# Model 1 Equations

#### Core World Stochastic Trends

$$\rho_t^{\omega, \mathrm{tr}} = \rho_{t-1}^{\omega, \mathrm{tr}} + \epsilon_t^{\rho, \omega} \tag{1}$$

$$\pi_t^{\omega, \text{tr}} = \pi_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\pi, \omega} \tag{2}$$

#### **Core Common Factors for Deviations**

$$f_t^{\rho,\text{dev}} = f_{t-1}^{\rho,\text{dev}} + \epsilon_t^{f,\rho} \tag{3}$$

$$f_t^{\pi,\text{dev}} = f_{t-1}^{\pi,\text{dev}} + \epsilon_t^{f,\pi} \tag{4}$$

## Core Idiosyncratic Deviation Trends

$$\rho_t^{\text{US, idio, tr}} = \rho_{t-1}^{\text{US, idio, tr}} + \epsilon_t^{\rho, \text{US, idio}}$$
 (5)

$$\pi_t^{\text{US, idio, tr}} = \pi_{t-1}^{\text{US, idio, tr}} + \epsilon_t^{\pi, \text{US, idio}}$$
 (6)

$$\rho_t^{\text{EA, idio, tr}} = \rho_{t-1}^{\text{EA, idio, tr}} + \epsilon_t^{\rho, \text{EA, idio}} \tag{7}$$

$$\pi_t^{\text{EA, idio, tr}} = \pi_{t-1}^{\text{EA, idio, tr}} + \epsilon_t^{\pi, \text{EA, idio}}$$
 (8)

$$\rho_t^{\text{JP, idio, tr}} = \rho_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\rho, \text{JP, idio}}$$
(9)

$$\pi_t^{\text{JP, idio, tr}} = \pi_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\pi, \text{JP, idio}} \tag{10}$$

## Derived Full Deviation Trends (Factor model structure)

$$\rho_t^{\rm US, \ dev, \ tr} = 1.0 \cdot f_t^{\rho, \rm dev} + \rho_t^{\rm US, \ idio, \ tr} \tag{11} \label{eq:total_state}$$

$$\pi_t^{\text{US, dev, tr}} = 1.0 \cdot f_t^{\pi, \text{dev}} + \pi_t^{\text{US, idio, tr}}$$

$$\tag{12}$$

$$\rho_t^{\text{EA, dev, tr}} = \lambda_{f\rho}^{\rho, \text{EA}} f_t^{\rho, \text{dev}} + \rho_t^{\text{EA, idio, tr}}$$
(13)

$$\pi_t^{\text{EA, dev, tr}} = \lambda_{f^{\pi}}^{\pi, \text{EA}} f_t^{\pi, \text{dev}} + \pi_t^{\text{EA, idio, tr}}$$
(14)

$$\rho_t^{\rm JP, \; dev, \; tr} = \lambda_{f\rho}^{\rho, \rm JP} f_t^{\rho, \rm dev} + \rho_t^{\rm JP, \; idio, \; tr} \tag{15}$$

$$\pi_t^{\text{JP, dev, tr}} = \lambda_{f^{\pi}}^{\pi, \text{JP}} f_t^{\pi, \text{dev}} + \pi_t^{\text{JP, idio, tr}}$$
(16)

#### Derived Full Real Rate and Inflation Trends

$$\rho_t^{\rm US, \; full, \; tr} = \rho_t^{\omega, \rm tr} + \rho_t^{\rm US, \; dev, \; tr} \tag{17}$$

$$\pi_t^{\text{US, full, tr}} = \lambda^{\pi, \text{US}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{US, dev, tr}}$$
(18)

$$\rho_t^{\text{EA, full, tr}} = \rho_t^{\omega, \text{tr}} + \rho_t^{\text{EA, dev, tr}} \tag{19}$$

$$\pi_t^{\text{EA, full, tr}} = \lambda^{\pi, \text{EA}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{EA, dev, tr}}$$
(20)

$$\rho_t^{\mathrm{JP,\;full,\;tr}} = \rho_t^{\omega,\mathrm{tr}} + \rho_t^{\mathrm{JP,\;dev,\;tr}} \tag{21}$$

$$\pi_t^{\rm JP, \, full, \, tr} = \lambda^{\pi, \rm JP} \pi_t^{\omega, \rm tr} + \pi_t^{\rm JP, \, dev, \, tr} \tag{22}$$

## Core Country-Specific Output Growth Trends (Euler Equations)

$$\gamma_t^{\rm US, \; tr} = \frac{1}{\phi_{\rm US}} \rho_t^{\rm US, \; full, \; tr} + \epsilon_t^{\gamma, \rm US} \tag{23}$$

$$\gamma_t^{\text{EA, tr}} = \frac{1}{\phi_{\text{EA}}} \rho_t^{\text{EA, full, tr}} + \epsilon_t^{\gamma, \text{EA}}$$
(24)

$$\gamma_t^{\text{JP, tr}} = \frac{1}{\phi_{\text{JP}}} \rho_t^{\text{JP, full, tr}} + \epsilon_t^{\gamma, \text{JP}}$$
(25)

# Derived Full Nominal Short Rate Trends

$$\begin{split} R_t^{\text{US, sh, tr}} &= \rho_t^{\text{US, full, tr}} + \pi_t^{\text{US, full, tr}} \\ R_t^{\text{EA, sh, tr}} &= \rho_t^{\text{EA, full, tr}} + \pi_t^{\text{EA, full, tr}} \\ R_t^{\text{JP, sh, tr}} &= \rho_t^{\text{JP, full, tr}} + \pi_t^{\text{JP, full, tr}} \end{split} \tag{26}$$

$$R_t^{\text{EA, sh, tr}} = \rho_t^{\text{EA, full, tr}} + \pi_t^{\text{EA, full, tr}}$$
(27)

$$R_t^{\text{JP, sh, tr}} = \rho_t^{\text{JP, full, tr}} + \pi_t^{\text{JP, full, tr}}$$
(28)

# Model 2 Equations

#### Core World Stochastic Trends

$$\rho_t^{\omega, \text{tr}} = \rho_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\rho, \omega} \tag{29}$$

$$\pi_t^{\omega, \text{tr}} = \pi_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\pi, \omega} \tag{30}$$

$$\tau_t^{\omega, \text{tr}} = \tau_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\tau, \omega} \tag{31}$$

#### **Core Common Factors for Deviations**

$$f_t^{\rho,\text{dev}} = f_{t-1}^{\rho,\text{dev}} + \epsilon_t^{f,\rho} \tag{32}$$

$$f_t^{\pi,\text{dev}} = f_{t-1}^{\pi,\text{dev}} + \epsilon_t^{f,\pi}$$
(33)

$$f_t^{\tau,\text{dev}} = f_{t-1}^{\tau,\text{dev}} + \epsilon_t^{f,\tau} \tag{34}$$

#### Core Idiosyncratic Deviation Trends

$$\rho_t^{\rm US, \; idio, \; tr} = \rho_{t-1}^{\rm US, \; idio, \; tr} + \epsilon_t^{\rho, \rm US, \; idio} \tag{35}$$

$$\pi_t^{\text{US, idio, tr}} = \pi_{t-1}^{\text{US, idio, tr}} + \epsilon_t^{\pi, \text{US, idio}}$$
(36)

$$\tau_t^{\text{US, idio, tr}} = \tau_{t-1}^{\text{US, idio, tr}} + \epsilon_t^{\tau, \text{US, idio}}$$
(37)

$$\rho_t^{\text{EA, idio, tr}} = \rho_{t-1}^{\text{EA, idio, tr}} + \epsilon_t^{\rho, \text{EA, idio}}$$
(38)

$$\pi_t^{\text{EA, idio, tr}} = \pi_{t-1}^{\text{EA, idio, tr}} + \epsilon_t^{\pi, \text{EA, idio}}$$
(39)

$$\tau_t^{\text{EA, idio, tr}} = \tau_{t-1}^{\text{EA, idio, tr}} + \epsilon_t^{\tau, \text{EA, idio}}$$
(40)

$$\rho_t^{\mathrm{JP,\;idio,\;tr}} = \rho_{t-1}^{\mathrm{JP,\;idio,\;tr}} + \epsilon_t^{\rho,\mathrm{JP,\;idio}} \tag{41}$$

$$\pi_t^{\text{JP, idio, tr}} = \pi_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\pi, \text{JP, idio}}$$
 (42)

$$\pi_t^{\text{JP, idio, tr}} = \pi_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\pi, \text{JP, idio}}$$

$$\tau_t^{\text{JP, idio, tr}} = \tau_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\tau, \text{JP, idio}}$$

$$\tag{43}$$

# Derived Full Deviation Trends (Factor model structure)

$$\rho_t^{\text{US, dev, tr}} = 1.0 \cdot f_t^{\rho, \text{dev}} + \rho_t^{\text{US, idio, tr}}$$
(44)

$$\pi_t^{\text{US, dev, tr}} = 1.0 \cdot f_t^{\pi, \text{dev}} + \pi_t^{\text{US, idio, tr}}$$

$$\tag{45}$$

$$\tau_t^{\text{US, dev, tr}} = 1.0 \cdot f_t^{\tau, \text{dev}} + \tau_t^{\text{US, idio, tr}}$$
 (46)

$$\rho_t^{\mathrm{EA, \ dev, \ tr}} = \lambda_{f^{\rho}}^{\rho,\mathrm{EA}} f_t^{\rho,\mathrm{dev}} + \rho_t^{\mathrm{EA, \ idio, \ tr}} \tag{47}$$

$$\pi_t^{\text{EA, dev, tr}} = \lambda_{f^{\pi}}^{\pi, \text{EA}} f_t^{\pi, \text{dev}} + \pi_t^{\text{EA, idio, tr}}$$
(48)

$$\tau_t^{\text{EA, dev, tr}} = \lambda_{f^{\tau}}^{\tau, \text{EA}} f_t^{\tau, \text{dev}} + \tau_t^{\text{EA, idio, tr}}$$
(49)

$$\rho_t^{\rm JP, \; dev, \; tr} = \lambda_{f^\rho}^{\rho, \rm JP} f_t^{\rho, \rm dev} + \rho_t^{\rm JP, \; idio, \; tr} \tag{50}$$

$$\pi_t^{\mathrm{JP, \, dev, \, tr}} = \lambda_{f^\pi}^{\pi,\mathrm{JP}} f_t^{\pi,\mathrm{dev}} + \pi_t^{\mathrm{JP, \, idio, \, tr}} \tag{51}$$

$$\tau_t^{\text{JP, dev, tr}} = \lambda_{t\tau}^{\tau, \text{JP}} f_t^{\tau, \text{dev}} + \tau_t^{\text{JP, idio, tr}}$$
(52)

#### Derived Full Real Rate and Inflation Trends

$$\rho_t^{\text{US, full, tr}} = \rho_t^{\omega, \text{tr}} + \rho_t^{\text{US, dev, tr}}$$
(53)

$$\pi_t^{\text{US, full, tr}} = \lambda^{\pi, \text{US}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{US, dev, tr}}$$
(54)

$$\rho_t^{\text{EA, full, tr}} = \rho_t^{\omega, \text{tr}} + \rho_t^{\text{EA, dev, tr}}$$
(55)

$$\pi_t^{\text{EA, full, tr}} = \lambda^{\pi, \text{EA}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{EA, dev, tr}}$$
(56)

$$\rho_t^{\mathrm{JP, full, tr}} = \rho_t^{\omega, \mathrm{tr}} + \rho_t^{\mathrm{JP, dev, tr}} \tag{57}$$

$$\pi_t^{\text{JP, full, tr}} = \lambda^{\pi, \text{JP}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{JP, dev, tr}}$$
(58)

# Core Country-Specific Output Growth Trends (Euler Equations)

$$\gamma_t^{\text{US, tr}} = \frac{1}{\phi_{\text{US}}} \rho_t^{\text{US, full, tr}} + \epsilon_t^{\gamma, \text{US}}$$
(59)

$$\gamma_t^{\text{EA, tr}} = \frac{1}{\phi_{\text{EA}}} \rho_t^{\text{EA, full, tr}} + \epsilon_t^{\gamma, \text{EA}}$$
(60)

$$\gamma_t^{\text{JP, tr}} = \frac{1}{\phi_{\text{JP}}} \rho_t^{\text{JP, full, tr}} + \epsilon_t^{\gamma, \text{JP}}$$
(61)

## Derived Full Nominal Short & Long Rate Trends

$$R_t^{\text{US, sh, tr}} = \rho_t^{\text{US, full, tr}} + \pi_t^{\text{US, full, tr}}$$

$$\tag{62}$$

$$R_t^{\text{US, lg, tr}} = R_t^{\text{US, sh, tr}} + \tau_t^{\text{US, tr}} + \tau_t^{\text{US, dev, tr}}$$
(63)

$$R_t^{\text{EA, sh, tr}} = \rho_t^{\text{EA, full, tr}} + \pi_t^{\text{EA, full, tr}}$$
(64)

$$R_{t}^{\text{US, lg, tr}} = P_{t}^{\text{US, sh, tr}} + \tau_{t}^{\omega, \text{tr}} + \tau_{t}^{\text{US, dev, tr}}$$

$$R_{t}^{\text{EA, sh, tr}} = R_{t}^{\text{US, sh, tr}} + \tau_{t}^{\omega, \text{tr}} + \tau_{t}^{\text{US, dev, tr}}$$

$$R_{t}^{\text{EA, sh, tr}} = \rho_{t}^{\text{EA, full, tr}} + \pi_{t}^{\text{EA, full, tr}}$$

$$R_{t}^{\text{EA, lg, tr}} = R_{t}^{\text{EA, sh, tr}} + \tau_{t}^{\omega, \text{tr}} + \tau_{t}^{\text{EA, dev, tr}}$$

$$R_{t}^{\text{JP, sh, tr}} = \rho_{t}^{\text{JP, full, tr}} + \pi_{t}^{\text{JP, full, tr}}$$

$$R_{t}^{\text{JP, lg, tr}} = R_{t}^{\text{JP, sh, tr}} + \tau_{t}^{\omega, \text{tr}} + \tau_{t}^{\text{JP, dev, tr}}$$

$$(65)$$

$$R_t^{\text{JP, sh, tr}} = \rho_t^{\text{JP, full, tr}} + \pi_t^{\text{JP, full, tr}}$$
(66)

$$R_t^{\text{JP, lg, tr}} = R_t^{\text{JP, sh, tr}} + \tau_t^{\text{J,tr}} + \tau_t^{\text{JP, dev, tr}}$$

$$\tag{67}$$

# Model 3 Equations

#### Core World Stochastic Trends (Random Walks)

$$\rho_t^{\omega, \text{tr}} = \rho_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\rho, \omega} \tag{68}$$

$$\pi_t^{\omega, \text{tr}} = \pi_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\pi, \omega}$$

$$\tau_t^{\omega, \text{tr}} = \tau_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\tau, \omega}$$

$$(69)$$

$$\tau_t^{\omega, \text{tr}} = \tau_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\tau, \omega} \tag{70}$$

#### Core Country-Specific Stochastic Deviation Trends (Random Walks)

$$\rho_t^{\rm US, \ dev, \ tr} = \rho_{t-1}^{\rm US, \ dev, \ tr} + \epsilon_t^{\rho, \rm US, \ dev} \tag{71} \label{eq:71}$$

$$\pi_t^{\text{US, dev, tr}} = \pi_{t-1}^{\text{US, dev, tr}} + \epsilon_t^{\pi, \text{US, dev}}$$

$$\tau_t^{\text{US, dev, tr}} = \pi_{t-1}^{\text{US, dev, tr}} + \epsilon_t^{\pi, \text{US, dev}}$$

$$\tau_t^{\text{US, dev, tr}} = \tau_{t-1}^{\text{US, dev, tr}} + \epsilon_t^{\tau, \text{US, dev}}$$

$$\rho_t^{\text{EA, dev, tr}} = \rho_{t-1}^{\text{EA, dev, tr}} + \epsilon_t^{\rho, \text{EA, dev}}$$

$$(73)$$

$$\tau_t^{\text{US, dev, tr}} = \tau_{t-1}^{\text{US, dev, tr}} + \epsilon_t^{\tau, \text{US, dev}}$$
(73)

$$\rho_t^{\text{EA, dev, tr}} = \rho_{t-1}^{\text{EA, dev, tr}} + \epsilon_t^{\rho, \text{EA, dev}}$$
(74)

$$\pi_t^{\text{EA, dev, tr}} = \pi_{t-1}^{\text{EA, dev, tr}} + \epsilon_t^{\pi, \text{EA, dev}}$$
 (75)

$$\pi_t^{\text{EA, dev, tr}} = \pi_{t-1}^{\text{EA, dev, tr}} + \epsilon_t^{\pi, \text{EA, dev}}$$

$$\tau_t^{\text{EA, dev, tr}} = \tau_{t-1}^{\text{EA, dev, tr}} + \epsilon_t^{\tau, \text{EA, dev}}$$

$$(75)$$

$$\rho_t^{\rm JP,\; dev,\; tr} = \rho_{t-1}^{\rm JP,\; dev,\; tr} + \epsilon_t^{\rho,\rm JP,\; dev} \tag{77}$$

$$\pi_t^{\text{JP, dev, tr}} = \pi_{t-1}^{\text{JP, dev, tr}} + \epsilon_t^{\pi, \text{JP, dev}}$$
(78)

$$\tau_t^{\text{JP, dev, tr}} = \tau_{t-1}^{\text{JP, dev, tr}} + \epsilon_t^{\tau, \text{JP, dev}} \tag{79}$$

#### Derived Full Real Rate and Inflation Trends

$$\rho_t^{\text{US, full, tr}} = \rho_t^{\omega, \text{tr}} + \rho_t^{\text{US, dev, tr}}$$
(80)

$$\pi_t^{\text{US, full, tr}} = \lambda^{\pi, \text{US}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{US, dev, tr}}$$
 (81)

$$\rho_t^{\text{EA, full, tr}} = \rho_t^{\omega, \text{tr}} + \rho_t^{\text{EA, dev, tr}}$$
(82)

$$\pi_t^{\text{EA, full, tr}} = \lambda^{\pi, \text{EA}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{EA, dev, tr}}$$
(83)

$$\rho_t^{\rm JP,\;full,\;tr} = \rho_t^{\omega,\rm tr} + \rho_t^{\rm JP,\;dev,\;tr} \eqno(84)$$

$$\pi_t^{\text{JP, full, tr}} = \lambda^{\pi, \text{JP}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{JP, dev, tr}}$$
(85)

#### Core Country-Specific Output Growth Trends (Shocked Euler Equations)

$$\gamma_t^{\text{US, tr}} = \frac{1}{\phi_{\text{US}}} \rho_t^{\text{US, full, tr}} + \epsilon_t^{\gamma, \text{US}}$$
(86)

$$\gamma_t^{\text{EA, tr}} = \frac{1}{\phi_{\text{EA}}} \rho_t^{\text{EA, full, tr}} + \epsilon_t^{\gamma, \text{EA}} \tag{87}$$

$$\gamma_t^{\text{JP, tr}} = \frac{1}{\phi_{\text{JP}}} \rho_t^{\text{JP, full, tr}} + \epsilon_t^{\gamma, \text{JP}}$$
(88)

#### Derived Full Nominal Short & Long Rate Trends

$$R_t^{\text{US, sh, tr}} = \rho_t^{\text{US, full, tr}} + \pi_t^{\text{US, full, tr}}$$
(89)

$$R_t^{\text{US, lg, tr}} = R_t^{\text{US, sh, tr}} + \tau_t^{\omega, \text{tr}} + \tau_t^{\text{US, dev, tr}}$$
(90)

$$R_t^{\text{EA, sh, tr}} = \rho_t^{\text{EA, full, tr}} + \pi_t^{\text{EA, full, tr}}$$
(91)

$$R_t^{\text{EA, lg, tr}} = R_t^{\text{EA, sh, tr}} + \tau_t^{\omega, \text{tr}} + \tau_t^{\text{EA, dev, tr}}$$
(92)

$$R_t^{\text{JP, sh, tr}} = \rho_t^{\text{JP, full, tr}} + \pi_t^{\text{JP, full, tr}}$$
(93)

$$R_t^{\text{JP, lg, tr}} = R_t^{\text{JP, sh, tr}} + \tau_t^{\omega, \text{tr}} + \tau_t^{\text{JP, dev, tr}}$$
(94)

# Model 4 Equations

#### Core World Stochastic Trends

$$m_t^{\omega, \text{tr}} = m_{t-1}^{\omega, \text{tr}} + \epsilon_t^{m, \omega} \tag{95}$$

$$\xi_t^{\omega, \text{tr}} = \xi_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\xi, \omega} \tag{96}$$

$$\pi_t^{\omega, \text{tr}} = \pi_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\pi, \omega} \tag{97}$$

$$\tau_t^{\omega, \text{tr}} = \tau_{t-1}^{\omega, \text{tr}} + \epsilon_t^{\tau, \omega} \tag{98}$$

#### Implicit world SAFE real rate trend

$$r_t^{\omega,\text{safe, tr}} = m_t^{\omega,\text{tr}} - \xi_t^{\omega,\text{tr}} \tag{99}$$

#### **Core Common Factors for Deviations**

$$f_t^{\xi,\text{dev}} = f_{t-1}^{\xi,\text{dev}} + \epsilon_t^{f,\xi} \tag{100}$$

$$f_t^{\pi,\text{dev}} = f_{t-1}^{\pi,\text{dev}} + \epsilon_t^{f,\pi}$$
(101)

$$f_t^{\tau,\text{dev}} = f_{t-1}^{\tau,\text{dev}} + \epsilon_t^{f,\tau} \tag{102}$$

## Core Idiosyncratic Deviation Trends

$$\xi_t^{\rm US,\;idio,\;tr} = \xi_{t-1}^{\rm US,\;idio,\;tr} + \epsilon_t^{\xi,{\rm US,\;idio}} \tag{103}$$

$$\pi_t^{\text{US, idio, tr}} = \pi_{t-1}^{\text{US, idio, tr}} + \epsilon_t^{\pi, \text{US, idio}}$$
 (104)

$$\begin{split} \pi_t^{\text{US, idio, tr}} &= \pi_{t-1}^{\text{US, idio, tr}} + \epsilon_t^{\pi, \text{US, idio}} \\ \tau_t^{\text{US, idio, tr}} &= \tau_{t-1}^{\text{US, idio, tr}} + \epsilon_t^{\tau, \text{US, idio}} \end{split} \tag{104}$$

$$\xi_t^{\text{EA, idio, tr}} = \xi_{t-1}^{\text{EA, idio, tr}} + \epsilon_t^{\xi, \text{EA, idio}} \tag{106}$$

$$\pi_t^{\mathrm{EA, idio, tr}} = \pi_{t-1}^{\mathrm{EA, idio, tr}} + \epsilon_t^{\pi,\mathrm{EA, idio}} \tag{107}$$

$$\tau_t^{\mathrm{EA, idio, tr}} = \tau_{t-1}^{\mathrm{EA, idio, tr}} + \epsilon_t^{\tau,\mathrm{EA, idio}} \tag{108}$$

$$\xi_t^{\text{JP, idio, tr}} = \xi_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\xi, \text{JP, idio}}$$
(109)

$$\pi_t^{\text{JP, idio, tr}} = \pi_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\pi, \text{JP, idio}}$$
(110)

$$\begin{split} \pi_t^{\text{JP, idio, tr}} &= \pi_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\pi, \text{JP, idio}} \\ \tau_t^{\text{JP, idio, tr}} &= \tau_{t-1}^{\text{JP, idio, tr}} + \epsilon_t^{\tau, \text{JP, idio}} \end{split} \tag{110}$$

#### Derived Full Deviation Trends (Factor model structure)

$$\xi_t^{\text{US, dev, tr}} = 1.0 \cdot f_t^{\xi, \text{dev}} + \xi_t^{\text{US, idio, tr}} \tag{112} \label{eq:transformation}$$

$$\pi_t^{\text{US, dev, tr}} = 1.0 \cdot f_t^{\pi, \text{dev}} + \pi_t^{\text{US, idio, tr}} \tag{113} \label{eq:transform}$$

$$\tau_t^{\rm US, \ dev, \ tr} = 1.0 \cdot f_t^{\tau, \rm dev} + \tau_t^{\rm US, \ idio, \ tr} \tag{114} \label{eq:tts}$$

$$\xi_t^{\mathrm{EA, \ dev, \ tr}} = \lambda_{f^\xi}^{\xi,\mathrm{EA}} f_t^{\xi,\mathrm{dev}} + \xi_t^{\mathrm{EA, \ idio, \ tr}} \tag{115}$$

$$\pi_t^{\text{EA, dev, tr}} = \lambda_{f^{\pi}}^{\pi, \text{EA}} f_t^{\pi, \text{dev}} + \pi_t^{\text{EA, idio, tr}}$$
(116)

$$\tau_t^{\text{EA, dev, tr}} = \lambda_{f^{\tau}}^{\tau, \text{EA}} f_t^{\tau, \text{dev}} + \tau_t^{\text{EA, idio, tr}}$$
(117)

$$\xi_t^{\text{JP, dev, tr}} = \lambda_{f^{\xi}}^{\xi, \text{JP}} f_t^{\xi, \text{dev}} + \xi_t^{\text{JP, idio, tr}}$$
(118)

$$\pi_t^{\mathrm{JP, \, dev, \, tr}} = \lambda_{f^\pi}^{\pi,\mathrm{JP}} f_t^{\pi,\mathrm{dev}} + \pi_t^{\mathrm{JP, \, idio, \, tr}} \tag{119}$$

$$\tau_t^{\rm JP,\; dev,\; tr} = \lambda_{f^\tau}^{\tau, \rm JP} f_t^{\tau, \rm dev} + \tau_t^{\rm JP,\; idio,\; tr} \tag{120}$$

#### Derived Full SAFE Real Rate and Inflation Trends

$$r_t^{\text{US, safe, full, tr}} = r_t^{\omega, \text{safe, tr}} + \xi_t^{\text{US, dev, tr}}$$
 (121)

$$\pi_t^{\text{US, full, tr}} = \lambda^{\pi, \text{US}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{US, dev, tr}}$$
(122)

$$r_t^{\text{EA, safe, full, tr}} = r_t^{\omega, \text{safe, tr}} + \xi_t^{\text{EA, dev, tr}}$$
(123)

$$\pi_t^{\text{EA, full, tr}} = \lambda^{\pi, \text{EA}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{EA, dev, tr}}$$
(124)

$$r_t^{\mathrm{JP, \, safe, \, full, \, tr}} = r_t^{\omega, \mathrm{safe, \, tr}} + \xi_t^{\mathrm{JP, \, dev, \, tr}} \tag{125}$$

$$\pi_t^{\text{JP, full, tr}} = \lambda^{\pi, \text{JP}} \pi_t^{\omega, \text{tr}} + \pi_t^{\text{JP, dev, tr}}$$
(126)

# Core Country-Specific Output Growth Trends (Euler Equations, use SAFE real rates)

$$\gamma_t^{\text{US, tr}} = \frac{1}{\phi_{\text{US}}} r_t^{\text{US, safe, full, tr}} + \epsilon_t^{\gamma, \text{US}}$$
(127)

$$\gamma_t^{\text{EA, tr}} = \frac{1}{\phi_{\text{EA}}} r_t^{\text{EA, safe, full, tr}} + \epsilon_t^{\gamma, \text{EA}}$$
(128)

$$\gamma_t^{\rm JP, \, tr} = \frac{1}{\phi_{\rm JP}} r_t^{\rm JP, \, safe, \, full, \, tr} + \epsilon_t^{\gamma, \rm JP} \tag{129}$$

#### Derived Full Nominal SAFE Short & Long Rate Trends

$$R_t^{\rm US, \ safe, \ sh, \ tr} = r_t^{\rm US, \ safe, \ full, \ tr} + \pi_t^{\rm US, \ full, \ tr} \tag{130}$$

$$R_t^{\text{US, safe, lg, tr}} = R_t^{\text{US, safe, sh, tr}} + \tau_t^{\omega, \text{tr}} + \tau_t^{\text{US, dev, tr}}$$
(131)

$$R_t^{\text{EA, safe, sh, tr}} = r_t^{\text{EA, safe, full, tr}} + \pi_t^{\text{EA, full, tr}}$$
 (132)

$$R_t^{\text{EA, safe, lg, tr}} = R_t^{\text{EA, safe, sh, tr}} + \tau_t^{\omega, \text{tr}} + \tau_t^{\text{EA, dev, tr}}$$
(133)

$$R_t^{\mathrm{JP, \, safe, \, sh, \, tr}} = r_t^{\mathrm{JP, \, safe, \, full, \, tr}} + \pi_t^{\mathrm{JP, \, full, \, tr}} \tag{134}$$

$$R_t^{\text{JP, safe, lg, tr}} = R_t^{\text{JP, safe, sh, tr}} + \tau_t^{\omega, \text{tr}} + \tau_t^{\text{JP, dev, tr}}$$
(135)

#### US Baa Corporate Bond Rate Trend

$$\rho_t^{\rm US,\;Baa,\;tr} = m_t^{\omega,{\rm tr}} \tag{136}$$

$$R_t^{\text{US, Baa, tr}} = \rho_t^{\text{US, Baa, tr}} + \pi_t^{\text{US, full, tr}}$$
 (137)