$= 2\sin(3x+0).(-1.\frac{3}{2}) = -3\sin 3x$

3(c) $\cos ax$ [রা.'০১]

মনে করি, $f(x) = \cos ax$.

$f(x+h) = \cos a(x+h) = \cos(ax+ah)$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{f(x)\} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\cos ax) = \lim_{h\to 0}\frac{\cos(ax+ah)-\cos ax}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}[2\sin\frac{ax+ah+ax}{2}\sin\frac{ax-ah-ax}{2}]$$

$$= 2\lim_{h\to 0}\sin(ax+\frac{ah}{2})\times -\lim_{h\to 0}\frac{\sin(ah/2)}{ah/2}\times\frac{a}{2}$$

$$= 2\sin(ax+0).(-1.\frac{a}{2}) = -a\sin ax$$

3(d) $\tan 2x$ [চ.'০১]

মনে করি, $f(x) = \tan 2x$.

$f(x+h) = \tan 2(x+h) = \tan(2x+2h)$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{f(x)\} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\tan 2x) = \lim_{h\to 0}\frac{\tan(2x+2h)-\tan 2x}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}[\frac{\sin(2x+2h)}{\cos(2x+2h)}-\frac{\sin 2x}{\cos 2x}]$$

$$= \lim_{h\to 0}\frac{1}{h}[\frac{\sin(2x+2h)\cos 2x-\sin 2x\cos(2x+2h)}{\cos(2x+2h)\cos 2x}]$$

$$= \lim_{h\to 0}\frac{1}{h}\frac{\sin(2x+2h-2x)}{\cos(2x+2h)\cos 2x}$$

$$= \lim_{h\to 0}\frac{\sin 2h}{2h}\times 2\times\lim_{h\to 0}\frac{1}{\cos(2x+2h)\cos 2x}$$

$$= 1\times 2\times\frac{1}{\cos(2x+0)\cos 2x} = \frac{2}{\cos^2 x}$$

$$= 2\sec^2 2x$$

3(e) $\sec 2x$ [য. '০২,'০৭; চ.'০৭,'১০]

মনে করি, $f(x) = \sec 2x$.

$f(x+h) = \sec 2(x+h) = \sec(2x+2h)$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{f(x)\} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\sec 2x) = \lim_{h\to 0}\frac{\sec(2x+2h)-\sec 2x}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}[\frac{1}{\cos(2x+2h)}-\frac{1}{\cos 2x}]$$

$$= \lim_{h\to 0}\frac{\cos 2x-\cos(2x+2h)}{h\cos(2x+2h)\cos 2x}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{2x+2x+2h}{2}\sin\frac{2x+2h-2x}{2}}{h\cos(2x+2h)\cos 2x}$$

$$= 2\lim_{h\to 0}\frac{\sin(2x+h)}{\cos(2x+2h)\cos 2x}\times\lim_{h\to 0}\frac{\sin h}{h}$$

$$= 2\frac{\sin(2x+0)}{\cos(2x+0)\cos 2x}\times 1$$

$$= \frac{2\sin 2x}{\cos 2x\cos 2x} = 2\tan 2x\sec 2x$$

3(f) e^{2x} [রা.'০৩]

মনে করি, $f(x) = e^{2x}$.

$f(x+h) = e^{2(x+h)} = e^{2x+2h}$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{f(x)\} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(e^{2x}) = \lim_{h\to 0}\frac{e^{2x+2h}-e^{2x}}{h}$$

$$= \lim_{h\to 0}\frac{e^{2x}.e^{2h}-e^{2x}}{h} = \lim_{h\to 0}\frac{e^{2x}}{h}(e^{2h}-1)$$

$$= e^{2x}\lim_{h\to 0}\frac{e^{2h}-1}{2h}\times 2$$

$$= e^{2x}\times 1\times 2 = 2e^{2x}, [\because \lim_{x\to 0}\frac{e^x-1}{x} = 1]$$

3. (g) cosec ax

মনে করি, $f(x) = \text{cosec } ax$.

$f(x+h) = \text{cosec }(ax+ah)$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{ f(x) \} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\text{cosec } ax) = \lim_{h\to 0}\frac{\text{cosec}(ax+ah)-\text{cosec}\, ax}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{1}{\sin(ax+ah)}-\frac{1}{\sin ax}\right]$$

$$= \lim_{h\to 0}\frac{\sin ax-\sin(ax+ah)}{h\sin(ax+ah)\sin ax}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{ax-ax-ah}{2}\cos\frac{ax+ax+ah}{2}}{h\sin(ax+ah)\sin ax}$$

$$= \lim_{h\to 0}\frac{2\sin(-h)\cos(ax+h)}{h\sin(ax+ah)\sin ax}$$

$$= -2\lim_{h\to 0}\frac{\sin h}{h}\times\lim_{h\to 0}\frac{\cos(ax+h)}{\sin(ax+ah)\sin ax}$$

$$= -2\times 1\times\frac{\cos(ax+0)}{\sin(ax+0)\sin ax}$$

$$= -2\times\frac{\cos ax}{\sin ax\sin ax}$$

$$= -2\cot ax\ \text{cosec}\ ax$$

3(h) $\cos 2x$ **[মা.বো.'০৪; ব.'১১]**

মনে করি, $f(x) = \cos 2x$.

$f(x+h) = \cos 2(x+h) = \cos(2x+2h)$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{ f(x) \} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\cos 2x) = \lim_{h\to 0}\frac{\cos(2x+2h)-\cos 2x}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left[2\sin\frac{2x+2h+2x}{2}\sin\frac{2x-2h-2x}{2}\right]$$

$$= 2\lim_{h\to 0}\sin(2x+h)\times -\lim_{h\to 0}\frac{\sin h}{h}$$

$$= 2\sin(2x+0).(-1) = -2\sin 2x$$

3(i) e^{ax} **[ব'০৫,'০৯; ঢা.'০৬; য., দি.'১১; কু.'১৩]**

মনে করি, $f(x) = e^{ax}$.

$f(x+h) = e^{a(x+h)} = e^{ax+ah}$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{ f(x) \} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(e^{ax}) = \lim_{h\to 0}\frac{e^{ax+ah}-e^{ax}}{h}$$

$$= \lim_{h\to 0}\frac{e^{ax}.e^{ah}-e^{ax}}{h} = \lim_{h\to 0}\frac{e^{ax}}{h}(e^{ah}-1)$$

$$= e^{ax}\lim_{h\to 0}\frac{1}{h}\left[\left\{\left(1+ah+\frac{(ah)^2}{2!}+\frac{(ah)^3}{3!}+\cdots\right\}-1\right]\right. =$$

$$e^{ax}\lim_{h\to 0}\frac{1}{h}\left(ah+\frac{a^2h^2}{2!}+\frac{a^3h^3}{3!}+\cdots\cdots\right)$$

$= e^{ax}\lim_{h\to 0}\left(a+\frac{a^2h}{2!}+\frac{a^3h^2}{3!}+\right.$ h- এর উচ্চঘাত সম্বলিত পদসমূহ)

$$= e^{ax}(a+0+0+\cdots\cdots) = ae^{ax}$$

3(j) $\log_a x$ **[চ.'০৮; ঢা.'১১; য.'১২,'১৪; দি.'১৪]**

ধরি, $f(x) = \log_a x = \log_a e\times\log_e x$

$$= \frac{\ln x}{\log_e a} = \frac{\ln x}{\ln a}$$

$$f(x+h) = \frac{\ln(x+h)}{\ln a}$$

অন্তরক সহগের সংজ্ঞা হতে পাই,

$$\frac{d}{dx}\{f(x)\} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(\log_a x) = \lim_{h\to 0}\frac{1}{h}\left[\frac{\ln(x+h)}{\ln a}-\frac{\ln x}{\ln a}\right]$$

$$= \lim_{h\to 0}\frac{1}{h\ln a}ln\frac{x+h}{x} = \lim_{h\to 0}\frac{1}{h\ln a}ln\left(1+\frac{h}{x}\right)$$

$$= \frac{1}{\ln a}\lim_{h\to 0}\frac{1}{h}\left[\frac{h}{x}-\frac{1}{2}\frac{h^2}{x^2}+\frac{1}{3}\frac{h^3}{x^3}-\cdots\cdots\right]$$

$= \frac{1}{\ln a}\lim_{h\to 0}\left[\frac{1}{x}-\frac{1}{2}\frac{h}{x^2}+\right.$ h-এর উচ্চঘাত সম্বলিত পদসমূহ]

$$= \frac{1}{\ln a}\frac{1}{x}-0 = \frac{1}{x\ln a}$$

4.(a) মূল নিয়মে $x = 2$ -তে x^5 এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = x^5$. $\quad f(2) = 2^5$

$$f'(2) = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{x^5-2^5}{x-2}$$

$$= 5\times(2)^4 \quad [\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}]$$

$$= 5\times 16 = 80$$

4(b) মূল নিয়মে $x = a$ -তে e^{mx} এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = e^{mx}$ $\quad f(a) = e^{ma}$

$$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a}\frac{e^{mx}-e^{ma}}{x-a} = \lim_{x\to a}\frac{e^{ma}(e^{mx-ma}-1)}{x-a}$$

$$= e^{ma}\lim_{x-a\to 0}\frac{e^{m(x-a)}-1}{m(x-a)}\times m$$

$$= me^{ma}.1 \qquad [\quad \lim_{x\to 0}\frac{e^x-1}{x} = 1]$$

$$= me^{ma}$$

4(c) মূল নিয়মে $x = \frac{\pi}{4}$-তে $\tan x$ এর অন্তরক সহগ নির্ণয়।

মনে করি, $f(x) = \tan x.$ $\quad f(\frac{\pi}{4}) = \tan\frac{\pi}{4}$

$$f'(\frac{\pi}{4}) = \lim_{x\to\frac{\pi}{4}}\frac{f(x)-f(\frac{\pi}{4})}{x-\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\tan x-\tan\frac{\pi}{4}}{x-\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}}{(x-\frac{\pi}{4})\cos x\cos\frac{\pi}{4}}$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{\sin(x-\frac{\pi}{4})}{(x-\frac{\pi}{4})\cos x\cos\frac{\pi}{4}}$$

$$= \lim_{x-\frac{\pi}{4}\to 0}\frac{\sin(x-\frac{\pi}{4})}{x-\frac{\pi}{4}}\times\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x\cos\frac{\pi}{4}}$$

$$= 1.\frac{1}{\cos\frac{\pi}{4}\cos\frac{\pi}{4}} = \frac{1}{(1/\sqrt{2})^2} = 2$$

প্রশ্নমালা IX D

x এর সাপেক্ষে অন্তরক সহগ নির্ণয় কর ঃ

1(a) $\frac{d}{dx}\{x^2\ln(x)\}$

$$= x^2\frac{d}{dx}\{ln(x)\} + ln(x)\frac{d}{dx}(x^2)$$

$$= x^2\frac{1}{x} + ln(x).(2x) = x + 2x\,ln(x)$$

1(b) $5e^x\log_a x$ [ব.'০৮;দি.'১৩]

মনে করি, $y = 5e^x\log_a x$

$$\frac{dy}{dx} = 5\{e^x\frac{d}{dx}(\log_a x)+\log_a x\frac{d}{dx}(e^x)\}$$

$$= 5\{e^x\frac{1}{x\ln a} + \log_a x.\,e^x\}$$

$$\therefore \frac{d}{dx}\{5e^x\log_a x\} = 5e^x\{\frac{1}{x\ln a}+\log_a x\}$$

1(c) $\log_{10}x$ [দি.'১১,'১৩]

মনে করি, $y = \log_{10}x = \log_{10}e\times\log_e x$

$$\Rightarrow y = \frac{1}{\log_e 10}\times\ln x = \frac{1}{\ln 10}\times\ln x$$

$$\frac{dy}{dx} = \frac{1}{\ln 10}\frac{d}{dx}(\ln x) = \frac{1}{\ln 10}\times\frac{1}{x}$$

$$\frac{d}{dx}(\log_{10}x) = \frac{1}{x\ln 10} \text{ (Ans.)}$$

1(d) $\log_a x$ [ঢা.'১৩]

মনে করি, $y = \log_a x = \log_a e\times\log_e x$

$$\Rightarrow y = \frac{1}{\log_e a}\times\ln x = \frac{1}{\ln a}\times\ln x$$

$$\frac{dy}{dx}=\frac{1}{\ln a}\frac{d}{dx}(\ln x)=\frac{1}{\ln a}\times\frac{1}{x}$$

$$\frac{d}{dx}(\log_a x)=\frac{1}{x\ln a} \text{ (Ans.)}$$

2. (a) $a^x \ln(x)+be^x \sin x$

$$\frac{d}{dx}\{a^x\ln(x)+be^x\sin x\}=a^x\frac{d}{dx}\{\ln(x)\}$$

$$+\ln(x)\frac{d}{dx}(a^x)+b\{e^x\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(e^x)\}$$

$$=a^x\frac{1}{x}+\ln(x)(a^x \ln a)+b\{e^x(\cos x)+\sin x(e^x)\}$$

$$=a^x\{\frac{1}{x}+\ln a\ \ln(x)\}+be^x(\cos x+\sin x)$$

2(b) $x^2\log_a x-x^3\ln a^x+6x e^x\ln x$

ধরি, $y=x^2\log_a x-x^3\ln a^x+6x e^x\ln x$

$$=x^2\log_a x-x^4\ln a+6x e^x\ln x$$

$$\frac{dy}{dx}=x^2\frac{d}{dx}(\log_a x)+\log_a x\frac{d}{dx}(x^2)-\ln a\frac{d}{dx}(x^4)+6\{x e^x\frac{d}{dx}(\ln x)+x\ \ln x\frac{d}{dx}(e^x)+e^x\ln x\frac{d}{dx}(x)\}$$

$$=x^2\frac{1}{x\ln a}+\log_a x.(2x)-\ln a.(4x^3)+6\{x e^x.\frac{1}{x}+x\ \ln x.e^x+e^x\ln x.1\}$$

$$=x(\frac{1}{\ln a}+2\log_a x-4x^2\ln a)+6e^x(1+x\ \ln x+\ln x)$$

3. (a) মনে করি, $y=\dfrac{x}{x^2+a^2}$

$$\frac{dy}{dx}=\frac{(x^2+a^2)\frac{d}{dx}(x)-x\frac{d}{dx}(x^2+a^2)}{(x^2+a^2)^2}$$

$$=\frac{(x^2+a^2).1-x(2x+0)}{(x^2+a^2)^2}=\frac{x^2+a^2-2x^2}{(x^2+a^2)^2}$$

$$\frac{d}{dx}(\frac{x}{x^2+a^2})=\frac{a^2-x^2}{(x^2+a^2)^2}$$

3(b) $\dfrac{d}{dx}(\dfrac{1-\tan x}{1+\tan x})$ [দি.'১০; ব.'১৩]

$$=\frac{(1+\tan x)\frac{d}{dx}(1-\tan x)-(1-\tan x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)(-\sec^2 x)-(1-\tan x)(\sec^2 x)}{(1+\tan x)^2}$$

$$=\frac{(-1-\tan x-1+\tan x)\sec^2 x}{(1+\tan x)^2}$$

$$=\frac{-2\sec^2 x}{(1+\tan x)^2} \text{ (Ans.)}$$

3(c) $\dfrac{d}{dx}(\dfrac{1+\sin x}{1+\cos x})=$ [কু.'০৪]

$$\frac{(1+\cos x)\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}=$$

$$\frac{(1+\cos x)(\cos x)-(1+\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$=\frac{\cos x+\cos^2 x+\sin x+\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{\cos x+\sin x+1}{(1+\cos x)^2} \text{ (Ans.)}$$

3(d) $\dfrac{1+\sin x}{1-\sin x}$

[ঢা.'১৩; ব. '০৭; রা.'০৯; চ.'১২; দি.'১৪]

$$\frac{d}{dx}(\frac{1+\sin x}{1-\sin x})=$$

$$\frac{(1-\sin x)\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$$

$$=\frac{(1-\sin x)(\cos x)-(1+\sin x)\frac{d}{dx}(-\cos x)}{(1-\sin x)^2}$$

$$=\frac{(1-\sin x+1+\sin x)\cos x}{(1-\sin x)^2}$$

$$=\frac{2\cos x}{(1-\sin x)^2} \text{ (Ans.)}$$

3(e) $\dfrac{\cos x-\cos 2x}{1-\cos x}$

[ব.'১০; রা., কু.'০৮; য.'১৩; ঢা.'১৪]

$$\frac{\cos x-\cos 2x}{1-\cos x}=\frac{\cos x-(2\cos^2 x-1)}{1-\cos x}$$

$$=\frac{1+\cos x-2\cos^2 x}{1-\cos x}$$

$$=\frac{(1-\cos x)(1+2\cos x)}{1-\cos x}=1+2\cos x$$

$$\frac{d}{dx}\left(\frac{\cos x-\cos 2x}{1-\cos x}\right)=-2\sin x$$

3(f) $\dfrac{\sin x+\cos x}{\sqrt{1+\sin 2x}}$ [ঢা.'০৯; ব.'০৯,'১১; য.'১৪]

$$\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}=\frac{\sin x+\cos x}{\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}}$$

$$=\frac{\sin x+\cos x}{\sqrt{(\sin x+\cos x)^2}}=\frac{\sin x+\cos x}{\sin x+\cos x}=1$$

$$\frac{d}{dx}\left(\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}\right)=0 \text{ (Ans.)}$$

3(g) ধরি, $y=\dfrac{x\ln x}{\sqrt{1+x^2}}$ [প্র.ভ.প.'০৫]

$$\frac{dy}{dx}=\frac{\sqrt{1+x^2}\frac{d}{dx}(x\ln x)-x\ln x\frac{d}{dx}(\sqrt{1+x^2})}{(\sqrt{1+x^2})^2}$$

$$=\frac{1}{1+x^2}\left[\sqrt{1+x^2}\left(x.\frac{1}{x}+\ln x\right)-x\ln x\frac{2x}{2\sqrt{1+x^2}}\right]$$

$$=\frac{1}{1+x^2}\left[\frac{(1+x^2)(1+\ln x)-x^2\ln x}{\sqrt{1+x^2}}\right]$$

$$\frac{d}{dx}\left(\frac{x\ln x}{\sqrt{1+x^2}}\right)=\frac{1+x^2+\ln x}{(\sqrt{1+x^2})^3}$$

বিকল্প পদ্ধতি ঃ ধরি, $y=\dfrac{x\ln x}{\sqrt{1+x^2}}$

$$\frac{dy}{dx}=\frac{x\ln x}{\sqrt{1+x^2}}\left[\frac{1}{x}\frac{d}{dx}(x)+\frac{1}{\ln x}\frac{d}{dx}(\ln x)-\frac{1}{\sqrt{1+x^2}}\frac{d}{dx}(\sqrt{1+x^2})\right]$$

$$=\frac{x\ln x}{\sqrt{1+x^2}}\left[\frac{1}{x}+\frac{1}{\ln x}.\frac{1}{x}-\frac{1}{\sqrt{1+x^2}}.\frac{2x}{2\sqrt{1+x^2}}\right]$$

$$=\frac{x\ln x}{\sqrt{1+x^2}}\;\frac{\ln x(1+x^2)+1+x^2-x^2\ln x}{x(1+x^2)\ln x}$$

$$\frac{d}{dx}\left(\frac{x\ln x}{\sqrt{1+x^2}}\right)=\frac{1+x^2+\ln x}{(\sqrt{1+x^2})^3} \text{ (Ans.)}$$

প্রশ্নমালা IX E

1.(a) $(1+\sin 2x)^2$ [চ.'০৪]

ধরি, $y=(1+\sin 2x)^2$

$$\frac{dy}{dx}=2(1+\sin 2x)\frac{d}{dx}(1+\sin 2x)$$

$$=2(1+\sin 2x)(0+\cos 2x)\frac{d}{dx}(2x)$$

$$=2(1+\sin 2x)\cos 2x\,(2.1)$$

$$\frac{d}{dx}\{(1+\sin 2x)^2\}=4\cos 2x(1+\sin 2x)$$

1(b) a^{px+q} [চ'০১]

ধরি, $y=a^{px+q}$

$$\frac{dy}{dx}=a^{px+q}.\ln a\frac{d}{dx}(px+q)$$

$$\left[\because \frac{d}{dx}(a^x)=a^x\ln a\right]$$

$$=a^{px+q}.\ln a\,(p.1+0)$$

$$\frac{d}{dx}(a^{px+q})=p\,a^{px+q}.\ln a \text{ (Ans.)}$$

1(c) $a^{\cos x}$ [চ.'০০]

$\frac{d}{dx}(a^{\cos x}) = a^{\cos x}.\ lna.\ \frac{d}{dx}(\cos x)$

$= a^{\cos x}.\ lna.(-\sin x)$

$= -a^{\cos x}\sin x.\ lna$

1(d) $10^{\ln(\sin x)}$ [সি.'০২ '০৫; চ.'০৭]

ধরি, $y = 10^{\ln(\sin x)}$

$\frac{dy}{dx} = 10^{\ln(\sin x)}.ln10\ \frac{d}{dx}\{\ln(\sin x)\}$

$= 10^{\ln(\sin x)}.ln10\ \frac{1}{\sin x}\ \frac{d}{dx}(\sin x)$

$= 10^{\ln(\sin x)}.ln10\ \frac{1}{\sin x}(\cos x)$

$\frac{d}{dx}\{10^{\ln(\sin x)}\} = 10^{\ln(\sin x)}.\ ln10.\cot x$

1(e) $10^{ln(\tan x)}$

ধরি, $y = 10^{\ln(\tan x)}$

$\frac{dy}{dx} = 10^{\ln(\tan x)}.ln10\ \frac{d}{dx}\{\ln(\tan x)\}$

$= 10^{\ln(\tan x)}.ln10\ \frac{1}{\tan x}\ \frac{d}{dx}(\tan x)$

$= 10^{\ln(\sin x)}.ln10\ \frac{\cos x}{\sin x}(\sec^2 x)$

$= 10^{\ln(\sin x)}.ln10\ \frac{\cos x}{\sin x}.\frac{1}{\cos^2 x}$

$= 10^{\ln(\sin x)}.ln10\ \frac{2}{2\sin x\cos x}$

$= 10^{\ln(\sin x)}.ln10\ \frac{2}{\sin 2x}$

$= 2\ \text{cosec}\ 2x.10^{\ln(\sin x)}.ln10$

1(f) $a^{ln(\cos x)}$ [রা.'০৫]

ধরি, $y = a^{\ln(\cos x)}$

$\frac{dy}{dx} = a^{\ln(\cos x)}.lna\ \frac{d}{dx}\{\ln(\cos x)\}$

$= a^{\ln(\cos x)}.lna\frac{1}{\cos x}\ \frac{d}{dx}(\cos x)$

$= a^{\ln(\cos x)}.ln\ a\ \frac{1}{\cos x}(-\sin x)$

$\frac{d}{dx}\{a^{\ln(\cos x)}\} = -\tan x\ a^{\ln(\cos x)}\ lna$

1(g) $e^{2\ln(\tan 5x)}$ [ব.'০৬,'১১; কু.'০৭; সি.'১০,'১৩]

$e^{2\ln(\tan 5x)} = e^{\ln(\tan 5x)^2} = (\tan 5x)^2$

$\frac{d}{dx}\{e^{2\ln(\tan 5x)}\} = 2\tan 5x\ \frac{d}{dx}(\tan 5x)$

$= 2\tan 5x(\sec^2 5x)\frac{d}{dx}(5x)$

$= 2\tan 5x\sec^2 5x(5)$

$= 10\tan 5x\sec^2 5x$

1(h) $(ln\sin x^2)^n$ [সি.'০৬; রা.'০৯]

ধরি, $y = (ln\sin x^2)^n$

$\frac{dy}{dx} = n(ln\sin x^2)^{n-1}\frac{d}{dx}(ln\sin x^2)$

$= n(ln\sin x^2)^{n-1}.\frac{1}{\sin x^2}\ \frac{d}{dx}(\sin x^2)$

$= n(ln\sin x^2)^{n-1}.\frac{1}{\sin x^2}(\cos x^2)(2x)$

$\frac{d}{dx}\{(ln\sin x^2)^n\} = nx\cot x^2(ln\sin x^2)^{n-1}$

1(i) $\cos(e^{\tan^2 2x})$

$\frac{d}{dx}\{\cos(e^{\tan^2 2x})\} = \frac{d\{\cos(e^{\tan^2 2x})\}}{d(e^{\tan^2 2x})}$

$\frac{d(e^{\tan^2 2x})}{d(\tan^2 2x)}\ \frac{d(\tan^2 2x)}{d(\tan 2x)}\ \frac{d(\tan 2x)}{d(2x)}\ \frac{d(2x)}{dx}$

$= -\sin(e^{\tan^2 2x}).e^{\tan^2 2x}.2\tan 2x.\sec^2 2x.2$

$= -4\tan 2x\sec^2 2x\sin(e^{\tan^2 2x})\ e^{\tan^2 2x}$

1(j) $\frac{d}{dx}(\sin^3 x^2)$ [চ.'০৯]

$= \frac{d(\sin x^2)^3}{d(\sin x^2)}\ \frac{d(\sin x^2)}{d(x^2)}\ \frac{d(x^2)}{dx}$

$= 3(\sin x^2)^2.\cos x^2.2x$

$= 6x\sin^2 x^2\cos x^2$ (Ans.)

1(k) $e^{5\ln(\tan x)}$ [চ.'১২]

$= e^{\ln(\tan x)^5} = (\tan x)^5$

$$\frac{d}{dx}\{e^{5\ln(\tan x)}\} = 5\tan^4 x\,\frac{d}{dx}(\tan x)$$
$$= 5\tan^4 x\sec^2 x$$

1(*l*) $x^n \ln(2x)$ [চ.'০৭]

মনে করি, $y = x^n \ln(2x)$

$$\frac{dy}{dx} = x^n\frac{d}{dx}\{\ln(2x)\} + \ln(2x)\frac{d}{dx}(x^n)$$
$$= x^n\frac{1}{2x}\frac{d}{dx}(2x) + \ln(2x).nx^{n-1}$$
$$= x^{n-1}\frac{1}{2}.(2) + nx^{n-1}\ln(2x)$$
$$\frac{d}{dx}\{x^n\ln(2x)\} = x^{n-1}\{1 + n\ln(2x)\}$$

1(m) $x\sqrt{\sin x}$ [ঢা.'০৮]

মনে করি, $y = x\sqrt{\sin x} = x(\sin x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = x\frac{d}{dx}\{(\sin x)^{\frac{1}{2}}\} + (\sin x)^{\frac{1}{2}}\frac{d}{dx}(x)$$
$$= x.\frac{1}{2}(\sin x)^{-\frac{1}{2}}\frac{d}{dx}(\sin x) + \sqrt{\sin x}.1$$
$$= \frac{1}{2}x\frac{1}{\sqrt{\sin x}}(\cos x) + \sqrt{\sin x}$$
$$\frac{d}{dx}(x\sqrt{\sin x}) = \frac{x\cos x + 2\sin x}{2\sqrt{\sin x}}$$

1(n) $e^{ax}\tan^2 x$ [ঢা.'০৯]

মনে করি, $y = e^{ax}\tan^2 x$

$$\frac{dy}{dx} = e^{ax}\frac{d}{dx}(\tan^2 x) + \tan^2 x\frac{d}{dx}(e^{ax})$$
$$= e^{ax}(2\tan x)\frac{d}{dx}(\tan x) + \tan^2 x.e^{ax}(a)$$
$$= e^{ax}\tan x(2\sec^2 x + a\tan x) \text{ (Ans.)}$$

2.(a) $ln(\cos x)$ [রা.'০৩,'০৫,'১০]

$$\frac{d}{dx}\{ln(\cos x)\} = \frac{1}{\cos x}\frac{d}{dx}(\cos x)$$
$$= \frac{1}{\cos x}(-\sin x) = -\tan x \text{ (Ans.)}$$

$$\frac{d}{dx}\{ln(e^x + e^{-x})\} = \frac{1}{e^x + e^{-x}}\frac{d}{dx}(e^x + e^{-x})$$
$$= \frac{1}{e^x + e^{-x}}(e^x - e^{-x}) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

2(c) $\log_x a$ [রা.'০১; চ.'০৬;'০৮]

$$\log_x a = \log_x e\times\log_e a = ln\,a\frac{1}{\log_e x}$$
$$= ln\,a\frac{1}{\ln x} = ln\,a\,(ln\,x)^{-1}$$
$$\therefore \frac{d}{dx}(\log_x a) = ln\,a\{-1(ln\,x)^{-2}\frac{d}{dx}(ln\,x)\}$$
$$= -ln\,a\frac{1}{(\ln x)^2}.\frac{1}{x} = -\frac{\ln a}{x(\ln x)^2}$$

2(d) $\log_{10} 3x$ [য.'০৬,'১৩]

$$\log_{10} 3x = \log_{10} e\times\log_e 3x = \frac{1}{\log_e 10}\ln(3x)$$
$$\frac{d}{dx}(\log_{10} 3x) = \frac{1}{\ln 10}\frac{1}{3x}\frac{d}{dx}(3x)$$
$$= \frac{1}{\ln 10}\frac{1}{3x}(3.1) = \frac{1}{x\ln 10} \text{ (Ans.)}$$

2(e) $\log_a x + \log_x a$

$$= \log_a e\times\log_e x + \log_x e\times\log_e a$$
$$= \frac{1}{\log_e a}\times\ln x + \frac{1}{\log_e x}\times\ln a$$
$$= \frac{1}{\ln a}\times\ln x + \ln a\times(\ln x)^{-1}$$
$$\frac{d}{dx}(\log_a x + \log_x a)$$
$$= \frac{1}{\ln a}\frac{1}{x} + \ln a\times\{-1(\ln x)^{-2}\frac{1}{x}\}$$
$$= \frac{1}{x\ln a} - \frac{\ln a}{x(\ln x)^2}$$

2(f) ধরি, $y = \log_x \tan x = \log_x e\times\log_e \tan x$

$$= \frac{1}{\log_e x}\times\ln(\tan x) = \frac{\ln(\tan x)}{\ln x}$$

$$\therefore \frac{dy}{dx} = \frac{\ln x \frac{d}{dx}\{\ln(\tan x)\} - \ln(\tan x)\frac{d}{dx}(\ln x)}{(\ln x)^2}$$

$$= \frac{\ln x \frac{1}{\tan x}\sec^2 x - \ln(\tan x).\frac{1}{x}}{(\ln x)^2}$$

$$= \frac{\ln x \frac{\cos x}{\sin x}\frac{1}{\cos^2 x} - \frac{1}{x}\ln(\tan x)}{(\ln x)^2}$$

$$= \frac{\ln x \frac{2}{\sin 2x} - \frac{1}{x}\ln(\tan x)}{(\ln x)^2}$$

$$= \frac{2x\ln x \operatorname{cosec} 2x - \ln(\tan x)}{x(\ln x)^2} \text{ (Ans.)}$$

2(g) $ln(\sin 2x)$ [ঢা.'১১; সি.'১৩]

$$\frac{d}{dx}\{ ln(\sin 2x)\} = \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x)$$

$$= \frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x) = 2\cot 2x$$

(h) $\ln(\sin x^2)$ [রা.'১২]

$$\frac{d}{dx}\{ \ln(\sin x^2)\} = \frac{1}{\sin x^2}\frac{d}{dx}(\sin x^2)$$

$$= \frac{1}{\sin x^2}(\cos x^2)\frac{d}{dx}(x^2) = 2x\cot x^2$$

3(a) $ln\ [x - \sqrt{x^2 - 1}]$ [রা. '০২; কু. '০৩; চ.'০৫]

$$\frac{d}{dx}\{ ln\ (x - \sqrt{x^2-1})\}$$

$$= \frac{1}{x-\sqrt{x^2-1}}\frac{d}{dx}(x - \sqrt{x^2-1})$$

$$= \frac{1}{x-\sqrt{x^2-1}}\{1 - \frac{1}{2\sqrt{x^2-1}}(2x)\}$$

$$= \frac{1}{x-\sqrt{x^2-1}}\{\frac{\sqrt{x^2-1}-x}{\sqrt{x^2-1}}\}$$

$$= -\frac{1}{\sqrt{x^2-1}} \text{ (Ans.)}$$

3(b) $ln\ [x - \sqrt{x^2 + 1}]$ [রা. '০২; কু. '০৩,'১০]

$$\frac{d}{dx}\{ ln\ (x - \sqrt{x^2+1})\}$$

$$= \frac{1}{x-\sqrt{x^2+1}}\frac{d}{dx}(x - \sqrt{x^2+1})$$

$$= \frac{1}{x-\sqrt{x^2+1}}\{1 - \frac{1}{2\sqrt{x^2+1}}(2x)\}$$

$$= \frac{1}{x-\sqrt{x^2+1}}\{\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\}$$

$$= -\frac{1}{\sqrt{x^2+1}} \text{ (Ans.)}$$

3(c) $ln\ (\sqrt{x-a} + \sqrt{x-b})$ [কু.'০১]

$$\frac{d}{dx}\{ ln\ (\sqrt{x-a} + \sqrt{x-b})\}$$

$$= \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\frac{d}{dx}(\sqrt{x-a}+\sqrt{x-b})$$

$$= \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\{\frac{1}{2\sqrt{x-a}} + \frac{1}{2\sqrt{x-b}}\}$$

$$= \frac{1}{\sqrt{x-a}+\sqrt{x-b}}\{\frac{\sqrt{x-b}+\sqrt{x-a}}{2\sqrt{x-a}\sqrt{x-b}}\}$$

$$= \frac{1}{2\sqrt{(x-a)(x-b)}} \text{ (Ans.)}$$

3(d) $ln\left\{e^x\left(\frac{x-1}{x+1}\right)^{3/2}\right\}$ [চ.'০০]

ধরি, $y = ln\left\{e^x\left(\frac{x-1}{x+1}\right)^{3/2}\right\}$

$$= ln\, e^x + \frac{3}{2}\{ ln\ (x-1) - ln\ (x+1)\}$$

$$= x + \frac{3}{2}\{ ln\ (x-1) - ln\ (x+1)\}$$

$$\frac{dy}{dx} = 1 + \frac{3}{2}\{\frac{1}{x-1} - \frac{1}{x+1}\}$$

$$= 1 + \frac{3}{2}\{\frac{x+1-x+1}{(x-1)(x+1)}\}$$

$= 1 + \frac{3}{2}\{\frac{2}{x^2-1}\} = \frac{x^2-1+3}{x^2-1}$

$= \frac{x^2+2}{x^2-1}$ (Ans.)

4. (a) $\frac{\tan x - \cot x}{\tan x + \cot x}$ [চ.'০৭; য.'০৬]

$\frac{\tan x - \cot x}{\tan x + \cot x} = \frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$

$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x} = \frac{-\cos 2x}{1} = -\cos 2x$

$\frac{d}{dx}(\frac{\tan x - \cot x}{\tan x + \cot x}) = \sin 2x.2 = 2\sin 2x$

4(b) $\left(\frac{\sin 2x}{1+\cos 2x}\right)^2$ [কু.'০৩]

$= \left(\frac{2\sin x\cos x}{2\cos^2 x}\right)^2 = \left(\frac{\sin x}{\cos x}\right)^2 = \tan^2 x$

$\frac{d}{dx}\left(\frac{\sin 2x}{1+\cos 2x}\right)^2 = 2\tan x \frac{d}{dx}(\tan x)$

$= 2\tan x \sec^2 x$

4(c) $ln\sqrt{\frac{1-\cos x}{1+\cos x}}$ [ঢা.'০৭,'১৩; রা.'১১; কু.'১৪]

$= ln\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}} = ln\sqrt{\tan^2\frac{x}{2}} = ln\tan\frac{x}{2}$

$\frac{d}{dx}\{ln\sqrt{\frac{1-\cos x}{1+\cos x}}\} = \frac{1}{\tan\frac{x}{2}}\sec^2\frac{x}{2}.\frac{1}{2}$

$= \frac{1}{2}\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\frac{1}{\cos^2\frac{x}{2}} = \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{1}{\sin x} = \text{cosec } x$ (Ans.)

4(d) $\sqrt{\frac{1+x}{1-x}}$ [প্র.ভ.প.'৮৩; রা.'১১]

ধরি, $y = \sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}}$

$\frac{dy}{dx} = \frac{\sqrt{1-x}\frac{d}{dx}(\sqrt{1+x}) - \sqrt{1+x}\frac{d}{dx}(\sqrt{1-x})}{(\sqrt{1-x})^2}$

$= \frac{\sqrt{1-x}\frac{1}{2\sqrt{1+x}}.1 - \sqrt{1+x}\frac{1}{2\sqrt{1-x}}(-1)}{1-x}$

$= \frac{\sqrt{1-x}\frac{1}{2\sqrt{1+x}}.1 - \sqrt{1+x}\frac{1}{2\sqrt{1-x}}(-1)}{1-x}$

$= \frac{1-x+1+x}{2(1-x)\sqrt{(1+x)(1-x)}} = \frac{2}{2(1-x)\sqrt{1-x^2}}$

$\frac{d}{dx}(\sqrt{\frac{1+x}{1-x}}) = \frac{1}{(1-x)\sqrt{1-x^2}}$

4.(e) $\ln\sqrt[3]{\frac{1-\cos x}{1+\cos x}}$ [দি.'১২; প্র.ভ.প.'০৫]

$= \ln\left(\frac{2\sin^2(x/2)}{2\cos^2(x/2)}\right)^{1/3} = \frac{1}{3}\ln\tan^2\frac{x}{2}$

$= \frac{2}{3}\ln\tan\frac{x}{2}$

$\frac{d}{dx}(\ln\sqrt[3]{\frac{1-\cos x}{1+\cos x}}) = \frac{2}{3}\frac{\sec^2(x/2)}{\tan(x/2)}.\frac{1}{2}$

$= \frac{1}{3}\frac{\cos\frac{x}{2}}{\cos^2\frac{x}{2}\sin\frac{x}{2}} = \frac{2}{3}\frac{1}{2\cos\frac{x}{2}\sin\frac{x}{2}}$

$= \frac{2}{3}\frac{1}{\sin x} = \frac{2}{3}\text{cosec}x$

5. (a) $\sin^2[ln(\sec x)]$ [রা.'০৭,'১৩; কু.,সি., মা.বো.'০৯; চ.'১১; ঢা.'১২; য., দি.'১৩]

ধরি, $y = \sin^2[ln(\sec x)]$

$\therefore \frac{dy}{dx} = \frac{d\{\sin[\ln(\sec x)]\}^2}{d\{\sin[\ln(\sec x)]\}}\frac{d\{\sin[\ln(\sec x)]\}}{d\{\ln(\sec x)\}}$

$$\frac{d\{\ln(\sec x)\}}{d(\sec x)}\frac{d(\sec x)}{dx}$$

$$= 2\sin[\ln(\sec x)]\cos[\ln(\sec x)]\frac{1}{\sec x}\sec x\tan x$$

$$= \tan x\ \sin[2\ln(\sec x)]$$

5(b) $\sin^2\{ln(x^2)\}$

[য.'০৭,'০৮; চ.'০৬,'১৩; ঢা.,সি,'১৪]

$$\frac{d}{dx}[\sin^2\{ln(x^2)\}] = \frac{d[\sin\{\ln(x^2)\}]^2}{d[\sin\{\ln(x^2)\}]}\frac{d[\sin\{\ln(x^2)\}]}{d[\ln(x^2)]}\frac{d[\ln(x^2)]}{d(x^2)}\frac{d(x^2)}{dx}$$

$$= 2\sin\{\ln(x^2)\}\cos\{\ln(x^2)\}\frac{1}{x^2}.2x$$

$$= \frac{2}{x}\sin\{2\ln(x^2)\} = \frac{2}{x}\sin\{4\ln(x)\}$$

5(c) $\sqrt{\sin\sqrt{x}}$ [চ.'০১; ঢা.'০৫,'০৭]

$$\frac{d}{dx}(\sqrt{\sin\sqrt{x}})$$

$$= \frac{d(\sqrt{\sin\sqrt{x}})}{d(\sin\sqrt{x})}\frac{d(\sin\sqrt{x})}{d(\sqrt{x})}\frac{d(\sqrt{x})}{dx}$$

$$= \frac{1}{2\sqrt{\sin\sqrt{x}}}\cos\sqrt{x}.\frac{1}{2\sqrt{x}}$$

$$= \frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{\sin\sqrt{x}}} \text{ (Ans.)}$$

5(d) $\cos(lnx) + ln(\tan x)$

[ব.'০৩; সি.'০৬]

$$\frac{d}{dx}\{\cos(lnx) + ln(\tan x)\}$$

$$= \frac{d}{dx}\{\cos(lnx)\} + \frac{d}{dx}\{ln(\tan x)\}$$

$$= -\sin(lnx).\frac{1}{x} + \frac{1}{\tan x}.\sec^2 x$$

$$= -\frac{1}{x}\sin(lnx) + \frac{\cos x}{\sin x}\frac{1}{\cos^2 x}$$

$$= \frac{2}{2\sin x\cos x} - \frac{1}{x}\sin(lnx)$$

$$= 2\ \text{cesec}\ 2x - \frac{1}{x}\sin(lnx)$$

5(e) 2cosec2x cos (ln tan$x$) [রা.'০৬]

$$\frac{d}{dx}\{2\ \text{cosec}\ 2x\cos(ln\tan x)\}$$

$$= 2\left[\text{cosec}\ 2x\frac{d}{dx}\{\cos(ln\tan x)\} + \cos(ln\tan x)\frac{d}{dx}(\text{cosec}\ 2x)\right]$$

$$= 2\left[\text{cosec}\ 2x\{-\sin(ln\tan x)\}.\frac{1}{\tan x}.\sec^2 x + \cos(ln\tan x)(-\text{cosec}2x\cot 2x.\ .2)\right]$$

$$= 2\left[-\text{cosec}\ 2x\ \sin(ln\tan x)\}.\frac{\cos x}{\sin x}.\frac{1}{\cos^2 x} - 2\text{cosec}2x\cot 2x\cos(ln\tan x)\right]$$

$$= 2[-\text{cosec}\ 2x\ \sin(ln\tan x)\}\frac{2}{2\sin x\cos x} - 2\text{cosec}2x\cot 2x\cos(ln\tan x)]$$

$$= -4[\text{cosec}^2 2x\ \sin(ln\tan x)\} + \text{cosec}\ 2x\cot 2x\cos(ln\tan x)]$$

5(f) $\frac{d}{dx}\{1+\tan(1+\sqrt{x})\}^{1/3}$

$$= \frac{1}{3}\{1+\tan(1+\sqrt{x})\}^{\frac{1}{3}-1}\{0+\sec^2(1+\sqrt{x})\}\left(0+\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{6\sqrt{x}}\{1+\tan(1+\sqrt{x})\}^{-\frac{2}{3}}\sec^2(1+\sqrt{x})$$

5(g) $\frac{d}{dx}(\sqrt{\tan e^{x^2}})$ [য.'০৯]

$$= \frac{d(\sqrt{\tan e^{x^2}})}{d(\tan e^{x^2})}\frac{d(\tan e^{x^2})}{d(e^{x^2})}\frac{d(e^{x^2})}{d(x^2)}\frac{d(x^2)}{dx}$$

$$=\frac{1}{2\sqrt{\tan e^{x^2}}}\sec^2 e^x \cdot e^x .2x=\frac{xe^x\sec^2 e^x}{\sqrt{\tan e^x}}$$

5(h) $\frac{d}{dx}\{\sin^2\log(\sec x)\}$ [সি.'১২]

$= 2\sin\{\log(\sec x)\}.\cos\{\log(\sec x)\}\times \frac{d}{dx}\{\log(\sec x)\}$

$=\sin\{2\log(\sec x)\}\times\frac{1}{\sec x\ln 10}\frac{d}{dx}(\sec x)$

$=\frac{\sin\{2\log(\sec x)\}}{\sec x\ln 10}\sec x\,.\,\tan x$

$=\frac{\sin\{2\log(\sec x)\}.\tan x}{\ln 10}$

5(i) $\frac{d}{dx}(\sin\sqrt{x})$ [সি.'১২; কু.'১৩]

$=\cos\sqrt{x}\,\frac{d}{dx}(\sqrt{x})$

$=\cos\sqrt{x}\,.\frac{1}{2\sqrt{x}}=\frac{\cos\sqrt{x}}{2\sqrt{x}}$

6.(a) ধরি, $y=x^2\sqrt{\frac{1+x}{1-x}}$ [রা.'০১]

$\therefore \ln y=2\ln x+\frac{1}{2}[\ln(1+x)-\ln(1-x)]$

ইহাকে এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{1}{y}\frac{dy}{dx}=2.\frac{1}{x}+\frac{1}{2}[\frac{1}{1+x}-\frac{1}{1-x}(-1)]$

$\Rightarrow \frac{dy}{dx}=y[\frac{2}{x}+\frac{1}{2}\{\frac{1-x+1+x}{(1+x)(1-x)}\}]$

$\Rightarrow \frac{dy}{dx}=x^2\frac{\sqrt{1+x}}{\sqrt{1-x}}[\frac{2}{x}+\frac{1}{2}\{\frac{1-x+1+x}{(1+x)(1-x)}\}]$

$=2x\sqrt{\frac{1+x}{1-x}}+\frac{x^2}{\sqrt{(1+x)(1-x)^{3/2}}}$

6(b) $\sqrt{e^{\sqrt{x}}}$ [কু.'০৪; ঢা.'০৬,'০৯; য.'১৩]

$\frac{d}{dx}(\sqrt{e^{\sqrt{x}}})=\frac{1}{2\sqrt{e^{\sqrt{x}}}}\frac{d}{dx}(e^{\sqrt{x}})$

$=\frac{1}{2\sqrt{e^{\sqrt{x}}}}e^{\sqrt{x}}\frac{d}{dx}(\sqrt{x})$

$=\frac{(e^{\sqrt{x}})^{1-\frac{1}{2}}}{2}.\frac{1}{2\sqrt{x}}=\frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$ (Ans.)

6.(c) $\frac{1}{\sqrt{x+1}+\sqrt{x+2}}$ [চ.'০০]

$=\frac{\sqrt{x+1}-\sqrt{x+2}}{(\sqrt{x+1}+\sqrt{x+2})(\sqrt{x+1}-\sqrt{x+2})}$

$=\frac{\sqrt{x+1}-\sqrt{x+2}}{x+1-x-2}=\sqrt{x+2}-\sqrt{x+1}$

$\therefore \frac{d}{dx}(\frac{1}{\sqrt{x+1}+\sqrt{x+2}})=\frac{1}{2\sqrt{x+2}}-\frac{1}{2\sqrt{x+1}}$

$=-\frac{\sqrt{x+2}-\sqrt{x+1}}{2\sqrt{(x+2)(x+1)}}$ (Ans.)

6(d) $\frac{d}{dx}\{\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}\}$ [কু.'০৯]

$=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}[\frac{1}{(x+1)^2}\frac{d}{dx}(x+1)^2+\frac{1}{\sqrt{x-1}}\frac{d}{dx}(\sqrt{x-1})-\frac{1}{(x+4)^3}\frac{d}{dx}(x+4)^3-\frac{1}{e^x}\frac{d}{dx}(e^x)]$

$=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}[\frac{2(x+1)}{(x+1)^2}+\frac{1}{\sqrt{x-1}}\frac{1}{2\sqrt{x-1}}-\frac{3(x+4)^2}{(x+4)^3}-\frac{1}{e^x}(e^x)]$

$=\frac{(x+1)^2\sqrt{x-1}}{(x+4)^3e^x}[\frac{2}{x+1}+\frac{1}{2(x-1)}-\frac{3}{x+4}-1]$

7.(a) $\frac{\ln(\cos x)}{x}$ [ঢা.'০৬; সি.'০৭;'০৯,'১১; ব.'১০]

$\frac{d}{dx}\{\frac{\ln(\cos x)}{x}\}$

$$= \frac{x\frac{d}{dx}\{\ln(\cos x) - \ln(\cos x)\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\frac{1}{\cos x}(-\sin x) - \ln(\cos x).1}{x^2}$$

$$= \frac{\{x\tan x + \ln(\cos x)}{x^2}$$

7(b) ধরি , $y = \dfrac{e^{-3x}(3x+5)}{7x-1}$ [য.'০৫]

$\ln y = \ln e^{-3x} + \ln(3x+5) - \ln(7x-1)$

$= -3x + \ln(3x+5) - \ln(7x-1)$

ইহাকে এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{y}\frac{dy}{dx} = -3 + \frac{1}{3x+5}(3) - \frac{1}{7x-1}(7)$$

$$= \frac{-3(21x^2+32x-5)+21x-3-21x-35}{(3x+5)(7x-1)}$$

$$\Rightarrow \frac{dy}{dx} = y\frac{-63x^2-96x+15-38}{(3x+5)(7x-1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{-3x}(3x+5)}{7x-1}\cdot\frac{-(63x^2+96x+23)}{(3x+5)(7x-1)}$$

$$= \frac{-(63x^2+96x+23)e^{-3x}}{(7x-1)^2}$$

7. (c) $\dfrac{x^4}{\ln x}$ [ঢা.'০৪]

$$\frac{d}{dx}\left(\frac{x^4}{\ln x}\right) = \frac{\ln x\frac{d}{dx}(x^4) - x^4\frac{d}{dx}(\ln x)}{(\ln x)^2}$$

$$= \frac{\ln x(4x^3) - x^4\frac{1}{x}}{(\ln x)^2} = \frac{x^3(4\ln x - 1)}{(\ln x)^2}$$

8. (a) $\cos x°$ [রা.'০৪]

$$\cos x° = \cos\frac{\pi x}{180}$$

$$\frac{d}{dx}(\cos x°) = -\sin\frac{\pi x}{180}\cdot\frac{d}{dx}\left(\frac{\pi x}{180}\right)$$

$$= -\sin x°\cdot\frac{\pi}{180} = -\frac{\pi}{180}\sin x°$$

8(b) $e^{5x}\sin x°$ [সি.'০২]

$$= e^{5x}\sin\frac{\pi x}{180}$$

$$\frac{d}{dx}\left(e^{5x}\sin\frac{\pi x}{180}\right) = e^{5x}.\cos\frac{\pi x}{180}\frac{d}{dx}\left(\frac{\pi x}{180}\right) + \sin\frac{\pi x}{180}.e^{5x}\frac{d}{dx}(5x)$$

$$= e^{5x}.\cos x°.\left(\frac{\pi}{180}\right) + \sin x°.e^{5x}.5$$

$$= e^{5x}\left(\frac{\pi}{180}\cos x° + 5\sin x°\right)$$

8(c) $2x°\cos 3x°$ [চ.'০৩; য.'০৫; কু.'১০,'১৩; সি. '০৬,'০৮,'১১; ব., রা.'০৭,'১৪; দি.'০৯,'১১]

$$2x°\cos 3x° = 2\frac{\pi x}{180}\cos\frac{3\pi x}{180}$$

$$\frac{d}{dx}(2x°\cos 3x°) = \frac{\pi}{90}\left[x\left(-\sin\frac{3\pi x}{180}\right).\frac{d}{dx}\left(\frac{3\pi x}{180}\right) + \cos\frac{3\pi x}{180}\frac{d}{dx}(x)\right]$$

$$= \frac{\pi}{90}\left[x(-\sin 3x°).\left(\frac{3\pi}{180}\right) + \cos 3x°.1\right]$$

$$= \frac{\pi}{90}\left(\cos 3x° - \frac{\pi}{60}x\sin 3x°\right)$$

প্রশ্নমালা IX F

1. (a) $\sqrt{\sin^{-1}x^5}$ [রা.'০৪, '০৬]

$$\frac{d}{dx}\left(\sqrt{\sin^{-1}x^5}\right) = \frac{1}{2\sqrt{\sin^{-1}x^5}}\frac{d}{dx}(\sin^{-1}x^5)$$

$$= \frac{1}{2\sqrt{\sin^{-1}x^5}}\frac{1}{\sqrt{1-(x^5)^2}}\frac{d}{dx}(x^5)$$

$$= \frac{1}{2\sqrt{\sin^{-1}x^5}\sqrt{1-x^{10}}}(5x^4)$$

$$= \frac{5x^4}{2\sqrt{\sin^{-1}x^5}\sqrt{1-x^{10}}}$$

1.(b) $\tan^{-1}(\sin e^x)$ [চ. '০৫; ব. '০৫;য.'০৯]

$$\frac{d}{dx}\{\tan^{-1}(\sin e^x)\} = \frac{d\{\tan^{-1}(\sin e^x)\}}{d(\sin e^x)}$$