1 Initial model

var
// Main variables
L_GDP_GAP // Output Gap
DLA_CPI // QoQ Core Inflation
RS // MP Rate
RR_GAP // Real Interest Rate Gap
RES_L_GDP_GAP // exogenous states
RES_DLA_CPI
RES_RS
;
varexo
SHK_L_GDP_GAP // Output gap shock
SHK_DLA_CPI
SHK_RS // Foreign interest rate shock
;

```
// Aggregate demand
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*RR_GAP(+1) + RES_L_GDP_GAP;
// Core Inflation
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
// Monetary policy reaction function
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*DLA_CPI(+3) + g3*L_GDP_GAP) + RES_RS;
RR_GAP = RS - DLA_CPI(+1);
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*RES_RS(-2) + SHK_RS;
```

2 Clean Equations

```
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*(RR_GAP(+1)) + RES_L_GDP_GAP;
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
RR_GAP = RS - DLA_CPI(+1)
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*DLA_CPI(+3) + g3*L_GDP_GAP) + RES_RS;
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*RES_RS + SHK_RS;
```

3 Identify lead and lag structure.

Variables in the system

```
L_GDP_GAP enters in t, t+1, t-1
```

```
DLA_CPI  enters in t, t+1, t+3, t-1
RS: enters  t, t-1, t+1
RES_L_GDP_GAP: enters in t, t-1
RES_DLA_CPI: enters in t, t-1
RES_RS: enters in t,t-1, t-2

The system should be written for variables in t+1, t and t-1. So variables with longer leads

aux_DLA_CPI_lead(t) = DLA_CPI(t+1)
aux_DLA_CPI_lead2(t) = aux_DLA_CPI_lead(t+1)
aux_DLA_CPI_lead3(t) = aux_DLA_CPI_lead2(t+1)

Similar longer lags (longer than (-1)) should also imply auxiliary variables. This will impl
RES_RS_lag = RES_RS(-1)
RES_RS_lag2 = RES_RS_lag(-1)
```

4 Rewrite the system of equations in terms of the auxiliary variables

```
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*(RR_GAP(+1)) + RES_L_GDP_GAP;
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*aux_DLA_CPI_lead(+1) + g3*L_GDP_GAP) + RES_RS;
RR_GAP = RS - DLA_CPI(+1);
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*aux_RES_RS_lag(-1) + SHK_RS;
aux_DLA_CPI_lead = DLA_CPI(+1)
aux_DLA_CPI_lead2 = aux_DLA_CPI_lead(+1)
aux_RES_RS_lag = RES_RS(-1)
```

5 Replace by _mk and pk.

```
L_GDP_GAP = (1-b1)*L_GDP_GAP_p1 + b1*L_GDP_GAP_m1 - b4*(RR_GAP_p1) + RES_L_GDP_GAP;
DLA_CPI = a1*DLA_CPI_m1 + (1-a1)*DLA_CPI_p1 + a2*L_GDP_GAP + RES_DLA_CPI;
RS = g1*RS_m1 + (1-g1)*(DLA_CPI_p1 + g2*aux_DLA_CPI_lead_p1 + g3*L_GDP_GAP) + RES_RS;
RR_GAP = RS - DLA_CPI_p1;
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS_m1 + rho_rs2*aux_RES_RS_lag_m1 + SHK_RS;
aux_DLA_CPI_lead = DLA_CPI_p1
```

```
aux_DLA_CPI_lead2 = aux_DLA_CPI_lead_p1
aux_RES_RS_lag = RES_RS_m1

10 equations on 10 variables

List of contempotaneous variables
L_GDP_GAP
DLA_CPI
RS
RR_GAP
RES_L_GDP_GAP
RES_DLA_CPI
RES_RS
aux_DLA_CPI_lead
aux_DLA_CPI_lead2
aux_RES_RS_lag

The order of the variables should be
forward_looking =
L_GDP_GA
DLA_CPI
RS
aux_DLA_CPI_lead
aux_DLA_CPI_lead2


backward_looking_list
RES_L_GDP_GAP
RES_DLA_CPI
RES_RS
aux_RES_RS_lag

exo_var_list
SHK_L_GDP_GAP
SHK_DLA_CPI
SHK_RS
```

6 How to clasify variables

model;

y = a*y(-1) + b*y(+2) - c*w(+1) + zy;
x = alpha*x(+1) + beta*y + zx;
w = x(+1) - zw;
zy = rhozy*zy(-1) + shk_zy;

zx = rhozx*zx(-1) + shk_zx;
zw = rhozw*zw(-1) + rhozw*zw(-2) + shk_zw;
end;
parsed model
0 = a*y_m1 + b*aux_y_lead_p1 - c*w_p1 + zy-y;
0 = alpha*x_1 + beta*y + zx -x;
0 = x_p1 - zw-w;
0 = rhozy*zy_m1 + shk_zy -zy;
0 = rhozx*zx_m1 + shk_zx -zx;
0 = rhozw*zw_m1 + rhozw1*aux_zw_lag_m1 + shk_zw-zw;
0 = -aux_zw_lag + zw_m1;
0 = -aux_y_lead + y_p1;
Variables initial order y,w, x, zy, zx, zw, aux_zw_lag, aux_y_lead,

$$A = \begin{bmatrix} b & -c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \beta & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix};$$

$$C = \begin{bmatrix} a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{zy} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{zx} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{zw} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

How to order the variables:
Backward looking exogenous variables:
All columns in A for these variables are zero, and
All columns of other variables on these variables are zero in B and C.

Backward looking endogenous variables:
All columns in A for those variables are zero.
Forward looking variables:
All columns in C for those variables are zero.
Static variables:
All columns in A, B for those variables are zero.
The idea is then
1) Eliminate static variables by substituting out them:
2) On the remaining variables order them as: backward looking exogenous states zy, zx, zw follow by forward/backward endogenous variables y,w, x.
3) Order equations for the backward exogenous variables first. In this case, we don;t have the other equations variables in any way. Just move the equations for the exogenous variables first.

7 Jacoobians of the ordered equations and variables

$x_t = (mix_t, back_t)$,eps_t=exo_var_list

$$A = \frac{\partial F}{\partial x_{t+1}}$$

$$B = \frac{\partial F}{\partial x_t}$$

$$C = \frac{\partial F}{\partial x_{t-1}}$$

$$D = \frac{\partial F}{\partial \epsilon_t}$$

8 Build the complete state space

8.1 elements

The dyn files has sections

```
varexo_trends
SHK_L_GDP_TREND,
SHK_G_TREND,
SHP_PI_TREND,
SHK_RS_TREND,
SHK_RR_TREND
;
```

```
trends_vars
L_GDP_TREND,
PI_TREND,
RS_TREND,
RR_TREND,
G_TREND
;
trend_model;
L_GDP_TREND = L_GDP_TREND(-1) + G_TREND(-1) + SHK_L_GDP_TREND;
G_TREND = G_TREND(-1) + SHK_G_TREND;
PI_TREND = PI_TREND(-1) + SHP_PI_TREND;
RS_TREND = RR_TREND + PI_TREND;
RR_TREND = RR_TREND(-1) + SHK_RR_TREND;
end;
varobs
L_GDP_OBS
DLA_CPI_OBS
PI_TREND_OBS
RS_OBS
;
measument_equations;
L_GDP_OBS = L_GDP_TREND + L_GDP_GAP;
DLA_CPI_OBS = DLA_CPI + PI_TREND;
PI_TREND_OBS = PI_TREND;
RS_OBS = RS_TREND + RS;
end;
```

In this section you have defined: The list of trends in trends_vars, the trend_model, and the varexo_trends that are the list of shocks to the stochastic trends. You also have varobs with the list of observable variables and measument_equations; end; that links how observable variables are related with the statee vector.

8.2 Building the state space

The SPD algorithm gives you the solution

$$y_t = Py_{t-1} + Q\epsilon_t$$

$$\epsilon_t \sim N(0, \Sigma)$$

this soluction should be augmented to include the trends and the link with the observable variables.

Let's define an augmented state vector $y_t^a = (y_t, trends_t)$. The the augmented state-space would be

$$y_t^a = \left[\begin{array}{cc} P & 0 \\ 0 & P_{trends} \end{array} \right] y_{t-1}^a + \left[\begin{array}{cc} Q & 0 \\ 0 & Q_{trends} \end{array} \right] \epsilon_t^a$$

shocks ϵ_t^a include shocks to trends.

The measurement equation would be

$$y_t^{obs} = \Omega y_t^a + H\xi_t$$

$$\xi_t \sim N(0, I)$$

and H is a diagonal matrix with measurement errors standard errors. Ω is the measurement matrix/observation matrix an relects the relationships presented in measument_equations.

8.3 How to build Q_{trends}, Σ and P_trends,Ω?

If $trend_t$is a vector with L_GDP_TREND, PI_TREND,RS_TREND,RR_TREND,G_TREND and then the parsed

```
L_GDP_TREND = L_GDP_TREND_m1 + G_TREND_m1 + SHK_L_GDP_TREND;
G_TREND = G_TREND_m1 + SHK_G_TREND;
PI_TREND = PI_TREND_m1 + SHP_PI_TREND;
RS_TREND = RR_TREND + PI_TREND;
RR_TREND = RR_TREND_m1 + SHK_RR_TREND;
```

```
the P_trend is the derivarive of the RHS of each equation with respect to _m coefficients
```

$$trend_t = P_{trend} trend_{t-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} trend_{t-1}$$

$$Q_{trends} = \operatorname{diag}(\sigma_{trend1}, \ldots, \sigma_{trendx})$$

$$\Sigma = \operatorname{diag}(\sigma_{sh1}, \ldots, \sigma_{shx})$$

- P_trend: Could depend on parameters also.
- The vector of parameters (full) should include sigma's for shocks (place holders), parameters for trends.

SImilarly Ω is the derivative (jacobbian) of the observation measument_equations with respect to y_t^{obs}