$$\frac{d(\sin e^x)}{d(e^x)}\frac{d(e^x)}{dx}$$

$$= \frac{1}{1+(\sin e^x)^2}(\cos e^x).e^x = \frac{e^x\cos e^x}{1+\sin^2 e^x}$$

1(c) $\sin^{-1}(\sin e^x) = e^x$ [চ.'০৪]

$$\frac{d}{dx}\{\sin^{-1}(\sin e^x)\} = \frac{d}{dx}(e^x) = e^x$$

1(d) $\frac{d}{dx}(\sin^{-1}\sqrt{xe^x})$

$$= \frac{1}{\sqrt{1-(\sqrt{xe^x})^2}}\frac{d}{dx}(\sqrt{xe^x})$$

$$= \frac{1}{\sqrt{1-xe^x}}\frac{1}{2\sqrt{xe^x}}\frac{d}{dx}(xe^x)$$

$$= \frac{1}{2\sqrt{xe^x(1-xe^x)}}(xe^x+e^x)$$

$$= \frac{e^x(1+x)}{2\sqrt{xe^x(1-xe^x)}} \text{ (Ans.)}$$

1(e) $\sin^{-1}(\tan^{-1}x)$ [সি.'০১]

$$\frac{d}{dx}\{\sin^{-1}(\tan^{-1}x)\}$$

$$= \frac{1}{\sqrt{1-(\tan^{-1}x)^2}}\frac{d}{dx}(\tan^{-1}x)$$

$$= \frac{1}{\sqrt{1-(\tan^{-1}x)^2}}\frac{1}{1+x^2}$$

$$= \frac{1}{(1+x^2)\sqrt{1-(\tan^{-1}x)^2}}$$

1(f) $\frac{d}{dx}\{\tan^{-1}(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2})\}$

$$= \frac{1}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\sqrt{\frac{a-b}{a+b}}\frac{d}{dx}(\tan\frac{x}{2})$$

$$= \frac{1}{1+\frac{(a-b)\sin^2(x/2)}{(a+b)\cos^2(x/2)}}\sqrt{\frac{a-b}{a+b}}\sec^2\frac{x}{2}.\frac{1}{2}$$

$$= \frac{(a+b)\cos^2(x/2)}{a(\cos^2\frac{x}{2}+\sin^2\frac{x}{2})+b(\cos^2\frac{x}{2}-\sin^2\frac{x}{2})}$$

$$.\frac{1}{2}\frac{\sqrt{a-b}}{\sqrt{a+b}}\frac{1}{\cos^2(x/2)}$$

$$= \frac{\sqrt{(a-b)(a+b)}}{2(a+b\cos x)} = \frac{\sqrt{a^2-b^2}}{2(a+b\cos x)}$$

1.(g) $\frac{d}{dx}\{\sin^{-1}(\frac{a+b\cos x}{b+a\cos x})\}$

$$= \frac{1}{\sqrt{1-(\frac{a+b\cos x}{b+a\cos x})^2}}$$

$$\frac{(b+a\cos x)(-b\sin x)-(a+b\cos x)(-a\sin x)}{(b+a\cos x)^2}$$

$$= \frac{b+a\cos x}{\sqrt{(b+a\cos x)^2-(a+b\cos x)^2}}$$

$$\frac{(-b^2+a^2)\sin x}{(b+a\cos x)^2}$$

$$= \frac{(a^2-b^2)\sin x}{(b+a\cos x)\sqrt{b^2+a^2\cos^2 x-a^2-b^2\cos^2 x}}$$

$$= \frac{-(b^2-a^2)\sin x}{(b+a\cos x)\sqrt{(b^2-a^2)(1-\cos^2 x)}}$$

$$= \frac{-(b^2-a^2)\sin x}{(b+a\cos x)\sqrt{(b^2-a^2)\sin^2 x}}$$

$$= \frac{-\sqrt{b^2-a^2}}{b+a\cos x}$$

1(h) ধরি, $y = \sec^{-1}(\frac{x^2+1}{x^2-1}) = -\sec^{-1}\frac{1+x^2}{1-x^2}$

$= -\cos^{-1}\frac{1-x^2}{1+x^2} = -2\tan^{-1}x$

$\frac{dy}{dx} = -2\frac{d}{dx}(\tan^{-1}x) = \frac{-2}{1+x^2}$

2. (a) $x\sin^{-1}x$ [সি.'০১]

$\frac{d}{dx}(x\sin^{-1}x) = x\frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x\frac{d}{dx}(x)$

$= x\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x.1$

$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x$

2(b) $x^2\sin^{-1}(1-x)$ [রা.'০৬; ব.'০৮; ঢা.'১৪]

$\frac{d}{dx}\{x^2\sin^{-1}(1-x)\}$

$= x^2\frac{d}{dx}\{\sin^{-1}(1-x)\} + \sin^{-1}(1-x)\frac{d}{dx}(x^2)$

$= x^2\frac{1}{\sqrt{1-(1-x)^2}}(-1) + \sin^{-1}(1-x).2x$

$= -\frac{x^2}{\sqrt{1-1+2x-x^2}} + 2x\sin^{-1}(1-x)$

$= 2x\sin^{-1}(1-x) - \frac{x^2}{\sqrt{2x-x^2}}$

2(c) $\frac{d}{dx}\{e^x\sin^{-1}x\}$ [য.'০৪]

$= e^x\frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x\frac{d}{dx}(e^x)$

$= e^x\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x.\, e^x$

$= e^x\left(\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\right)$

2.(d) $\tan^{-1}\left(\frac{x^2}{e^x}\right) + \tan^{-1}\left(\frac{e^x}{x^2}\right)$

$= \tan^{-1}\frac{\frac{x^2}{e^x}+\frac{e^x}{x^2}}{1-\frac{x^2}{e^x}.\frac{e^x}{x^2}} = \tan^{-1}\frac{\frac{x^2}{e^x}+\frac{e^x}{x^2}}{1-1}$

$= \cot^{-1}\frac{1-1}{\frac{x^2}{e^x}+\frac{e^x}{x^2}} = \cot^{-1}0 = \frac{\pi}{2}$

$\therefore \frac{d}{dx}\left\{\tan^{-1}\left(\frac{x^2}{e^x}\right) + \tan^{-1}\left(\frac{e^x}{x^2}\right)\right\} = \frac{d}{dx}\left(\frac{\pi}{2}\right) = 0$

2(e) $\frac{d}{dx}(\tan x\sin^{-1}x)$ [ঢা.'০৫]

$= \tan x\frac{d}{dx}(\sin^{-1}x) + \sin^{-1}x\frac{d}{dx}(\tan x)$

$= \tan x\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x.(\sec^2x)$

$= \frac{\tan x}{\sqrt{1-x^2}} + \sec^2x\sin^{-1}x$

2(f) $(x^2+1)\tan^{-1}x - x$ [ঢা.,য.'১১; কু.,দি.'১২]

মনে করি, $y = (x^2+1)\tan^{-1}x - x$

$\frac{dy}{dx} = (x^2+1)\frac{d}{dx}(\tan^{-1}x) +$

$\tan^{-1}x\frac{d}{dx}(x^2+1) - \frac{d}{dx}(x)$

$= (x^2+1)\frac{1}{1+x^2} + \tan^{-1}x\times(2x) - 1$

$= 1 + 2x\tan^{-1}x - 1$

$\frac{d}{dx}\{(x^2+1)\tan^{-1}x - x\} = 2x\tan^{-1}x$

3.(a) $\tan^{-1}\frac{1-x}{1+x}$ [কু.'০৩]

$= \tan^{-1}\frac{1-x}{1+1.x} = \tan^{-1}(1) - \tan^{-1}x$

$= \frac{\pi}{4} - \tan^{-1}x$

$\frac{d}{dx}\left(\tan^{-1}\frac{1-x}{1+x}\right) = \frac{d}{dx}\left(\frac{\pi}{4} - \tan^{-1}x\right)$

$= 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$ (Ans.)

3(b) $\cot^{-1}\frac{1-x}{1+x}$ [চ.'০১,'১০; য.'০৫]

$$= \tan^{-1}\frac{1+x}{1-x} = \tan^{-1}\frac{1+x}{1-1.x}$$

$$= \tan^{-1}(1) + \tan^{-1}x = \frac{\pi}{4} + \tan^{-1}x$$

$$\therefore \frac{d}{dx}\{\cot^{-1}\frac{1-x}{1+x}\} = \frac{d}{dx}(\frac{\pi}{4} + \tan^{-1}x)$$

$$= 0 + \frac{1}{1+x^2} = \frac{1}{1+x^2}$$

3(c) $\tan^{-1}\frac{1-\sqrt{x}}{1+\sqrt{x}}$ [কু.'০০]

$$= \tan^{-1}\frac{1-\sqrt{x}}{1+1.\sqrt{x}} = \tan^{-1}(1) - \tan^{-1}\sqrt{x}$$

$$= \frac{\pi}{4} - \tan^{-1}\sqrt{x}$$

$$\frac{d}{dx}\{\tan^{-1}\frac{1-\sqrt{x}}{1+\sqrt{x}}\} = \frac{d}{dx}(\frac{\pi}{4} - \tan^{-1}\sqrt{x})$$

$$= 0 - \frac{1}{1+(\sqrt{x})^2}\frac{d}{dx}(\sqrt{x})$$

$$= -\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}(1+x)}$$

3(d) $\tan^{-1}\frac{a+bx}{a-bx}$ [য.'০২,'১১; ঢা.'০৯,'১১; ব.'০৯; চ.'১২; কু.'১৩ প্র.ভ.প. '০৬]

$$= \tan^{-1}\frac{a(1+\frac{b}{a}x)}{a(1-\frac{b}{a}x)} = \tan^{-1}\frac{1+\frac{b}{a}x}{1-1.\frac{b}{a}x}$$

$$= \tan^{-1}(1) - \tan^{-1}(\frac{b}{a}x) = \frac{\pi}{4} - \tan^{-1}(\frac{b}{a}x)$$

$$\therefore \frac{d}{dx}\{\tan^{-1}\frac{a+bx}{a-bx}\} = \frac{d}{dx}\{\frac{\pi}{4} - \tan^{-1}(\frac{b}{a}x)\}$$

$$= 0 - \frac{1}{1+(\frac{b}{a}x)^2}\frac{d}{dx}(\frac{b}{a}x)$$

$$= \frac{a^2}{a^2+b^2x^2}\cdot\frac{b}{a} = \frac{ab}{a^2+b^2x^2}$$

3(e) $\tan^{-1}\frac{a\cos x - b\sin x}{b\cos x + a\sin x}$ [প্র.ভ.প. '১৬]

$$= \tan^{-1}\frac{\frac{a\cos x}{b\cos x} - \frac{b\sin x}{b\cos x}}{\frac{b\cos x}{b\cos x} + \frac{a\sin x}{b\cos x}} = \tan^{-1}\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}.\tan x}$$

$$= \tan^{-1}\frac{a}{b} - \tan^{-1}\tan x = \tan^{-1}\frac{a}{b} - x$$

$$\frac{d}{dx}\{\tan^{-1}\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\} = 0 - 1 = -1$$

3(f) $\cot^{-1}\frac{1+x}{1-x}$ [চ. '০৬; সি. '০৪; রা. য. ০৭]

$$= \tan^{-1}\frac{1-x}{1+x} = \tan^{-1}\frac{1-x}{1+1.x}$$

$$= \tan^{-1}(1) - \tan^{-1}x = \frac{\pi}{4} - \tan^{-1}x$$

$$\frac{d}{dx}\{\cot^{-1}\frac{1+x}{1-x}\} = \frac{d}{dx}(\frac{\pi}{4} - \tan^{-1}x)$$

$$= 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$$ ((Ans.)

3(g) ধরি, $y = \cos^{-1}(\frac{1+x}{2})^{1/2}$ [চ.'০৯]

এবং $x = \cos\theta$. তাহলে, $\theta = \cos^{-1}x$ এবং

$$y = \cos^{-1}\{\frac{1}{2}(1+\cos\theta)\}^{1/2} = \cos^{-1}(\cos^2\frac{\theta}{2})^{1/2}$$

$$= \cos^{-1}\cos\frac{\theta}{2} = \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$$

$$\frac{d}{dx}\{\cos^{-1}(\frac{1+x}{2})^{1/2}\} = \frac{d}{dx}(\frac{1}{2}\cos^{-1}x)$$

$$= \frac{1}{-2\sqrt{1-x^2}}$$

3(h) $\tan^{-1}\frac{a+bx}{b-ax}$ [ব.'১৩; বুয়েট.'০৯]

$$= \tan^{-1}\frac{b(\frac{a}{b}+x)}{b(1-\frac{a}{b}.x)} = \tan^{-1}(\frac{a}{b}) + \tan^{-1}(x)$$

$$\frac{d}{dx}\{\tan^{-1}\frac{a+bx}{b-ax}\} = \frac{d}{dx}\{\tan^{-1}(\frac{a}{b})\} + \frac{d}{dx}\{\tan^{-1}(x)\}$$

$$= 0 + \frac{1}{1+x^2} = \frac{1}{1+x^2}$$

4.(a) ধরি, $y = \sin^{-1}\frac{1-x^2}{1+x^2}$ [য.'০২,'১২,'১৪]

এবং $x = \tan\theta$. তাহলে, $\theta = \tan^{-1}x$ এবং

$$y = \sin^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta} = \sin^{-1}\cos 2\theta$$

$$= \sin^{-1}\sin(\frac{\pi}{2} - 2\theta) = \frac{\pi}{2} - 2\theta$$

$$= \frac{\pi}{2} - 2\tan^{-1}x$$

$$\frac{dy}{dx} = \frac{d}{dx}(\frac{\pi}{2} - 2\tan^{-1}x) = 0 - 2\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sin^{-1}\frac{1-x^2}{1+x^2}) = \frac{-2}{1+x^2}$$

4(b) $\cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1}x$ [য.'০৬; চ.'০৭]

$$\frac{d}{dx}(\cos^{-1}\frac{1-x^2}{1+x^2}) = \frac{d}{dx}(2\tan^{-1}x)$$

$$= 2\frac{1}{1+x^2} = \frac{2}{1+x^2} \text{ (Ans.)}$$

4(c) $\sec^{-1}\frac{1+x^2}{1-x^2}$ [য.'০৬; কু. ০৯; সি.'১০]

$$= \cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1}x$$

$$\frac{d}{dx}(\sec^{-1}\frac{1+x^2}{1-x^2}) = \frac{d}{dx}(2\tan^{-1}x)$$

$$= 2\frac{1}{1+x^2} = \frac{2}{1+x^2} \text{ (Ans.)}$$

4(d) $\tan^{-1}\frac{4x}{1-4x^2}$ [ব.'০৪]

$$= \tan^{-1}\frac{2.2x}{1-(2x)^2} = 2\tan^{-1}(2x)$$

$$[\quad \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x]$$

$$\frac{d}{dx}(\tan^{-1}\frac{4x}{1-4x^2}) = \frac{d}{dx}\{2\tan^{-1}(2x)\}$$

$$= 2\frac{1}{1+(2x)^2}.2 = \frac{4}{1+4x^2} \text{ (Ans.)}$$

4(e) $\tan^{-1}\frac{4\sqrt{x}}{1-4x}$

[চ.'০৯; রা.'০৬; সি.'০৯,'১২; ব.'১১; দি.'১৩]

$$= \tan^{-1}\frac{2.2\sqrt{x}}{1-(2\sqrt{x})^2} = 2\tan^{-1}(2\sqrt{x})$$

$$[\because \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x]$$

$$\frac{d}{dx}(\tan^{-1}\frac{4\sqrt{x}}{1-4x}) = \frac{d}{dx}\{2\tan^{-1}(2\sqrt{x})\}$$

$$= 2\frac{1}{1+(2\sqrt{x})^2}\frac{d}{dx}(2\sqrt{x})$$

$$= \frac{2}{1+4x}.2.\frac{1}{2\sqrt{x}} = \frac{2}{\sqrt{x}(1+4x)} \text{ (Ans.)}$$

4(f) $\sin^{-1}\frac{4x}{1+4x^2}$ [সি.'০২]

$$= \sin^{-1}\frac{2.2x}{1+(2x)^2} = 2\tan^{-1}(2x).$$

$$\frac{d}{dx}(\sin^{-1}\frac{4x}{1+4x^2}) = \frac{d}{dx}\{2\tan^{-1}(2x)\}$$

$$= 2\frac{1}{1+(2x)^2}\frac{d}{dx}(2x) = \frac{4}{1+4x^2} \text{ (Ans.)}$$

4(g) $\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

$$\frac{d}{dx}(\sin^{-1}\frac{2x}{1+x^2}) = \frac{d}{dx}(2\tan^{-1}x)$$

$$= \frac{2}{1+x^2} \text{ (Ans.)}$$

4(h) $\sin^{-1}\frac{6x}{1+9x^2}$ [ঢা.'০১]

$$= \sin^{-1}\frac{2.3x}{1+(3x)^2} = 2\tan^{-1}(3x)$$

$$[\because \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x]$$

$$\frac{d}{dx}(\sin^{-1}\frac{6x}{1+9x^2}) = \frac{d}{dx}\{2\tan^{-1}(3x)\}$$

$$= 2\frac{1}{1+(3x)^2}\frac{d}{dx}(3x) = \frac{2}{1+9x^2}.3$$

$$= \frac{9}{1+9x^2} \text{ (Ans.)}$$

4.(i) $\tan^{-1}\frac{2\sqrt{x}}{1-x}$ [চ'০৬,'১১; ঢা.'০৭; সি.'১১]

$$= \tan^{-1}\frac{2\sqrt{x}}{1-(\sqrt{x})^2} = 2\tan^{-1}\sqrt{x}$$

$$\frac{d}{dx}(\tan^{-1}\frac{2\sqrt{x}}{1-x}) = \frac{d}{dx}\{2\tan^{-1}(\sqrt{x})\}$$

$$= 2\frac{1}{1+(\sqrt{x})^2}\frac{d}{dx}(\sqrt{x}) = \frac{2}{1+x}\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{(1+x)\sqrt{x}} \text{ (Ans.)}$$

5.(a) ধরি, $y = \cos^{-1}(2x\sqrt{1-x^2}\}$

[য.'০১,'১০; কু.'১০]

এবং $x = \sin\theta$. তাহলে, $\theta = \sin^{-1}x$ এবং

$$y = \cos^{-1}(2\cos\theta\sqrt{1-\cos^2\theta}\}$$

$$= \cos^{-1}(2\cos\theta\sin\theta) = \cos^{-1}\sin 2\theta$$

$$= \cos^{-1}\cos(\frac{\pi}{2}-2\theta) = \frac{\pi}{2}-2\theta$$

$$= \frac{\pi}{2}-2\sin^{-1}x$$

$$\frac{dy}{dx} = \frac{d}{dx}(\frac{\pi}{2}-2\sin^{-1}x)$$

$$= 0-2\frac{1}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}} \text{ (Ans.)}$$

5(b) ধরি, $y = \sin^{-1}\{2ax\sqrt{1-a^2x^2}\}$ [কু.'০৮; সি.'১৩]

এবং $ax = \sin\theta$. তাহলে, $\theta = \sin^{-1}(ax)$ এবং

$$y = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$$

$$= \sin^{-1}\{2\sin\theta\cos\theta\} = \sin^{-1}\sin 2\theta$$

$$= 2\theta = 2\sin^{-1}(ax)$$

$$\frac{dy}{dx} = 2\frac{1}{\sqrt{1-(ax)^2}}\frac{d}{dx}(ax)$$

$$= \frac{2a}{\sqrt{1-a^2x^2}}$$

5(c)) ধরি, $y = \tan^{-1}\frac{4x}{\sqrt{1-4x^2}}$ [রা.'০২]

এবং $2x = \sin\theta$.

$$y = \tan^{-1}\frac{2\sin\theta}{\sqrt{1-\sin^2\theta}} = \tan^{-1}\frac{2\sin\theta}{\cos\theta}$$

$$= \tan^{-1}(2\tan\theta)$$

$$\frac{dy}{dx} = \frac{1}{1+(2\tan\theta)^2}\frac{d}{dx}(2\tan\theta)$$

$$= \frac{2\sec^2\theta}{1+4\tan^2\theta} = \frac{2/\cos^2\theta}{1+\frac{4\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2}{\cos^2\theta+4\sin^2\theta} = \frac{2}{1+3\sin^2\theta}$$

$$= \frac{2}{1+3(2x)^2} = \frac{2}{1+12x^2}$$

5(d) ধরি, $y = \sin^{-1}\frac{x+\sqrt{1-x^2}}{\sqrt{2}}$ এবং

$x = \sin\theta$. তাহলে, $\theta = \sin^{-1}x$ এবং

$$y = \sin^{-1}\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}$$

$$= \sin^{-1}(\sin\theta.\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cos\theta)$$

$$= \sin^{-1}(\sin\theta.\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos\theta)$$

$$= \sin^{-1}\sin(\theta+\frac{\pi}{4}) = \theta+\frac{\pi}{4} = \sin^{-1}x+\frac{\pi}{4}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}x+\frac{\pi}{4}) = \frac{1}{\sqrt{1-x^2}} \text{ (Ans.)}$$

6.(a) ধরি, $y = \tan^{-1}\frac{1}{\sqrt{x^2-1}}$ [রা.'০৩]

এবং $x = \sec\theta$. তাহলে, $\theta = \sec^{-1}x$ এবং

$y=\tan^{-1}\frac{1}{\sqrt{\sec^2\theta-1}}=\tan^{-1}\frac{1}{\sqrt{\tan^2\theta}}$

$=\tan^{-1}\frac{1}{\tan\theta}=\tan^{-1}\cot\theta=\tan^{-1}\tan(\frac{\pi}{2}-\theta)=$

$\frac{\pi}{2}-\theta=\frac{\pi}{2}-\sec^{-1}x$

$\frac{dy}{dx}=\frac{d}{dx}(\frac{\pi}{2}-\sec^{-1}x)=0-\frac{1}{x\sqrt{x^2-1}}$

অর্থাৎ, $\frac{d}{dx}(\tan^{-1}\frac{1}{\sqrt{x^2-1}})=-\frac{1}{x\sqrt{x^2-1}}$

6.(b) $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$ [সি.'০৫,'০৭; প্র.ভ.প.'১০]

ধরি, $y=\tan^{-1}\sqrt{\frac{1-x}{1+x}}$ এবং $x=\cos\theta$.

তাহলে, $\theta=\cos^{-1}x$ এবং

$y=\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\tan^{-1}\sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}}$

$=\tan^{-1}\sqrt{\tan^2\frac{\theta}{2}}=\tan^{-1}\tan\frac{\theta}{2}$

$=\frac{\theta}{2}=\frac{1}{2}\cos^{-1}x$

$\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(\cos^{-1}x)=\frac{1}{2}\frac{-1}{\sqrt{1-x^2}}$

অর্থাৎ, $\frac{d}{dx}(\tan^{-1}\sqrt{\frac{1-x}{1+x}})=\frac{-1}{2\sqrt{1-x^2}}$

6(e) $\sin^4\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ [বুয়েট,'০৯]

ধরি, $y=\sin^4\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$ এবং $x=\cos\theta$

$y=\sin^4\left(\cot^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right)$

$=\sin^4\left(\cot^{-1}\sqrt{\frac{2\cos^2(\theta/2)}{2\sin^2(\theta/2)}}\right)$

$=\sin^4(\cot^{-1}\cot\frac{\theta}{2})=\sin^4\frac{\theta}{2}=\{\frac{1}{2}(2\sin^2\frac{\theta}{2})\}^2$

$=\{\frac{1}{2}(1-\cos\theta)\}^2=\frac{1}{4}(1-x)^2$

$\frac{dy}{dx}=\frac{1}{4}\times2(1-x)\times(-1)=-\frac{1}{2}(1-x)$

6(f) $\tan(\sin^{-1}x)$ [চ.'০২,'০৯;কু.'০৮,'১১; রা.'০৮; ব.'০৯,'১২; ঢা.,য.,সি.'১০; ঢা.'১২; দি.'১৩]

$\frac{d}{dx}\{\tan(\sin^{-1}x)\}=\sec^2(\sin^{-1}x).\frac{d}{dx}(\sin^{-1}x)$

$=\frac{1}{\cos^2(\sin^{-1}x)}.\frac{1}{\sqrt{1-x^2}}$

$=\frac{1}{1-\sin^2(\sin^{-1}x)}.\frac{1}{\sqrt{1-x^2}}$

$=\frac{1}{1-\{\sin(\sin^{-1}x)\}^2}.\frac{1}{\sqrt{1-x^2}}$

$=\frac{1}{1-x^2}.\frac{1}{\sqrt{1-x^2}}=\frac{1}{(1-x^2)^{3/2}}$ (Ans.)

7.(a) $\tan^{-1}(\sec x+\tan x)$ [সি.'১৪;য.'০৭; চ.'১৩]

$=\tan^{-1}(\frac{1}{\cos x}+\frac{\sin x}{\cos x})=\tan^{-1}(\frac{1+\sin x}{\cos x})$

$=\tan^{-1}\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$

$=\tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})}$

$=\tan^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$

$=\tan^{-1}\frac{\cos\frac{x}{2}(1+\tan\frac{x}{2})}{\cos\frac{x}{2}(1-\tan\frac{x}{2})}=\tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

$=\tan^{-1}(1)+\tan^{-1}\tan(\frac{x}{2})=\frac{\pi}{4}+\frac{x}{2}$

$\therefore \frac{d}{dx}\{\tan^{-1}(\sec x+\tan x)\} = \frac{d}{dx}(\frac{\pi}{4}+\frac{x}{2})$

$= \frac{1}{2}$ (Ans.)

7(b) $\tan^{-1}\frac{\cos x}{1+\sin x}$ [ঢা.'০৫,'১৩]

$= \tan^{-1}\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})}{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}$

$=\tan^{-1}\frac{\cos\frac{x}{2}(1-\tan\frac{x}{2})}{\cos\frac{x}{2}(1+\tan\frac{x}{2})} = \tan^{-1}\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

$= \tan^{-1}(1)-\tan^{-1}\tan(\frac{x}{2}) = \frac{\pi}{4}-\frac{x}{2}$

$\frac{d}{dx}(\tan^{-1}\frac{\cos x}{1+\sin x}) = \frac{d}{dx}(\frac{\pi}{4}-\frac{x}{2})$

$= 0-\frac{1}{2} = -\frac{1}{2}$

7(c) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ [রা.'১০; কু.'১১; ব.'১২]

$= \tan^{-1}\sqrt{\frac{2\sin^2(x/2)}{2\cos^2(x/2)}} = \tan^{-1}\sqrt{\tan^2\frac{x}{2}}$

$= \tan^{-1}\tan\frac{x}{2} = \frac{x}{2}$

$\frac{d}{dx}(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}) = \frac{d}{dx}(\frac{x}{2}) = \frac{1}{2}$

7(d) $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

[ব.'০২; চ. '০৮; রা. '০৯,'১১; দি.'০৯,'১১]

ধরি, $y = \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ এবং $x = \cos\theta$

তাহলে, $\theta = \cos^{-1}x$ এবং

$y = \sin\left(2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)$

$= \sin\left(2\tan^{-1}\sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}}\right)$

$= \sin(2\tan^{-1}\tan\frac{\theta}{2}) = \sin(2.\frac{\theta}{2}) = \sin\theta$

$= \sin(\cos^{-1}x) = \sin\ \sin^{-1}\sqrt{1-x^2}$

$= \sqrt{1-x^2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sqrt{1-x^2}) = \frac{1}{2\sqrt{1-x^2}}.(-2x)$

$\frac{d}{dx}\{\sin(2\tan^{-1}\sqrt{\frac{1-x}{1+x}})\} = \frac{-x}{\sqrt{1-x^2}}$

প্রশ্নমালা IX G

$\frac{dy}{dx}$ নির্ণয় কর ঃ **1. (a)** $x=\sqrt{t},\ y=t-\frac{1}{\sqrt{t}}$

$\frac{dx}{dt} = \frac{d}{dt}(\sqrt{t}) = \frac{1}{2\sqrt{t}}$ এবং

$\frac{dy}{dt} = \frac{d}{dt}(t-\frac{1}{\sqrt{t}}) = \frac{d}{dt}(t-t^{-\frac{1}{2}})$

$= 1-(-\frac{1}{2})t^{-\frac{1}{2}-1} = 1+\frac{1}{2t\sqrt{t}}$

$= \frac{1}{2\sqrt{t}}(2\sqrt{t}+\frac{1}{t})$

$\therefore \frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx} = \frac{1}{2\sqrt{t}}(2\sqrt{t}+\frac{1}{t})\times\frac{2\sqrt{t}}{1}$

$= 2\sqrt{t}+\frac{1}{t}$

1.(b) $x=\frac{3at}{1+t^3}\cdots\cdots(1),\ y=\frac{3at^2}{1+t^3}\cdots(2)$

$(2)\div(1) \Rightarrow \frac{y}{x}=t$

(1) হতে পাই, $x = \dfrac{3a\dfrac{y}{x}}{1+(\dfrac{y}{x})^3} = \dfrac{3ay}{x} \times \dfrac{x^3}{x^3+y^3}$

$\Rightarrow\ x = \dfrac{3ax^2y}{x^3+y^3} \Rightarrow x^3+y^3 = 3axy$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$3x^2 + 3y^2\dfrac{dy}{dx} = 3a(x\dfrac{dy}{dx}+y)$

$\Rightarrow\ (y^2-ax)\dfrac{dy}{dx} = ay - x^2\ \therefore \dfrac{dy}{dx} = \dfrac{ay-x^2}{y^2-ax}$

1(c) $x = a(\cos\phi + \phi\sin\phi),\ y = a(\sin\phi - \phi\cos\phi)$

$\dfrac{dx}{d\phi} = \dfrac{d}{d\phi}\{a(\cos\phi + \phi\sin\phi)\}$

$= a(-\sin\phi + \phi\cos\phi + \sin\phi) = a\phi\cos\phi$

$\dfrac{dy}{d\phi} = \dfrac{d}{d\phi}\{a(\sin\phi - \phi\cos\phi)\}$

$= a(\cos\phi + \phi\sin\phi - \cos\phi) = a\phi\sin\phi$

$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{d\phi}}{\dfrac{dx}{d\phi}} = \dfrac{a\phi\sin\phi}{a\phi\cos\phi} = \tan\phi$

1(*d*) $x = \sqrt{a^{\sin^{-1}t}}\ ,y = \sqrt{a^{\cos^{-1}t}}$

$= \dfrac{1}{2\sqrt{a^{\sin^{-1}t}}}a^{\sin^{-1}t}\ln a\dfrac{1}{\sqrt{1-t^2}}$

$= \dfrac{\ln a\sqrt{a^{\sin^{-1}t}}}{2\sqrt{1-t^2}} = \dfrac{x\ln a}{2\sqrt{1-t^2}}$

$\dfrac{dy}{dt} = \dfrac{d}{dt}(\sqrt{a^{\cos^{-1}t}})$

$= \dfrac{1}{2\sqrt{a^{\cos^{-1}t}}}a^{\cos^{-1}t}\ln a\dfrac{1}{-\sqrt{1-t^2}}$

$= -\dfrac{\ln a\sqrt{a^{\cos^{-1}t}}}{2\sqrt{1-t^2}} = -\dfrac{y\ln a}{2\sqrt{1-t^2}}$

$\therefore \dfrac{dy}{dx} = \dfrac{dy}{dt}\times\dfrac{dt}{dx} = -\dfrac{y\ln a}{2\sqrt{1-t^2}}\times\dfrac{2\sqrt{1-t^2}}{x\ln a}$

$= -\dfrac{y}{x}$

2. (a) $x^{\frac{1}{x}}$ [ব.'০৪; চ.'১৩;সি.'০৭,'০৯; ঢা., য.'০৮]

$\dfrac{d}{dx}(x^{\frac{1}{x}}) = x^{\frac{1}{x}}[\dfrac{1}{x}\dfrac{d}{dx}(\ln x) + \ln x\dfrac{d}{dx}(\dfrac{1}{x})]$

$[\quad \dfrac{d}{dx}(u^v) = u^v\{v\dfrac{d}{dx}(\ln u) + \ln u\dfrac{dv}{dx}\}]$

$= x^{\frac{1}{x}}[\dfrac{1}{x}.\dfrac{1}{x} + \ln x\dfrac{d}{dx}(x^{-1})]$

$= x^{\frac{1}{x}}[\dfrac{1}{x^2} + \ln x.(-x^{-2})] = x^{\frac{1}{x}}(\dfrac{1}{x^2} - \dfrac{\ln x}{x^2})$

$= x^{\frac{1}{x}}.\dfrac{1-\ln x}{x^2} = x^{\frac{1}{x}-2}(1-\ln x)$ (Ans.)

2. (b) $\dfrac{d}{dx}(1+x)^x$ [ব.'১৩]

$= (1+x)^x[x\dfrac{d}{dx}\{\ln(1+x)\} + \ln(1+x)\dfrac{d}{dx}(x)]$

$[\because \dfrac{d}{dx}(u^v) = u^v\{v\dfrac{d}{dx}(\ln u) + \ln u\dfrac{dv}{dx}\}]$

$= (1+x)^x[x\dfrac{1}{1+x} + \ln(1+x).1]$

$= (1+x)^x\{\dfrac{x}{1+x} + \ln(1+x)\}$

2(c) $(1+x^2)^{2x}$ [য.'০৬]

$\dfrac{d}{dx}\{(1+x^2)^{2x}\} = (1+x^2)^{2x}$

$[2x\dfrac{d}{dx}\{\ln(1+x^2)\} + \ln(1+x^2)\dfrac{d}{dx}(2x)]$

$= (1+x^2)^{2x}[\dfrac{2x}{1+x^2}(2x) + \ln(1+x^2).(2)]$

$= 2(1+x^2)^{2x}[\dfrac{2x^2}{1+x^2} + \ln(1+x^2)]$

2(d) $(1+x^2)^{x^2}$ [সি.'০১]

$\dfrac{d}{dx}(1+x^2)^{x^2} = (1+x^2)^{x^2}$