Trend Model Equations

Core World Stochastic Trends

$$r_{w,t}^{\tau} = r_{w,t-1}^{\tau} + \epsilon_{r_w,t} \tag{1}$$

$$\pi_{w,t}^{\tau} = \pi_{w,t-1}^{\tau} + \epsilon_{\pi_w,t} \tag{2}$$

Core Common Factors for Deviations

$$f_{r,t}^{\tau} = f_{r,t-1}^{\tau} + \epsilon_{f_r,t} \tag{3}$$

$$f_{\pi,t}^{\tau} = f_{\pi,t-1}^{\tau} + \epsilon_{f_\pi,t} \tag{4}$$

Core Idiosyncratic Deviation Trends

For each country $C \in \{US, EA, JP\}$:

$$r_{C,idio,t}^{\tau} = r_{C,idio,t-1}^{\tau} + \epsilon_{r_C,idio,t} \tag{5}$$

$$\pi_{C,idio,t}^{\tau} = \pi_{C,idio,t-1}^{\tau} + \epsilon_{\pi_C,idio,t} \tag{6}$$

Derived Full Deviation Trends

$$r_{US,dev,t}^{\tau} = 1.0 \cdot f_{r,t}^{\tau} + r_{US,idio,t}^{\tau} \tag{7}$$

$$\pi_{US,dev,t}^{\tau} = 1.0 \cdot f_{\pi,t}^{\tau} + \pi_{US,idio,t}^{\tau} \tag{8}$$

$$r_{EA,dev,t}^{\tau} = \lambda_{r,EA} \cdot f_{r,t}^{\tau} + r_{EA,idio,t}^{\tau} \tag{9}$$

$$\pi_{EA,dev,t}^{\tau} = \lambda_{\pi,EA} \cdot f_{\pi,t}^{\tau} + \pi_{EA,idio,t}^{\tau} \tag{10}$$

$$r_{JP,dev,t}^{\tau} = \lambda_{r,JP} \cdot f_{r,t}^{\tau} + r_{JP,idio,t}^{\tau} \tag{11}$$

$$\pi_{JP,dev,t}^{\tau} = \lambda_{\pi,JP} \cdot f_{\pi,t}^{\tau} + \pi_{JP,idio,t}^{\tau} \tag{12}$$

where $\lambda_{r,C}$ and $\lambda_{\pi,C}$ are country-specific factor loadings for real rate and inflation deviations, respectively.

Derived Full Real Rate and Inflation Trends

For each country $C \in \{US, EA, JP\}$:

$$rr_{C,t}^{\tau} = r_{w,t}^{\tau} + r_{C,dev,t}^{\tau} \tag{13}$$

$$\pi_{C,t}^{\tau} = \beta_{\pi,C} \cdot \pi_{w,t}^{\tau} + \pi_{C,dev,t}^{\tau} \tag{14}$$

where $rr_{C,t}^{\tau}$ is the full real interest rate trend, $\pi_{C,t}^{\tau}$ is the full inflation trend, and $\beta_{\pi,C}$ is the pass-through coefficient from world inflation to country C's inflation trend.

Core Country-Specific Output Growth Trends

For each country $C \in \{US, EA, JP\}$:

$$g_{C,t}^{\tau} = \frac{1}{\phi_C} \cdot rr_{C,t}^{\tau} + \epsilon_{g_C,t} \tag{15}$$

where $g_{C,t}^{\tau}$ is the output growth trend and ϕ_C is the coefficient of relative risk aversion (or intertemporal elasticity of substitution, depending on the model setup).

Derived Full Nominal Short Rate Trends

For each country $C \in \{US, EA, JP\}$:

$$i_{C,t}^{\tau} = rr_{C,t}^{\tau} + \pi_{C,t}^{\tau} \tag{16}$$

where $i_{C,t}^{\tau}$ is the nominal short-term interest rate trend.