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Study of Dynamic Biped Locomotion on Rugged Terrain
— Derivation and Application of the Linear Inverted Pendulum Mode —

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Abstract

This paper introduces a new control method for biped locomotion on rugged terrain. We assume the ideal robot model which has massless legs. By applying constraint control to the robot body so that it moves on a particular straight line and rotates at a constant angular velocity, the dynamics of the center of mass of the body becomes completely linear. We call such motion of the ideal model the *Linear Inverted Pendulum Mode* and use it to develop the control scheme of the biped walking on rugged terrain. Under the control in the linear inverted pendulum mode, walking on a particular rugged ground is shown to be equivalent to walking on a level ground. It is also shown that the additional use of the ankle torque makes our control scheme robust and applicable to a real biped robot with mass legs. Simulation result is presented.

1. Introduction

A lot of studies of a biped locomotion have been conducted worldwide[1]-[12]. One of the important approaches is to extract a dominant feature of its dynamical behavior. Because biped dynamics is high-order and nonlinear, it is difficult to understand unless we make some kind of simplification.

A good analyzing technique helps us understand the dynamics of biped locomotion. Sometimes, it also helps us establish the control law. Golliday and Hemami decoupled the high-order system dynamics of biped into independent low-order subsystems using state feedback [2]. Miyazaki and Arimoto used a singular perturbation technique, and showed that biped locomotion can be divided into two modes: a fast mode and a slow mode [6]. Furusho and Masubuchi derived a reduced order model as a dominant subsystem which approximates the original high-order model very well by the application of local feedback at each articular joint of the biped robot [1]. Raibert used symmetry to analyze his one, two, and four legged robots which are controlled by the algorithm named three parts control [8].

In this paper, we introduce a new technique for analyzing the dynamic walk of a biped. First, we consider applying constraint control to an ideal robot so that the body of the robot moves on a particular straight line and rotates at a constant angular velocity. As a result of such control, we find that the dynamics of the center of mass of the body becomes completely linear. In addition, the dynamics of the constrained system has many advantages in designing biped walking motion. We call such motion of the ideal model the *Linear Inverted Pendulum Mode.*

Based on the dynamics of the linear inverted pendulum mode, we develop the control scheme of the biped walk on rugged terrain. Additional use of the ankle torque is considered to make the control scheme robust and applicable to a real biped robot with mass legs. Simulation is carried out to ascertain the entire theory.

2. Linear Inverted Pendulum Mode

2.1. Motion Equation of Support Phase

We assume a biped robot which has a mass body and two massless legs and is restricted to move in a sagittal plane (that is defined by a vertical axis and the direction of locomotion). We also consider the state in which only one leg supports the body. Figure 1 shows such robot model. As a generalized structure of the leg, we assume the first link connected to the ankle joint and any kind of two D.O.F. mechanism that drives the body with respect to the first link.

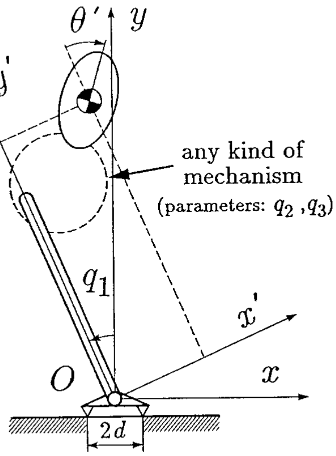


Fig.1 Generalized biped robot

The angle of the first link, q_1 is measured from a vertical axis. q_2 and q_3 are the joint angles of the mechanism that drives the body. With respect to the base coordinate frame $O-xy$, the position and the attitude of the body are represented as follows.

$$x = x'(q_2,q_3)\cos q_1 + y'(q_2,q_3)\sin q_1 \quad (1)$$

$$y = -x'(q_2,q_3)\sin q_1 + y'(q_2,q_3)\cos q_1 \quad (2)$$

$$\theta = q_1 + \theta'(q_2,q_3) \quad (3)$$

where $x'(q_2,q_3)$ and $y'(q_2,q_3)$ denote the body position and $\theta'(q_2,q_3)$ denotes the attitude with respect to the coordinate frame that is embedded in the first link, $O-x'y'$. From these equations, Jacobian matrix $\boldsymbol{J}$ is determined as follows.

$$\boldsymbol{J} = \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{q}}$$

$$\boldsymbol{p} = (x,\, y,\, \theta)^{\mathrm{T}}$$

$$\boldsymbol{q} = (q_1,q_2,q_3)^{\mathrm{T}}$$

We assume there exist no singular points in the work space of the leg. Then the motion equation of Fig. 1 is represented as follows.

$$\boldsymbol{M}\ddot{\boldsymbol{p}} = (\boldsymbol{J}^{\mathrm{T}})^{-1}\boldsymbol{u} + \boldsymbol{M}\boldsymbol{g} \quad (4)$$

where

$$\boldsymbol{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\boldsymbol{u} = (u_1,u_2,u_3)^{\mathrm{T}}$$

$$\boldsymbol{g} = (0,-g,0)^{\mathrm{T}}$$

m : mass of the body
I : moment of inertia of the body
g : gravity acceleration

u_i is the generalized force that corresponds to q_i, and u_1 is the ankle torque of the support leg. Now, we premultiply eq. (4) by $\boldsymbol{J}^{\mathrm{T}}$ to obtain

$$\boldsymbol{J}^{\mathrm{T}}\boldsymbol{M}\ddot{\boldsymbol{p}} = \boldsymbol{u} + \boldsymbol{J}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{g} \ . \quad (5)$$

The first row of eq. (5) is as follows.

$$m(\frac{\partial x}{\partial q_1}\ddot{x} + \frac{\partial y}{\partial q_1}\ddot{y}) + I\frac{\partial \theta}{\partial q_1}\ddot{\theta} = u_1 - \frac{\partial y}{\partial q_1}mg \quad (6)$$

From eqs. (1) , (2) and (3), we can derive following equations.

$$\frac{\partial x}{\partial q_1} = -x'(q_2,q_3)\sin q_1 + y'(q_2,q_3)\cos q_1 = \quad y$$

$$\frac{\partial y}{\partial q_1} = -x'(q_2,q_3)\cos q_1 - y'(q_2,q_3)\sin q_1 = -x$$

$$\frac{\partial \theta}{\partial q_1} = 1$$

By substituting these equations into eq. (6) , we obtain

$$m(y\ddot{x} - x\ddot{y}) + I\ddot{\theta} = u_1 + mgx \quad . \quad (7)$$

Note that this equation does not depend on the structure of the mechanism that drives the body.

2.2. Derivation of the Linear Inverted Pendulum

Now, let us assume the motion of the body is controlled to be constrained by the following equations.

$$y = kx + y_H \quad (y_H \neq 0) \quad (8)$$

$$\dot{\theta} = \omega_c \quad (9)$$

Equation (8) is a constraint that keeps the center of the gravity of the body to move on a straight line that is defined by its intersection with y-axis, y_H and the slope, k. Equation (9) is a constraint that keeps the rotation rate of the body constant, ω_c. By differentiating eqs. (8) and (9), we obtain the constraints in terms of acceleration.

$$\ddot{y} = k\ddot{x} \quad (8)'$$

$$\ddot{\theta} = 0 \quad (9)'$$

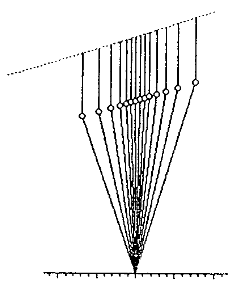
By substituting eqs. (8)', (9)' and (8) into eq. (7), we obtain

$$\ddot{x} = \frac{g}{y_H}x + \frac{1}{my_H}u_1 \tag{10}$$

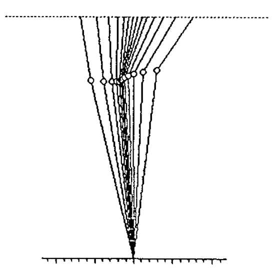
Equation (10) is a motion equation of one D.O.F., because we impose two constraints upon the three D.O.F. mechanism. The forces of constraints are calculated from the second and the third raws of eq. (5).

$$u_2 = m(\frac{\partial x}{\partial q_2} + \frac{\partial y}{\partial q_3})\ddot{x} + \frac{\partial y}{\partial q_2}mg \tag{11}$$

$$u_3 = m(\frac{\partial x}{\partial q_3} + \frac{\partial y}{\partial q_3})\ddot{x} + \frac{\partial y}{\partial q_3}mg \tag{12}$$



(a) Slope trajectory
k=0.3, y_H=30.0[cm], ω_c=0.0[rad/sec], $x(0)$=−7.0[cm], $\dot{x}(0)$=41.0[cm]



(b) Horizontal trajectory, rotating body
k=0.0, y_H=30.0[cm], ω_c=1.0[rad/sec], $x(0)$=−7.0[cm], $\dot{x}(0)$=41.0[cm]

Fig.2 Linear inverted pendulum mode
m=2.0[kg], I=1.7×10^4[g·cm^2]
distance between hip joint and center of mass: 8.0[cm]

Figure 2 (a) and (b) show examples of constrained motions of the model which contains a variable length leg, a rigid body and a hip joint. The constraint line is sloped in Fig. 2(a) and the body rotates at a constant rate in Fig. 2(b). Both motions do without the ankle torque (u_1=0). Note that the horizontal components of the both motions of the center of mass of the body are the same, because both the examples have same y_H and the same initial condition of horizontal components of the motion.

Equation (10) represents a class of the solution of the nonlinear equation (4). The features of eq. (10) are:

1. Linear differential equation in the Cartesian coordinate frame.
2. Does not depend on the structure of the leg.
3. Does not depend on the constraint parameters except y_H.

We call such motion the *Linear Inverted Pendulum Mode* and propose it for the design and the control of dynamic walking motion of a biped.

Equation (10) may look like the linealized motion equation of a simple inverted pendulum. Note, however, that eq. (10) can be used in the whole state space, while the linearized equation can be used only in the neighborhood of the equilibrium point. Therefore, the linear inverted pendulum mode is useful to design the biped locomotion of wide stride.

3. Design of Walking Motion

3.1. Change of the Slope of the Constraint Line

Equation (10) does not include the parameter k, so it is possible to change the slope of the constraint line without affecting the horizontal component of the body motion. To keep the parameter y_H constant, the slope is changed at the point of x=0. When the body passes x=0, the body must be driven by an impulse along a vertical axis.

$$f_y\Delta t = m(k_2-k_1)\dot{x} \ . \tag{13}$$

where k_1 and k_2 are the slopes of the constraint line before and after the change. If the constraint is realized by using local feedback control, the impulse is generated by the servo system, and we do not have to take the impulse into account in the design of walking motion.

3.2. Walking Motion and Support Leg Exchange

To design the nominal walking motion, we consider the motion that does not use the ankle torque (u_1=0), because the ankle torque is limited in magnitude and we want to reserve it in order to cope with the disturbances as shown in Section 4.

Figure 3 shows a support leg exchange. We specify that the body is constrained to the same line (same k, same y_H) over the leg exchange, and the support leg is assumed to be exchanged instantaneously. We assume that the horizontal component of the body velocity changes from v_- to v_+ at the instant of exchange as follows.

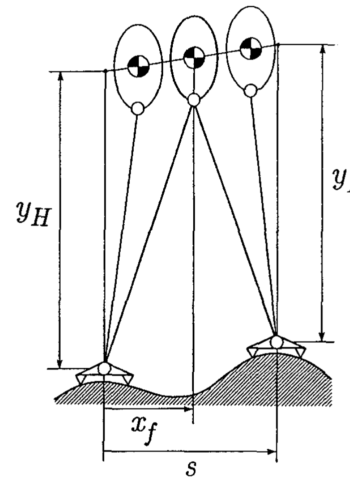


Fig.3 Support leg exchange

$$v_+ = ev_- \quad (0<e\leq 1) \tag{14}$$

where e is the parameter which depends on the velocity of the foot of the swing leg at the exchange, the compliance of the ground and so on.

When the ankle torque u_1 is zero, the first integral of eq. (10) is

$$E = -\frac{g}{2y_H}x^2+\frac{1}{2}\dot{x}^2 \ . \tag{15}$$

E is conserved during each step, and characterizes the particular motion of the step. We call this the *orbital energy* [4]. Considering Fig. 5, we get equations of E_1 and E_2, the orbital energy before and after the support leg exchange.

$$E_1 = -\frac{g}{2y_H}x_f^2+\frac{1}{2}v_-^2 \tag{16}$$

$$E_2 = -\frac{g}{2y_H}(x_f-s)^2+\frac{1}{2}v_+^2 \tag{17}$$

where x_f is the horizontal position of the body at the support leg exchange and s the stride. When we specify the nominal motion by E_2, the leg exchange condition can be calculated from eqs. (14), (16) and (17).

$$x_f = \frac{s-\sqrt{s^2-(1-e^2)(s^2+(2y_H/g)(E_2-e^2E_1))}}{1-e^2} \quad (0<e<1)$$

$$x_f = \frac{y_H}{gs}(E_2-E_1)+\frac{s}{2} \qquad (e=1) \tag{18}$$

3.3. Design of Walking Motion on Rugged Terrain

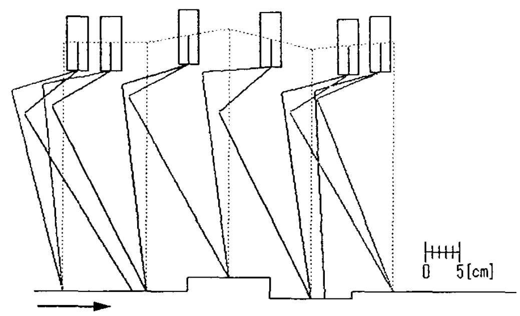
The algorithm for designing the walking motion on a rugged ground is as follows.

step 1. Decide the landing points on the ground.

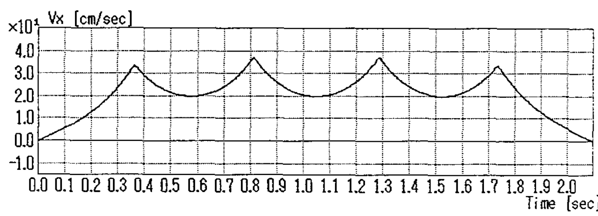
step 2. Define the constraint line by connecting the points that are at the height y_H from the landing points.

step 3. Calculate the orbital energy E for each support phase from the nominal motion and calculate the body position, x_f at the support leg exchange using eq. (18).

Figure 4 shows an example of the walking motion on a rugged ground.



(a) Stick diagram



(b) Horizontal component of body velocity

Fig.4 Designed walk on a rugged ground

4. Robust Walk using the Ankle Torque

The biped robot suffers many kinds of disturbances during walking. If we regard the disturbances in terms of forces, they affect the robot motion in two ways: deviation from the constraint line and deviation from the nominal horizontal trajectory. The former can be minimized by making constraint control stiff and robust against disturbances using local feedback. So we will consider the disturbance affecting the horizontal trajectory of the body.

$$\ddot{x} = \frac{g}{y_H}x + \frac{1}{my_H}u_1 + d \qquad (19)$$

where d is the disturbance force.

To consider the control of the real biped robot, we assume d also includes the effect of the masses of the legs. Under the disturbances, the motion without control using u_1 never follows the trajectory that we designed in Section 3. Moreover, if there is an error between the state of the system and the nominal state, it will grow up exponentially because the system of eq. (19) is unstable. It is especially serious when the duration of each support phase is long compared with the time constant of the system $\sqrt{y_H/g}$.

This problem is solved when we use the ankle torque to control the horizontal motion of the body. For example, we can use following simple control law.

$$u_1 = f_1(x-x^*)+f_2(\dot{x}-\dot{x}^*) \qquad (20)$$

where x^* and $\dot{x}^*$ give the nominal trajectory of an ideal biped as shown in Section 3, and f_1 and f_2 are feedback gains.

In eq. (20) the ankle torque u_1 is used to cancel the disturbance force d. The available magnitude of the ankle torque is limited because the actuator torque limited and because the ankle torque the robot exerts to the ground must satisfy the condition that the foot is kept in secure contact with the ground. Therefore, it is necessary that the masses of the legs are small enough compared with the mass of the body.

5. Simulation

Simulation is carried out to verify the control method discussed above. We use the model based on our experimental biped robot (Fig. 5, Table 1, 2) [4].

Each leg is a parallel link structure made of four links. The two links of each leg that are shown by broken lines are only used to transmit the forces to the leg. To make easier the derivation of the motion equation, we assume these two links of each leg are massless. The other links are assumed to have masses. As the result, our simulation model is regarded as a five-link model.

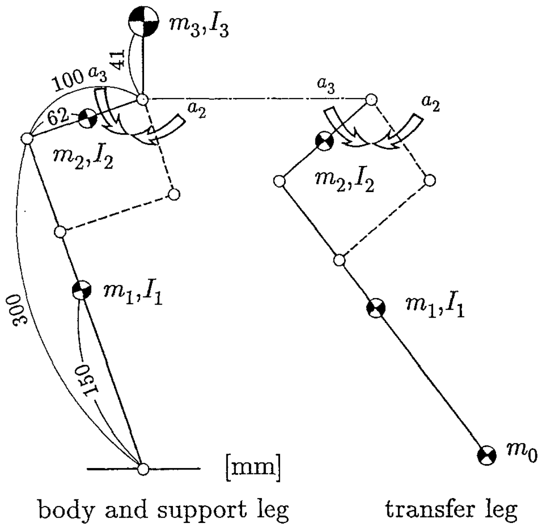


Fig.5 Five-link model (in a sagittal plane)

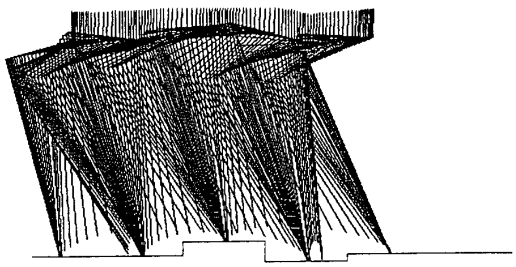
Part	Mass [g]	Moment of inertia [g·cm^2]
Pedal	m_0 = 40	—
Link1	m_1 = 170	$I_1 = 1.7\times10^4$
Link2	m_2 = 140	$I_2 = 2.3\times10^3$
Body	m_3 = 1720	$I_3 = 1.7\times10^4$

Table 1 Values of parameters of biped

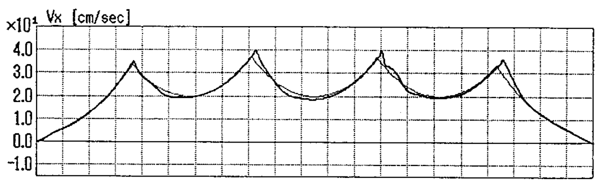
DC servo motor (armature)	
Rated power	10 W
Resistance	3.1 Ω
Torque constant	2.58×10^2 g·cm/A
Voltage constant	2.60×10^{-3} V/rpm
Moment of inertia	12.0 g·cm^2
Reduction gear ratio	
a_2	1/60
a_3	1/40

Table 2 Values of parameters of actuators

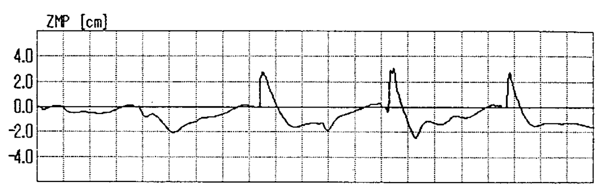
Local PD feedback control is applied to each joint except the ankle joint. The reference angles of these joints are calculated from the angle of the ankle joint to maintain the constraint condition. The ankle torque is calculated from eq. (20). As the nominal trajectory, we use the walking motion of the ideal model shown in Fig. 4. The swing leg is controlled to reach the next landing point at the time of support leg exchange. We design a smooth trajectory without large acceleration to avoid the influence of the inertial force of the swing leg to the body motion.



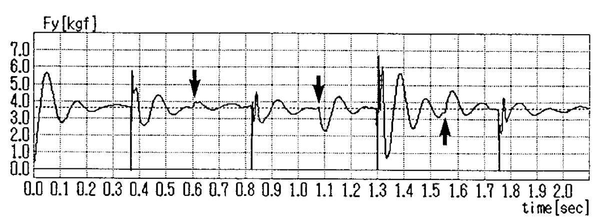
(a) Stick diagram (20 msec intervals)



(b) Horizontal component of body velocity



(c) Zero moment point



(d) Vertical reaction force from ground

Fig.6 Simulation result

Figure 6 (a) shows the stick picture of simulated biped walking. In Fig. 6 (b) the bold line shows the horizontal component of the body velocity of the simulated controlled model. The thin line shows the reference velocity calculated from the massless leg model for comparison. The velocity of the body is perturbed from the reference at leg support exchange. But the error of the trajectory converges within each support phase.

Fig. 6 (c) shows the position of zero moment point (ZMP) with respect to each landing point. The ZMP is the point on the ground around which the moment caused the ground reaction forces equals zero. It can simply be calculated dividing the ankle torque by the total vertical reaction force from the ground. When the ZMP exists within the length of the foot, the toe (or the heel) does not lift from the ground and stable biped walk is possible [10][11]. From this figure, we know the foot of the robot needs a length about 4 cm forward and 3.5 cm backward from the ankle. This is an adequate size of the foot for the robot that walks with 12 cm strides.

In Fig. 6 (d), the solid line shows the vertical reaction force from the ground, and the broken line shows the total weight of the robot. The arrows in the figure indicate the points where the slope of the constraint line changes. The vertical reaction force from the floor is always positive except for the impulsive force at the leg exchanges. Therefore, the foot of the robot keeps in contact with ground during walking.

In the simulation we assumed $e=1$ in eq. (14). In the reality, however, it is difficult to evaluate e, for it varies depending on the collision phenomena of the link mechanism and the ground, and the variation leads to the instability. For the stable control of the biped, three countermeasures can be considered:

(1) to make the control law robust,

(2) to soften the collision using compliance control at the foot touch down,

and, (3) to analyze the collision phenomena to make more precise model. All of these belong to our future work.

6. Conclusion

From the generalized model of the biped, we derived a simple dynamics that is useful to design and control biped locomotion. We name this the

Linear Inverted Pendulum Mode, and presented the method to design the walking motion on rugged terrain.

The linear inverted pendulum mode is derived from the model whose legs have no masses. However, it was shown that the robot with masses of legs can be controlled in a same sense using the ankle torque. We carried out simulation of the biped walking on rugged terrain to ascertain our control strategy.

In this paper, we assumed the robot is restricted to move in a two-dimensional sagittal plane. However, we would also like to control the robot to walk in a three-dimensional space. A prospective control method is to decompose the robot motion into the sagittal plane component and the lateral plane component and to control each motion component in the Linear Inverted Pendulum Mode with synchronized support leg exchange. The story of this method is included in our future work.

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