2022-2023 James E Davis Trimester 1 Algebra 1 Week 1 Class Notes

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Intersections and Unions

For compound inequalities, we want to deal with phrases like

"$7 \leq x \leq 10$" and phrases like "$7 \geq x$ or $x \leq 10$"

What we want to do is graph compound inequalities that utilize "or" and "and", and that's where understanding intersection and union comes in.

In mathematics, we call "sets" collections of objects, such as the set of numbers that satisfy an inequality, or a circle in a venn diagram that contains ideas (so that's all to say that a venn diagram is a good way to visualize sets).

Union

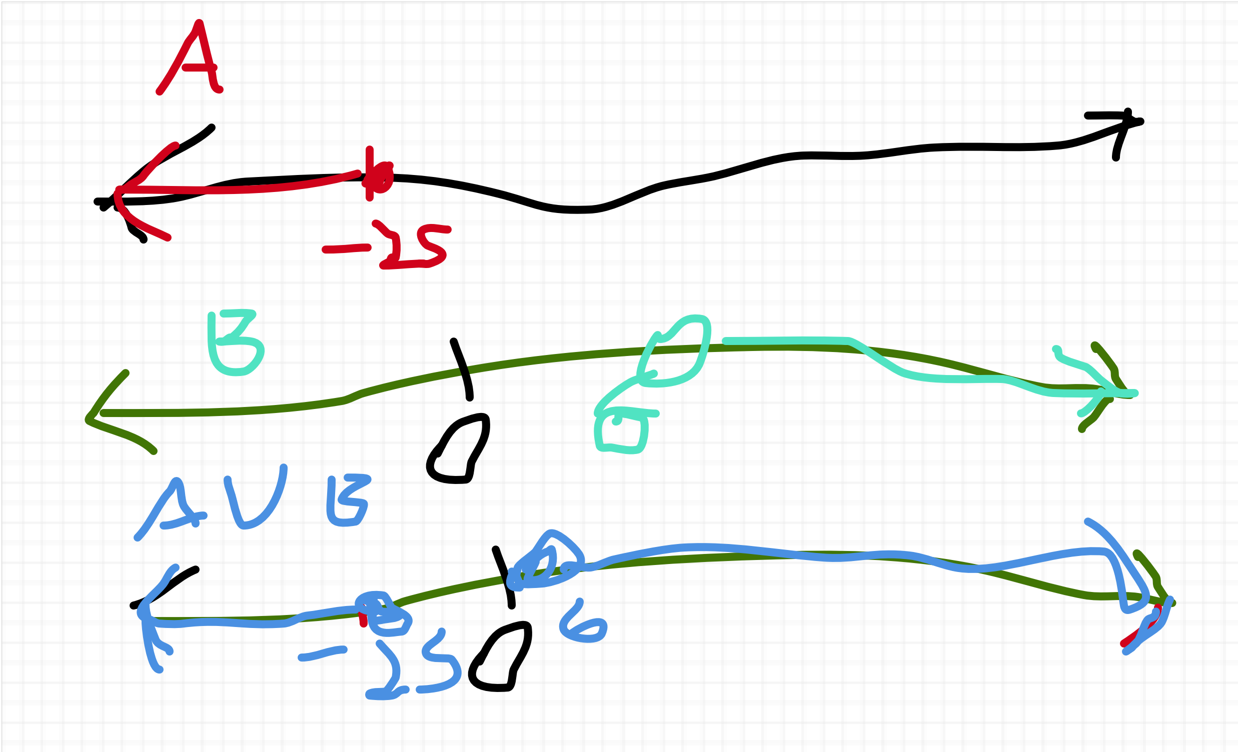
The **union** of sets A and B, abbreviated $A \cup B$ is the set of numbers that are either in set A or a set B

Example 1. Let's say

A is the set of all numbers x such that $x \leq -25$
B is the set of all numbers x such that $x > 6$.

What is $A \cup B$?

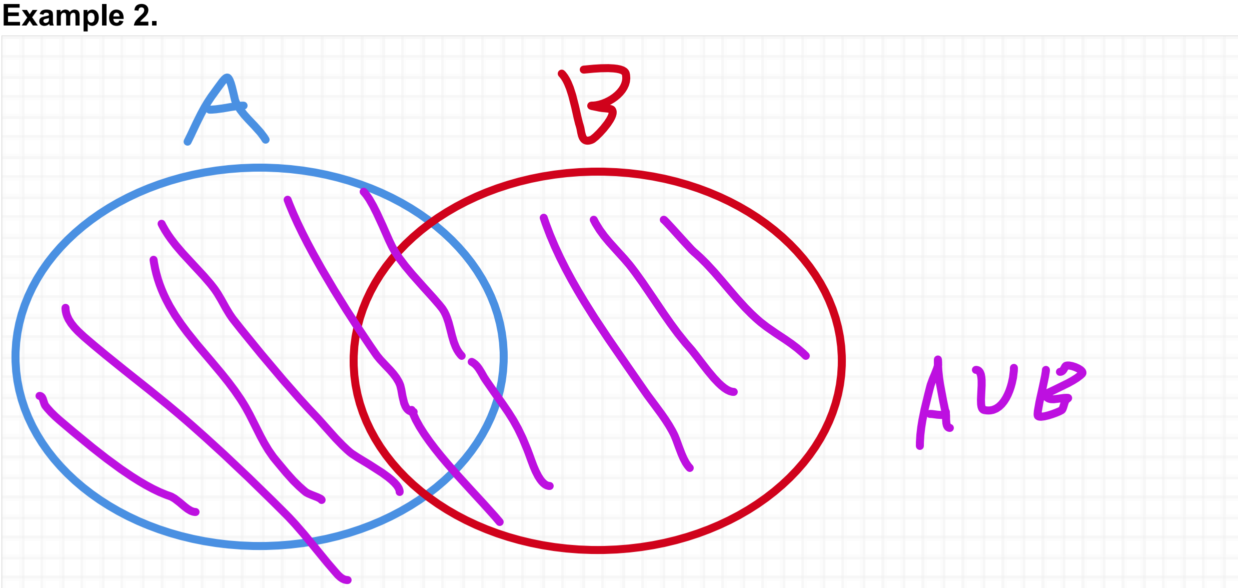
Well if we draw the inequalities, on a number line, we get



So when we draw the region of a union of sets, we draw both regions, since both regions drawn together consist of everything in both A and B.

For the next example, let's take a little bit of a break from inequalities and talk about venn diagram examples

Example 2.

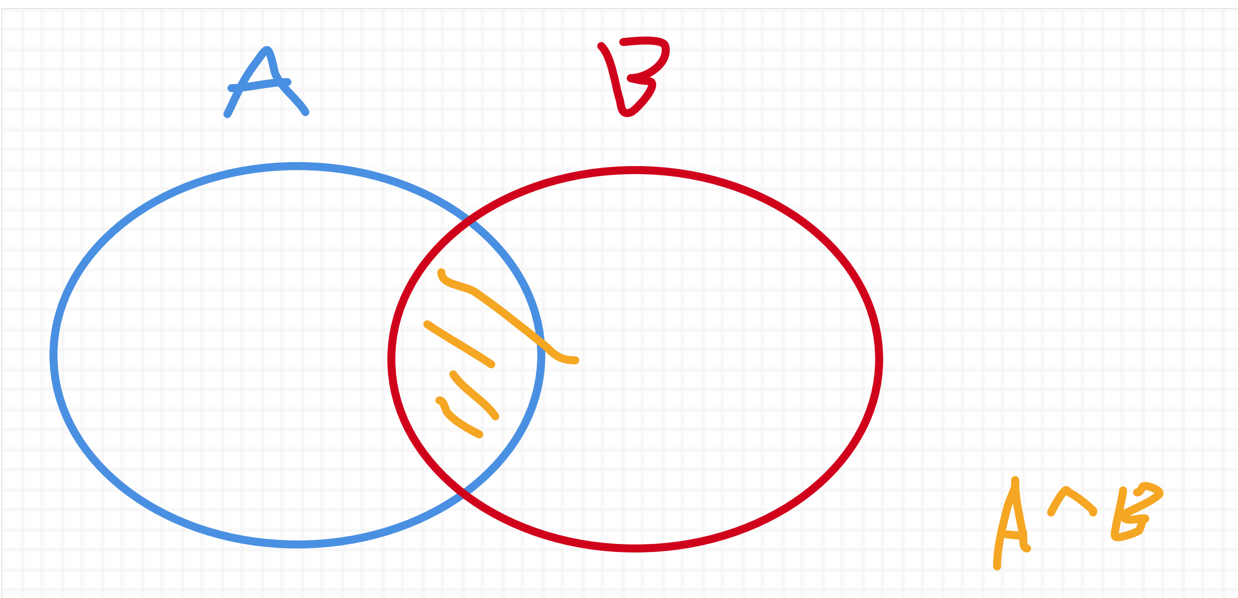


The union $A \cup B$ is the group of the sets A and B so we shade everything that is in A and also everything that is in B, since $A \cup B$ consists of everything that is either in A OR in B.

Intersection

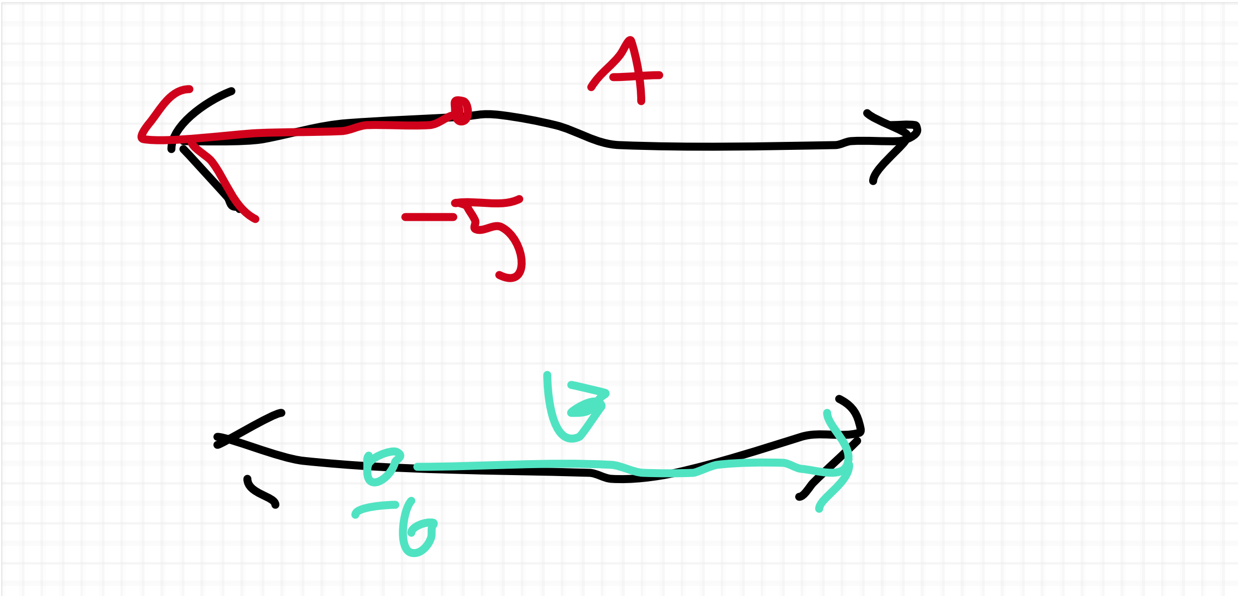
The **intersection** of sets A and B, abbreviated $A \cap B$, is the set of objects that are in both sets.

Example 3. The intersection of two circles in a venn diagram in the overlap of the two circles since that is everything that is in both circles, which makes up the intersection.

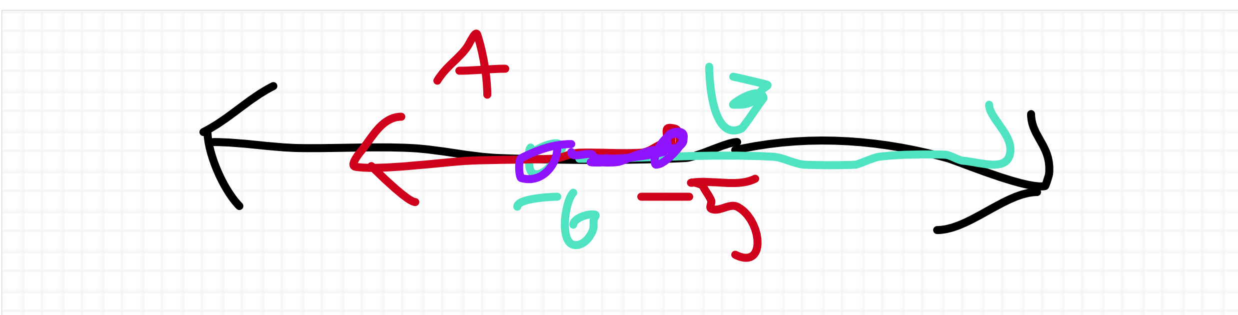


Example 4. Let's look at the intersection of the following

A is the set of all numbers x such that $x \leq -5$
B is the set of all numbers x such that $x > -6$.



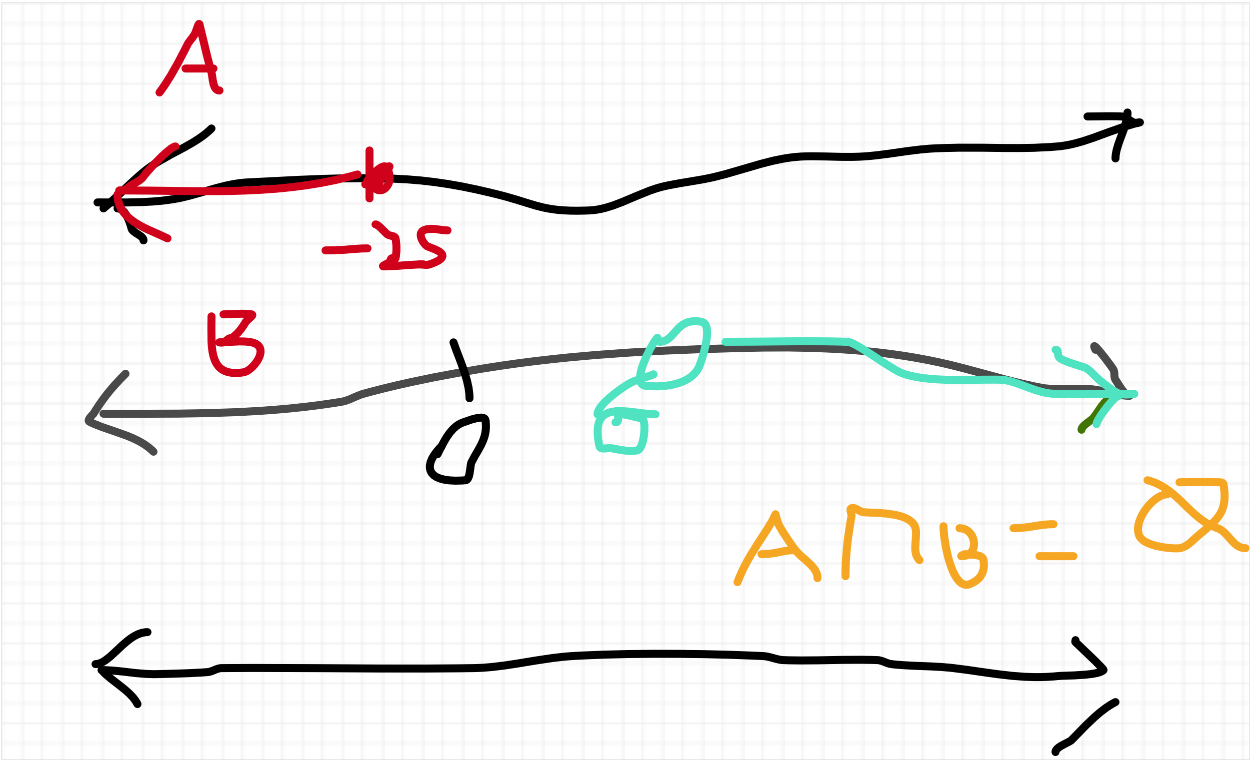
How do we draw $A \cap B$? We first draw A and B, and instead of drawing both as one region (remember that doing such is the union), we draw the overlap of the two regions, since that consists of everything that is in the A region and the B region.



Example 5. Let's say we had the same setup as in **Example 1**, i.e.

A is the set of all numbers x such that $x \leq -25$
B is the set of all numbers x such that $x > 6$.

What is $A \cap B$?



For $A \cap B$, we have nothing, since we have no overlap between those regions. For the set of nothing, we like to use the symbol $\varnothing$, which we call the "empty set" or "no solution symbol", i.e., the set of nothing.

Preview for Tomorrow

N/A

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Previously...

We did Venn Diagrams and through Venn Diagrams, we learned intersections and unions.

Intersection $\cap$ is the overlap of two sets, i.e. the set of everything in one set AND the other set.

union $\cup$ is a group of sets, i.e. the set of everything in one set OR the other

The Warm-up

Graph the following:

1. $A \cap B$, where A is the set of all $x \geq 5$ and B is the set of all $x < 9$

2. $A \cup B$, where A is the set of all $x \geq 8$ and B is the set of all $x <$

Compound Inequalities

The statement "A and B is true" means BOTH of them must be true

The statement "A or B is true" means at least ONE of them must be true (not necessarily both are true, but both could be true as well)

So what we did with intersections and unions corresponds to compound inequalities in the sense that inequalities that involve one inequality or the other corresponds to unions and inequalities that involve one inequality and the the other corresponds to intersections.

One thing to understand is when we talk about when x is "between" two inequalities.

"x is between a and b" is equivalent to $a \leq x \leq b$, i.e. $x \geq a$ and $x \leq b$.

Note: Sometimes the compound statement we get, we have to solve for the x value in question.

Example 1. Sketch the graph of $-6 < 5x - 2 < 13$

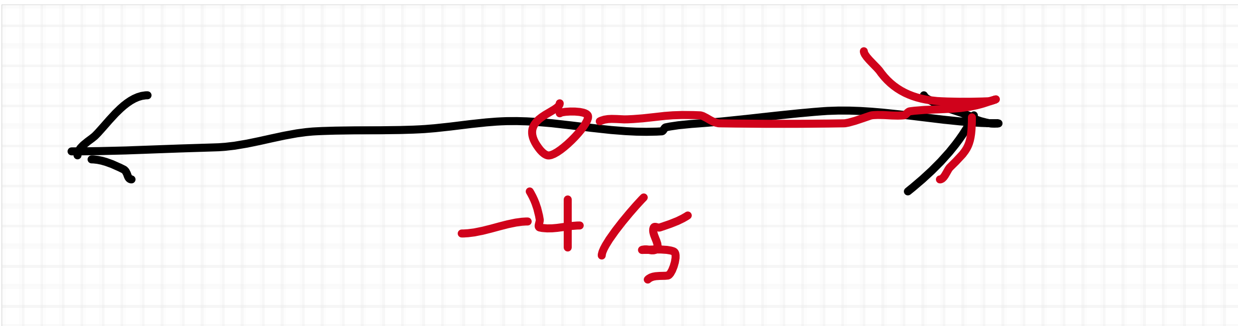
$-6 < 5x - 2$ and $5x - 2 < 13$

Let's solve for x

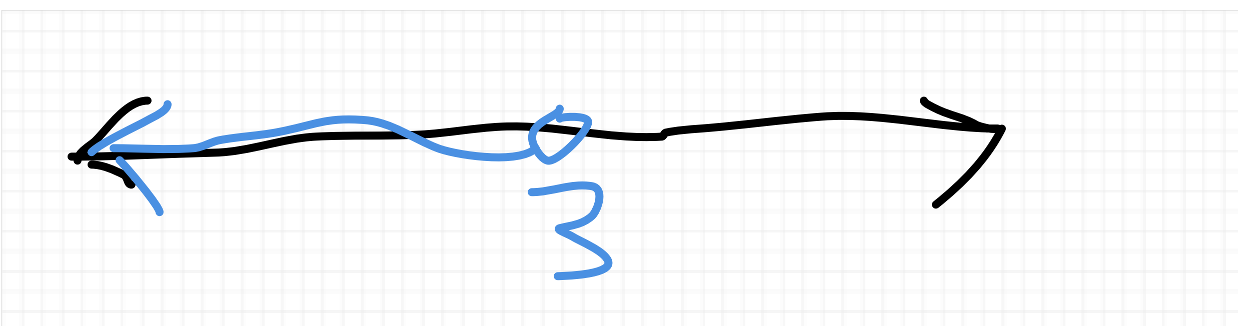
$-6 < 5x - 2$
$+2 \qquad +2$
$-4 < 5x$
$\div 5 \quad \div 5$

$-\frac{4}{5} < x$

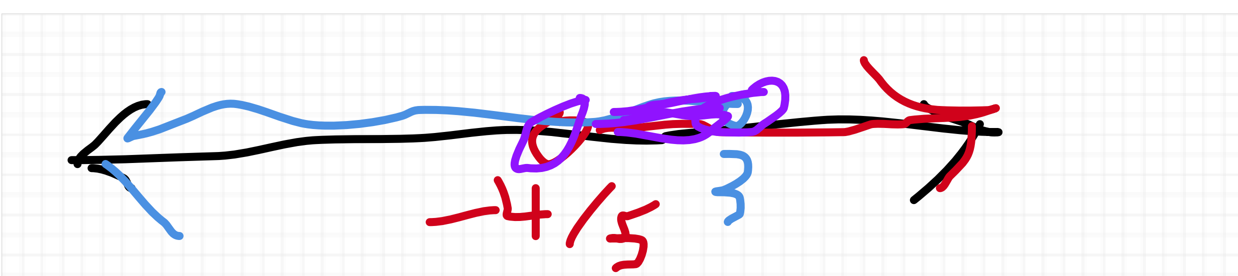
$x > -\frac{4}{5}$



$5x - 2 < 13$
$\quad +2 \qquad +2$
$5x \qquad < 15$
$\div 5 \qquad \div 5$
$x \qquad < 3$



So our solution is "$x > -\frac{4}{5}$ and $x < 3$"



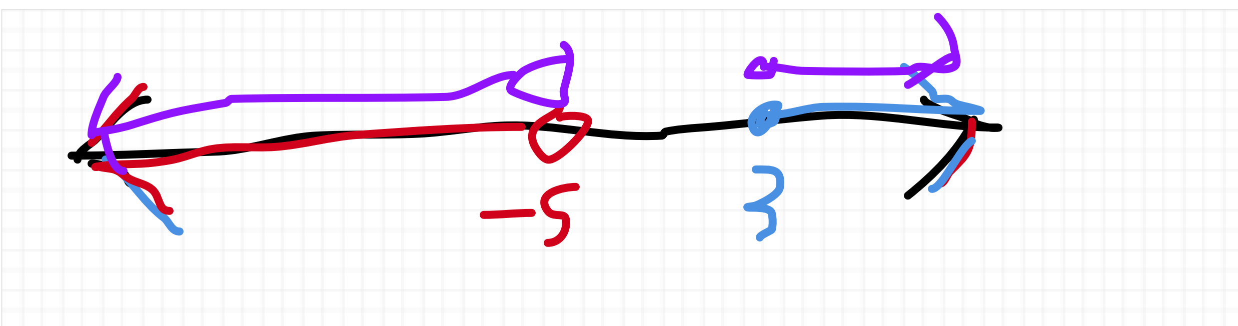
Example 2. Sketch of $3x + 7 < \ -8$ or $5 - 2x \leq -9$.

Now we need to solve for x for both inequalities

$$\begin{array}{lll} 3x + 7 & < & -8 \\ \quad -7 & & -7 \\ 3x & < & -15 \\ \div 3 & & \div 3 \\ x & < & -5 \end{array}$$

$$\begin{array}{lll} 5 - 2x & \leq & -9 \\ -5 & & -5 \\ -2x & \leq & -14 \\ \div -2 & & \div -2 \\ x & \geq & 7 \end{array}$$

The whole solution is $x < \ -5$ or $x \geq 7$



We want to think about it as $x \leftarrow 5 \cup x \geq 7$

$3x + 5 < 20$ or $2x - 1 > 13$

$x < 5$ or $x > 7$

Preview for Tomorrow

Next time, we'll do algebra with absolute value.

what is the absolute value? "The distance from 0"

$$|a| = \begin{cases} a & \text{if } a \text{ is positive} \\ -a & \text{if } a \text{ is negative} \end{cases}$$

Example. Let's solve for x in the equation

$$|5x - 2| = 3$$

This equality gives us two cases

$5x - 2$ is positive

$$|5x - 2| = 5x - 2$$

$$\begin{aligned} 5x - 2 &= 3 \\ +2 \quad &\quad +2 \\ 5x \quad &= 5 \\ \div 5 \quad &\quad \div 5 \\ x \quad &= 1 \end{aligned}$$

$5x - 2$ is negative

$$|5x - 2| = -(5x - 2)$$

$$\begin{aligned} -(5x - 2) &= 3 \\ -5x + 2 &= 3 \\ -2 \quad &\quad -2 \\ -5x \quad &= 1 \\ \div -5 \quad &\quad \div -5 \\ x \quad &= -\frac{1}{5} \end{aligned}$$

So we have two solutions of $x = 1$ and $x = -\frac{1}{5}$

Tomorrow we'll learn how to do inequalities with absolute value.

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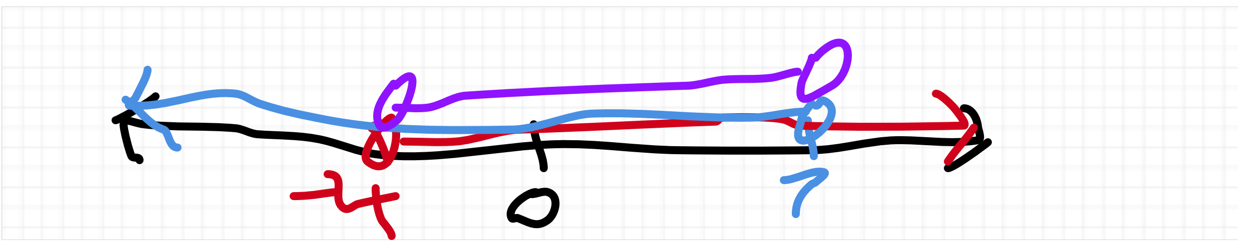
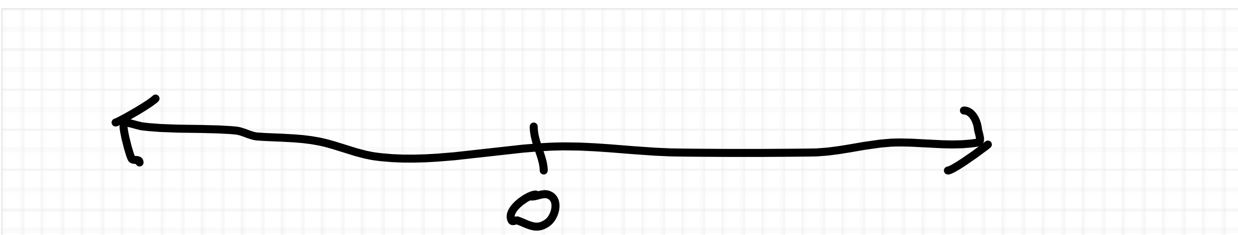
The Entrace Pass

Graph and trace the following:

1. $-4 < x < 7$

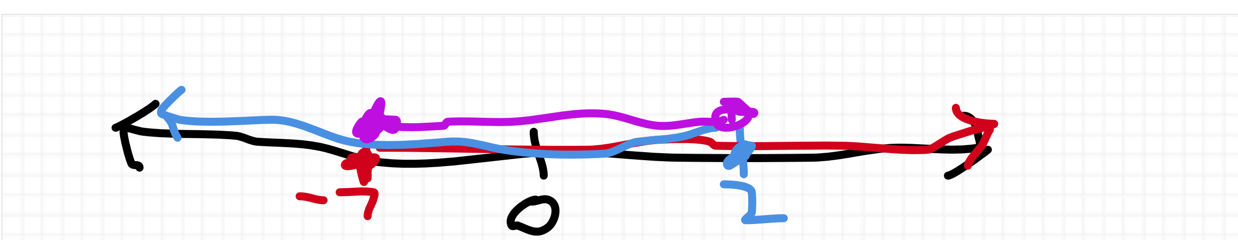
$x > -4$ and $x < 7$





2. $-7 \leq x \leq 2$

$x \geq -7$ and $x \leq 2$



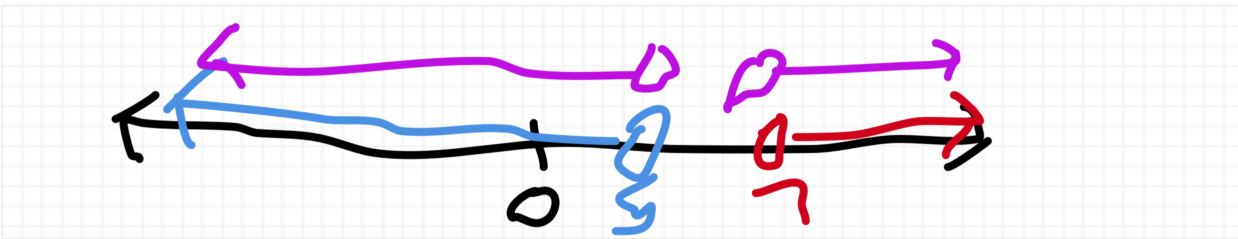
3. $3x + 5 < 20$ or $2x - 1 > 13$

$3x + 5 < 20$

$\quad -5 \quad -5$

$3x \quad < 15$
$\div 3 \quad \div 3$
$x \quad < 5$

$2x - 1 > 13$
$+1 \quad + 1$
$2x \quad > 14$
$\div 2 \quad \div 2$
$x \quad > 7$



Previously...

Refer to entrance pass for what we did

Algebra With Absolute Values

Example 1. Find the solution to

$|x + 5| = -7$

Answer is no solution

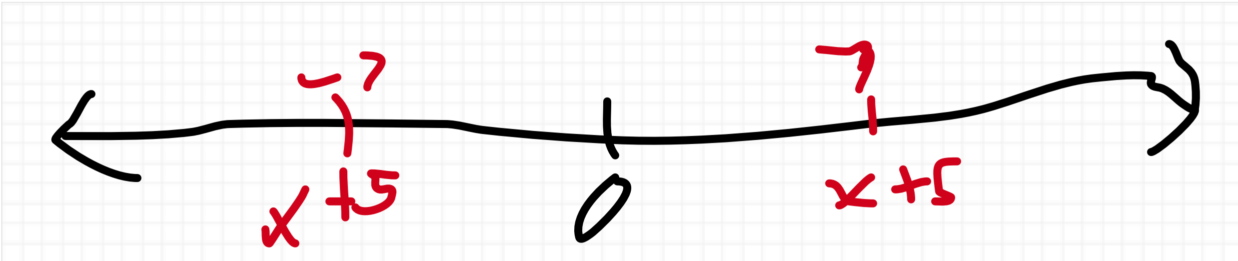
Example 2.

$|x + 5| = 7$

If the absolute value of $x + 5$ is equal to 7, what does $x + 5$ eqaul

So when we solve for a x in an absolute value statement, we want to account for two solutions, since it could be that

$x + 5 = 7$ or $x + 5 = -7$



We could write the solution as follows:

$x = 2$ or $x = -12$
$x = 2, -12$

Example 4. Find the solutions to $42 - |x + 3| = 15$

Preview for Tomorrow

N/A

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Algebra With Absolute Values (Cont.)

Three Steps to Solving Absolute Value Equations

Step 1. Get the absolute value portion of the equality by itself; that way, we'll be able to find out what the distance from zero of the absolute value portion of the equality is, allowing us to do the next step.

In other words, you first, before doing anything else, want to SOLVE FOR the absolute value sign.

Step 2. Split the equality into the two cases.

Step 3. Solve for x in each of those cases.

Example 1.

$2 \cdot |3x + 4| = 16$

First, let's do step 1, i.e., we will set $a = |3x + 4|$ and solve for a. Now we have

$2a = 16$
$\div 2 \quad \div 2$
$a = 8$

Next, we'll do step 2, and then split it into cases. We have

$|3x + 4| = 8$

giving us the following cases:

$3x + 4 = 8$ or $3x + 4 = -8$

$3x + 4 = 8$
$\quad -4 \quad -4$
$3x \quad = 4$
$\div 3 \quad \div 3$
$x \quad = \frac{4}{3}$

$3x + 4 = -8$
$\quad -4 \quad -4$
$3x \quad = -12$
$\div 3 \quad \div 3$
$x \quad = -4$

$x = \frac{4}{3}, -4$

Example 2.

$3|5x - 4| + 3 = 6x$

Set $a = |5x - 4|$ and solve for a

$3a + 3 = 6x$

$-3 \quad -3$
$3a = 6x - 3$
$\div 3 \quad \div 3$
$a = 2x - 1$

$|5x - 4| = 2x - 1$

We have the following cases

$5x - 4 = 2x - 1$ or $5x - 4 = -(2x - 1) = -2x + 1$

$5x - 4 = 2x - 1$
$+4 \quad +4$
$5x = 2x + 3$
$-2x \quad -2x$
$3x = 3$
$\div 3 \quad \div 3$
$x = 1$

$5x - 4 = -2x + 1$
$+4 \quad +4$
$5x = -2x + 5$
$+2x \quad +2x$
$7x = 5$
$\div 7 \quad \div 7$
$x = \frac{5}{7}$

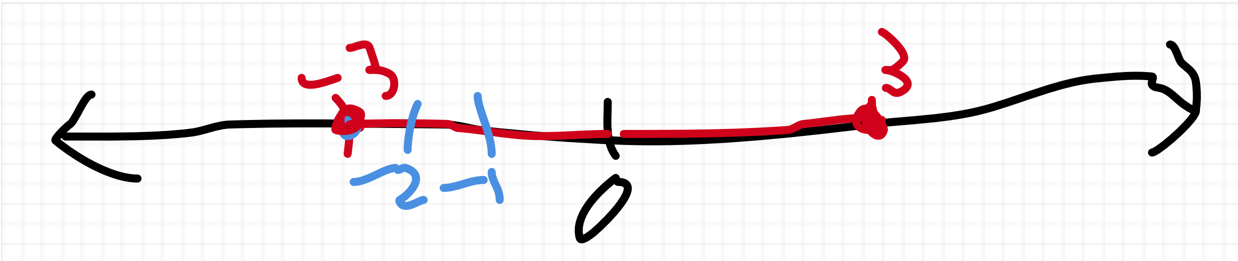
$x = 1, \frac{5}{7}$

Upcoming Preview

We will talk about Inequalities with absolute value

Take for example the inequality $|x| \leq 3$

Note that this inequality includes all possible values of x whose absolute value is less than or equal to 3



The way to talk about it algebraically is to--as we did with equalities--split the possibilities into cases

$|x| \leq 3$, that means either x is positive and $0 \leq x \leq 3$, or negative and we have the case $-3 \leq x < 0$

With those two cases, we have

$-3 \leq x < 0$ or $0 \leq x \leq 3$, which gives us the number line above and the solution

$-3 \leq x \leq 3$

Exit Pass

Solve for x.

1. $|2x - 7| = 43$

2. $95 - |x| = 81$

3. $|3 + 8x| = -1$

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Previously...

We shall ratify the five step process for solving absolute value equations as follows as follows:

$95 - |2x| = 81$

Step 1. Check if the absolute value on one side or not. If it's on one side, then we proceed to step 3. If not, we have to set absolute value to a and proceed to the next step

$a = |2x|$

Step 2. Solve for a

$95 - a = 81$
$-95 \quad\quad -95$
$-a = -14$
$\div -1 \quad \div -1$
$a = 14$

Step 3. Check if the absolute value makes sense, in other words, make sure the absolute is not negative.

$|2x| = 14$ is positive.

Note: It may be that the absolute value is given in terms of x. For that, I'll have you assume that all cases, the answers will be positive.

Step 4. If the absolute value problem makes sense, then split the problem into cases

$2x = 14$ or $2x = -14$

Step 5. Solve for x in both cases, and each case will be a solution (so the answer will usually have two solutions).

$2x = 14$ or $2x = -14$
$\div 2 \quad \div 2 \quad\quad \div 2 \quad\quad \div 2$
$x = 7 \quad\quad\quad x = -7$

$x = 7, -7$

Going over 9/16 and 9/22 Exit Passes

9/16 Exit Pass Solutions

Solve for x for the following inequalities

1. $5x + 4 < 39$

$5x + 4 < 39$
$\quad -4 \qquad -4$
$5x \qquad < 35$
$\div 5 \qquad \div 5$
$x \qquad < 7$

2. $71 > 4 - x$

$71 > 4 - x$
$-4 \quad -4$
$67 > -x$
$\div -1 \quad \div -1$
$-67 < x$

3. $5(x + 3) - 2x \geq -21$

$5(x + 3) - 2x \geq -21$
$5x + 15 - 2x \geq -21$
$3x + 15 \qquad \geq -21$
$\quad -15 \qquad\quad -15$
$3x \qquad\quad \geq -36$
$\div 3 \qquad\quad \div 3$
$x \qquad\quad \geq -12$

9/22 Exit Pass Solutions

1. $|2x - 7| = 43$

$|2x - 7| = 43$

$2x - 7 = 43$ or $2x - 7 = -43$

$$
\begin{array}{llll}
2x - 7 = 43 & & 2x - 7 = -43 & \\
+7 & +7 & +7 & +7 \\
2x & = 50 & 2x & = -36 \\
\div 2 & \div 2 & \div 2 & \div 2 \\
x & = 25 & x & = -18
\end{array}
$$

2. $95 - |x| = 81$

We first have to solve for $a = |x|$ before we split it into cases

$$
\begin{array}{ll}
95 - a = 81 & \\
-95 & -95 \\
-a = -14 & \\
\div -1 & \div -1 \\
a & = 14
\end{array}
$$

$|x| = 14$

Then we can do the cases

$x = 14$ or $x = -14$

3. $|3 + 8x| = -1$

No solution, since no absolute value can be negative