1 Initial model

```
// Aggregate demand
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*RR_GAP(+1) + RES_L_GDP_GAP;
// Core Inflation
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
// Monetary policy reaction function
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*DLA_CPI(+3) + g3*L_GDP_GAP) + RES_RS;
RR_GAP = RS - DLA_CPI(+1);
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*RES_RS(-2) + SHK_RS;
```

2 Identification of equations/variables

1. Dynamic: A variable that contains leads and lags of its own variable.
2. Static: Equations/variables that at time t are defined purely by other variables at time t, or the expected value of other variables. The variable itself does not appear with a lead or lag in its own definition.

identification

```
L_GDP_GAP -> Dynamic (depends on L_GDP_GAP(+1) and L_GDP_GAP(-1));
DLA_CPI -> Dynamic (depends on DLA_CPI(-1), DLA_CPI(+1));
RS -> (Depends on  RS(-1) and DLA_CPI(+1), DLA_CPI(+3));
RR_GAP -> Static (at time t depends on RS and the expected value of DLA_CPI, that are kno
RES_L_GDP_GAP(+1) ->  dynamic (exogenous state), depends on RES_L_GDP_GAP and SHK_L_GDP_G
RES_DLA_CPI(+1) ->  dynamic (exogenous state), depends on RES_DLA_CPI and SHK_L_GDP_GAP;
RES_RS(+1) ->  dynamic (exogenous state), depends on RES_RS(-1) and SHK_L_GDP_GAP;
```

3 Eliminate Static Equations

```
// Aggregate demand
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*(RS(+1) - DLA_CPI(+2)) + RES_L_G
// Core Inflation
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
// Monetary policy reaction function
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*DLA_CPI(+3) + g3*L_GDP_GAP) + RES_RS;
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*RES_RS + SHK_RS;
```

4 Identify lead and lag structure.

Variables in the system

```
L_GDP_GAP enters in t, t+1, t-1
DLA_CPI  enters in t, t+1, t+3, t-1
RS: enters  t, t-1, t+1
RES_L_GDP_GAP: enters in t, t-1
RES_DLA_CPI: enters in t+1, t-1
RES_RS: enters in t,t-1, t-2

The system should be written for variables in t+1, t and t-1. So variables with longer le

aux_DLA_CPI_lead(t) = DLA_CPI(t+1)
aux_DLA_CPI_lead2(t) = aux_DLA_CPI_lead(t+1)
aux_DLA_CPI_lead3(t) = aux_DLA_CPI_lead2(t+1)

Similar longer lags (longer than (-1)) should also imply auxiliary variables. This will i
RES_RS_lag = RES_RS(-1)
RES_RS_lag2 = RES_RS_lag(-1)
```

5 Rewrite the system of equations in terms of the auxiliary variables

```
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*(RS(+1) - aux_DLA_CPI_lead2(t))
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*aux_DLA_CPI_lead3(t) + g3*L_GDP_GAP) + RES_RS;
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI + SHK_DLA_CPI;
RES_RS = rho_rs*RES_RS(-1) + rho_rs2*aux_RES_RS_lag(-1) + SHK_RS;
aux_DLA_CPI_lead(t) = DLA_CPI(t+1)
aux_DLA_CPI_lead2(t) = aux_DLA_CPI_lead(t+1)
aux_DLA_CPI_lead3(t) = aux_DLA_CPI_lead(t+1)
aux_RES_RS_lag = RES_RS(-1)
```

6 Reduce the system again.

```
L_GDP_GAP = (1-b1)*L_GDP_GAP(+1) + b1*L_GDP_GAP(-1) - b4*(RS(+1) - aux_DLA_CPI_lead(t+1))
DLA_CPI = a1*DLA_CPI(-1) + (1-a1)*DLA_CPI(+1) + a2*L_GDP_GAP + RES_DLA_CPI;
RS = g1*RS(-1) + (1-g1)*(DLA_CPI(+1) + g2*aux_DLA_CPI_lead2(t+1) + g3*L_GDP_GAP) + RES_RS
```

```
RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
RES_DLA_CPI = rho_DLA_CPI*RES_DLA_CPI(-1) + SHK_DLA_CPI;
RES_RS  = rho_rs*RES_RS(-1) + rho_rs2*RES_RS(-2) + SHK_RS;
aux_DLA_CPI_lead(t) = DLA_CPI(t+1)
aux_DLA_CPI_lead2(t) = aux_DLA_CPI_lead(t+1)
aux_RES_RS_lag = RES_RS(-1)

10 equations on 10 variables

List of contempotaneous variables
L_GDP_GAP
DLA_CPI
RS
RES_L_GDP_GAP
RES_DLA_CPI
RES_RS
aux_DLA_CPI_lead
aux_DLA_CPI_lead2
aux_DLA_CPI_lead3
aux_RES_RS_lag

List of lagged variables
L_GDP_GAP_m1
DLA_CPI_m1
RS_m1
RES_L_GDP_GAP_m1
RES_DLA_CPI_m1
RES_RS_m1
aux_DLA_CPI_lead_m1
aux_DLA_CPI_lead2_m1
aux_DLA_CPI_lead3_m1
aux_RES_RS_lag_m1

List of forward variables
L_GDP_GAP_p1
DLA_CPI_p1
RS_p1
RES_L_GDP_GAP_p1
RES_DLA_CPI_p1
RES_RS_p1
aux_DLA_CPI_lead_p1
aux_DLA_CPI_lead2_p1
aux_DLA_CPI_lead3_p1
aux_RES_RS_lag_p1

Final set of equations:
```

```
0 = (1-b1)*L_GDP_GAP_p1 + b1*L_GDP_GAP_m1 - b4*(RS_p1 - aux_DLA_CPI_lead2) + RES_L_GDP_GA
0 = a1*DLA_CPI_m1 + (1-a1)*DLA_CPI_p1 + a2*L_GDP_GAP + RES_DLA_CPI -DLA_CPI;
0 = g1*RS_m1 + (1-g1)*(DLA_CPI_p1 + g2*aux_DLA_CPI_lead3 + g3*L_GDP_GAP) + RES_RS -RS ;
0 = RES_L_GDP_GAP = rho_L_GDP_GAP*RES_L_GDP_GAP(-1) + SHK_L_GDP_GAP;
0 = rho_DLA_CPI*RES_DLA_CPI_m1 + SHK_DLA_CPI - RES_DLA_CPI;
0 = rho_rs*RES_RS_m1 + rho_rs2*aux_RES_RS_lag_m1 + SHK_RS - RES_RS;
0 = DLA_CPI_p1 - aux_DLA_CPI_lead
0 = aux_DLA_CPI_lead_p1 - aux_DLA_CPI_lead2
0 = aux_DLA_CPI_lead2_p1 - aux_DLA_CPI_lead3
0 = RES_RS_m1 - aux_RES_RS_lag
```