

$$1(a) \sin(-1230^\circ) - \cos\{(2n+1)\pi + \frac{\pi}{3}\}$$

$$= -\sin 1230^\circ - \cos\{2n\pi + (\pi + \frac{\pi}{3})\}$$

$$= -\sin(3.360^\circ + 150^\circ) - \cos(\pi + \frac{\pi}{3})$$

$$= -\sin 150^\circ - (-\cos \frac{\pi}{3})$$

$$= -\sin(180^\circ - 30^\circ) + \cos \frac{\pi}{3}$$

$$= -\sin 30^\circ + \cos \frac{\pi}{3} = -\frac{1}{2} + \frac{1}{2} = 0 \text{ (Ans.)}$$

$$1(b) \sin 780^\circ \cos 390^\circ +$$

$$\sin(-330^\circ) \cos(-300^\circ) \quad [\text{চ. '০১}]$$

$$= \sin 780^\circ \cos 390^\circ - \sin 330^\circ \cos 300^\circ$$

$$= \sin(2.360^\circ + 60^\circ) \cos(360^\circ + 30^\circ) -$$

$$\sin(360^\circ - 30^\circ) \cos(360^\circ - 60^\circ)$$

$$= \sin 60^\circ \cos 30^\circ - (-\sin 30^\circ) \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \text{ (Ans.)}$$

2. মান নির্ণয় কর :

$$(a) \sin^2 \frac{\pi}{7} + \sin^2 \frac{5\pi}{14} + \sin^2 \frac{8\pi}{7} + \sin^2 \frac{9\pi}{14}$$

$$[\text{চ. '০২; সি. '০৯; মা.বো. '০৯; ব. '১০; য. '১১}]$$

$$= \sin^2 \frac{\pi}{7} + \sin^2(\frac{\pi}{2} - \frac{\pi}{7}) + \sin^2(\pi + \frac{\pi}{7}) +$$

$$\sin^2(\frac{\pi}{2} + \frac{\pi}{7})$$

$$= \sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7}$$

$$= 2(\sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7}) = 2.1 = 2 \text{ (Ans.)}$$

$$2(b) \sin^2 \frac{\pi}{12} + \sin^2 \frac{3\pi}{12} + \sin^2 \frac{5\pi}{12} + \sin^2 \frac{7\pi}{12} +$$

$$\sin^2 \frac{9\pi}{12} + \sin^2 \frac{11\pi}{12}$$

$$= \sin^2 \frac{\pi}{12} + \sin^2 \frac{3\pi}{12} + \sin^2 \frac{5\pi}{12} + \sin^2(\frac{\pi}{2} + \frac{\pi}{12})$$

$$+ \sin^2(\frac{\pi}{2} + \frac{3\pi}{12}) + \sin^2(\frac{\pi}{2} + \frac{5\pi}{12})$$

$$= \sin^2 \frac{\pi}{12} + \sin^2 \frac{3\pi}{12} + \sin^2 \frac{5\pi}{12} + \cos^2 \frac{\pi}{12}$$

$$+ \cos^2 \frac{3\pi}{12} + \cos^2 \frac{5\pi}{12}$$

$$= (\sin^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{12}) + (\sin^2 \frac{3\pi}{12} + \cos^2 \frac{3\pi}{12})$$

$$+ (\sin^2 \frac{5\pi}{12} + \cos^2 \frac{5\pi}{12})$$

$$= 1 + 1 + 1 = 3 \text{ (Ans.)}$$

$$2.(c) \sin^2 \frac{17\pi}{18} + \sin^2 \frac{5\pi}{8} + \cos^2 \frac{37\pi}{18} + \cos^2 \frac{3\pi}{8}$$

$$= \sin^2(\pi - \frac{\pi}{18}) + \sin^2(\pi - \frac{3\pi}{8}) +$$

$$\cos^2(2\pi + \frac{\pi}{18}) + \cos^2 \frac{3\pi}{8}$$

$$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{3\pi}{8} + \cos^2 \frac{\pi}{18} + \cos^2 \frac{3\pi}{8}$$

$$= (\sin^2 \frac{\pi}{18} + \cos^2 \frac{\pi}{18}) + (\sin^2 \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8})$$

$$= 1 + 1 = 2 \text{ (Ans.)}$$

$$3.(a) \sec^2 \frac{14\pi}{17} - \sec^2 \frac{39\pi}{17} + \cot^2 \frac{41\pi}{34} - \cot^2 \frac{23\pi}{34}$$

$$= \sec^2(\pi - \frac{3\pi}{17}) - \sec^2(2\pi + \frac{5\pi}{17}) +$$

$$\cot^2(\pi + \frac{7\pi}{34}) - \cot^2(\pi - \frac{11\pi}{34})$$

$$= \sec^2 \frac{3\pi}{17} - \sec^2 \frac{5\pi}{17} + \cot^2 \frac{7\pi}{34} - \cot^2 \frac{11\pi}{34}$$

$$= \sec^2 \frac{3\pi}{17} - \sec^2 \frac{5\pi}{17} + \cot^2(\frac{\pi}{2} - \frac{5\pi}{17}) -$$

$$\cot^2(\frac{\pi}{2} - \frac{3\pi}{17})$$

$$= \sec^2 \frac{3\pi}{17} - \sec^2 \frac{5\pi}{17} + \tan^2 \frac{5\pi}{17} - \tan^2 \frac{3\pi}{17}$$

$$= (\sec^2 \frac{3\pi}{17} - \tan^2 \frac{3\pi}{17}) - (\sec^2 \frac{5\pi}{17} - \tan^2 \frac{5\pi}{17})$$

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$$= 1 - 1 = 0 \text{ (Ans.)}$$

$$\begin{aligned} 3(b) & \tan 15^\circ + \tan 45^\circ + \tan 75^\circ + \dots + \tan 165^\circ \\ &= \tan 15^\circ + \tan 45^\circ + \tan 75^\circ + \tan 105^\circ + \\ & \quad \tan 135^\circ + \tan 165^\circ \\ &= \tan 15^\circ + \tan 45^\circ + \tan(90^\circ - 15^\circ) + \\ & \quad \tan(90^\circ + 15^\circ) + \tan(180^\circ - 45^\circ) + \\ & \quad \tan(180^\circ - 15^\circ) \\ &= \tan 15^\circ + \tan 45^\circ + \cot 15^\circ - \cot 15^\circ - \\ & \quad \tan 45^\circ - \tan 15^\circ = 0 \text{ (Ans.)} \end{aligned}$$

$$\begin{aligned} 3(c) & \cos^2 15^\circ + \cos^2 25^\circ + \cos^2 35^\circ + \dots + \cos^2 75^\circ \\ &= \cos^2 15^\circ + \cos^2 25^\circ + \cos^2 35^\circ + \cos^2 45^\circ \\ & \quad + \cos^2 55^\circ + \cos^2 65^\circ + \cos^2 75^\circ \\ &= \cos^2 15^\circ + \cos^2 25^\circ + \cos^2 35^\circ + \left(\frac{1}{\sqrt{2}}\right)^2 \\ & \quad + \cos^2(90^\circ - 35^\circ) + \cos^2(90^\circ - 25^\circ) + \cos^2(90^\circ - 15^\circ) \\ &= \cos^2 15^\circ + \cos^2 25^\circ + \cos^2 35^\circ + \frac{1}{2} + \\ & \quad \sin^2 35^\circ + \sin^2 25^\circ + \sin^2 15^\circ \\ &= (\sin^2 5^\circ + \cos^2 5^\circ) + (\sin^2 25^\circ + \cos^2 25^\circ) \\ & \quad + (\sin^2 35^\circ + \cos^2 35^\circ) + \frac{1}{2} \\ &= 1 + 1 + 1 + \frac{1}{2} = 3 + \frac{1}{2} = \frac{7}{2} \text{ (Ans.)} \end{aligned}$$

4(a) প্রমাণ : দেওয়া আছে, [দি.'১৪; '১২; চ.'০৯]

$$\sin \theta = \frac{5}{13} \text{ এবং } \frac{\pi}{2} < \theta < \pi$$

$$\begin{aligned} \operatorname{cosec} \theta &= \frac{13}{5}, \cos \theta = -\sqrt{1 - \sin^2 \theta} \\ &= -\sqrt{1 - \frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13} \end{aligned}$$

$$\sec \theta = -\frac{13}{12} \text{ এবং}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{5}{13} \times \left(-\frac{13}{12}\right) = -\frac{5}{12}$$

$$\Rightarrow \cot \theta = -\frac{12}{5}$$

$$\text{এখন, } \frac{\tan \theta + \sec(-\theta)}{\cot \theta + \operatorname{cosec}(-\theta)} = \frac{\tan \theta + \sec \theta}{\cot \theta - \operatorname{cosec} \theta}$$

$$\begin{aligned} &= \frac{-\frac{5}{12} + \frac{-13}{12}}{-\frac{12}{5} - \frac{13}{5}} = \frac{-\frac{5+13}{12}}{-\frac{12+13}{5}} \\ &= \frac{-\frac{18}{12}}{-\frac{25}{5}} = \frac{3}{2} \times \frac{1}{5} = \frac{3}{10} \end{aligned}$$

$$\begin{aligned} &= \left(-\frac{18}{12}\right) \times \left(-\frac{5}{25}\right) = \frac{3}{2} \times \frac{1}{5} = \frac{3}{10} \\ \therefore \frac{\tan \theta + \sec(-\theta)}{\cot \theta + \operatorname{cosec}(-\theta)} &= \frac{3}{10} \end{aligned}$$

$$4(b) \text{ যেহেতু } \cot \theta = \frac{3}{4} \Rightarrow \tan \theta = \frac{4}{3} \text{ এবং } \cos \theta$$

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$$\therefore \sec \theta = -\sqrt{1 + \tan^2 \theta} = -\sqrt{1 + \frac{16}{9}}$$

$$= -\sqrt{\frac{25}{9}} = -\frac{5}{3}$$

$$\therefore \cos \theta = -\frac{3}{5} \text{ এবং}$$

$$\sin \theta = \tan \theta \cos \theta = \frac{4}{3} \times \left(-\frac{3}{5}\right) = -\frac{4}{5}$$

$$\therefore \operatorname{cosec} \theta = -\frac{5}{4}$$

$$\text{এখন, } \frac{\cot(-\theta) + \operatorname{cosec} \theta}{\cos \theta + \sin(-\theta)} = \frac{-\cot \theta + \operatorname{cosec} \theta}{\cos \theta - \sin \theta}$$

$$\begin{aligned} &= \frac{-\frac{3}{4} + \left(-\frac{5}{4}\right)}{-\frac{3}{5} - \frac{-4}{5}} = \frac{-\frac{3+5}{4}}{\frac{-3+4}{5}} = \frac{-\frac{8}{4}}{\frac{1}{5}} = -8 \times 5 = -40 \\ &= -\frac{40}{1} = -40 \text{ (Ans.)} \end{aligned}$$

5. সমাধান :

$$(a) \sin x + \sin(\pi + x) + \sin(2\pi + x) + \dots$$

(n+1)তম পদ পর্যন্ত

$$= \sin x - \sin x + \sin x - \sin x + \dots$$

(n+1) তম পদ পর্যন্ত

$n = 1$  হলে,  $(1 + 1)$  বা ২য় পদ পর্যন্ত যোগফল  
 $= \sin x - \sin x = 0$

$n = 3$  হলে,  $(3 + 1)$  বা ৪র্থ পদ পর্যন্ত

যোগফল  $= \sin x - \sin x + \sin x - \sin x = 0$

তদুপ,  $n$  যেকোন বিজোড় সংখ্যা হলে নির্ণেয় যোগফল  $= 0$

আবার,  $n = 2$  হলে  $(2 + 1)$  বা ৩য় পদ পর্যন্ত যোগফল

$$= \sin x - \sin x + \sin x = \sin x$$

$n = 4$  হলে,  $(4 + 1)$  বা ৫ম পদ পর্যন্ত যোগফল

$$= \sin x - \sin x + \sin x - \sin x + \sin x$$

$$= \sin x$$

তদুপ,  $n$  যেকোন জোড় সংখ্যা হলে নির্ণেয় যোগফল  $= \sin x$

$$5(b) \tan \theta + \tan(\pi + \theta) + \tan(2\pi + \theta) + \tan(n\pi + \theta)$$

$$= \tan \theta + \tan \theta + \tan \theta + \dots n \text{ তম পদ পর্যন্ত}$$

$$= (n + 1) \tan \theta \text{ (Ans.)}$$

$$6(a) \text{ দেওয়া আছে, } \theta = \frac{\pi}{20} \Rightarrow \frac{\pi}{2} = 10\theta$$

$$\text{L.H.S.} = \cot \theta \cot 3\theta \cot 5\theta \cot 7\theta \cot 9\theta \cot 11\theta \cot 13\theta \cot 15\theta \cot 17\theta \cot 19\theta$$

$$= \cot \theta \cot 3\theta \cot 5\theta \cot 7\theta \cot 9\theta$$

$$\cot(10\theta + \theta) \cot(10\theta + 3\theta)$$

$$\cot(10\theta + 5\theta) \cot(10\theta + 7\theta)$$

$$\cot(10\theta + 9\theta)$$

$$= \cot \theta \cot 3\theta \cot 5\theta \cot 7\theta \cot 9\theta$$

$$\cot\left(\frac{\pi}{2} + \theta\right) \cot\left(\frac{\pi}{2} + 3\theta\right) \cot\left(\frac{\pi}{2} + 5\theta\right)$$

$$\cot\left(\frac{\pi}{2} + 7\theta\right) \cot\left(\frac{\pi}{2} + 9\theta\right)$$

$$= \frac{1}{\tan \theta \tan 3\theta \tan 5\theta \tan 7\theta \tan 9\theta} (-\tan \theta)$$

$$(-\tan 3\theta) (-\tan 5\theta) (-\tan 7\theta) (-\tan 9\theta)$$

$$= -1 = \text{R.H.S.}$$

$$6. (b) \text{ দেওয়া আছে, } \theta = \frac{\pi}{28} \Rightarrow \frac{\pi}{2} = 14\theta$$

$$\text{L.H.S.} = \tan \theta \tan 3\theta \tan 5\theta \tan 7\theta$$

$$\tan 9\theta \tan 11\theta \tan 13\theta$$

$$= \tan \theta \tan 3\theta \tan 5\theta \tan 7\theta$$

$$\tan(14\theta - 5\theta) \tan(14\theta - 3\theta)$$

$$\tan(14\theta - \theta)$$

$$= \frac{1}{\tan \theta \tan 3\theta \tan 5\theta} \tan \frac{\pi}{4}$$

$$\tan\left(\frac{\pi}{2} - 5\theta\right) \tan\left(\frac{\pi}{2} - 3\theta\right) \tan\left(\frac{\pi}{2} - \theta\right)$$

$$= \frac{1}{\tan \theta \tan 3\theta \tan 5\theta} .1. \tan 5\theta. \tan 3\theta. \tan \theta$$

$$= 1 = \text{R.H.S.}$$

$$6(c) \tan \theta. \tan 2\theta. \tan 3\theta. \dots \tan (2n-1)\theta$$

এখানে, পদসংখ্যা  $= 2n-1$ , যা বিজোড় সংখ্যা।

$$\frac{2n-1+1}{2} \text{ অর্থাৎ } n \text{ তম পদ মধ্যপদ।}$$

$$\therefore \text{ মধ্যপদ} = \tan n\theta = \tan \frac{\pi}{4} = 1 \quad [\because 4n\theta = \pi]$$

$$\tan \theta. \tan (2n-1)\theta = \tan \theta. \tan (2n\theta - \theta)$$

$$= \tan \theta. \tan \left(\frac{\pi}{2} - \theta\right) \quad [\because 4n\theta = \pi]$$

$$= \tan \theta. \cot \theta = 1$$

$$\tan 2\theta. \tan (2n-2)\theta = \tan 2\theta. \tan (2n\theta - 2\theta)$$

$$= \tan 2\theta. \tan \left(\frac{\pi}{2} - 2\theta\right)$$

$$= \tan 2\theta. \cot 2\theta = 1$$

$$\text{অনুরূপভাবে, } \tan 3\theta. \tan (2n-3)\theta = 1$$

$$\tan 4\theta. \tan (2n-4)\theta = 1, \dots \text{ ইত্যাদি।}$$

$$\text{অর্থাৎ, মধ্যপদ হতে সমদূরবর্তী পদ দুইটির গুণফল} = 1$$

$$\therefore \tan \theta. \tan 2\theta. \tan 3\theta. \dots \dots \tan (2n-1)\theta = 1$$

### অতিরিক্ত প্রশ্ন (সমাধানসহ)

1. মান নির্ণয় কর :

$$(a) \tan(-1590^\circ) = -\tan(1590^\circ)$$

$$= -\tan(4.360^\circ + 150^\circ) = -\tan 150^\circ$$

$$= -\tan(180^\circ - 30^\circ) = +\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$(b) \cos 420^\circ \sin(-300^\circ) - \sin 870^\circ \cos 570^\circ$$

$$= \cos 420^\circ (-\sin 300^\circ) - \sin 870^\circ \cos 570^\circ$$

$$= -\cos(360^\circ + 60^\circ) \sin(360^\circ - 60^\circ)$$

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$$\begin{aligned} & -\sin(2.360^\circ + 150^\circ) \cos(2.360^\circ - 150^\circ) \\ & = -\cos 60^\circ (-\sin 60^\circ) - \sin 150^\circ \cos 150^\circ \\ & = \cos 60^\circ \sin 60^\circ - \sin(180^\circ - 30^\circ) \\ & \quad \cos(180^\circ - 30^\circ) \end{aligned}$$

$$\begin{aligned} & = \cos 60^\circ \sin 60^\circ - \sin 30^\circ (-\cos 30^\circ) \\ & = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \text{ (Ans.)} \end{aligned}$$

$$\begin{aligned} 2. \quad & \cos^2 \frac{\pi}{24} + \cos^2 \frac{19\pi}{24} + \cos^2 \frac{31\pi}{24} + \cos^2 \frac{37\pi}{24} \\ & = \cos^2 \frac{\pi}{24} + \cos^2 \frac{19\pi}{24} + \cos^2 \left( \frac{\pi}{2} + \frac{19\pi}{24} \right) \\ & \quad + \cos^2 \left( 3 \cdot \frac{\pi}{2} + \frac{\pi}{24} \right) \end{aligned}$$

$$\begin{aligned} & = \cos^2 \frac{\pi}{24} + \cos^2 \frac{19\pi}{24} + \sin^2 \frac{\pi}{24} + \sin^2 \frac{19\pi}{24} \\ & = (\sin^2 \frac{\pi}{24} + \cos^2 \frac{\pi}{24}) + (\sin^2 \frac{19\pi}{24} + \cos^2 \frac{19\pi}{24}) \\ & = 1 + 1 = 2 \text{ (Ans.)} \end{aligned}$$

$$\begin{aligned} 3a) \quad & \cos^2 25^\circ + \cos^2 35^\circ + \cos^2 45^\circ + \cos^2 55^\circ + \cos^2 65^\circ \\ & = \cos^2 25^\circ + \cos^2 35^\circ + \left( \frac{1}{\sqrt{2}} \right)^2 + \cos^2(90^\circ - 35^\circ) + \cos^2(90^\circ - 25^\circ) \\ & = \cos^2 25^\circ + \cos^2 35^\circ + \frac{1}{2} + \sin^2 35^\circ + \sin^2 25^\circ \\ & = (\sin^2 25^\circ + \cos^2 25^\circ) + \frac{1}{2} + (\sin^2 25^\circ + \cos^2 25^\circ) \\ & = 1 + \frac{1}{2} + 1 = \frac{5}{2} \text{ (Ans.)} \end{aligned}$$

$$\begin{aligned} 3b) \quad & \sin^2 10^\circ + \sin^2 20^\circ + \sin^2 30^\circ + \sin^2 40^\circ + \sin^2 50^\circ + \sin^2 60^\circ \\ & + \sin^2 70^\circ + \sin^2 80^\circ \\ & = \sin^2 10^\circ + \sin^2 20^\circ + \sin^2 30^\circ + \end{aligned}$$

$$\begin{aligned} & \sin^2 40^\circ + \sin^2(90^\circ - 40^\circ) + \sin^2(90^\circ - 30^\circ) + \sin^2(90^\circ - 20^\circ) \\ & + \sin^2(90^\circ - 10^\circ) \\ & = \sin^2 10^\circ + \sin^2 20^\circ + \sin^2 30^\circ \\ & + \sin^2 40^\circ + \cos^2 40^\circ + \cos^2 30^\circ \\ & + \cos^2 20^\circ + \cos^2 10^\circ \\ & = (\sin^2 10^\circ + \cos^2 10^\circ) + (\sin^2 20^\circ + \cos^2 20^\circ) \\ & + (\sin^2 30^\circ + \cos^2 30^\circ) + (\sin^2 40^\circ + \cos^2 40^\circ) \\ & = 1 + 1 + 1 + 1 = 4 \text{ (Ans.)} \end{aligned}$$

$$4. \quad \tan \theta = \frac{3}{4} \text{ এবং } \cos \theta \text{ ঋণাত্মক হলে,}$$

$$\frac{\sin \theta + \cos \theta}{\sec \theta + \tan \theta} \text{ এর মান নির্ণয় কর।}$$

সমাধান : দেওয়া আছে,

$$\tan \theta = \frac{3}{4} \text{ এবং } \cos \theta \text{ ঋণাত্মক}$$

$$\therefore \sec \theta = -\sqrt{1 + \tan^2 \theta} = -\sqrt{1 + \frac{9}{16}}$$

$$= -\sqrt{\frac{25}{16}} = -\frac{5}{4} \therefore \cos \theta = -\frac{4}{5} \text{ এবং}$$

$$\sin \theta = \tan \theta \cos \theta = \frac{3}{4} \left( -\frac{4}{5} \right) = -\frac{3}{5}$$

$$\begin{aligned} \text{এখন, } \frac{\sin \theta + \cos \theta}{\sec \theta + \tan \theta} &= \frac{-\frac{3}{5} - \frac{4}{5}}{-\frac{5}{4} + \frac{3}{4}} \\ &= -\frac{3+4}{5} \times \frac{4}{-5+3} = -\frac{7}{5} \times \frac{4}{-2} = \frac{14}{5} \text{ (Ans.)} \end{aligned}$$

$$5. \quad \sin \theta = \frac{12}{13} \text{ এবং } 90^\circ < \theta < 180^\circ \text{ হলে}$$

$$\text{দেখাও যে, } \frac{\tan \theta + \sec(-\theta)}{\cot \theta + \operatorname{cosec}(-\theta)} = \frac{10}{3}$$

$$\text{প্রমাণ : যেহেতু } \sin \theta = \frac{12}{13} \Rightarrow \operatorname{cosec} \theta = \frac{13}{12}$$

$$\text{এবং } 90^\circ < \theta < 180^\circ,$$

$$\therefore \cos \theta = -\sqrt{1 - \sin^2 \theta}$$

$$= -\sqrt{1 - \frac{144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$$

$$\sec \theta = -\frac{13}{5}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{12}{13} \times \left(-\frac{13}{5}\right) = -\frac{12}{5}$$

$$\Rightarrow \cot \theta = -\frac{5}{12}$$

$$\text{এখন, } \frac{\tan \theta + \sec(-\theta)}{\cot \theta + \operatorname{cosec}(-\theta)} = \frac{\tan \theta + \sec \theta}{\cot \theta - \operatorname{cosec} \theta}$$

$$= \frac{-\frac{12}{5} - \frac{13}{5}}{-\frac{5}{12} - \frac{13}{12}} = \frac{-25}{5} \times \frac{12}{-5-13}$$

$$= 5 \times \frac{12}{18} = \frac{10}{3}$$

6. যোগফল নির্ণয় কর :  $\cos \theta + \cos (\pi + \theta) + \cos (2\pi + \theta) + \dots + \cos (n\pi + \theta)$

সমাধান:  $\cos \theta + \cos (\pi + \theta) + \cos (2\pi + \theta) + \dots + \cos (n\pi + \theta)$

$$= \cos \theta + \{-\cos \theta + \cos \theta - \cos \theta + \dots + (-1)^n \cos \theta\}$$

$$n = 2 \text{ হলে যোগফল} = \cos \theta + \{-\cos \theta + \cos \theta\} = \cos \theta$$

$$n = 4 \text{ হলে যোগফল} = \cos \theta + \{-\cos \theta + \cos \theta - \cos \theta + \cos \theta\} = \cos \theta$$

তদুপ,  $n$  যেকোন জোড় হলে নির্ণেয় যোগফল  $= \cos \theta$

$$n = 1 \text{ হলে যোগফল} = \cos \theta + (-\cos \theta) = 0$$

$$n = 3 \text{ হলে যোগফল} = \cos \theta + \{-\cos \theta + \cos \theta - \cos \theta\} = 0$$

তদুপ,  $n$  যেকোন বিজোড় হলে নির্ণেয় যোগফল  $= 0$

7.  $n \in \mathbb{Z}$  হলে,  $\sin \{ n\pi + (-1)^n \frac{\pi}{4} \}$  এর মান নির্ণয় কর।

সমাধান : (a)  $\sin \{ n\pi + (-1)^n \frac{\pi}{4} \}$

$n$  জোড় সংখ্যা হলে মনে করি,  $n = 2m$ , যেখানে  $m \in \mathbb{N}$ .

$$\therefore \sin \{ n\pi + (-1)^n \frac{\pi}{4} \}$$

$$= \sin \{ 2m\pi + (-1)^{2m} \frac{\pi}{4} \}$$

$$= \sin (2m\pi + \frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$n$  বিজোড় সংখ্যা হলে মনে করি,  $n = 2m + 1$ ;  $m \in \mathbb{N}$ .

$$\therefore \sin \{ n\pi + (-1)^n \frac{\pi}{4} \}$$

$$= \sin \{ (2m + 1)\pi + (-1)^{2m+1} \frac{\pi}{4} \}$$

$$= \sin \{ 2m\pi + (\pi - \frac{\pi}{4}) \}$$

$$= \sin (\pi - \frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ (Ans.)}$$

8. দেখাও যে,  $\tan \frac{\pi}{12} \tan \frac{5\pi}{12} \tan \frac{7\pi}{12} \tan \frac{11\pi}{12} = 1$

$$\text{প্রমাণ: } \tan \frac{\pi}{12} \tan \frac{5\pi}{12} \tan \frac{7\pi}{12} \tan \frac{11\pi}{12}$$

$$= \tan \frac{\pi}{12} \tan \frac{5\pi}{12} \tan (\frac{\pi}{2} - \frac{\pi}{12}) \tan (\frac{\pi}{2} - \frac{5\pi}{12})$$

$$= \tan \frac{\pi}{12} \tan \frac{5\pi}{12} \cot \frac{\pi}{12} \cot \frac{5\pi}{12}$$

$$= (\tan \frac{\pi}{12} \cdot \cot \frac{\pi}{12}) (\tan \frac{5\pi}{12} \cdot \cot \frac{5\pi}{12})$$

$$= 1 \cdot 1 = 1 \quad [\because \tan \theta \cdot \cot \theta = 1]$$

### প্রশ্নমালা VII B

1. মান নির্ণয় কর : (a)  $\tan 105^\circ$  (b)  $\cot 165^\circ$  (c)  $\operatorname{cosec} 165^\circ$

$$(a) \tan 105^\circ = \tan (60^\circ + 45^\circ)$$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1}$$

$$= \frac{(1 + \sqrt{3})^2}{(1 - \sqrt{3})(1 + \sqrt{3})} = \frac{1 + 2\sqrt{3} + 3}{1 - 3}$$

$$= \frac{2(\sqrt{3} + 2)}{-2} = -(\sqrt{3} + 2)$$