)

2. $\frac{d}{dx}(2x^3 - 4x^{\frac{5}{2}} + \frac{7}{2}x^{-\frac{2}{3}} + 7)$

$= 2(3x^2) - 4(\frac{5}{2}x^{\frac{5}{2}-1}) + \frac{7}{2}(-\frac{2}{3}x^{-\frac{2}{3}-1}) + 0$

$= 6x^2 - 10x^{\frac{3}{2}} - \frac{7}{3}x^{-\frac{5}{3}}$ (Ans.)

3(a) মূল নিয়মে $x = 2$ -তে $\sqrt[3]{x}$ এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = \sqrt[3]{x} = x^{1/3}$

$\therefore f'(2) = \lim\limits_{x\to 2}\frac{f(x) - f(2)}{x - 2} = \lim\limits_{x\to 2}\frac{x^{1/3} - 2^{1/3}}{x - 2}$

$= \frac{1}{3} \times 2^{\frac{1}{3}-1}$ [$\lim\limits_{x\to a}\frac{x^n - a^n}{x - a} = na^{n-1}$]

$= \frac{1}{3} \times 2^{-\frac{2}{3}} = \frac{1}{3} \times 4^{-\frac{1}{3}} = \frac{1}{3\sqrt[3]{4}}$

3(b) মূল নিয়মে $x = a$ -তে $\cos^2 x$ এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = \cos^2 x$. $\quad f(a) = \cos^2 a$

$f'(a) = \lim\limits_{x\to a}\frac{f(x) - f(a)}{x - a}$

$= \lim\limits_{x\to a}\frac{\cos^2 x - \cos^2 a}{x - a}$

$= \lim\limits_{x\to a}\frac{\sin(x + a)\sin(a - x)}{x - a}$

[$\because \cos^2 B - \cos^2 A = \sin(A + B)\sin(A - B)$]

$= -\lim\limits_{x-a\to 0}\frac{\sin(x - a)}{x - a}.\lim\limits_{x\to a}\sin(x + a)$

$= -1.\sin(a + a) = -\sin 2a$ (Ans.)

4. $(2x)^n - b^n$ [চ.'০২]

$(2x)^n - b^n = 2^n x^n - b^n$

$\therefore \frac{d}{dx}\{(2x)^n - b^n\} = 2^n\frac{d}{dx}(x^n) - \frac{d}{dx}(b^n)$

$= 2^n n x^{n-1} - 0 = 2^n n x^{n-1}$

5(a) $x^2 \log_a x + 7e^x \cos x$ [সি.'০৪]

$\frac{d}{dx}(x^2 \log_a x + 7e^x \cos x) = x^2\frac{d}{dx}(\log_a x)$

$+ \log_a x\frac{d}{dx}(x^2) + 7\{e^x\frac{d}{dx}(\cos x) +$

$\cos x\frac{d}{dx}(e^x)\}$

$= x^2\frac{1}{x\ln a} + \log_a x\ (2x) +$

$7\{e^x(-\sin x) + \cos x\ .\ e^x\}$

$= x(\frac{1}{\ln a} + 2\log_a x) + 7e^x(\cos x - \sin x)$

5(b) $\sin^2 2x + e^{2\ln(\cos 2x)}$ [প্র.ভ.প.'৯৩]

$\sin^2 2x + e^{2\ln(\cos 2x)} = \sin^2 2x + e^{\ln(\cos 2x)^2}$

$= \sin^2 2x + (\cos 2x)^2$

$= \sin^2 2x + \cos^2 2x = 1$

$\frac{d}{dx}\{\sin^2 2x + e^{2\ln(\cos 2x)}\} = \frac{d}{dx}(1) = 0$

5(c) $5e^x \ln x$ [য.'০৪]

মনে করি, $y = 5e^x \ln x$

$$\frac{dy}{dx} = 5\{\, e^x \frac{d}{dx}(\ln x) + \ln x \frac{d}{dx}(e^x)\,\}$$

$$= 5\{e^x.\frac{1}{x} + \ln x\,(e^x)\}$$

$$\frac{d}{dx}(5e^x \ln x) = 5e^x(\frac{1}{x} + \ln x)$$

6.(a) $\frac{d}{dx}(\frac{x^n + \tan x}{e^x - \cot x}) =$

$$\frac{(e^x - \cot x)\frac{d}{dx}(x^n + \tan x) - (x^n + \tan x)\frac{d}{dx}(e^x - \cot x)}{(e^x - \cot x)^2}$$

$$= \frac{(e^x - \cot x)(nx^{n-1} + \sec^2 x) - (x^n + \tan x)\frac{d}{dx}(e^x + \operatorname{cosec}^2 x)}{(e^x - \cot x)^2}$$

6(b) $\frac{d}{dx}(\frac{1-\cos x}{1+\cos x})$

$$= \frac{(1+\cos x)\frac{d}{dx}(1-\cos x) - (1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x)(\sin x) - (1-\cos x)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\sin x(1+\cos x + 1 - \cos x)}{(1+\cos x)^2}$$

$$= \frac{2\sin x}{(1+\cos x)^2}$$

6(c) $\frac{x\sin x}{x+\cos x}$ [রা.'০০]

$$\frac{d}{dx}(\frac{x\sin x}{x+\cos x}) = \frac{1}{(x+\cos x)^2}[(x+\cos x)$$

$$\frac{d}{dx}(x\sin x) - x\sin x\frac{d}{dx}(x+\cos x)]$$

$$= \frac{1}{(x+\cos x)^2}[(x+\cos x)(x\cos x + \sin x.1)$$

$$- x\sin x\,(1-\sin x)]$$

$$= \frac{1}{(x+\cos x)^2}[(x^2\cos x + x\sin x + x\cos^2 x +$$

$$\cos x\sin x - x\sin x + x\sin^2 x\,]$$

$$= \frac{x(\sin^2 x + \cos^2 x) + x^2\cos x + \cos x\sin x}{(x+\cos x)^2}$$

$$= \frac{x + (x^2 + \sin x)\cos x}{(x+\cos x)^2} \text{ (Ans.)}$$

6.(d) $\frac{\sin^2 x}{1+\cos x}$ [দি.'০১]

$$\frac{\sin^2 x}{1+\cos x} = \frac{1-\cos^2 x}{1+\cos x} = \frac{(1-\cos x)(1+\cos x)}{1+\cos x}$$

$$= 1 - \cos x \qquad \frac{d}{dx}(\frac{\sin^2 x}{1+\cos x}) = \sin x$$

6(e) $\frac{\cos x}{1+\sin^2 x}$ [য.'০৯]

$$\frac{d}{dx}(\frac{\cos x}{1+\sin^2 x}) =$$

$$\frac{(1+\sin^2 x)\frac{d}{dx}(\cos x) - \cos x\frac{d}{dx}(1+\sin^2 x)}{(1+\sin^2 x)^2}$$

$$= \frac{(1+\sin^2 x)(-\sin x) - \cos x(2\sin x\cos x)}{(1+\sin^2 x)^2}$$

$$= \frac{-\sin x(1+\sin^2 x + 2\cos^2 x)}{(1+\sin^2 x)^2}$$

$$= \frac{-\sin x(2+\cos^2 x)}{(1+\sin^2 x)^2}$$

7(a) ধরি, $y = (x+\sqrt{1+x^2})^n$

$$\therefore \frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\frac{d}{dx}(x+\sqrt{1+x^2})$$

$$= n(x+\sqrt{1+x^2})^{n-1}\{1 + \frac{1}{2\sqrt{1+x^2}}.2x\}$$

$$= n(x+\sqrt{1+x^2})^{n-1}\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}((x+\sqrt{1+x^2})^n) = \frac{n(x+\sqrt{1+x^2})^n}{\sqrt{1+x^2}}$$

7(b) $\frac{d}{dx}\{\text{cosec}(e^{x^2})\}$

$= \frac{d\{\text{cosec}(e^{x^2})\}}{d(e^{x^2})}\frac{d(e^{x^2})}{d(x^2)}\frac{d(x^2)}{dx}$

$= -\text{cosec}(e^{x^2})\cot(e^{x^2}).(e^{x^2}).2x$

$= -2x\ e^{x^2}\ \text{cosec}(e^{x^2})\cot(e^{x^2})$ (Ans.)

8(a) $\log_x 5$ **[প্র.ভ.প. '৮৮]**

$\log_x 5 = \log_x e \times \log_e 5 = ln\ 5\frac{1}{\log_e x}$

$= ln\ 5\frac{1}{\ln x} = ln\ 5\ (ln\ x)^{-1}$

$\therefore \frac{d}{dx}(\log_x a) = ln5\ \{-1(ln\ x)^{-2}\frac{d}{dx}(ln\ x)\}$

$= -ln\ 5\ \frac{1}{(\ln x)^2}.\frac{1}{x} = -\frac{\ln 5}{x(\ln x)^2}$

8(b) $\ln(\sin e^{x^2})$ **[প্র.ভ.প. '৯৫]**

$\frac{d}{dx}\{\ln(\sin e^{x^2})\}$

$= \frac{1}{\sin(e^{x^2})}\{\cos(e^{x^2})\}e^{x^2}.2x$

$= 2xe^{x^2}\cot(e^{x^2})$

8(c) $\frac{d}{dx}\{ln\ (\tan\frac{x}{2})\}$

$= \frac{d\{\ln(\tan\frac{x}{2})\}}{d(\tan\frac{x}{2})}\frac{d(\tan\frac{x}{2})}{d(\frac{x}{2})}\frac{d(\frac{x}{2})}{dx}$

$= \frac{1}{\tan\frac{x}{2}}\sec^2\frac{x}{2}.\frac{1}{2} = \frac{1}{2}\frac{\cos(x/2)}{\sin(x/2)}\frac{1}{\cos^2(x/2)}$

$= \frac{1}{2\sin(x/2)\cos(x/2)} = \frac{1}{\sin x} = \cos ecx$

9. (a) $\frac{d}{dx}\{ln\ (a x^2 + b x + c)\}$

$= \frac{1}{ax^2+bx+c}\frac{d}{dx}(ax^2+bx+c)$

$= \frac{2ax+b}{ax^2+bx+c}$ (Ans.)

9(b) $\frac{d}{dx}\{ln\ (x + \sqrt{x^2 \pm a^2})\}$

$= \frac{1}{x+\sqrt{x^2\pm a^2}}\frac{d}{dx}(x+\sqrt{x^2\pm a^2})$

$= \frac{1}{x+\sqrt{x^2\pm a^2}}\{1 + \frac{1}{2\sqrt{x^2\pm a^2}}(2x)\}$

$= \frac{1}{x+\sqrt{x^2\pm a^2}}\{\frac{\sqrt{x^2\pm a^2}+x}{\sqrt{x^2\pm a^2}}\}$

$= \frac{1}{\sqrt{x^2\pm a^2}}$ (Ans.)

9.(c) $ln\ \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$

$= ln\ (\sqrt{x+1}-1) - ln\ (\sqrt{x+1}+1)$

$\frac{d}{dx}\{ln\ \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\}$

$= \frac{1}{\sqrt{x+1}-1}\frac{1}{2\sqrt{x+1}} - \frac{1}{\sqrt{x+1}+1}\frac{1}{2\sqrt{x+1}}$

$= \frac{\sqrt{x+1}+1-\sqrt{x+1}+1}{2\sqrt{x+1}(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$

$= \frac{2}{2\sqrt{x+1}(x+1-1)} = \frac{1}{x\sqrt{x+1}}$ (Ans.)

10(a) $\left(\frac{\sin 2x}{1+\cos 2x}\right)^2 = \left(\frac{2\sin x\cos x}{2\cos^2 x}\right)^2$

$= \left(\frac{\sin x}{\cos x}\right)^2 = \tan^2 x$

$\frac{d}{dx}\left(\frac{\sin 2x}{1+\cos 2x}\right)^2 = 2\ \tan x\frac{d}{dx}(\tan x)$

$= 2\ \tan x.\ \sec^2 x$

10(b) $[\frac{x}{\sqrt{1-x^2}}]^n$ [প্র.ভ.প.'০৫]

$$\frac{d}{dx}[\frac{x}{\sqrt{1-x^2}}]^n = n[\frac{x}{\sqrt{1-x^2}}]^{n-1} \frac{\sqrt{1-x^2}.1 - x\frac{1}{2\sqrt{1-x^2}}(-2x)}{(\sqrt{1-x^2})^2}$$

$$= n[\frac{x}{\sqrt{1-x^2}}]^{n-1} \frac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}}$$

$$= n[\frac{x}{\sqrt{1-x^2}}]^{n-1} \frac{1}{(1-x^2)^{3/2}}$$

10(c) $\frac{d}{dx}\{ x\, lnx\, ln(lnx) \}$

$$= x\, lnx \frac{d}{dx}\{ln(lnx)\} + x\, ln(lnx)\frac{d}{dx}(lnx) + lnx\, ln(lnx)\frac{d}{dx}(x)$$

$$= x\, lnx \frac{1}{lnx}.\frac{1}{x} + x\, ln(lnx)\frac{1}{x} + lnx\, ln(lnx).1$$

$$= 1 + ln(lnx)(1+lnx)$$

10(d) $\frac{d}{dx}(\sin x \ \sin 2x \ \sin 3x)$

$$= \sin x \ \sin 2x\frac{d}{dx}(\sin 3x) + \sin x \ \sin 3x \frac{d}{dx}(\sin 2x) + \sin 2x \ \sin 3x\frac{d}{dx}(\sin x)$$

$= \sin x \ \sin 2x\ (\cos 3x).3 + \sin x \sin 3x\ (\cos 2x).2 + \sin 2x \ \sin 3x\ (\cos x).\ 1$

$= 3\sin x \sin 2x \cos 3x + 2\sin x \sin 3x \cos 2x + \sin 2x \ \sin 3x \cos x$

11(a) $\frac{d}{dx}(e^{\sqrt{x}} + e^{-\sqrt{x}})$

$$= e^{\sqrt{x}}\frac{d}{dx}(\sqrt{x}) + e^{-\sqrt{x}}\frac{d}{dx}(-\sqrt{x})$$

$$= e^{\sqrt{x}}\frac{1}{2\sqrt{x}} - e^{-\sqrt{x}}\frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$$

11(a) $\frac{d}{dx}(e^{-x} + e^{\frac{1}{x}})$

$$= e^{-x}\frac{d}{dx}(-x) + e^{\frac{1}{x}}\frac{d}{dx}(\frac{1}{x})$$

$$= -e^{-x}.1 + e^{\frac{1}{x}}(-\frac{1}{x^2}) = -(e^{-x} + \frac{1}{x^2}e^{\frac{1}{x}})$$

12(a) ধরি , $y = ln\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}$

$$= \frac{1}{2}\{\ln(1+\sin x) - \ln(1-\sin x)\}$$

$$\frac{dy}{dx} = \frac{1}{2}\{\frac{\cos x}{1+\sin x} - \frac{(-\cos x)}{1-\sin x}\}$$

$$= \frac{1}{2}\frac{\cos x(1-\sin x + 1 + \sin x)}{(1+\sin x)(1-\sin x)}$$

$$= \frac{1}{2}\frac{2\cos x}{1-\sin^2 x} = \frac{\cos x}{\cos^2 x} = \sec x$$

12(b) ধরি, $y = \cos\frac{x^{-1}-x}{x^{-1}+x}$ [প্র.ভ.প.৮৯]

$$\frac{dy}{dx} = -\sin\frac{x^{-1}-x}{x^{-1}+x}\frac{d}{dx}(\frac{x^{-1}-x}{x^{-1}+x})$$

$$= -\sin\frac{x^{-1}-x}{x^{-1}+x}\frac{d}{dx}(\frac{1-x^2}{1+x^2})$$

$$= -\sin\frac{x^{-1}-x}{x^{-1}+x}\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$$

$$= -\sin\frac{x^{-1}-x}{x^{-1}+x}\frac{2x(-1-x^2-1+x^2)}{(1+x^2)^2}$$

$$= \frac{4x}{(1+x^2)^2}\sin\frac{x^{-1}-x}{x^{-1}+x}$$

12(c) $e^{3x}\cos x° = e^{3x}\cos\frac{\pi x}{180}$

$$\frac{d}{dx}(e^{3x} \cos x°) = e^{3x}(-\sin\frac{\pi x}{180})\frac{d}{dx}(\frac{\pi x}{180}) + \cos\frac{\pi x}{180}.e^{3x}\frac{d}{dx}(3x)$$

$$= -e^{3x}.\sin x°.(\frac{\pi}{180}) + \cos x°.e^{3x}.3$$

$$= e^{3x}(3\cos x^\circ - \frac{\pi}{180}\sin x^\circ)$$

13(a) $\frac{d}{dx}\{\sin^{-1}(e^{\tan^{-1}x})\}$

$$= \frac{1}{\sqrt{1-(e^{\tan^{-1}x})^2}}\frac{d}{dx}(e^{\tan^{-1}x})$$

$$= \frac{1}{\sqrt{1-e^{2\tan^{-1}x}}}e^{\tan^{-1}x}\frac{1}{1+x^2}$$

$$= \frac{e^{\tan^{-1}x}}{(1+x^2)\sqrt{1-e^{2\tan^{-1}x}}}$$

13(b) $\frac{d}{dx}\{\cos^{-1}(\frac{a+b\cos x}{b+a\cos x})\}$

$$= -\frac{1}{\sqrt{1-(\frac{a+b\cos x}{b+a\cos x})^2}}$$

$$\frac{(b+a\cos x)(-b\sin x)-(a+b\cos x)(-a\sin x)}{(b+a\cos x)^2}$$

$$= -\frac{b+a\cos x}{\sqrt{(b+a\cos x)^2-(a+b\cos x)^2}}$$

$$\frac{(-b^2+a^2)\sin x}{(b+a\cos x)^2}$$

$$= \frac{-(a^2-b^2)\sin x}{(b+a\cos x)\sqrt{b^2+a^2\cos^2 x-a^2-b^2\cos^2 x}}$$

$$= \frac{(b^2-a^2)\sin x}{(b+a\cos x)\sqrt{(b^2-a^2)(1-\cos^2 x)}}$$

$$= \frac{(b^2-a^2)\sin x}{(b+a\cos x)\sqrt{(b^2-a^2)\sin^2 x}}$$

$$= \frac{\sqrt{b^2-a^2}}{b+a\cos x}$$

13(c) $\sin^{-1}(\frac{2x^{-1}}{x+x^{-1}}) = \sin^{-1}(\frac{2/x}{x+1/x})$

$$= \sin^{-1}(\frac{2}{x^2+1})$$

$$\therefore \frac{d}{dx}\{\sin^{-1}(\frac{2x^{-1}}{x+x^{-1}})\}$$

$$= \frac{1}{\sqrt{1-\frac{4}{(x^2+1)^2}}}2\frac{d}{dx}(x^2+1)^{-1}$$

$$= \frac{x^2+1}{\sqrt{x^4+2x^2+1-4}}2(-1)(x^2+1)^{-2}.2x$$

$$= \frac{-4x(x^2+1)^{-1}}{\sqrt{x^4+2x^2-3}} = \frac{-4x}{(x^2+1)\sqrt{x^4+2x^2-3}}$$

13(d) $\frac{d}{dx}\{\cos^{-1}x\ \ln(\sin^{-1}x)\}$ **[প্র.ভ.প.'০৪]**

$$= \cos^{-1}x\frac{d}{dx}\{\ln(\sin^{-1}x)\} + \ln(\sin^{-1}x)\frac{d}{dx}(\cos^{-1}x)$$

$$= \cos^{-1}x\frac{1}{\sin^{-1}x}\frac{1}{\sqrt{1-x^2}} + \frac{\ln(\sin^{-1}x)}{-\sqrt{1-x^2}}$$

$$= \frac{1}{\sqrt{1-x^2}}\{\frac{\cos^{-1}x}{\sin^{-1}x} - \ln(\sin^{-1}x)\}$$

13(e) $\cot^{-1}(\frac{x^2}{e^x})+\cot^{-1}(\frac{e^x}{x^2})$ **[প্র.ভ.প.'০৫]**

$$= \tan^{-1}(\frac{e^x}{x^2})+\tan^{-1}(\frac{x^2}{e^x})$$

$$= \tan^{-1}\frac{\frac{e^x}{x^2}+\frac{x^2}{e^x}}{1-\frac{e^x}{x^2}.\frac{x^2}{e^x}} = \tan^{-1}\frac{\frac{e^x}{x^2}+\frac{x^2}{e^x}}{1-1}$$

$$= \cot^{-1}\frac{1-1}{\frac{e^x}{x^2}+\frac{x^2}{e^x}} = \cot^{-1}0 = \frac{\pi}{2}$$

$$\therefore \frac{d}{dx}\{\cot^{-1}(\frac{x^2}{e^x})+\cot^{-1}(\frac{e^x}{x^2})\} = \frac{d}{dx}(\frac{\pi}{2}) = 0$$

13(f) $\tan^{-1}\dfrac{\sqrt{x}+\sqrt{a}}{1-\sqrt{ax}}$ [প্র.ভ.প. '১৬]

$= \tan^{-1}\dfrac{\sqrt{x}+\sqrt{a}}{1-\sqrt{x}\sqrt{a}} = \tan^{-1}\sqrt{x}+\tan^{-1}\sqrt{a}$

$\therefore \dfrac{d}{dx}\{\tan^{-1}\dfrac{\sqrt{x}+\sqrt{a}}{1-\sqrt{ax}}\}$

$= \dfrac{d}{dx}(\tan^{-1}\sqrt{x}) + \dfrac{d}{dx}(\tan^{-1}\sqrt{a})$

$= \dfrac{1}{1+(\sqrt{x})^2}\dfrac{d}{dx}(\sqrt{x})+0$

$= \dfrac{1}{1+x}\cdot\dfrac{1}{2\sqrt{x}} = \dfrac{1}{2\sqrt{x}(1+x)}$

14(a) ধরি, $y=\tan^{-1}\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$ এবং

$x^2=\cos\theta$. তাহলে, $\theta=\cos^{-1}x^2$ এবং

$y = \tan^{-1}\dfrac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}$

$= \tan^{-1}\dfrac{\sqrt{2\cos^2(\theta/2)}-\sqrt{2\sin^2(\theta/2)}}{\sqrt{2\cos^2(\theta/2)}+\sqrt{2\sin^2(\theta/2)}}$

$= \tan^{-1}\dfrac{\sqrt{2}\{\cos(\theta/2)-\sin(\theta/2)\}}{\sqrt{2}\{\cos(\theta/2)+\sin(\theta/2)\}}$

$= \tan^{-1}\dfrac{\cos(\theta/2)\{1-\tan(\theta/2)\}}{\cos(\theta/2)\{1+\tan(\theta/2)\}}$

$= \tan^{-1}\dfrac{1-\tan(\theta/2)}{1+\tan(\theta/2)} = \tan^{-1}(1)-\tan^{-1}\tan\dfrac{\theta}{2}$

$= \dfrac{\pi}{4}-\dfrac{\theta}{2} = \dfrac{\pi}{4}-\dfrac{1}{2}\tan^{-1}x^2$

$\dfrac{dy}{dx} = 0-\dfrac{1}{2}\{-\dfrac{1}{1+(x^2)^2}\}(2x) = \dfrac{x}{\sqrt{1-x^4}}$

14(b) ধরি, $y=\sec^{-1}\dfrac{1}{2x^2-1}$ এবং $x=\cos\Theta$

তাহলে, $\theta=\cos^{-1}x$ এবং

$y = \sec^{-1}\dfrac{1}{2\cos^2\theta-1} = \sec^{-1}\dfrac{1}{\cos 2\theta}$

$= \sec^{-1}\sec 2\theta = 2\Theta = 2\cos^{-1}x$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(2\cos^{-1}x) = \dfrac{-2}{\sqrt{1-x^2}}$ (Ans.)

14(c) $\dfrac{d}{dx}\{\sin^{-1}(\tan^{-1}x)\}$ [সি.'০১]

$= \dfrac{1}{\sqrt{1-(\tan^{-1}x)^2}}\dfrac{d}{dx}(\tan^{-1}x)$

$= \dfrac{1}{\sqrt{1-(\tan^{-1}x)^2}}\dfrac{1}{1+x^2}$

$= \dfrac{1}{(1+x^2)\sqrt{1-(\tan^{-1}x)^2}}$ (Ans.)

14(d) $\tan^{-1}\dfrac{\cos x-\sin x}{\cos x+\sin x}$ [প্র.ভ.প. '০৫]

$= \tan^{-1}\dfrac{\cos x(1-\tan x)}{\cos x(1+\tan x)} = \tan^{-1}\dfrac{1-\tan x}{1+1.\tan x}$

$= \tan^{-1}1-\tan^{-1}(\tan x) = \dfrac{\pi}{4}-x$

$\therefore \dfrac{d}{dx}\{\tan^{-1}\dfrac{\cos x-\sin x}{\cos x+\sin x}\} = \dfrac{d}{dx}(\dfrac{\pi}{4}-x)$

$= 0-1 = -1$ ((Ans.)

$\dfrac{dy}{dx}$ নির্ণয় কর :

15(a) $x=a(\theta-\sin\theta), y=a(1+\cos\theta)$ [প্র.ভ.প. '০৬]

$\dfrac{dx}{d\theta} = \dfrac{d}{d\theta}\{a(\theta-\sin\theta)\} = a(1-\cos\theta)$

$\dfrac{dy}{d\theta} = \dfrac{d}{d\theta}\{a(1+\cos\theta)\} = a(0-\sin\theta)$

$\dfrac{dy}{dx} = \dfrac{dy}{d\theta}\times\dfrac{d\theta}{dx} = \dfrac{-a\sin\theta}{a(1-\cos\theta)}$

$= \dfrac{-2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{2\sin^2\dfrac{\theta}{2}} = -\cot\dfrac{\theta}{2}$

15(b) $\dfrac{d}{dx}(\sin x)^{\ln x} = (\sin x)^{\ln x}$

$$[\ln x\frac{d}{dx}\{\ln(\sin x)\}+\ln(\sin x)\frac{d}{dx}(\ln x)]$$

$$= (\sin x)^{\ln x}[\ln x\frac{1}{\sin x}.\cos x+\ln(\sin x).\frac{1}{x}]$$

$$= (\sin x)^{\ln x}[\ln x.\cot x+\frac{\ln(\sin x)}{x}]$$

15(c) $\frac{d}{dx}(\sin x)^{\tan x} = (\sin x)^{\tan x}$

$$[\tan x\frac{d}{dx}\{\ln(\sin x)\}+\ln(\sin x)\frac{d}{dx}(\tan x)]$$

$$=(\sin x)^{\tan x}[\frac{\sin x}{\cos x}\frac{\cos x}{\sin x}+\ln(\sin x).\sec^2 x]$$

$$= (\sin x)^{\tan x}[1+\sec^2 x.\ln(\sin x)]$$

15(d) $\frac{d}{dx}(\tan x)^{\ln x} = (\tan x)^{\ln x}$

$$[\ln x\frac{d}{dx}\{\ln(\tan x)\}+\ln(\tan x)\frac{d}{dx}(\ln x)]$$

$$= (\tan x)^{\ln x}[\ln x\frac{1}{\tan x}\sec^2 x+\ln(\tan x).\frac{1}{x}]$$

$$= (\tan x)^{\ln x}[\ln x\frac{\cos x}{\sin x}.\frac{1}{\cos^2 x}+\frac{\ln(\tan x)}{x}]$$

$$= (\tan x)^{\ln x}[\ln x\frac{2}{2\sin x\cos x}+\frac{\ln(\tan x)}{x}]$$

$$= (\tan x)^{\ln x}[2\ln x.\operatorname{cosec}2x+\frac{\ln(\tan x)}{x}]$$

15(e) $\frac{d}{dx}(\ln x)^{\ln x} =(\ln x)^{\ln x}$

$$[\ln x\frac{d}{dx}\{\ln(\ln x)\}+\ln(\ln x)\frac{d}{dx}(\ln x)]$$

$$= (\ln x)^{\ln x}[\ln x\frac{1}{\ln x}.\frac{1}{x}+\ln(\ln x).\frac{1}{x}]$$

$$= \frac{1}{x}(\ln x)^{\ln x}[1+\ln(\ln x)]$$

15.(f) $\frac{d}{dx}(\ln x)^{\tan^{-1}x} = (\ln x)^{\tan^{-1}x}$

$$[\tan^{-1}x\frac{d}{dx}\{\ln(\ln x)\}+\ln(\ln x)\frac{d}{dx}(\tan^{-1}x)]$$

$$= (\ln x)^{\tan^{-1}x}[\tan^{-1}x\frac{1}{\ln x}.\frac{1}{x}+\frac{\ln(\ln x)}{1+x^2}]$$

$$= (\ln x)^{\tan^{-1}x}[\frac{\tan^{-1}x}{x\ln x}+\frac{\ln(\ln x)}{1+x^2}]$$

(g) $\frac{d}{dx}(\tan x)^{\cos^{-1}x} = (\tan x)^{\cos^{-1}x}$

$$[\cos^{-1}x\frac{d}{dx}\{\ln(\tan x)\}+\ln(\tan x)\frac{d}{dx}(\cos^{-1}x)]$$

$$= (\tan x)^{\cos^{-1}x}[\frac{\sec^2 x.\cos^{-1}x}{\tan x}-\frac{\ln(\tan x)}{\sqrt{1-x^2}}]$$

(h) $(\sin^{-1}x)^{\ln x}$ **[প্র.ভ.প. '১৬]**

$$\frac{d}{dx}(\sin^{-1}x)^{\ln x}= (\sin^{-1}x)^{\ln x}$$

$$[\ln x\frac{d}{dx}\{\ln(\sin^{-1}x)\}+\ln(\sin^{-1}x)\frac{d}{dx}(\ln x)]$$

$$=(\sin^{-1}x)^{\ln x}[\frac{\ln x}{\sin^{-1}x}\frac{1}{\sqrt{1-x^2}}+\frac{\ln(\sin^{-1}x)}{x}]$$

$$= (\sin^{-1}x)^{\ln x}[\frac{\ln x}{\sqrt{1-x^2}\sin^{-1}x}+\frac{\ln(\sin^{-1}x)}{x}]$$

16.(a) $\frac{d}{dx}(x^x+x^{1/x})=\frac{d}{dx}(x^x)+\frac{d}{dx}(x^{1/x})$

$$= x^x\{x\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(x)\}+$$

$$x^{1/x}\{\frac{1}{x}\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}(\frac{1}{x})\}$$

$$=x^x\{x.\frac{1}{x}+\ln x.1\}+x^{1/x}\{\frac{1}{x}.\frac{1}{x}+\ln x.(-\frac{1}{x^2})\}$$

$$= x^x(1+\ln x)+x^{1/x}.\frac{1}{x^2}(1-\ln x)$$

$$= x^x(1+\ln x)+x^{\frac{1}{x}-2}(1-\ln x)$$

16(b) $\frac{d}{dx}(x^x.\ x^{\cos^{-1}x})$

$$= x^x\frac{d}{dx}(x^{\cos^{-1}x})+x^{\cos^{-1}x}\frac{d}{dx}(x^x)$$

$$= x^x.x^{\cos^{-1}x}[\cos^{-1}x\frac{d}{dx}(\ln x)$$

$+\ln x\dfrac{d}{dx}(\cos^{-1}x)]+x^{\cos^{-1}x}.x^x[x\dfrac{d}{dx}(\ln x)$

$+\ln x\dfrac{d}{dx}(x)]$

$= x^x.x^{\cos^{-1}x}[\dfrac{\cos^{-1}x}{x}+\dfrac{-\ln x}{\sqrt{1-x^2}}]$

$+x^{\cos^{-1}x}.x^x[x.\dfrac{1}{x}+\ln x.1]$

$= x^x.x^{\cos^{-1}x}[\dfrac{\cos^{-1}x}{x}-\dfrac{\ln x}{\sqrt{1-x^2}}+1+lnx]$

17(a) $x=y.\ln(xy)\Rightarrow\dfrac{x}{y}=\ln x+\ln y$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{y.1-x.\frac{dy}{dx}}{y^2}=\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}$$

$\Rightarrow xy-x^2\dfrac{dy}{dx}=y^2+xy\dfrac{dy}{dx}$

$\Rightarrow\ y(x-y)=x(x+y)\dfrac{dy}{dx}$

$$\frac{dy}{dx}=\frac{y(x-y)}{x(x+y)}$$

17(b) $y=\cot(x+y)\Rightarrow\cot^{-1}y=x+y$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$-\frac{1}{1+y^2}\frac{dy}{dx}=1+\frac{dy}{dx}$$

$\Rightarrow(-\dfrac{1}{1+y^2}-1)\dfrac{dy}{dx}=1$

$\Rightarrow-\dfrac{1+1+y^2}{1+y^2}\dfrac{dy}{dx}=1$

$$\frac{dy}{dx}=-\frac{1+y^2}{2+y^2}\quad\text{(Ans.)}$$

17(c) $y=\tan(x+y)$ **[প্র.ভ.প. '৮৯]**

$\Rightarrow\tan^{-1}y=x+y$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{1+y^2}\frac{dy}{dx}=1+\frac{dy}{dx}$$

$\Rightarrow(\dfrac{1}{1+y^2}-1)\dfrac{dy}{dx}=1$

$\Rightarrow\dfrac{1-1-y^2}{1+y^2}\dfrac{dy}{dx}=1\ \therefore\dfrac{dy}{dx}=-\dfrac{1+y^2}{y^2}$

17(d) $x^2+y^2=\sin(xy)$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$2x+2y\frac{dy}{dx}=\cos(xy)\ (x\frac{dy}{dx}+y)$$

$\Rightarrow\{2y-x\cos(xy)\}\dfrac{dy}{dx}=y\cos(xy)-2x$

$$\frac{dy}{dx}=\frac{y\cos(xy)-2x}{2y-x\cos(xy)}$$

(e) $\cos y=x\cos(a+y)\Rightarrow x=\dfrac{\cos y}{\cos(a+y)}$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$1=\frac{\cos(a+y)(-\sin y)\frac{dy}{dx}-\cos y\{-\sin(a+y)\}\frac{dy}{dx}}{\cos^2(a+y)}$$

$$1=\frac{\{\sin(a+y)\cos y-\cos(a+y)\sin y\}\frac{dy}{dx}}{\cos^2(a+y)}$$

$\cos^2(a+y)=\sin(a+y-y)\dfrac{dy}{dx}$

$$\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}\quad\text{(Ans.)}$$

17(f) $e^{2x}+5y^3=3\cos(xy)$ **[প্র.ভ.প. '৯৫]**

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$e^{2x}.2+15y^2\frac{dy}{dx}=3\{-\sin(xy)\}\frac{d}{dx}(xy)$$

$\Rightarrow 2e^{2x}+15y^2\dfrac{dy}{dx}=-3\sin(xy)(x\dfrac{dy}{dx}+y)$

$\Rightarrow\{15y^2+3x\sin(xy)\}\dfrac{dy}{dx}$

$=2e^{2x}+3y\sin(xy)$

$$\frac{dy}{dx}=\frac{2e^{2x}+3y\sin(xy)}{15y^2+3x\sin(xy)}$$

18(a) $y=x^y$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dx}=x^y[y\frac{d}{dx}(\ln x)+\ln x\frac{dy}{dx}]$$

$$\Rightarrow \frac{dy}{dx}=y[\frac{y}{x}+\ln x\frac{dy}{dx}] \quad [\because x^y=y]$$

$$\Rightarrow (1-y\ln x)\frac{dy}{dx}=\frac{y^2}{x}$$

$$\frac{dy}{dx}=\frac{y^2}{x(1-y\ln x)} \text{ (Ans.)}$$

18(b) $x^y y^x=1$ **[প্র.ভ.প. '০২]**

$$y\ln x + x\ln y=0$$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$y\frac{1}{x}+\ln x\frac{dy}{dx}+x.\frac{1}{y}\frac{dy}{dx}+\ln y=0$$

$$\Rightarrow y^2+xy\ln x\frac{dy}{dx}+x^2\frac{dy}{dx}+xy\ln y=0$$

$$\Rightarrow (xy\ln x+x^2)\frac{dy}{dx}=-(xy\ln y+y^2)$$

$$\frac{dy}{dx}=-\frac{y(x\ln y+y)}{x(y\ln x+x)}$$

18(c) $(\sin x)^{\cos y}+(\cos x)^{\sin y}=a$

উভয় পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$(\sin x)^{\cos y}[\cos y\frac{d}{dx}\{\ln(\sin x)\}+\ln(\sin x)$$

$$\frac{d}{dx}(\cos y)]+(\cos x)^{\sin y}[\sin y\frac{d}{dx}\{\ln(\cos x)\}$$

$$+\ln(\cos x)\frac{d}{dx}(\sin y)]=0$$

$$\Rightarrow (\sin x)^{\cos y}[\cos y\cot x+\ln(\sin x)$$

$$(-\sin y)\frac{dy}{dx}]+(\cos x)^{\sin y}[\sin y(-\tan x)+$$

$$\ln(\cos x).\cos y\frac{dy}{dx}]=0$$

$$\Rightarrow \{(\cos x)^{\sin y}\ln(\cos x).\cos y$$

$$-(\sin x)^{\cos y}\ln(\sin x)\sin y\}\frac{dy}{dx}=(\cos x)^{\sin y}$$

$$\sin y\tan x-(\sin x)^{\cos y}\cos y\cot x$$

$$\therefore \frac{dy}{dx}=$$

$$\frac{(\cos x)^{\sin y}\sin y\tan x-(\sin x)^{\cos y}\cos y\cot x}{(\cos x)^{\sin y}\ln(\cos x)\cos y-(\sin x)^{\cos y}\ln(\sin x)\sin y}$$

19. $y=\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}$ **হলে, দেখাও যে,**

$$\frac{dy}{dx}=-\frac{1}{2\sqrt{1-x^2}}$$

প্রমাণ : ধরি, $x=\cos\theta\Rightarrow\theta=\cos^{-1}x$

$$y=\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos\frac{\theta}{2}}}$$

$$=\tan^{-1}\tan\frac{\theta}{2}=\frac{\theta}{2}=\frac{1}{2}\cos^{-1}x$$

$$\frac{dy}{dx}=\frac{1}{2}\frac{-1}{\sqrt{1-x^2}}=-\frac{1}{2\sqrt{1-x^2}}$$

20. $x=1$ বিন্দুতে $y=x^2$ ফাংশনের অন্তরক আকার সমীকরণ থেকে dy এবং δy নির্ণয় কর যখন $dx=\delta x=2$.

সমাধান : ধরি, $f(x)=y=x^2$

$$\frac{dy}{dx}=2x\Rightarrow dy=2x\,dx$$

$$\Rightarrow dy=2\times1\times2,\ [\because x=1, dx=2]$$

$$\Rightarrow dy=4$$

আবার, $\delta y=f(x+\delta x)-f(x)$

$$=f(1+2)-f(1)=f(3)-f(1)$$

$$=3^2-1^2=9-1=8.$$

21. $x=3$ বিন্দুতে $y=\dfrac{x^2}{3}+1$ ফাংশনের অন্তরক আকার সমীকরণ থেকে dy এবং δy নির্ণয় কর যখন $dx=\delta x=3$.

সমাধান : ধরি, $f(x) = y = \frac{x^2}{3} + 1$

$$\frac{dy}{dx} = \frac{2}{3}x \Rightarrow dy = \frac{2}{3}x\ dx$$

$$\Rightarrow dy = \frac{2}{3} \times 3 \times 3,\ [\because x = 3, dx = 3]$$

$$dy = 6$$

আবার, $\delta y = f(x + \delta x) - f(x)$

$$= f(3 + 3) - f(3) = f(6) - f(3)$$

$$= (\frac{6^2}{3} + 1) - (\frac{3^2}{3} + 1)$$

$$= 12 - 3 = 9$$

ভর্তি পরীক্ষার MCQ :

1. $y = x^{-\frac{1}{x}}$ হলে $\frac{dy}{dx}$ এর মান- **[BUET 07-08]**

Sol^n : $\frac{dy}{dx} = x^{-\frac{1}{x}}[-\frac{1}{x}.\frac{1}{x} + \ln x(+\frac{1}{x^2})]$

$$= x^{-\frac{1}{x}}.\frac{1}{x^2}(\ln x - 1) = \frac{1}{x^{2+\frac{1}{x}}}(\ln x - 1)$$

SHIFT d/dx ALPHA X ALPHA X

SHIFT ∫dx ALPHA) ^ − 1 ab/c ALPHA)

X

÷ 1 ab/c 2) = 27.09

Option গুলোতে x = ½ বসালে $\frac{1}{x^{2+\frac{1}{x}}}(\ln x - 1)$

= 27.09 হয়।

2. $\frac{d}{dx}(\log_x e) = ?$ **[DU 08-09]**

Sol^n : $\frac{d}{dx}(\log_x e) = \frac{d}{dx}(\frac{1}{\ln x}) = -\frac{1}{x(\ln x)^2}$

3. $\frac{d}{dx}\{\ln(x + \sqrt{x^2 + a^2}) = ?$ **[DU 07-08]**

Sol^n : $\frac{d}{dx}\{\ln(x + \sqrt{x^2 + a^2})$

$$= \frac{1}{x + \sqrt{x^2 + a^2}}.(1 + \frac{2x}{2\sqrt{x^2 + a^2}}) = \frac{1}{\sqrt{x^2 + a^2}}$$

4. $y = \sqrt{\sec x}$ হলে, $\frac{dy}{dx} = ?$ **[DU 00-01]**

Sol^n : $\frac{dy}{dx} = \frac{1}{2\sqrt{\sec x}}.\sec x \tan x$

$$= \frac{\sqrt{\sec x}\tan x}{2} = \frac{y}{2}\tan x$$

5. $y = \cos\sqrt{x}$ হলে, $\frac{dy}{dx} = ?$ **[DU 03-04]**

Sol^n : $\frac{dy}{dx} = -\sin\sqrt{x}.\frac{1}{2\sqrt{x}} = -\frac{\sin\sqrt{x}}{2\sqrt{x}}$

6. $f(x) = \sqrt{1 - \sqrt{x}}$ হলে, $\frac{df}{dx} = ?$ **[DU 01-02]**

Sol^n : $\frac{df}{dx} = \frac{1}{2\sqrt{1 - \sqrt{x}}}\frac{-1}{2\sqrt{x}} = \frac{-1}{4\sqrt{x}\sqrt{1 - \sqrt{x}}}$

7. $y = \log_e(2x)^{1/3}$ হলে, $\frac{dy}{dx} = ?$ **[DU 98-99]**

Sol^n : $\frac{dy}{dx} = \frac{1}{3}\frac{d}{dx}\{\log_e(2x)\} = \frac{1}{3.2x}(2) = \frac{1}{3x}$

8. $y = \sin^{-1}\sin(x + 1)$ হলে, $\frac{dy}{dx} = ?$

[DU 97-98 ; SU 06-07]

Sol^n : $y = \sin^{-1}\sin(x + 1) = x + 1 \therefore \frac{dy}{dx} = 1$

9. $y = \frac{x}{\sqrt{x^2 + 1}}$ হলে, $\frac{dy}{dx} = ?$ **[NU 07-08]**

Sol^n : $\frac{dy}{dx} = \frac{\sqrt{x^2 + 1}.1 - x\frac{1}{2\sqrt{x^2 + 1}}.2x}{(\sqrt{x^2 + 1})^2}$

$$= \frac{x^2 + 1 - x^2}{(x^2 + 1)\sqrt{x^2 + 1}} = \frac{1}{(x^2 + 1)^{3/2}}$$

10. $\frac{d}{dx}(a^x) = ?$ **[KU,RU07-08;IU 02-03]**

Sol^n : $\frac{d}{dx}(a^x) = a^x \ln a$

11. $\frac{d}{dx}(\log_a m^2) = ?$ **[CU 07-08]**

Sol^n : $\frac{d}{dx}(\log_a m^2) = \theta$