2022-2023 James E Davis Trimester 1 Algebra 1 Week 2 Class Notes

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Previously...

We elaborated algebra on equalitiues with absolute value to five steps as so:

$95 - |2x| = 81$

Step 1. Check if the absolute value on one side or not. If it's on one side, then we proceed to step 3. If not, we have to set absolute value to a and proceed to the next step

$a = |2x|$

Step 2. Solve for a

$$\begin{aligned} 95 - a &= 81 \\ -95 \quad & \quad -95 \\ -a &= -14 \\ \div -1 \quad & \div -1 \\ a &= 14 \end{aligned}$$

Step 3. Check if the absolute value makes sense, in other words, make sure the absolute is not set equal to a negative. It may be that the absolute value is given in terms of x. For that, I'll have you assume that all cases, the answers will be positive.

Step 4. If the absolute value problem makes sense, then split the problem into cases

Step 5. Solve for x in both cases, and each case will be a solution (so the answer will usually have two solutions).

Warm up

1. $|2x - 5| = 29$

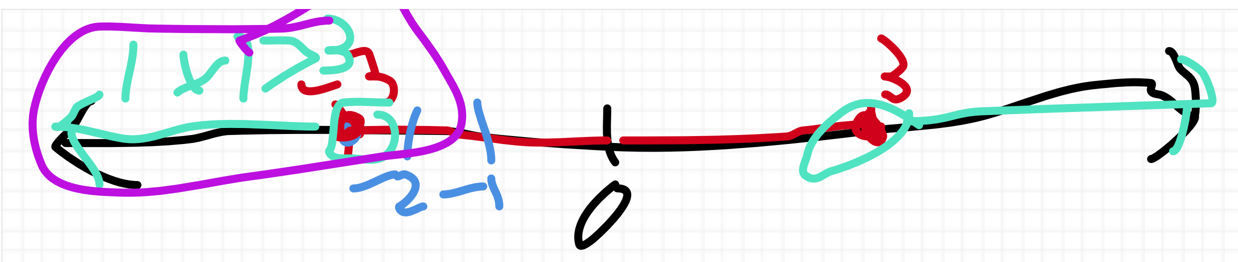
2. $-2|x + 3| = 2$

3. $5 - |x - 3| = 1$

Inequalities With Absolute Values

So now we'll solve for inequalities that involve the absolute value. Let's start with the inequality

What happens if we have $|x| > 3$? Remember that this is the shaded region for $|x| \geq 3$, and $|x| > 3$ is the opposite of $|x| \leq 3$



Another way to see this is dividing $|x| > 3$ into two cases (as follows):

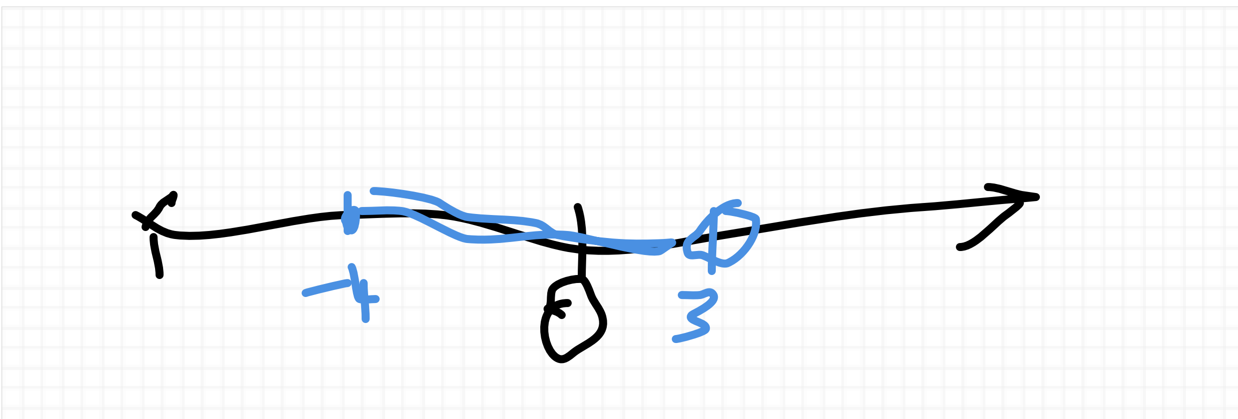
Case 1: x is positive, and when x is posiitive, we have $x > 3$

Case 2: x is negative, and when x is negaive, we have $x < -3$ since the absolute value is the distance from 0, and further negative we go (i.e., the "smaller" the value, the further away it is from zero).

Example 1. Solve and graph $|x+4| < 7$. To do this, we have to split it into cases, as we did with our previous thought exercise with $|x| \leq 3$ and $|x| > 3$. We have as before, two cases

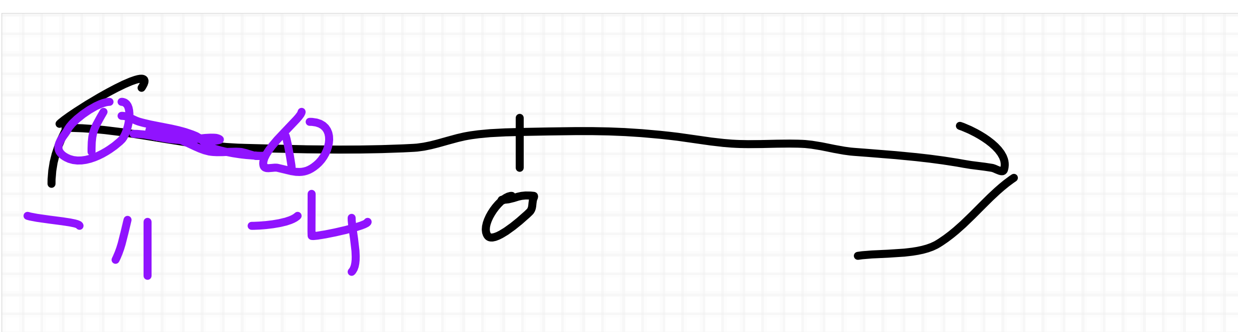
Case 1. $|x+4|$ is positive, assume $x+4 \geq 0$. In this case

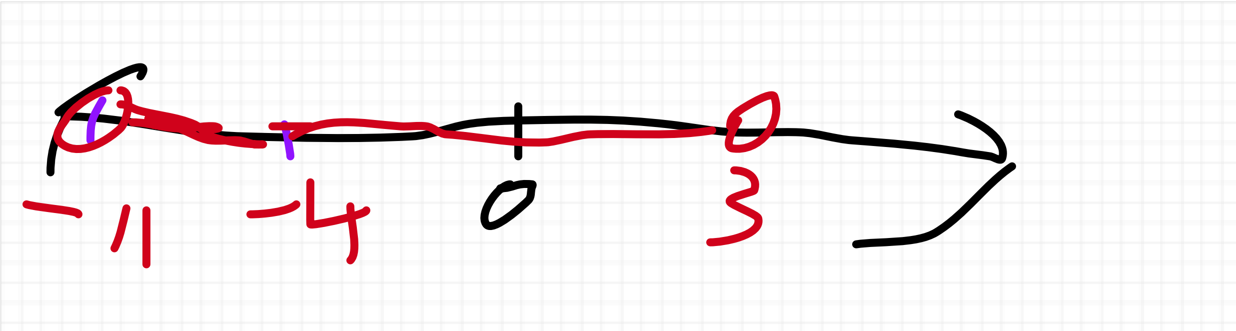
$0 \leq x+4 < 7$
$-4 \quad\quad -4 \quad -4$
$-4 \leq x < 3$



Case 2. $|x+4|$ is negative, i.e. $x+4 < 0$, we have

$-7 < x+4 < 0$
$-4 \quad\quad\quad -4 \quad -4$
$-11 < x < \; -4$





Since region for the $|x + 7| > 7$ includes both casaes, and hence we treat like a unon, i.e., we break it down into the statement

$-4 \leq x < 3$ or $-11 < x < 4$

Exit Pass:

Graph the following:

1. $|x| < 7$

2. $|x| > 10$

Little Hint: For both of these problems, we want to split it into cases. So for example, the first problem, either x is positve or negative. In the situation that x is positive, we view $|x| < 7$ through positive distance, i.e., the more right of the line is further from zero.

In the situation x is negative, we look at negative distance, i.e., the more left of the line, the further it is from zero.

Preview for Tomorrow

N/A

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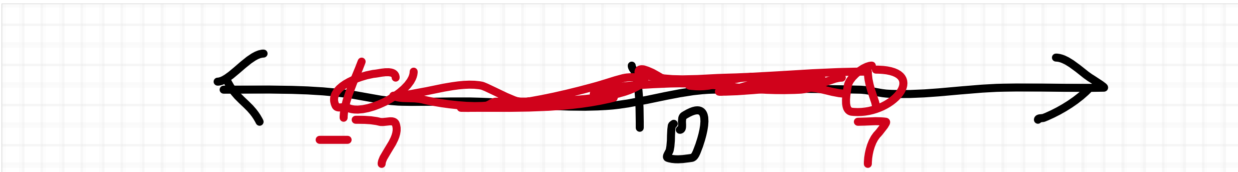
Previously...

We talked about absolute value inequalities and did this exit pass, which we'll go over

Graph the following:

1. $|x| < 7$



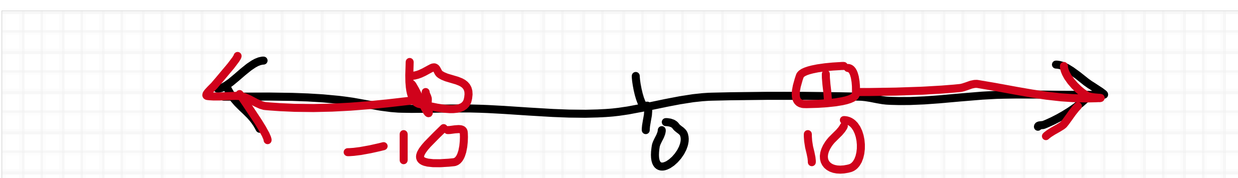
2. $|x| > 10$

In terms of cases, we have two:

1.

In this case, x is positive
$x > 10$

In this case x is negative
$x < -10$
$-x = |x| > 10$



Assignment Tuesday

Usually they'll be on a Monday, but they might be a Tuesday (as we're doing today)

Usually worth 25-40 points

Open note (in fact, I'll print out my notes for you to do this assignment)
Collaboration between is allowed
I'll be able to answer any questions you have (don't expect me to outright give you the answer)

[Algebra 1 Unit 3 Week 2 Main Assignment]

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Teacher PD, no class today

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Inequalities With Absolute Values (Cont.)

Example 1. Solve and graph $|x+4| < 7$

In order to solve for this inequality, we want to split it into cases

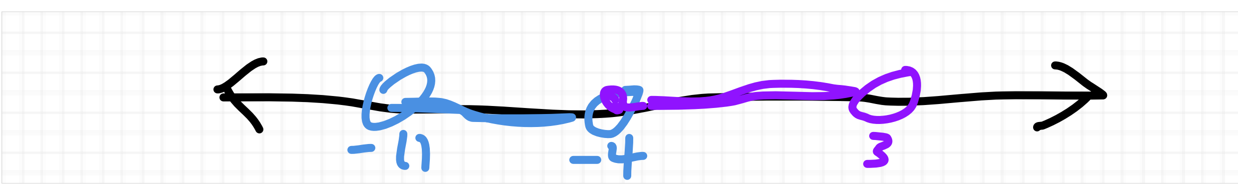
$0 > x + 4 > -7$

$0 \leq x + 4 < 7$

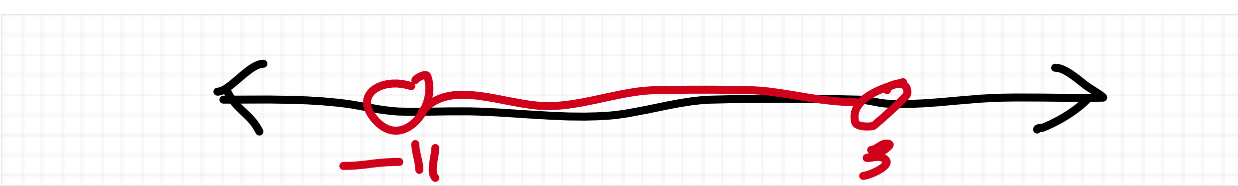
$0 > x + 4 > -7$
$-4 \quad -4 \quad -4$
$-4 > x \quad > -11$

$0 \leq x + 4 < 7$
$-4 \quad -4 \quad -4$
$-4 \leq x \quad < 3$

We found that $-11 < x < \; -4$ or $-4 \leq x < 3$



Since our inequalities are jointed by an or statement (since remember that an absolute is divided into cases, so we're solving for two problems factoring in the two possibilities)



Example 2. Solve and graph $|4x - 17| \geq 25$.

Like in the previous example, we want to split it into cases, and solve for the inequality in each case, though we'll end up with a different result (in terms of shading). The two cases are

$4x - 17 \geq 25$

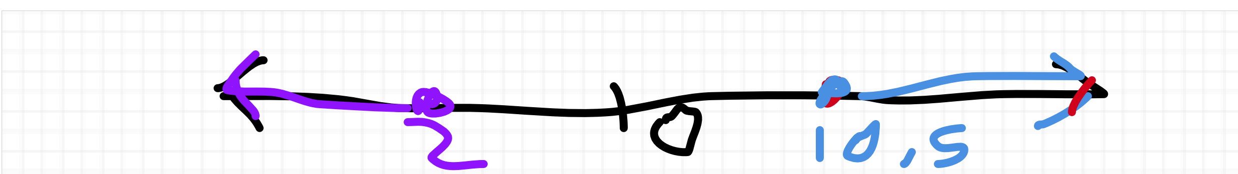
$4x - 17 \leq -25$

$$\begin{array}{ll} 4x - 17 & \geq 25 \\ \quad +17 & +17 \\ 4x & \geq 42 \\ \div 4 & \div 4 \\ x & \geq 10.5 \end{array}$$

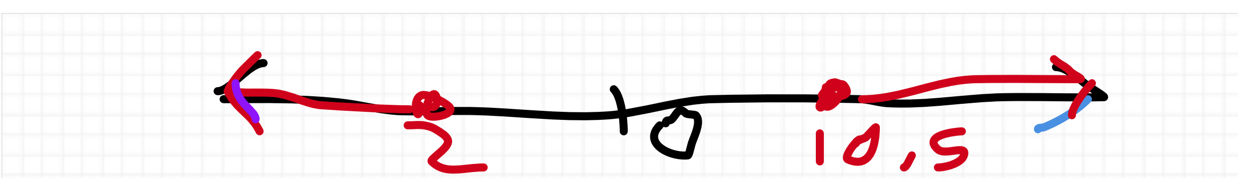
.

$$\begin{array}{ll} 4x - 17 & \leq -25 \\ \quad +17 & +17 \\ 4x & \leq 8 \\ \div 4 & \div 4 \\ x & \leq 2 \end{array}$$

$x \geq 10.5$ or $x \leq 2$



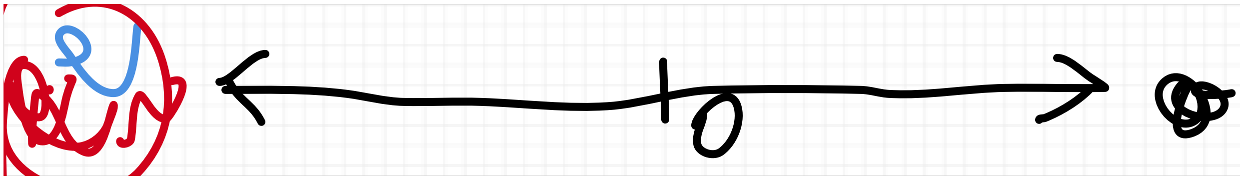
Since the statement is jointed by an "**OR**", we graph them together



Preview for Tomorrow

Next time, we'll do examples like $95 - |2x| < 81$, and figure out a way to generalize the five step process so that we can solve for absolute value inequalities in their most general form.

This is Dare'en's idea of using the pen for the lecture board and take advantage of the problem; THIS IS GENIUS! Dare'en deserves a shout out for this!



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Inequalities With Absolute Values (Cont.)

We elaborated algebra on equalitiues with absolute value to five steps as so:

$95 - |2x| < 81,\ 95 - |2x| \geq 81$

Step 1. Check if the absolute value on one side or not. If it's on one side, then we proceed to step 3. If not, we have to set absolute value to a and proceed to the next step.

$a = |2x|$

Step 2. Solve for a as an inequality

$$
\begin{array}{llll}
95 - a < 81 & & 95 - a \geq 81 & \\
-95 & -95 & 95 & -95 \\
-a < -14 & & -a \geq -14 & \\
\div -1 & \div -1 & \div -1 & \div -1 \\
a > 14 & & a \leq 4 &
\end{array}
$$

Step 3. Check if the absolute value has a positive or a negative on the other side. If it has a positive, carry on as normal. If it has a negative, replace it with zero.

NOTE: I will modify step 3 as follows:

Check if the absolute value has a positive or a negative on the other side. If it has a positive, carry on as normal. If it has a negative then we have two cases:

Case 1. If we have $a < b$ or $a \leq b$, for a the absolute value and b negative, then no solution exists (since we know $a \geq 0$) and we shade nothing.

Case 2. If we have $a > b$ or $a \geq b$, for a the absolute value and b negative, then every value of x is a solution since we know $a \geq 0$ automatically, regardless of what x is.

Step 4. Split the problem into cases (and then join it with an "**OR**" statement.

$|2x| > 14$ $\qquad$ $|2x| \leq 14$

$2x > 14$ or $2x < -14$ $\qquad$ $0 \leq 2x \leq -14$ or $0 > 2x \geq -14$

Step 5. Solve for x in both cases, and each case will be a solution (so the answer will usually have two solutions).

$2x > 14$ or $2x < -14$ $\qquad$ $0 \leq 2x \leq 14$ or $0 > 2x \geq -14$

$\div 2 \quad \div 2 \quad \div 2 \quad \div 2$ $\qquad$ $\div 2 \quad \div 2 \quad \div 2 \quad \div 2$

$x > 7 \qquad x < -7$ $\qquad$ $0 \leq x \leq 7 \qquad 0 > x \geq -7$

Example 3. Graph the following inequality

$2|5x + 3| - 3 \leq 11$

First and foremost, we will set $a = |5x + 3|$

$2a - 3 \leq 11$

$\quad +3 \quad +3$

$2a \quad \leq 14$

$\div 2 \quad \div 2$

$a \quad \leq 7$

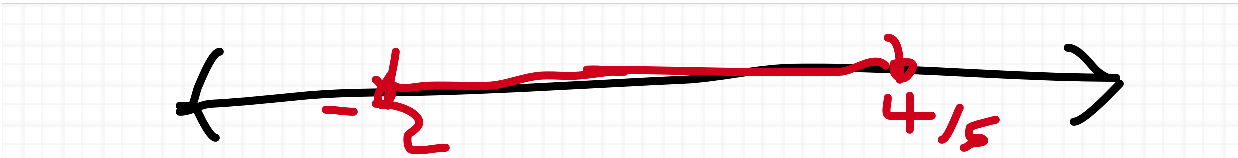
And now we have the absolute on one side and we're ready to split it into cases

$|5x + 3| \leq 7$

$0 > 5x + 3 \geq -7,$ $\qquad$ $0 \leq 5x + 3 \leq 7$

$-3 \quad -3 \quad -3$ $\qquad$ $-3 \quad -3 \quad -3$

$$-3 > 5x \geq -10 \qquad\qquad -3 \leq 5x \leq 4$$
$$\div 5 \quad \div 5 \qquad \div 5 \qquad\qquad \div 5 \quad \div 5 \quad \div 5$$
$$-\frac{3}{5} > x \geq -2 \qquad \text{or} \qquad -\frac{3}{5} \leq x \leq \frac{4}{5}$$



Example 5.

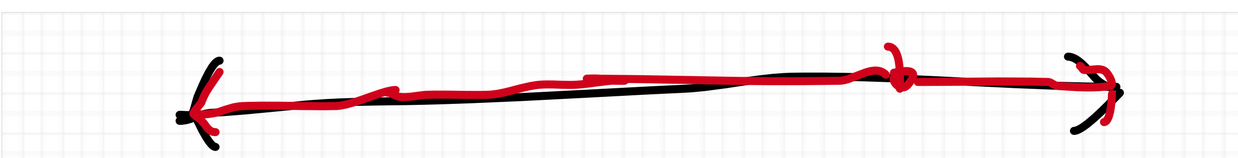
$|x+4| + 10 > -14$

Solve for $a = |x+4|$

$a + 10 > -14$
$\quad -10 \qquad -10$
$a \qquad > -24$

Recall that $|x+4| \geq 0$, that inequalify overrides $|x+4| > -24$, and now we divide it into cases

$x + 4 > 0 \qquad x + 4 \leq 0$
$\quad -4 \quad -4 \qquad -4 \quad -4$
$x \quad > -4 \qquad x \leq -4$



NOTE: This example illustrates how the revised version of Step 3 ends up working.

Exit Pass

Solve for the following:

1. $|x+7| \leq 5$

2. $|6x - 3| > 33$

3. $7 + |x| \geq 85$