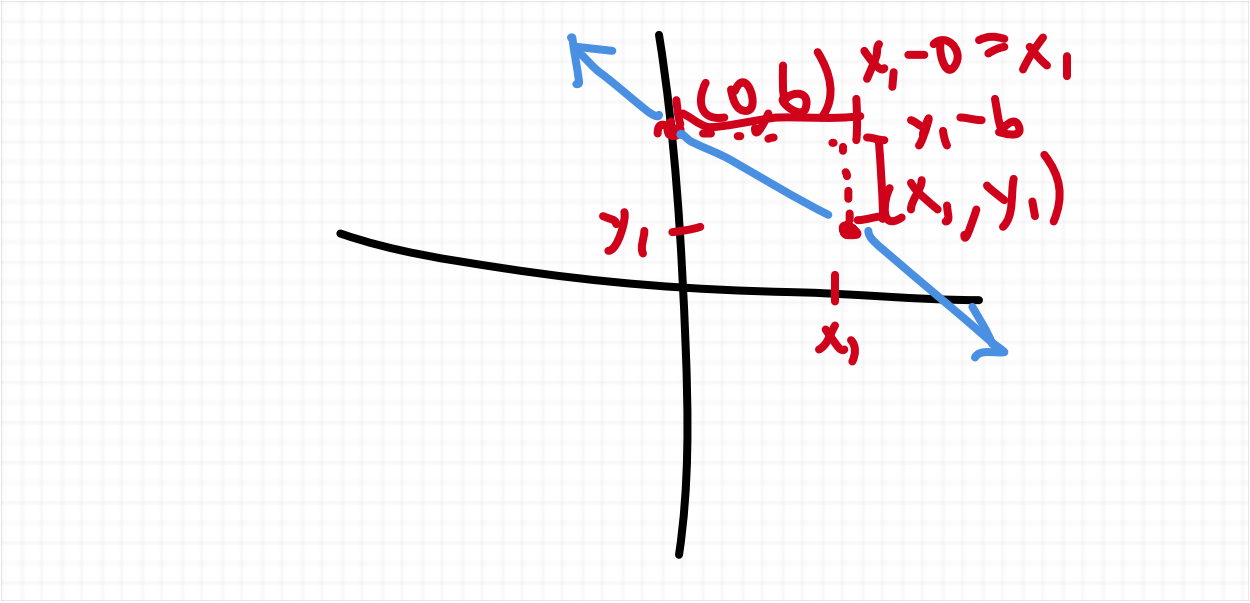
2022-2023 James E Davis Trimester 1 Geometry Week 5 Class Notes

10/17

Equations of Lines (Cont.)

1. slope-intercept formula

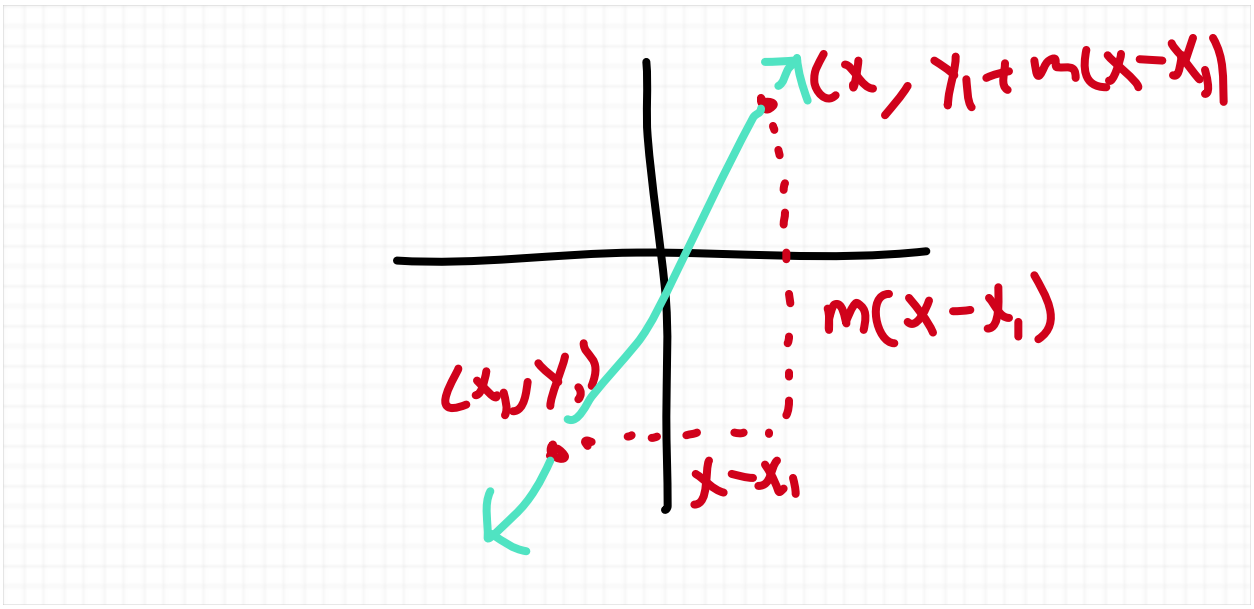
If we have the y-intercept $(0, b)$ and its slope m, then we have
$y = mx + b$



2. point slope formula

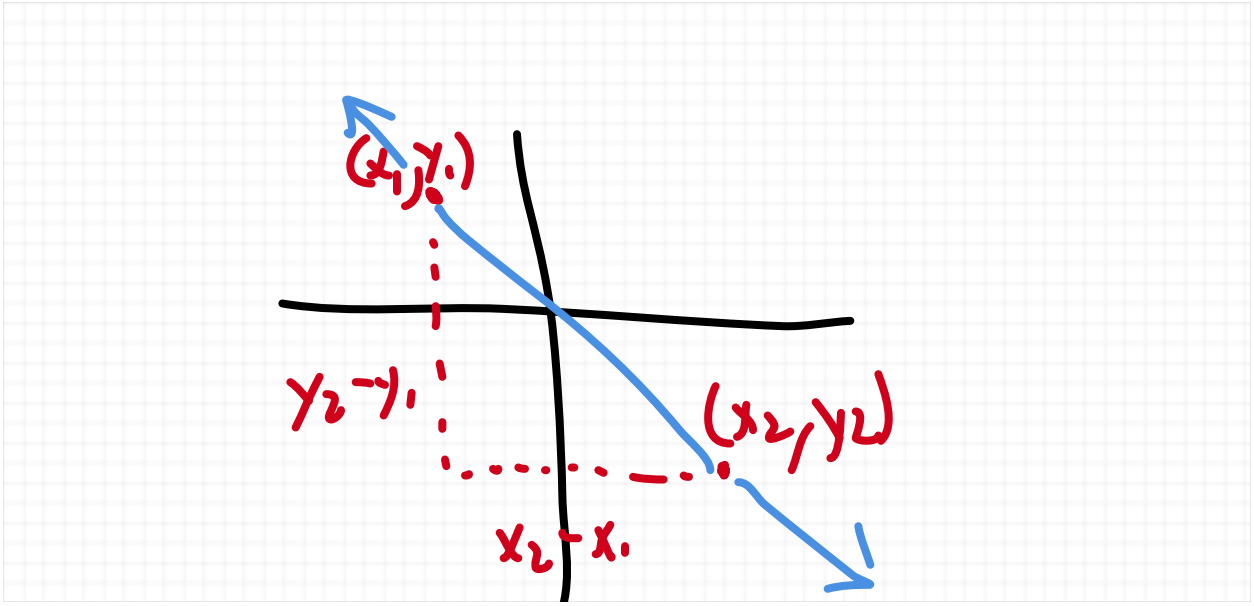
If we have a point (x_1, y_1) and its slope m, then we have

$y - y_1 = m(x - x_1)$
$y = y_1 + m(x - x_1)$



If we have two points (x_1, y_1) and (x_2, y_2), then we have

$y - y_1 = m(x - x_1)$ and $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.



3. the general form

If we know that the equation of the line is given of the fom

$ax + by = c,$

for constants a, b, c such that $b \neq 0$, then we know that the slope $m = -\frac{a}{b}$ and the y-intercept is $\left(0, \frac{c}{b}\right)$

$$
\begin{aligned}
ax + by &= c \\
-ax \qquad &\quad -ax \\
by &= c - ax \\
\div b \quad &\quad \div b \\
y &= \frac{c}{b} - \frac{a}{c}x
\end{aligned}
$$

$$y = -\frac{a}{c}x + \frac{c}{b}$$

$m = -\frac{a}{c}$, $\quad "b" = \frac{c}{b}$ (why $b \neq 0$, cookie monster would be very sad)

Example 1. Write the equation passing through $(-2, 5)$ and parallel to the line $y = 8x - 3$.

We know from the line being parallel that the slope of $y = 8x - 3$ is the same slope as the other line. The slope of the $y = 8x - 3$ is 8, since the equation is in slope-intercept form and we have $m = 8$.

The slope of the equation of the parallel line containing $(-2, 5)$ is also 8. So we ca then write the equation of the line using the point-slope form, plugging in $m = 8$ and $(x_1, y_1) = (-2, 5)$, so we get

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 5 &= 8(x - (-2))
\end{aligned}
$$

Note: In general, not mandatory to simplify linear equations unless the exercise wants you to put it in a specific form.

Example 2. Write the equation of the line passing through $(-2, 5)$ and perpendicular to the the line $y = 8x - 3$.

We know from the line being perpendicular that the slope of that line is the negative recriprical of the slope of the line $y = 8x - 3$ (which is $m_1 = 8$)

$$m_2 = -\frac{1}{m_1} = -\frac{1}{8}$$

The equation of the desired we can find by plugging in $m_2 = -\frac{1}{8}$ and $(x_1, y_1) = (-2, 5)$ to get

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -\frac{1}{8}(x - (-2))$$

Example 3. Show that lines represented by $4x + 3y = 7$ and $3x - 4y = 12$ are perpendicular.

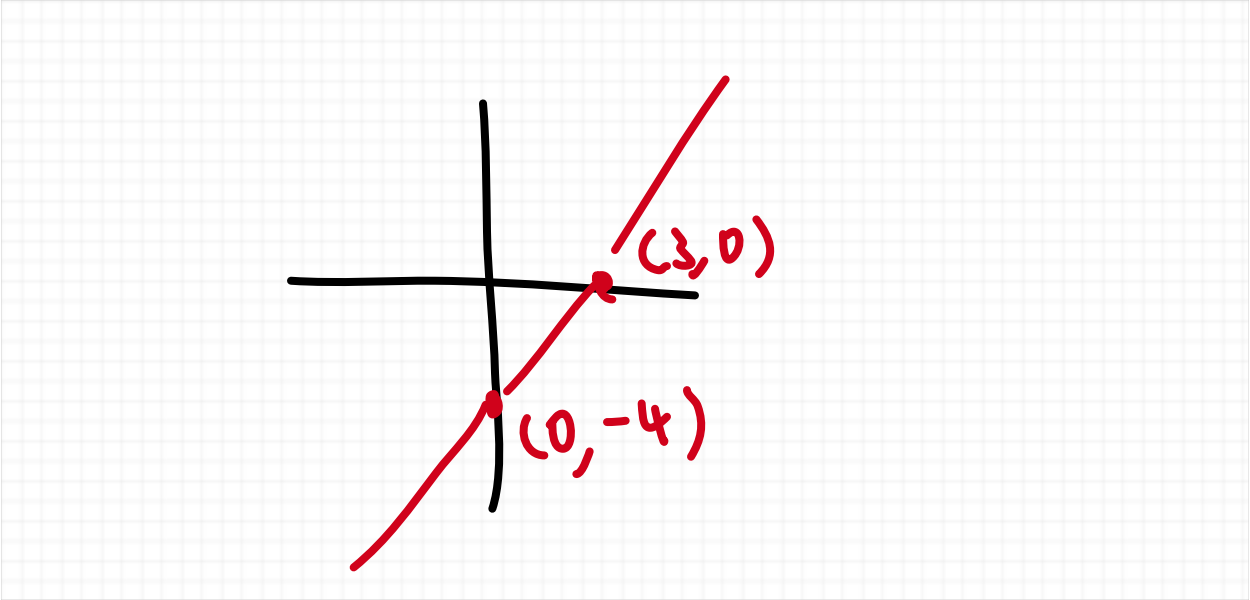
In any general equation $ax + bx = c$, we know that the slope is $-\frac{a}{b}$, so we know that the slope of $4x + 3y = 7$ $(a = 4,\ b = 3,\ c = 7)$ is $m_1 = -\frac{4}{3}$ and the slope of $3x - 4y = 12$ $(a = 3,\ b = -4,\ c = 12)$ is $m_2 = \frac{3}{-4} = \frac{3}{4}$, and we find

$$-\frac{1}{m_1} = -\frac{1}{-\frac{4}{3}} = -\left(\frac{-4}{3}\right)^{-1} = -\frac{3}{-4} = \frac{3}{4} = m_2.$$

10/18

Equations of Lines (Cont.)

Example 4.



$m = \frac{4}{3},\ (x_1 y_1) = (3, 0)$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{4}{3}(x - 3)$

$y = \frac{4}{3}(x - 3) = \frac{4}{3}x - \frac{4}{3} \cdot 3$

$= \frac{4}{3}x - 4$