

Strategies for Recovery and Maintain of A Biped Walking Robot Balance

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Abstract

Recovering and maintaining the balance of the biped walking robots play an important role in their operation. In this article we will analyze some strategies for balancing in the sagittal plane, in the presence of external disturbances and changing the proportions between leg's length and trunk's length (golden section), and/or adding weights (boot type) between the ankle and the knee so that the center of gravity is as low as possible. For equilibrium recovery, we suggest that the biped walking model be equipped with actuator that provides a torque at the hip. or/and at the ankle. The strategy of balance has a goal to move the disturbed system to the desired equilibrium state. We chose to study, the model of a double linear pendulum inverted under-actuated, with one passive and one active joint. Each case study and usage of these strategies is validated by Webots and is applied for NAO robot.

Keywords: walking robots; balance; under-actuated system; control law.

1. Introduction

The support area of the robot, is either the foot surface in case of one supporting leg or the minimum convex area containing both foot surfaces in case both feet are on the ground. These are referred to as single and double support phases, respectively.

The most used models and strategies for balancing of the biped walking robots are:

- CoG (or CoM) balancing which has goal to maintain the projection of the center of gravity (or center of mass) on the ground, inside of the foot support area. This strategy is used for static walk or only for slow walking speeds;

- ZMP (zero momentum point) balancing which has goal to maintain the point $P(x_{zmp}, y_{zmp}, z_{zmp})$ inside the surface defined by

BoS (boundary of support, this is the polygon described by the robot's foots). This strategy is used for dynamic walking. ZMP is a point where the horizontal moments are zero. In conclusion, if point P is inside the surface defined by BoS, all moments and forces exerted by the body on the ankle are compensated, which necessarily leads to a dynamic balance [1, 2, 10-12].

Combined strategy applies to both ankle and hip strategy to protect the system against external disturbances.

We specify that for the dynamic walking robots CoG (or CoM) can be outside of the BoS, but the ZMP, cannot. This paper uses a two-link inverted pendulum model in the sagittal plane, with actuator at the hip joint see Fig. 1.

2. The choice of the model

For analyzing of balance control for a biped walking can be modeled as one or multi-dimensional inverted pendulum chain. Thus, the walking biped can be modeled with one-dimensional inverted pendulum [3-5, 13-15], in this case the system will be described only by one variable: the angle of the ankle joint. This model is not sufficient to completely explain balance properties, even for standing balance. In many studies is used the double inverted pendulum model [6, 7]. In the best approximation of the human body, the biped walking robot can be modeled with multi-dimensional inverted pendulum chain that allows to study the responses to complex complex perturbations [16,17].

We chose between a simpler model and the model that can give as much information as possible to the balance recovery strategy of a biped walking robot. The two rigid links of the model are: both legs a link and the head, arms and torso, is the second. We chose to study, the model of a double inverted pendulum under-actuated, which one passive and one active joint, and who can approach balance in the case of a single phase, i.e. in case of one supporting leg [8, 9, 18-20]. This model also assumes that both legs move together at all times, therefore are modeled as a single link.

3. Mathematical modeling of two-link biped walking robot

The equations of motion for an two-link inverted pendulum where derived using the Newton-Euler equation and linearized by employing Taylor series expansion, evaluated around the equilibrium point $x=[1.57, 0, 0, 0]$, that is, vertical position and zero angular speeds, as follows:

$$M \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = -F \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - G \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + B_N \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (1)$$

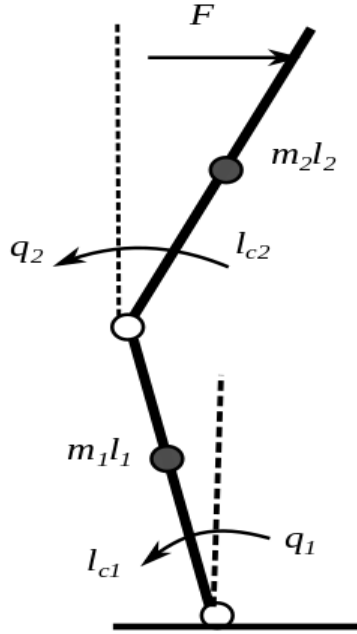


Fig.1: Two-link inverted pendulum model in the sagittal plane

Where the matrices can be written as follows,

$$M = \begin{bmatrix} J_1 + m_1 l_{c1}^2 + m_2 l_1^2 + 2m_2 l_1 l_{c2} + J_2 + m_2 l_{c2}^2 & J_2 + m_2 l_1 l_{c2} + m_2 l_{c2}^2 \\ J_2 + m_2 l_1 l_{c2} + m_2 l_{c2}^2 & J_2 + m_2 l_{c2}^2 \end{bmatrix}, \quad (2)$$

$$F = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix}, \quad (3)$$

is linearized friction model, with

$$f_1 = c_1 \frac{\alpha}{2} + v_1 \text{ and } f_2 = c_2 \frac{\alpha}{2} + v_2, \quad (4)$$

from nonlinear model:

$$F_i = c_i \operatorname{sgn}(\dot{q}_i) + v_i \dot{q}_i \approx c_i \operatorname{th}(\alpha \dot{q}_i) + v_i \dot{q}_i, \quad (5)$$

where c_i and v_i are Coulomb and viscous friction coefficients at the ankle joint ($i=1$) and at the hip joint ($i=2$), respectively. The function $\operatorname{sgn}(\cdot)$ is approximate with the hyperbolic tangent function $\operatorname{th}(\alpha \cdot)$, $\alpha=50$. Also,

$$G = \begin{bmatrix} m_1 g l_{c1} + m_2 g l_{c2} & m_2 g l_{c2} \\ m_2 g l_{c2} & m_2 g l_{c2} \end{bmatrix}, \quad (6)$$

$$B_N = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad (7)$$

in our case, i.e. actuator only at the hip joint, where m_1, m_2, l_1 and l_2 are the equivalent mass and length of each link, leg and torso respectively, l_{c1} and l_{c2} are mass centers relative to the lower joint and J_1 and J_2 are the moments of inertia about the CoM of the corresponding link (around pitch axis) and q_1 and q_2 are the ankle and hip joint angles, respectively τ_1 and τ_2 are the ankle joint torque and hip joint torque, respectively (in our case $\tau_1 = 0$).

After linearization, it is known that coriolis and centrifugal terms extant in the original non linear equations have been eliminated and do not contribute to the simplified model.

4. Linear control design

For formulating the feedback control model, we defined the state vector, x , of joint kinematics referenced as $x = [q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2]^T$, where q_1 and q_2 are the ankle and hip joint angles, respectively, and \dot{q}_1 and \dot{q}_2 represent respective angular velocities. The state model is determined by,

$$\dot{x} = Ax + Bu \quad (8)$$

where matrix A encapsulates the dynamical properties of the system that exist due to the particular chosen state, and B determines the effect an input will have on the respective states. The variable u is the control input and the system output or response function

$$y(t) = Cx(t) = [1 \ 0 \ 0 \ 0]x(t). \quad (9)$$

The system (8), represent two ordinary differential equations of the second order, is equivalent with system (1), which represents four ordinary equations of the first order. This equations are linearized around the equilibrium state which means a simplified model which is only valid within a close vicinity to this point. This assumption, for this control technique implies instability for large deviations from the desired position.

After we defined the state space model, it is important to determine appropriate control input which will guarantee stability and convergence of the system to the desired position. For this purpose, state feedback is employed which requires gain tuning to get convergence at the desired position, under constraints imposed by actuator and joint limitations.

If we note the matrices:

$$M^{-1} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, \quad (10)$$

$$M^{-1}G = \begin{bmatrix} mg_{11} & mg_{12} \\ mg_{21} & mg_{22} \end{bmatrix}, \quad (11)$$

$$M^{-1}F = \begin{bmatrix} mf_1 & 0 \\ 0 & mf_2 \end{bmatrix} \quad (12)$$

$$\text{and } M^{-1}B_N = \begin{bmatrix} 0 & m_{12} \\ 0 & m_{22} \end{bmatrix}, \quad (13)$$

form (1), the matrix A and B , from the system (8), gets:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -mg_{11} & -mg_{12} & -mf_1 & 0 \\ -mg_{21} & -mg_{22} & 0 & -mf_2 \end{bmatrix}, \quad (14)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & m_{12} \\ 0 & m_{22} \end{bmatrix}, \quad (15)$$

$$Bu = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & m_{12} \\ 0 & m_{22} \end{bmatrix} \begin{bmatrix} \tau_1 & 0 & 0 & 0 \\ \tau_2 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m_{12}\tau_2 & 0 & 0 & 0 \\ m_{22}\tau_2 & 0 & 0 & 0 \end{bmatrix} \quad (16)$$

and vector $\dot{x} = [\dot{q}_1 \ \dot{q}_2 \ \ddot{q}_1 \ \ddot{q}_2]^T$.

In the case studied, the ankle joint being stabilized, the hip joint is required to maintain its desired position $q_{2d}=0$. Optimality in this situation is determined by employing the linear quadratic regulator, described in the next section.

4.1. Linear Quadratic Regulator

To determine the optimal trajectory in recovering balance after, towards the vertical desired position, an optimal feedback controller needs to be designed. Optimality [4] has been defined in terms of a quadratic cost function as follows

$$J_{lqr} = \frac{1}{2} \int_0^\infty [x(t)^T Q x(t) + u(t)^T R u(t)] dt \quad (17)$$

where $x^T Q x$ is the *state cost* with weight $Q = Q^T > 0$, and $u^T R u$ is called the *control cost* with weight $R = R^T > 0$. The value of Q and R are randomly chosen and varied until the output of system does not get the desired value.

The linear feedback matrix u , is defined as:

$$u(t) = -K_{lqr} x(t). \quad (18)$$

The K_{lqr} matrix is responsible for defining optimality in the linear quadratic regulator, which is obtained by solving the Riccati equation, using MATLAB, is given as follows,

$$A^T P + PA + Q - PBR^{-1}B^T P = 0. \quad (19)$$

The solution of this equation is P , called the optimal matrix, with this solution, the gain matrix is determined by,

$$K_{lqr} = R^{-1}B^T P \quad (20)$$

The feedback control input u , i.e. joint torque, was assumed to be generated by full-state feedback in the following form:

$$u = [\tau_1 \ \tau_2]^T = -K_{lqr} x(t) \quad (21)$$

$$\text{and } K_{lqr} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix}. \quad (22)$$

By varying the gain matrix Q and R , the penalty error of the state x and the control effort u is controlled. The gain matrix used in experimentation are,

$$Q = I_{4 \times 4}, \quad R = (10e-12)I_{2 \times 2}, \quad (23)$$

where a higher penalty is placed on the control effort as compared to the state.

These gains can be determined by keeping in mind joint motor limitations for providing the control effort in terms of torque.

This approach is much faster compared to traditional pole placement technique, while the desired balance of priorities between the state and control effort can be regulated much easily. The system (8) will be equivalent with system:

$$\dot{x} = (A - BK_{lqr})x(t). \quad (24)$$

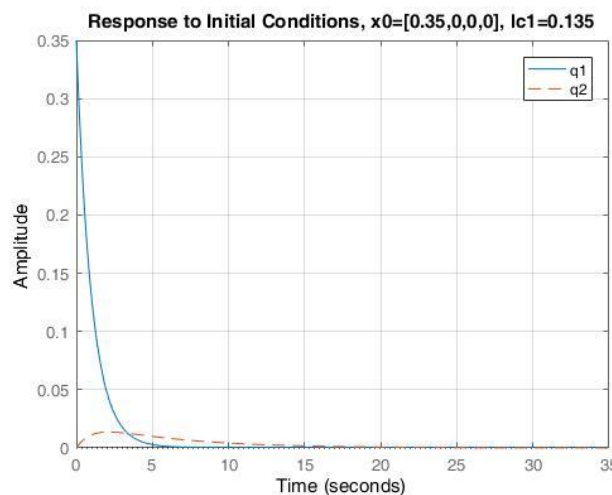


Fig. 2: Results for lower body disturbance $x_0=[0.35,0,0,0]$ and $l_{c1}=l_{c2}=0.135$

4.2. Numerical results using MATLAB

In order to highlight the efficiency of the LQR method in combination with redistribution of mass centers, some graphic results will be presented.

For this, we compared the return times to the equilibrium position for the same disturbance, in a case where the robot has the mass centers in the middle of the links and in another case when the robot has the mass centers placed in the ratio $(l_i - l_{ci}) / l_{ci} = 1.618, i = 1, 2$, i.e. the golden section.

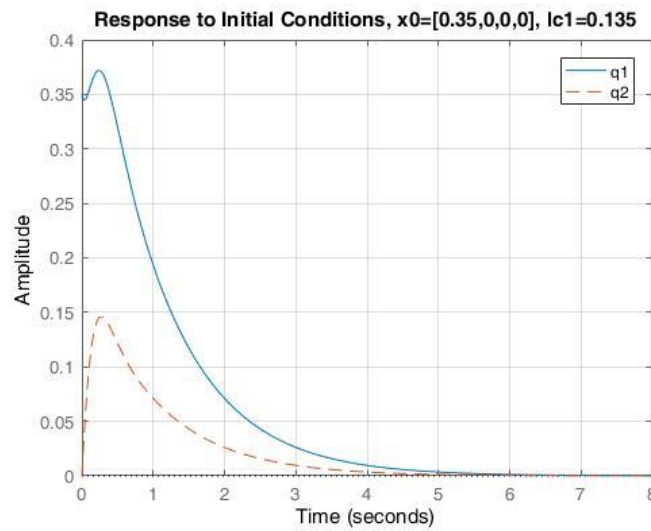


Fig. 3: Results for lower body disturbance $x_0=[0.35,0,0,0]$ and $l_{c1}=l_{c2}=0.103$ m

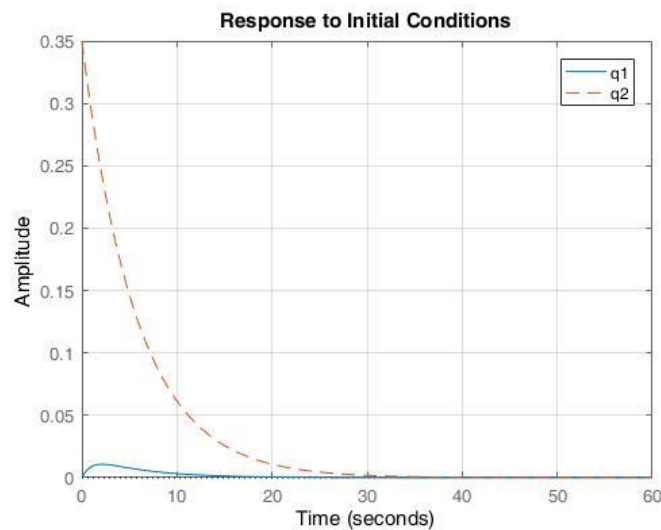


Fig. 4: Results for upper body disturbance $x_0=[0,0.35,0,0]$ and $l_{c1}=l_{c2}=0.135$ m

The numerical results justify the mathematical model, so when the mass centers are in the middle of the links, the return time to the equilibrium position is higher (about 7 times), and in the case where the mass centers are placed in the gold ratio, the difference

between the disturbing effect placed on the ankle (lower body) and that placed at the hip (upper body) is very small, compared to the first case, see Fig. 2 and Fig. 3, from 35 sec to 60 sec.

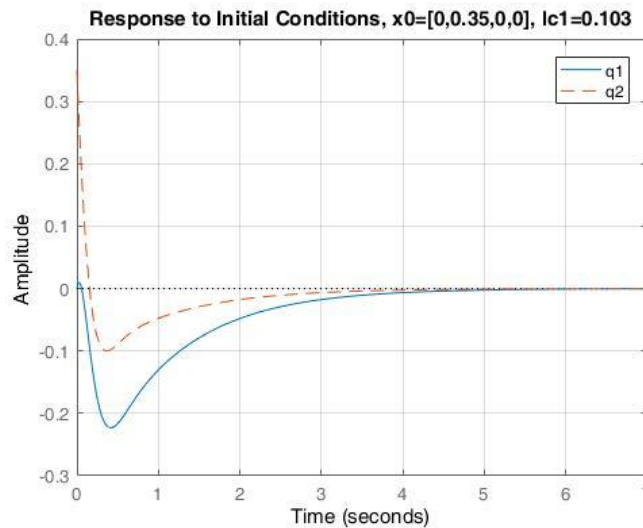


Fig. 5: Results for upper body disturbance $x_0=[0,0.35,0,0]$ and $l_{c1}=l_{c2}=0.103$ m

5. RESULTS AND DISCUSSION

The linear feedback is used as a faster means of convergence when the hip joint is close to the desired state. The control strategy formulated defines torque for the hip joint. This can be accompanied with a simple PD controller at the ankle,

$$\tau_1 = K_{p1}(1.57 - q_1) - K_{d1}\dot{q}_1$$

This algorithm completely removes any torque provided to the ankle and evaluates performance of various controllers under such conditions with zero ankle torque and a PD controller was proposed to cater friction components at the ankle joint.

6. CONCLUSIONS

The analyzed state space model, has been tested in several versions, under disturbance applied to system and has been compared the return times to the equilibrium position for the same disturbance, in a case where the robot has the mass centers in the middle of the links and in another case when the robot has the mass centers placed in the golden section, in this latter case the mass centers were lowered.

The obtained results were quantified by decreasing of spectral radius of the matrix $A - BK$, which means increasing stability of the biped walking robot. If the R gain value is increased to make a lower control effort, the stabilization is achieved after a few oscillations.

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