Fall 2021 Calculus I Recitation Lecture Notes

8/24 (Rewrite from Chalkboard Lecture)

We shall shall find the domain, and the sets

$Z := \{x \in \mathbb{R} : f(x) = 0\},$
$P := \{x \in \mathbb{R} : f(x) > 0\},$
$N := \{x \in \mathbb{R} : f(x) < 0\},$

of the functions

$f(x) = \sqrt{4 - |x+1|}$
$f(x) = \ln\left|\dfrac{2x+1}{x^2+x}\right|$

1. First we'll do this for the function $f(x) = \sqrt{4 - |x+1|}$. We find

$$\begin{aligned}
\sqrt{4-|x+1|} \text{ is defined } &\implies 4 - |x+1| \geq 0 \\
&\qquad +|x+1| \quad\ + |x+1| \\
&\implies 4 \geq |x+1| \\
&\implies 4 \geq x+1 \geq 0 \text{ or } -4 \leq x+1 < 0 \\
&\quad\ -1 \quad\ -1 \quad -1 \quad\ -1 \qquad -1 \quad -1 \\
&\implies 3 \geq x \geq -1 \text{ or } -5 \leq x < -1.
\end{aligned}$$

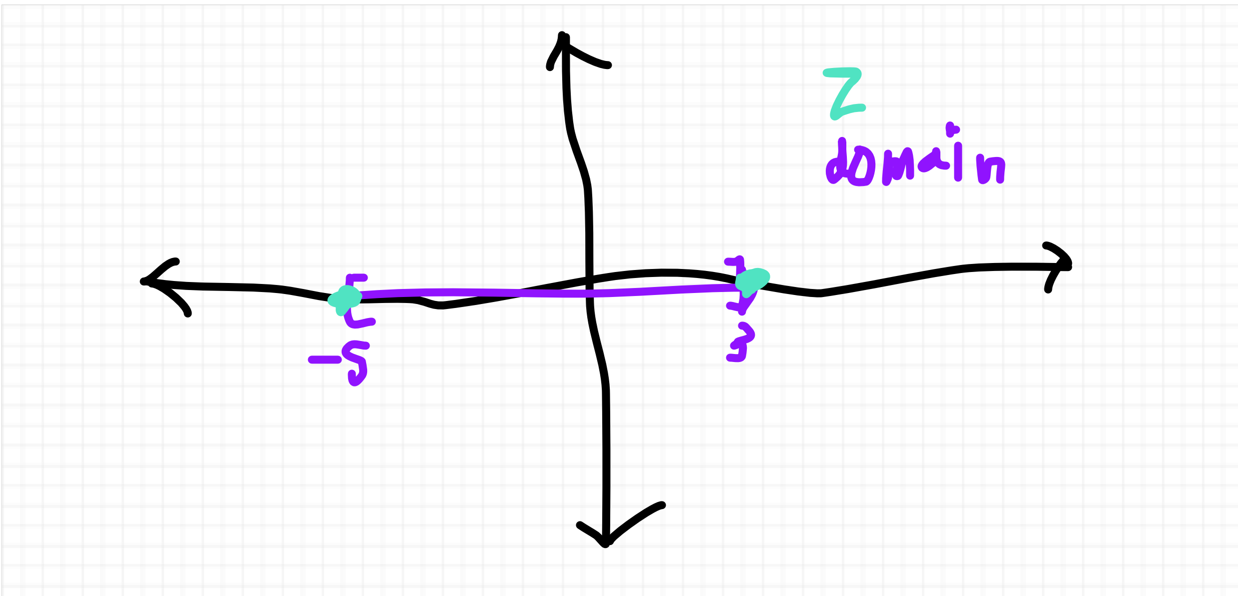
We conclude that $\sqrt{4-|x+1|}$ is defined on $[-5, 3]$

We find Z by solving for the equation $\sqrt{4-|x+1|} = 0$ as follows:

$$\begin{aligned}
\sqrt{4-|x+1|} = 0 &\implies 4 - |x+1| = 0 \\
&\qquad +|x+1| \quad\ + |x+1| \\
&\implies 4 = |x+1| \\
&\implies 4 = x+1 \text{ or } -4 = x+1 \\
&\quad\ -1 \quad\ -1 \quad -1 \quad\ -1 \\
&\implies 3 = x \quad \text{ or } -5 = x.
\end{aligned}$$

We conclude that $Z = \{3, -5\}$, i.e., $\sqrt{4-|x+1|} = 0$ has the solutions $x = 3, -5$.

Now, to find P and N, we resort to drawing a picture, which turns out to be the least tedious (though still somewhat tedious) way to do this problem. We start by drawing out the zeros and the domain of the function as follows:

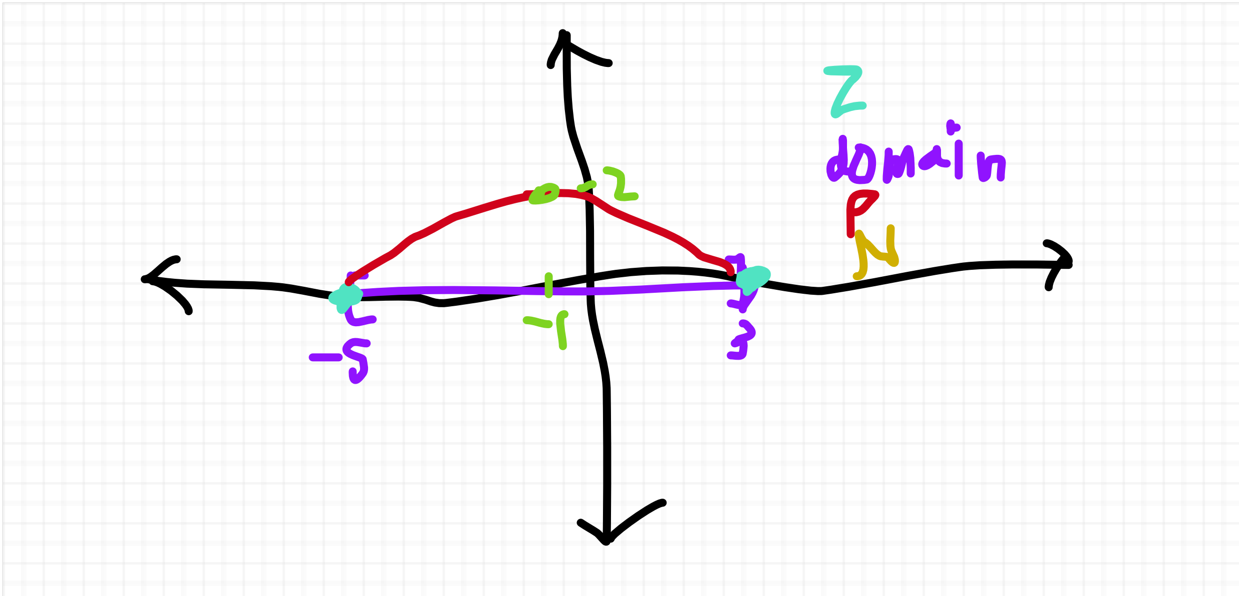


For the functions within their domain written as a single formula, we shall assume (correctly in this case) that the function is continuous (something we'll talk about in more detail in a few weeks), allowing us to conclude ALL values x such that $x_1 < x < x_2$ has the property $f(x) > 0$ (resp. $f(x) < 0$) if we plug in a SPECIFIC value a such that $x_1 < a < x_2$ and find that $f(a) > 0$ (resp. $f(a) < 0$).

So we plug in -1, which is in the interval $(-5, 3)$ and find that

$$f(-1) = \sqrt{4 - |(-1) + 1|} = \sqrt{4} = 2 > 0,$$

and conclude that $f(x) > 0$ for all $-5 < x < 3$, so $P = (-5, 3)$ (see the illustration below). This means that N has no values at all, i.e., $N = \varnothing$.



2. Next, we'll do this for the function $f(x) = \ln\left|\dfrac{2x+1}{x^2+x}\right|$.

To find the domain, we have

$$\begin{aligned}
\ln\left|\frac{2x+1}{x^2+x}\right| \text{ is defined} &\implies \frac{|2x+1|}{|x^2+1|} > 0 \\
&\implies |x^2+1| \neq 0 \text{ and } |2x+1| > 0 \\
&\implies x(x+1) = x^2+1 \neq 0 \text{ and} \\
&\quad\;\; 2x+1 > 0 \text{ or } 2x+1 < 0, \text{ i.e. } 2x+1 \neq 0 \\
&\implies x \neq 0, -1 \text{ and } x \neq \ -1/2.
\end{aligned}$$

We conclude that $\ln\left|\dfrac{2x+1}{x^2+x}\right|$ is defined on $\{x \in \mathbb{R} : x \neq 0, -1, -1/2\}$, i.e., all x except $0, -1, -1/2$.

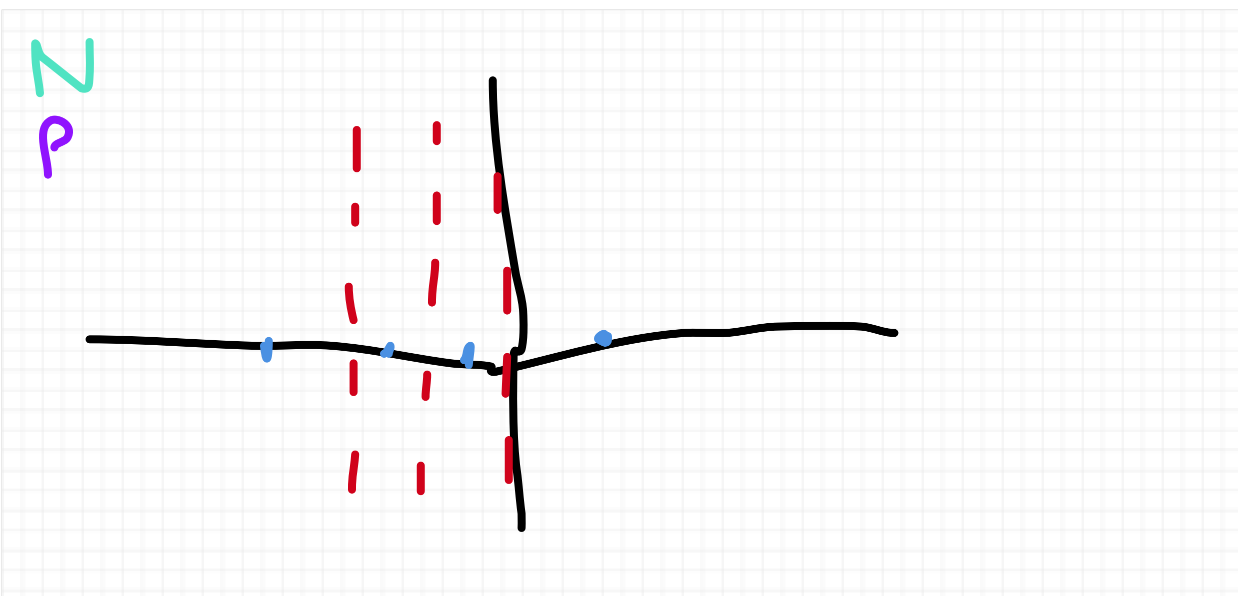
We find Z by solving for the equation $\ln\left|\dfrac{2x+1}{x^2+x}\right| = 0$ as follows:

$$\begin{aligned}
\ln\left|\frac{2x+1}{x^2+x}\right| = 0 &\implies \exp\left(\ln\left|\frac{2x+1}{x^2+x}\right|\right) = e^0 \\
&\implies \left|\frac{2x+1}{x^2+x}\right| = 1 \\
&\quad\; \cdot |x^2+x| \quad \cdot |x^2+x|
\end{aligned}$$

$$\begin{aligned}
&\implies |2x+1| = |x^2+x| \\
&\implies 2x+1 = x^2+x \text{ or} \\
&\quad\; 2x+1 = -\left(x^2+x\right) \\
&\implies 0 = x^2-x-1 \text{ or } 0 = x^2+3x+1 \\
&\implies x = \frac{1 \pm \sqrt{5}}{2} \text{ or } x = \frac{-3 \pm \sqrt{5}}{2}.
\end{aligned}$$

We conclude that $Z = \left\{\frac{1 \pm \sqrt{5}}{2}, \frac{-3 \pm \sqrt{5}}{2}\right\}$, i.e., $\ln\left|\frac{2x+1}{x^2+x}\right| = 0$ has the solutions $x = \frac{1 \pm \sqrt{5}}{2}, \frac{-3 \pm \sqrt{5}}{2}$..

Now, to find P and N, we again resort to drawing a picture. We start by drawing out the zeros, and the domain (including the vertical asymptotes where the function is undefined) of the function as follows:



This leaves us with four intervals to determine if they are in P or N. As before, we use the fact that if x_1, x_2 are zeros/undefined values, ALL values x such that $x_1 < x < x_2$ have the property $f(x) > 0$ (resp. $f(x) < 0$) if we plug in a SPECIFIC value a such that $x_1 < a < x_2$ and find that $f(a) > 0$ (resp. $f(a) < 0$).

We then plug in the following values for the following intervals:

$$f(x) = \ln\left|\frac{2x+1}{x^2+x}\right|$$

$$\left(-\infty, \frac{-3-\sqrt{5}}{2}\right) : x = -4 \implies f(x) < 0$$

$$\left(\frac{-3-\sqrt{5}}{2}, 1\right) : x = -2 \implies f(x) > 0$$

$$\left(1, \frac{-3+\sqrt{5}}{2}\right) : x = -3/4 \implies f(x) > 0$$

$$\left(\frac{-3+\sqrt{5}}{2}, 1/2\right) : x = -.578 \implies f(x) < 0$$

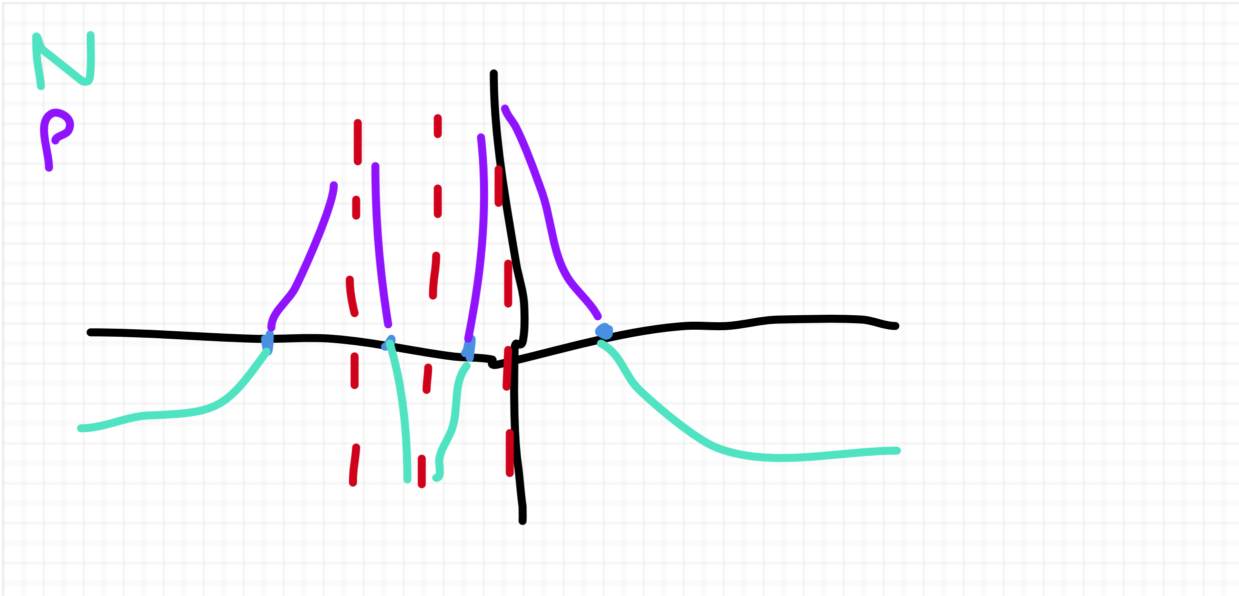
$$\left(1/2, \frac{1-\sqrt{5}}{2}\right) : x = -.425 \implies f(x) < 0$$

$$\left(\frac{1-\sqrt{5}}{2}, 0\right) : x = -1/5 \implies f(x) > 0$$

$$\left(0, \frac{1+\sqrt{5}}{2}\right) : x = 1 \implies f(x) > 0$$

$$\left(\frac{1+\sqrt{5}}{2}, +\infty\right) : x = 3 \implies f(x) < 0$$

which gives us the following picture:



and we conclude that

$$P = \left(\frac{-3-\sqrt{5}}{2}, 1\right) \cup \left(1, \frac{-3+\sqrt{5}}{2}\right) \cup \left(1, \frac{-3+\sqrt{5}}{2}\right) \cup \left(\frac{1-\sqrt{5}}{2}, 0\right) \cup \left(0, \frac{1+\sqrt{5}}{2}\right)$$

$$N = \left(-\infty, \frac{-3-\sqrt{5}}{2}\right) \cup \left(\frac{-3+\sqrt{5}}{2}, 1/2\right) \cup \left(1/2, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, +\infty\right),$$

i.e. P contains the intervals $\left(\frac{-3-\sqrt{5}}{2}, 1\right)$, $\left(1, \frac{-3+\sqrt{5}}{2}\right)$, $\left(1, \frac{-3+\sqrt{5}}{2}\right)$, $\left(\frac{1-\sqrt{5}}{2}, 0\right)$, $\left(\frac{1-\sqrt{5}}{2}, 0\right)$, and $\left(0, \frac{1+\sqrt{5}}{2}\right)$ and N contains the intervals $\left(-\infty, \frac{-3-\sqrt{5}}{2}\right)$, $\left(\frac{-3+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}\right)$, and $\left(\frac{1+\sqrt{5}}{2}, +\infty\right)$.

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Office Hours:

Monday, Thursday 4pm-5pm @Atwater Math Dept Building
Address is 1103 Atwater Ave.
Enter through the FRONT door opposite to the parking lot. (knock if locked)

I can also schedule office hours (in person or via zoom) by appointment.
email: agoodlad@iu.edu

Math Learning Commons (MLC) help is available:
My shift is after this class 10am-12pm
In general it's Monday-Thursday @Swain East 340 (specific times specified when I get them!)

assignment 2 exercise 1 item 7. Given the inequality

$$|5 - \sqrt{5x-1}| \geq 3$$

Solve for x:

Step 1: Rephrase this inequality in the form $f(x) \geq 0$, in this case

$$\begin{aligned} &|5 - \sqrt{5x-1}| \geq 3 \\ &-3 \qquad\qquad\quad -3 \\ &|5 - \sqrt{5x-1}| - 3 \geq 0 \end{aligned}$$

$$f(x) = |5 - \sqrt{5x-1}| - 3,$$

Step 2: Find the domain of $f(x)$.

$$\begin{aligned} |5 - \sqrt{5x-1}| - 3 \text{ is defined } \Longrightarrow\ & 5x - 1 \geq 0 \\ & \quad +1 \quad +1 \\ & 5x \qquad \geq 1 \\ & \div 5 \qquad \div 5 \\ & x \qquad \geq 1/5, \end{aligned}$$

so the domain is $[1/5, +\infty)$

Step 3: Find the zeros of $f(x)$ by solving for $f(x) = 0$. We find that

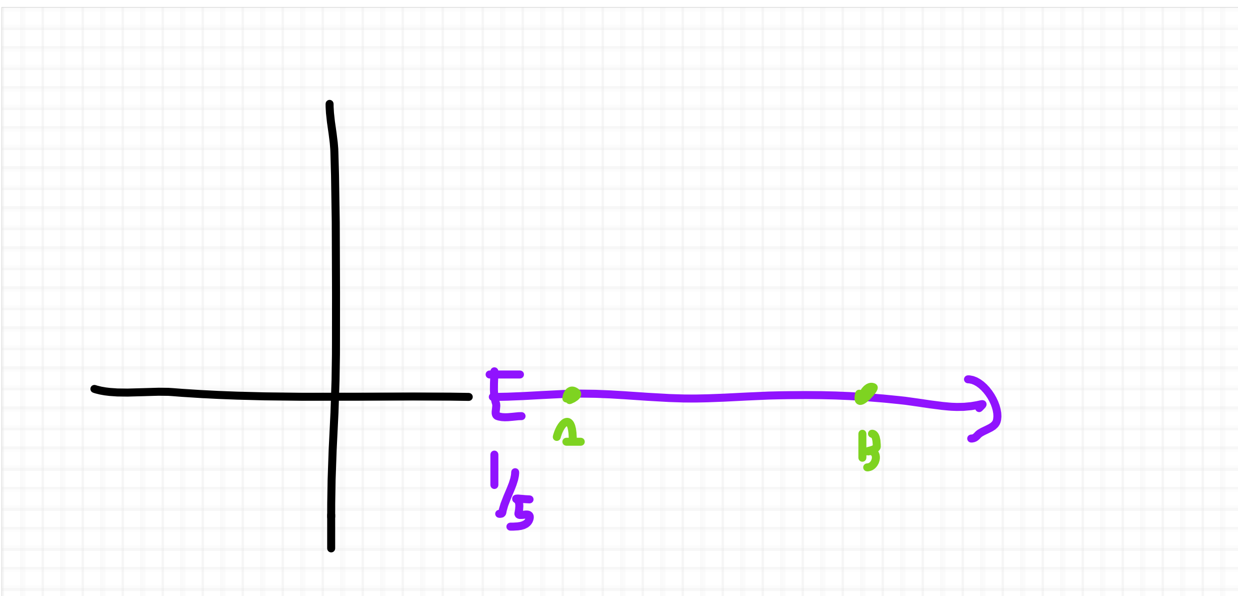
$$\begin{aligned} &|5 - \sqrt{5x-1}| - 3 = 0 \\ &\qquad\qquad\quad +3 \quad +3 \\ &|5 - \sqrt{5x-1}| \qquad = 3 \end{aligned}$$

$$\begin{aligned} &5 - \sqrt{5x-1} \quad = 3 \quad \text{OR} \quad 5 - \sqrt{5x-1} \quad = -3 \\ &-5 \qquad\qquad\quad -5 \qquad -5 \qquad\qquad\quad -5 \end{aligned}$$

$-\sqrt{5x-1} \quad = -2$ OR $-\sqrt{5x-1} \quad = -8$
square both sides
$5x-1 \quad = 4$ OR $5x-1 \quad = 64$
$\quad +1 \qquad\qquad\qquad +1$
$5x \quad = 5$ OR $5x \quad = 65$
$\div 5 \quad \div 5 \qquad \div 5 \qquad \div 5$
$x = 1$ OR $x = 13$

We get $x = 1, 13$ for our solution

Step 4: Sketch the function domain and zeros



Split the function into invervals $[1/5, 1)$, $(1, 13), (13, +\infty)$ and plug in values of each interval to find out whether $f(x) > 0$ or $f(x) < 0$

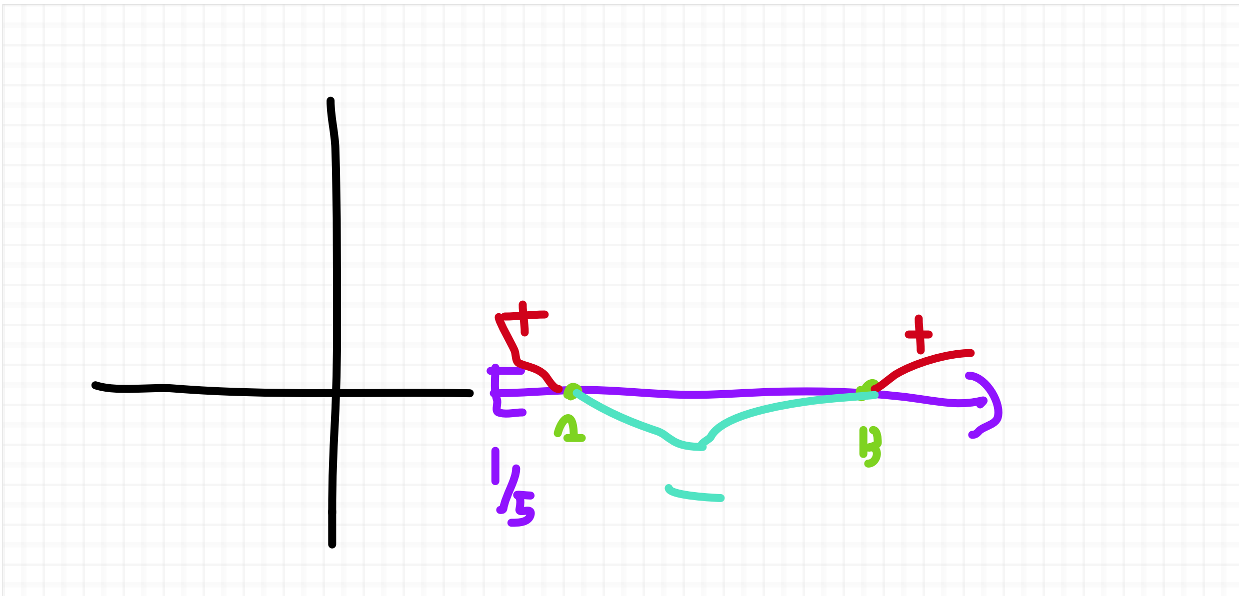
$f(x) = |5 - \sqrt{5x-1}| - 3$

$[1/5, 1)$ $x = 1/5$ $f(1/5) = 2 > 0$

$(1, 13)$ $x = 2$ $f(2) = -1 < 0$

$(13, +\infty)$ $x = 82/5$ $f(82/5) = 1 > 0$

Step 5: We find the intervals that satisfy the condition $f(x) \geq 0$ and state them in our solution (note that the zeros are included)



We include the intervals $[1/5, 1)$ and $(13, +\infty)$ in addition to the zeros $x = 1, 13,$, so our solution for $|5 - \sqrt{5x-1}| \geq 3$ is then

$[1/5, 1] \cup [13, +\infty)$,

i.e. all x such that $1/5 \leq x \leq 1$ or $13 \leq x$.

9/7

These notes can be accessed as a pdf in the following link:

https://agoodlad-instructor-notes.github.io/m211-fall-2021/recitation-notes.pdf

I will not be your recitator next week. Your next recitator will be Daniel Freese

For this week?

Thursday office this week still happening 4-5pm

Find me at the MLC today 10am-12pm (SE 325)

Exercise 2 item 5. Solve (for x) the following geometric inequality:

$$2\sin^2 x + \sin x - 1 < 0$$

First we find the domain for $f(x) = 2\sin^2 x + \sin x - 1$

Domain is all $\mathbb{R}$ (all x)

step 2. Find the zeros by soving for $2\sin^2 x + \sin x - 1 = 0$

$$2\sin^2 x + \sin x - 1 = 0$$

Not straightforward how to solve for x "directly", so let's solve for $y = \sin x$. Now we have

$$2y^2 + y - 1 = 0$$

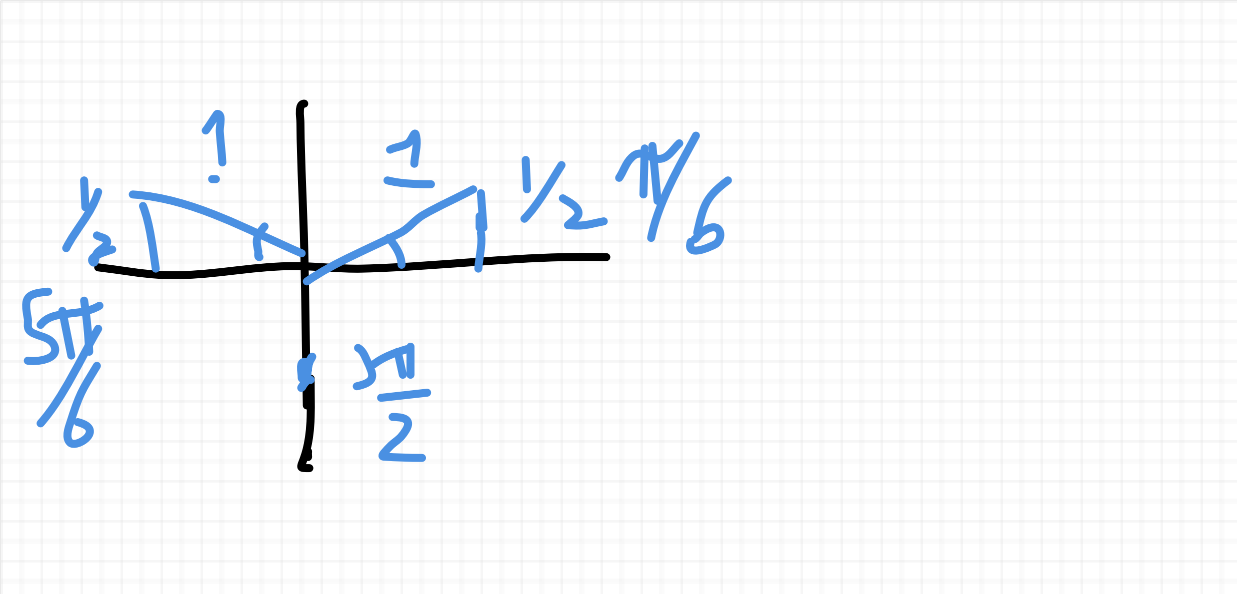
and we find the roots

$$(2y-1)(y+1) = 0 \Longrightarrow y = 1/2, -1$$

So that means

$$\sin x = 1/2, -1 \Longrightarrow x = \text{all } \theta \text{ such that } \sin\theta = 1/2, -1$$

How do we find such θ? Let's draw a unit circle



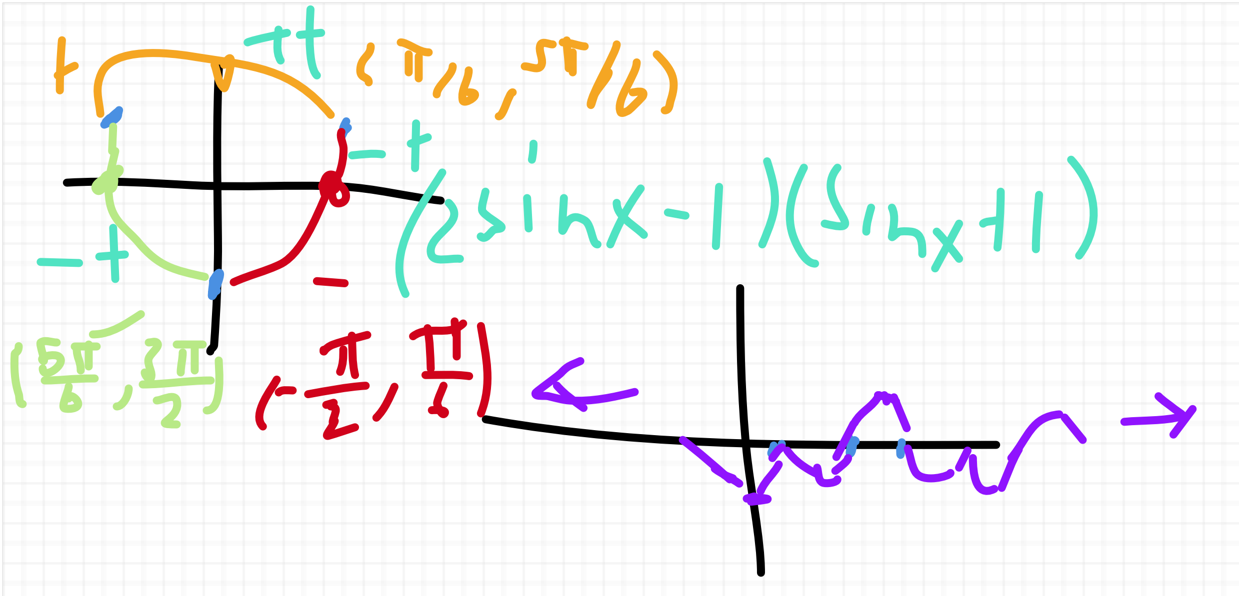
NOTE That every zero repeats for every $2\pi n$ for integer n, so the zeros are as follows:

$x = \pi/6 + 2\pi n,\ 5\pi/6 + 2\pi n,\ 3\pi/2 + 2\pi n \quad (n \text{ integer})$

To find the inequality, we again use the unit circle, and find where the cycles are positive and negative, and plug in the values $0,\ \pi/2,\ \pi$ on the respective intervals $(-\pi/2, \pi/6)$, $(\pi/6, 5\pi/6)$, and $(5\pi/6, 3\pi/2)$, respectively and we find

$f(x) = 2\sin^2 x + \sin x - 1$

$f(0) = -1,\ f(\pi/2) = 2,\ f(\pi) = -1$



Negative $(-\pi/2+2\pi n, \pi/6+2\pi n)$ and $(5\pi/6+2\pi n, 3\pi/2+2\pi n)$ (n integer)

Positive $(\pi/6+2\pi n, 5\pi/6+2\pi n)$ (n integer)

The solution is all intervals $(-\pi/2+2\pi n, \pi/6+2\pi n)$ and $(5\pi/6+2\pi n, 3\pi/2+2\pi n)$ (n integer)

Hint for Exercise 1 item 8.

$\sqrt{1-|x|} < |x|-2$

You want to make it in the form $f(x) < 0$ or $f(x) > 0$ (or respectively $\geq$ or $\leq$), and for this problem, we can do that as follows:

$$\sqrt{1-|x|} < |x|-2$$
$$-(|x|-2) \quad -(|x|-2)$$
$$\sqrt{1-|x|}-(|x|-2) < 0$$

and we can proceed in a similar way to previous examples where we find the negative (or positive part) (in this case you find the negative part).