2022-2023 James E Davis Trimester 1 Algebra 1 Week 4 Class Notes

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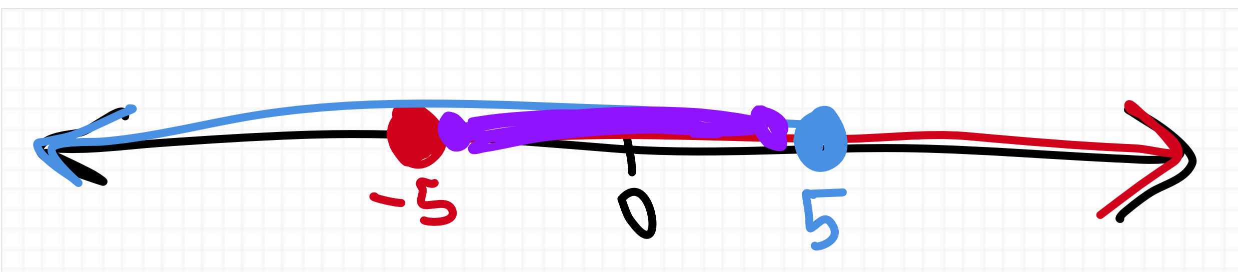
Unions and Intersections of Intervals (Cont.)

Example 5. Shade the following region:

$(-\infty, 5] \cap [-5, \infty)$

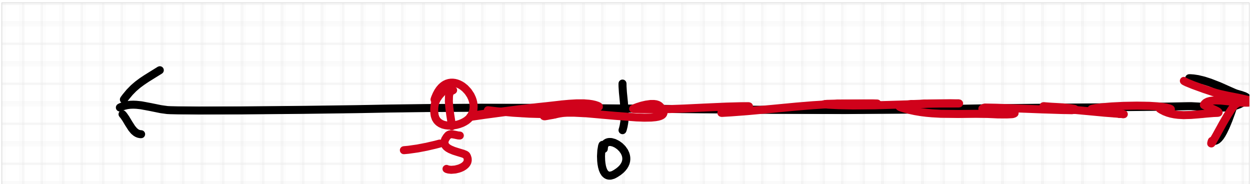


Another way we can express this in interval notation is $[-5, 5]$, so the main lesson with this example is there may be more than one way to express a compound inequality interval notation.

Example 6. Find the solution to $3x + 7 > -8$ in interval notation

$$\begin{aligned} 3x + 7 &> -8 \\ -7 \quad & \quad -7 \\ 3x \quad &> -15 \\ \div 3 \quad & \quad \div 3 \\ x \quad &> -5 \end{aligned}$$

Next, we want to take our solution we got, $x > -5$ and express it in interval notation. As we can infer from the number line below that the interval notation for solution is $(-5, \infty)$



Example 7. Let's say we had $4x + 7 > 2x + 13$ or $x + 3 < -10$. Express it in interval notation.

First, we'd solve for the compound inequality $4x + 7 > 2x + 13$ or $x + 3 < -10$ as before

$$\begin{array}{llll}
4x + 7 & > & 2x + 13 \\
\quad -7 & & \quad -7 \\
4x & > & 2x + 6 \\
-2x & & -2x \\
2x & > & 6 \\
\div 2 & & \div 2 \\
x & > & 3
\end{array}$$

$$\begin{array}{lll}
x + 3 & < & -10 \\
\quad -3 & & -3 \\
x & < & -13
\end{array}$$

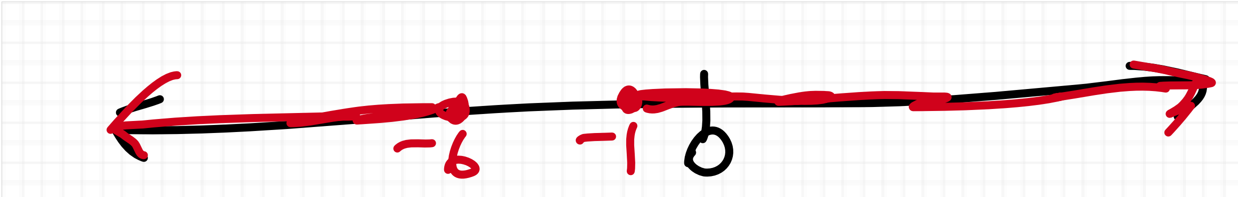
We then get $x > 3$ or $x < -13$ as a solution, so

$(3, \infty) \cup (-\infty, -13)$

Example 8. Expresss $|2x + 7| \geq 5$ in interval notation.

$2x + 7 \geq 5$ or $2x + 7 \leq -5$

$$\begin{array}{lll|lll}
2x + 7 & \geq & 5 & 2x + 7 & \leq & -5 \\
\quad -7 & & -7 & \quad -7 & & -7 \\
2x & \geq & -2 & 2x & \leq & -12 \\
\div 2 & & \div 2 & \div 2 & & \div 2 \\
x & \geq & -1 & x & \leq & -6
\end{array}$$



$(-\infty, -6] \cup [-1, \infty)$

Exit Pass

1.

a. Solve for the inequality $5 + 2|x| \geq 11$

b. Draw the number line and shade the region of the solution

c. Express the solution in interval notation.

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Unions and Intersections of Intervals (Cont.)

Example 9. Find the interval notation of the solution for the following

$5x + 7 > 9$ and $3x + 3 < -9x - 6$

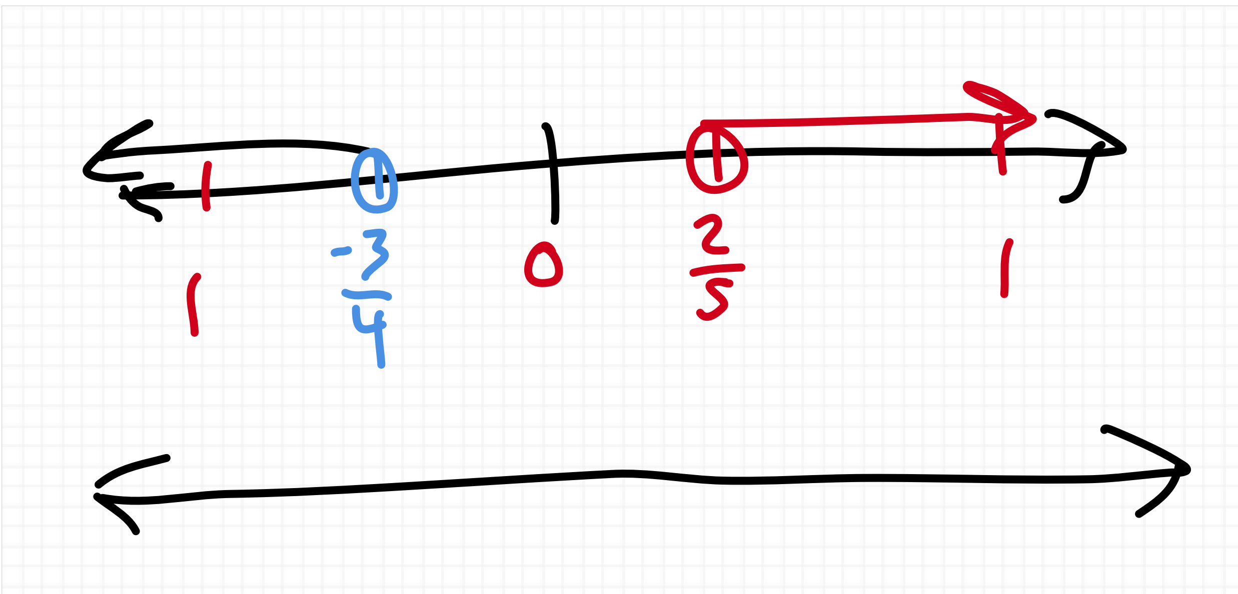
First, we need we need to take both equations and solve for it

$$
\begin{array}{lll}
5x + 7 & > & 9 \\
\;\; -7 & & -7 \\
5x & > & 2 \\
\div 5 & & \div 5 \\
x & > & \dfrac{2}{5}
\end{array}
$$

$$
\begin{array}{lll}
3x + 3 & < & -9x - 6 \\
+9x & & +9x \\
12x + 3 & < & -6
\end{array}
$$

$$
\begin{aligned}
&\quad -3 && -3 \\
&12x && < -9 \\
&\div 12 && \div 12 \\
&x && < -\frac{3}{4}
\end{aligned}
$$

$$\left(\frac{2}{5}, \infty\right) \cap \left(-\infty, -\frac{3}{4}\right)$$



Sometimes you have no overlap at all, and the intersection is just empty in that case.

Example 10. Find the interval notation for the solution to $|x-4|+3>6$

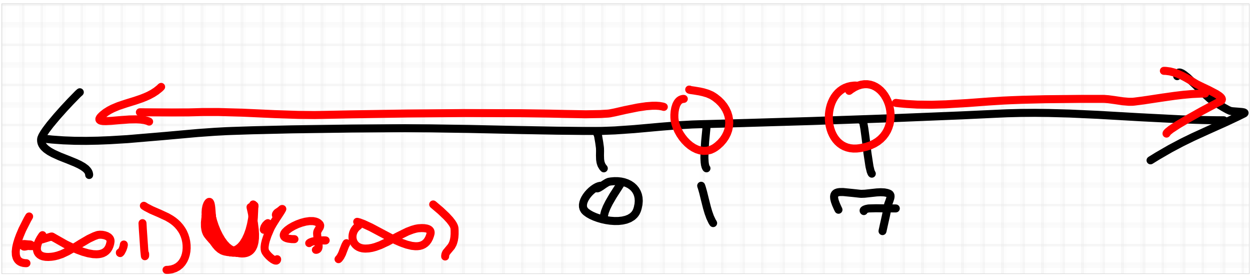
$a = |x-4|$

$$
\begin{aligned}
&a + 3 > 6 \\
&\ -3 \quad\ -3 \\
&a \quad\ > 3 \\
&|x-4| > 3
\end{aligned}
$$

Now we split it into cases

$x-4>3, \quad x-4< -3$

$$\begin{array}{llll} x-4>3 & & x-4< & -3 \\ \;\;+4 & +4 & \;\;+4 & +4 \\ x & >7 & x & <\;\;1 \end{array}$$



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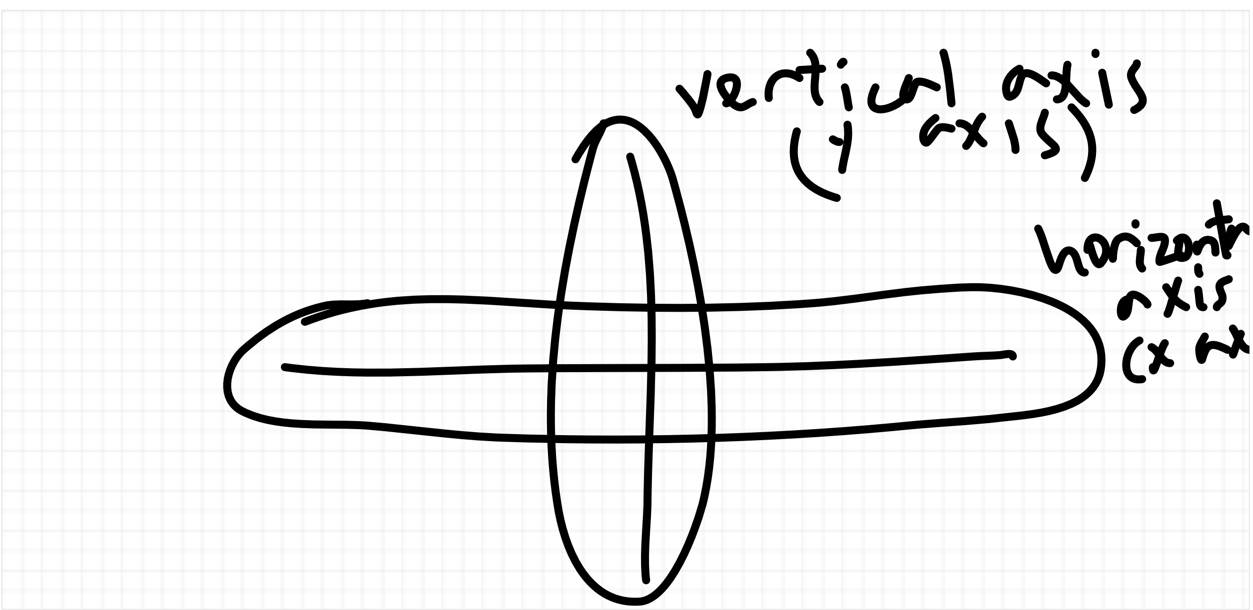
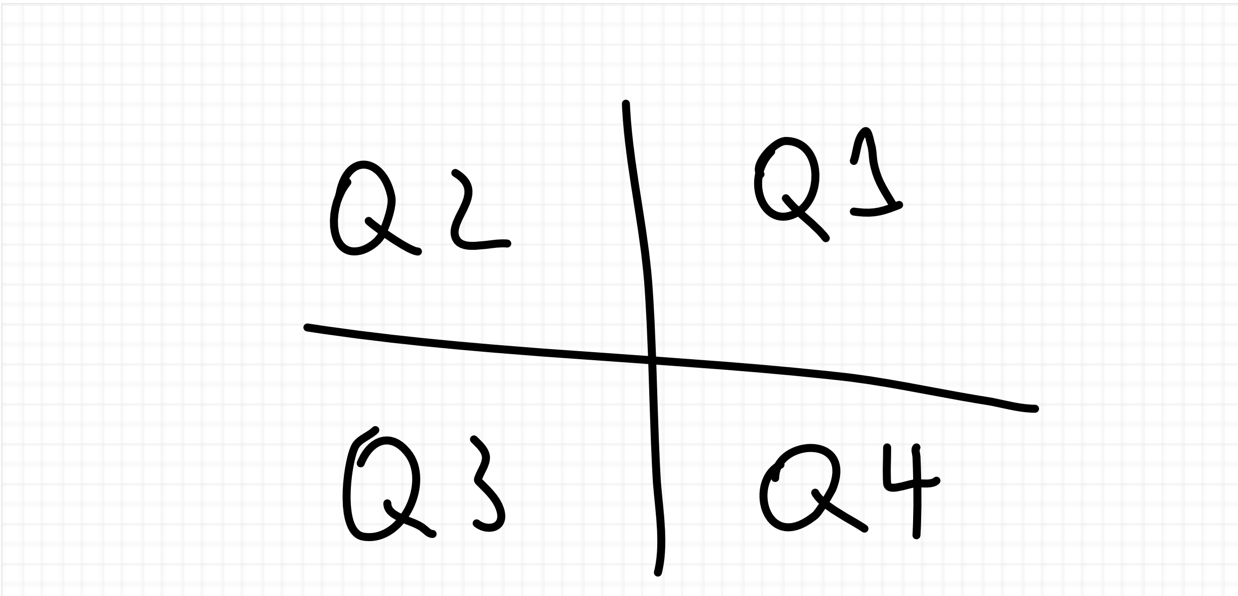
Lines and Slopes

In this course, we're going to graph a lot of lines, and we'll do it in 2 dimensional rectangular coordinates.

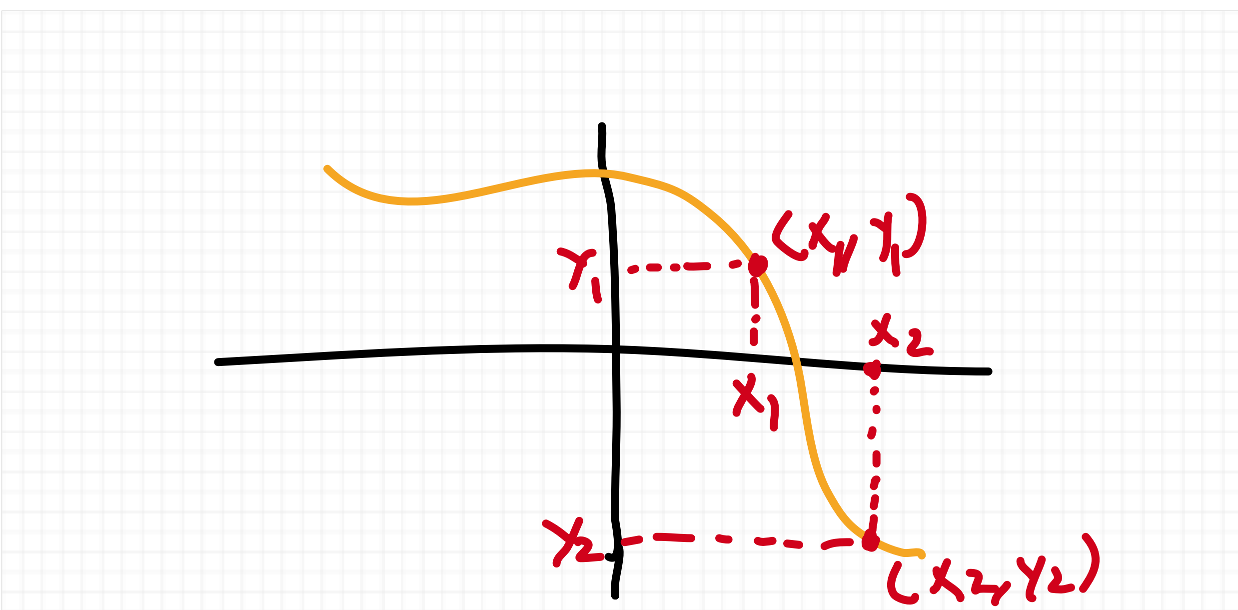
In a rectangular coordinate system (the cartesian plane), we describe two lines, one vertical and the other horizontal that we look at as axes



We have in the coordinate system four "quadrants" that are formed by the two axes of the plane



In general, we often take equations (of lines) and graph them. A graph of an equation is a drawing of all the points (x, y) that when plugged in satisfy the linear equation.



So let's say that we have some equation $f(x) = y$ and it represents the orange curve, then $f(x_1) = y_1$ and $f(x_2) = y_2$. So the general idea is to find some equation $f(x) = y$ that represents all the points in the curve, and that conversely, we can represent curves with an equation $f(x) = y$.

In this topic of linear equations, we're going to do this specifically for lines. Now we're going to graph linear equations, which are equations equivalent to an equation of the form

Recall that an equation is a "problem with an equals sign".

$ax + by = c$

Somes facts about lines:

1. The line connects any two points is the line in general
2. The "rate of change" (define that later) is constant

Example 1. Graph $3x + 2y = 12$

We want to able to find what y is given x (one variable in terms of the other). We want to solve for y in terms of x:

$$\begin{aligned} 3x + 2y &= 12 \\ -3x \quad\quad &\quad -3x \\ 2y &= 12 - 3x \\ \div 2 \quad &\quad \div 2 \\ y &= 6 - \frac{3}{2}x \end{aligned}$$

Now we can plug in values of x into the above equation and get output values of y

x	y
−2	9
−1	7.5
0	6
1	4.5
2	3

$y = 6 - \frac{3}{2}x$

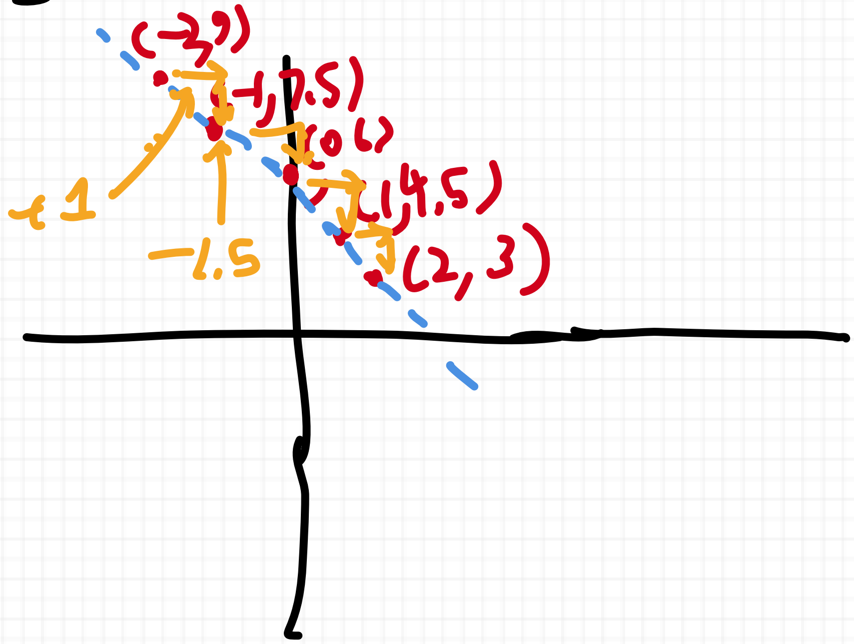
$y = 6 - \frac{3}{\cancel{2}}(-\cancel{2}) = 6 - 3($

$= 6 + 3$

$= 9$

$y = 6 - \frac{3}{2}(-1) = 7.5$

$y = 6 - \frac{3}{2}(0) = 6$



In general, you only need two points to connect the dots; you don't need a lot of points