

## 1 Initial model

```
// Aggregate demand
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*RR_GAP(+1) + RES_L_GDP_GAP;
// Core Inflation
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
// Monetary policy reaction function
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*DLA_CPI(+3) + g3*L_GDP_GAP) + RES_RS;
RR_GAP = RS - DLA_CPI(+1);
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*RES_RS(-2) + SHK_RS;
```

## 2 Identification of equations/variables

1. Dynamic: A variable that contains leads and lags of its own variable.
2. Static: Equations/variables that at time t are defined purely by other variables at time t, or the expected value of other variables. The variable itself does not appear with a lead or lag in its own definition.

### identification

```
L_GDP_GAP -> Dynamic (depends on L_GDP_GAP(+1) and L_GDP_GAP(-1));
DLA_CPI -> Dynamic (depends on DLA_CPI(-1), DLA_CPI(+1));
RS -> (Depends on RS(-1) and DLA_CPI(+1), DLA_CPI(+3));
RR_GAP -> Static (at time t depends on RS and the expected value of DLA_CPI, that are known at time t);
RES_L_GDP_GAP(+1) -> dynamic (exogenous state), depends on RES_L_GDP_GAP and SHK_L_GDP_GAP;
RES_DLA_CPI(+1) -> dynamic (exogenous state), depends on RES_DLA_CPI and SHK_DLA_CPI;
RES_RS(+1) -> dynamic (exogenous state), depends on RES_RS(-1) and SHK_RS;
```

## 3 Eliminate Static Equations

```
// Aggregate demand
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*(RS(+1) - DLA_CPI(+2)) + RES_L_GDP_GAP;
// Core Inflation
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
// Monetary policy reaction function
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*DLA_CPI(+3) + g3*L_GDP_GAP) + RES_RS;
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*RES_RS(-2) + SHK_RS;
```

## 4 Identify lead and lag structure.

Variables in the system

L\_GDP\_GAP enters in t, t+1, t-1  
DLA\_CPI enters in t, t+1, t+3, t-1  
RS: enters t, t-1, t+1  
RES\_L\_GDP\_GAP: enters in t, t-1  
RES\_DLA\_CPI: enters in t, t-1  
RES\_RS: enters in t, t-1, t-2

The system should be written for variables in t+1, t and t-1. So variables with longer lags

aux\_DLA\_CPI\_lead(t) = DLA\_CPI(t+1)  
aux\_DLA\_CPI\_lead2(t) = aux\_DLA\_CPI\_lead(t+1)  
aux\_DLA\_CPI\_lead3(t) = aux\_DLA\_CPI\_lead2(t+1)

Similar longer lags (longer than (-1)) should also imply auxiliary variables. This will imply  
RES\_RS\_lag = RES\_RS(-1)  
RES\_RS\_lag2 = RES\_RS\_lag(-1)

## 5 Rewrite the system of equations in terms of the auxiliary variables

L\_GDP\_GAP = (1-b1)\*L\_GDP\_GAP(+1) + b1\*L\_GDP\_GAP(-1) - b4\*(RS(+1) - aux\_DLA\_CPI\_lead2(t))  
DLA\_CPI = a1\*DLA\_CPI(-1) + (1-a1)\*DLA\_CPI(+1) + a2\*L\_GDP\_GAP + RES\_DLA\_CPI;  
RS = g1\*RS(-1) + (1-g1)\*(DLA\_CPI(+1) + g2\*aux\_DLA\_CPI\_lead3(t) + g3\*L\_GDP\_GAP) + RES\_RS;  
RES\_L\_GDP\_GAP = rho\_L\_GDP\_GAP\*RES\_L\_GDP\_GAP + SHK\_L\_GDP\_GAP;  
RES\_DLA\_CPI = rho\_DLA\_CPI\*RES\_DLA\_CPI + SHK\_DLA\_CPI;  
RES\_RS = rho\_rs\*RES\_RS(-1) + rho\_rs2\*aux\_RES\_RS\_lag(-1) + SHK\_RS;  
aux\_DLA\_CPI\_lead(t) = DLA\_CPI(t+1)  
aux\_DLA\_CPI\_lead2(t) = aux\_DLA\_CPI\_lead(t+1)  
aux\_DLA\_CPI\_lead3(t) = aux\_DLA\_CPI\_lead(t+1)  
aux\_RES\_RS\_lag = RES\_RS(-1)

## 6 Reduce the system again.

L\_GDP\_GAP = (1-b1)\*L\_GDP\_GAP(+1) + b1\*L\_GDP\_GAP(-1) - b4\*(RS(+1) - aux\_DLA\_CPI\_lead(t+1))  
DLA\_CPI = a1\*DLA\_CPI(-1) + (1-a1)\*DLA\_CPI(+1) + a2\*L\_GDP\_GAP + RES\_DLA\_CPI;  
RS = g1\*RS(-1) + (1-g1)\*(DLA\_CPI(+1) + g2\*aux\_DLA\_CPI\_lead2(t+1) + g3\*L\_GDP\_GAP) + RES\_RS

```

RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*RES_RS(-2) + SHK_RS;
aux_DLA_CPI_lead(t) = DLA_CPI(t+1)
aux_DLA_CPI_lead2(t) = aux_DLA_CPI_lead(t+1)
aux_RES_RS_lag = RES_RS(-1)

```

9 equations on 9 variables

List of contemporaneous variables

```

L_GDP_GAP
DLA_CPI
RS
RES_L_GDP_GAP
RES_DLA_CPI
RES_RS
aux_DLA_CPI_lead
aux_DLA_CPI_lead2
aux_RES_RS_lag

```

List of lagged variables

```

L_GDP_GAP_m1
DLA_CPI_m1
RS_m1
RES_L_GDP_GAP_m1
RES_DLA_CPI_m1
RES_RS_m1
aux_DLA_CPI_lead_m1
aux_DLA_CPI_lead2_m1
aux_RES_RS_lag_m1

```

List of forward variables

```

L_GDP_GAP_p1
DLA_CPI_p1
RS_p1
RES_L_GDP_GAP_p1
RES_DLA_CPI_p1
RES_RS_p1
aux_DLA_CPI_lead_p1
aux_DLA_CPI_lead2_p1
aux_RES_RS_lag_p1

```

Final set of equations (before final reordering)

```

0 = (1-b1)*L_GDP_GAP_p1 + b1*L_GDP_GAP_m1 - b4*(RS_p1 - aux_DLA_CPI_lead2) + RES_L_GDP_GAP
0 = a1*DLA_CPI_m1 + (1-a1)*DLA_CPI_p1 + a2*L_GDP_GAP + RES_DLA_CPI -DLA_CPI;
0 = g1*RS_m1 + (1-g1)*(DLA_CPI_p1 + g2*aux_DLA_CPI_lead3 + g3*L_GDP_GAP) + RES_RS -RS ;

```

$$\begin{aligned}
0 &= \text{rho\_L\_GDP\_GAP} * \text{RES\_L\_GDP\_GAP\_m1} + \text{SHK\_L\_GDP\_GAP} - \text{RES\_L\_GDP\_GAP}; \\
0 &= \text{rho\_DLA\_CPI} * \text{RES\_DLA\_CPI\_m1} + \text{SHK\_DLA\_CPI} - \text{RES\_DLA\_CPI}; \\
0 &= \text{rho\_rs} * \text{RES\_RS\_m1} + \text{rho\_rs2} * \text{aux\_RES\_RS\_lag\_m1} + \text{SHK\_RS} - \text{RES\_RS}; \\
0 &= \text{DLA\_CPI\_p1} - \text{aux\_DLA\_CPI\_lead} \\
0 &= \text{aux\_DLA\_CPI\_lead\_p1} - \text{aux\_DLA\_CPI\_lead2} \\
0 &= \text{aux\_DLA\_CPI\_lead2\_p1} - \text{aux\_DLA\_CPI\_lead3} \\
0 &= \text{RES\_RS\_m1} - \text{aux\_RES\_RS\_lag}
\end{aligned}$$

## 7 Final classification and ordering

Assume all variables are backward looking and eliminate variables that are mix or forward looking.

$0 = (1-b1)*L\_GDP\_GAP\_p1 + b1*L\_GDP\_GAP\_m1 - b4*(RS\_p1 - \text{aux\_DLA\_CPI\_lead2}) + \text{RES\_L\_GDP\_GAP}$   
 in this equation L\_GDP\_GAP enters with its own leads and lags so L\_GDP\_GAP is mix.  
 but RS, aux\_DLA\_CPI\_lead2, RES\_L\_GDP\_GAP that enter can still be anything.

From the second equations,

$0 = a1*DLA\_CPI\_m1 + (1-a1)*DLA\_CPI\_p1 + a2*L\_GDP\_GAP + \text{RES\_DLA\_CPI} - DLA\_CPI;$   
 DLA\_CPI\_m1 -> mix and RES\_ can be anything else.

$0 = g1*RS\_m1 + (1-g1)*(DLA\_CPI\_p1 + g2*aux\_DLA\_CPI\_lead3 + g3*L\_GDP\_GAP) + \text{RES\_RS} - RS ;$

RS -> Mix: Depends on its own lags but also depends on other variables lead and con, so mix.

The next equations:

$0 = \text{RES\_L\_GDP\_GAP} - \text{rho\_L\_GDP\_GAP} * \text{RES\_L\_GDP\_GAP\_m1} + \text{SHK\_L\_GDP\_GAP};$

$0 = \text{rho\_DLA\_CPI} * \text{RES\_DLA\_CPI\_m1} + \text{SHK\_DLA\_CPI} - \text{RES\_DLA\_CPI};$

$0 = \text{rho\_rs} * \text{RES\_RS\_m1} + \text{rho\_rs2} * \text{aux\_RES\_RS\_lag\_m1} + \text{SHK\_RS} - \text{RES\_RS};$

Depend on exogenous variables (SHK) and only on its own lags then

RES\_L\_GDP\_GAP, RES\_DLA\_CPI, RES\_DLA\_CPI -> Backward looking.

The next variables are forward looking

$0 = \text{DLA\_CPI\_p1} - \text{aux\_DLA\_CPI\_lead}$

$0 = \text{aux\_DLA\_CPI\_lead\_p1} - \text{aux\_DLA\_CPI\_lead2}$

$0 = \text{aux\_DLA\_CPI\_lead2\_p1} - \text{aux\_DLA\_CPI\_lead3}$

and finally

$0 = \text{RES\_RS\_m1} - \text{aux\_RES\_RS\_lag}$

aux\_RES\_RS\_lag -> Depend on only on a lag of a backward looking variable. So it is backward looking.

The finally ordered system of equations should be

Final set of equations (before final reordering)

$0 = (1-b1)*L\_GDP\_GAP\_p1 + b1*L\_GDP\_GAP\_m1 - b4*(RS\_p1 - \text{aux\_DLA\_CPI\_lead2}) + \text{RES\_L\_GDP\_GAP}$

$0 = a1*DLA\_CPI\_m1 + (1-a1)*DLA\_CPI\_p1 + a2*L\_GDP\_GAP + \text{RES\_DLA\_CPI} - DLA\_CPI;$

$0 = g1*RS\_m1 + (1-g1)*(DLA\_CPI\_p1 + g2*aux\_DLA\_CPI\_lead3 + g3*L\_GDP\_GAP) + \text{RES\_RS} - RS ;$

$0 = \text{DLA\_CPI\_p1} - \text{aux\_DLA\_CPI\_lead}$

$0 = \text{aux\_DLA\_CPI\_lead\_p1} - \text{aux\_DLA\_CPI\_lead2}$

$0 = \text{aux\_DLA\_CPI\_lead2\_p1} - \text{aux\_DLA\_CPI\_lead3}$

$0 = \text{RES\_RS\_m1} - \text{aux\_RES\_RS\_lag}$

$0 = \text{rho\_L\_GDP\_GAP} * \text{RES\_L\_GDP\_GAP\_m1} + \text{SHK\_L\_GDP\_GAP} - \text{RES\_L\_GDP\_GAP};$

```

0 = rho_DLA_CPI*RES_DLA_CPI_m1 + SHK_DLA_CPI - RES_DLA_CPI;
0 = rho_rs*RES_RS_m1 + rho_rs2*aux_RES_RS_lag_m1 + SHK_RS - RES_RS;

```

```

and the order of the variables should be
mix_list =
L_GDP_GA
DLA_CPI
RS
aux_DLA_CPI_lead
aux_DLA_CPI_lead2

```

```

backward_looking_list
RES_L_GDP_GAP
RES_DLA_CPI
RES_RS
aux_RES_RS_lag

```

```

exo_var_list
SHK_L_GDP_GAP
SHK_DLA_CPI
SHK_RS

```

## 8 Jacobians of the ordered equations and variables

$x_t = (mix_t, back_t), eps_t = exo\_var\_list$

$$A = \frac{\partial F}{\partial x_{t+1}}$$

$$B = \frac{\partial F}{\partial x_t}$$

$$C = \frac{\partial F}{\partial x_{t-1}}$$

$$D = \frac{\partial F}{\partial \epsilon_t}$$