2022-2023 James E Davis Trimester 1 Geometry Week 3 Class Notes

10/3

Briefly reviewed Ch. 1 and the beginning of Ch. 2 of the book

10/4

ASVAB Testing, no class

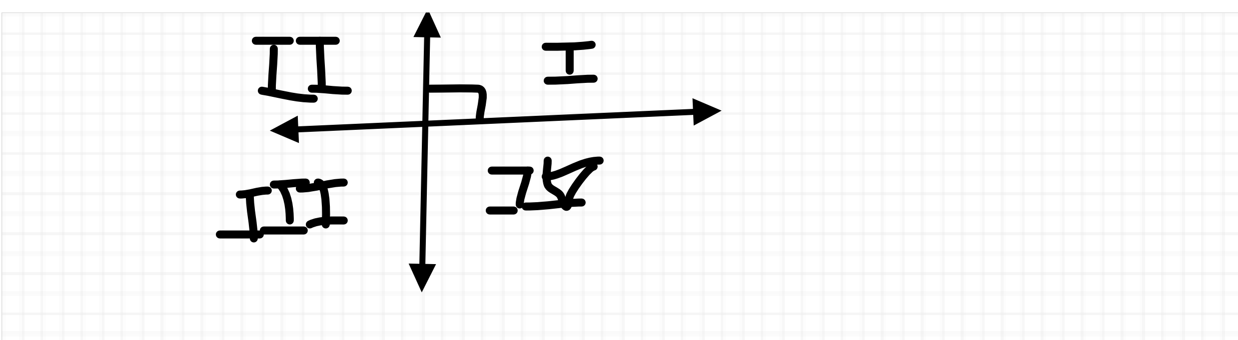
10/5

Previously...

We reviewed the book to go over important concepts. We'll continue by reviewing a little bit more before going on to starting section 3.6.

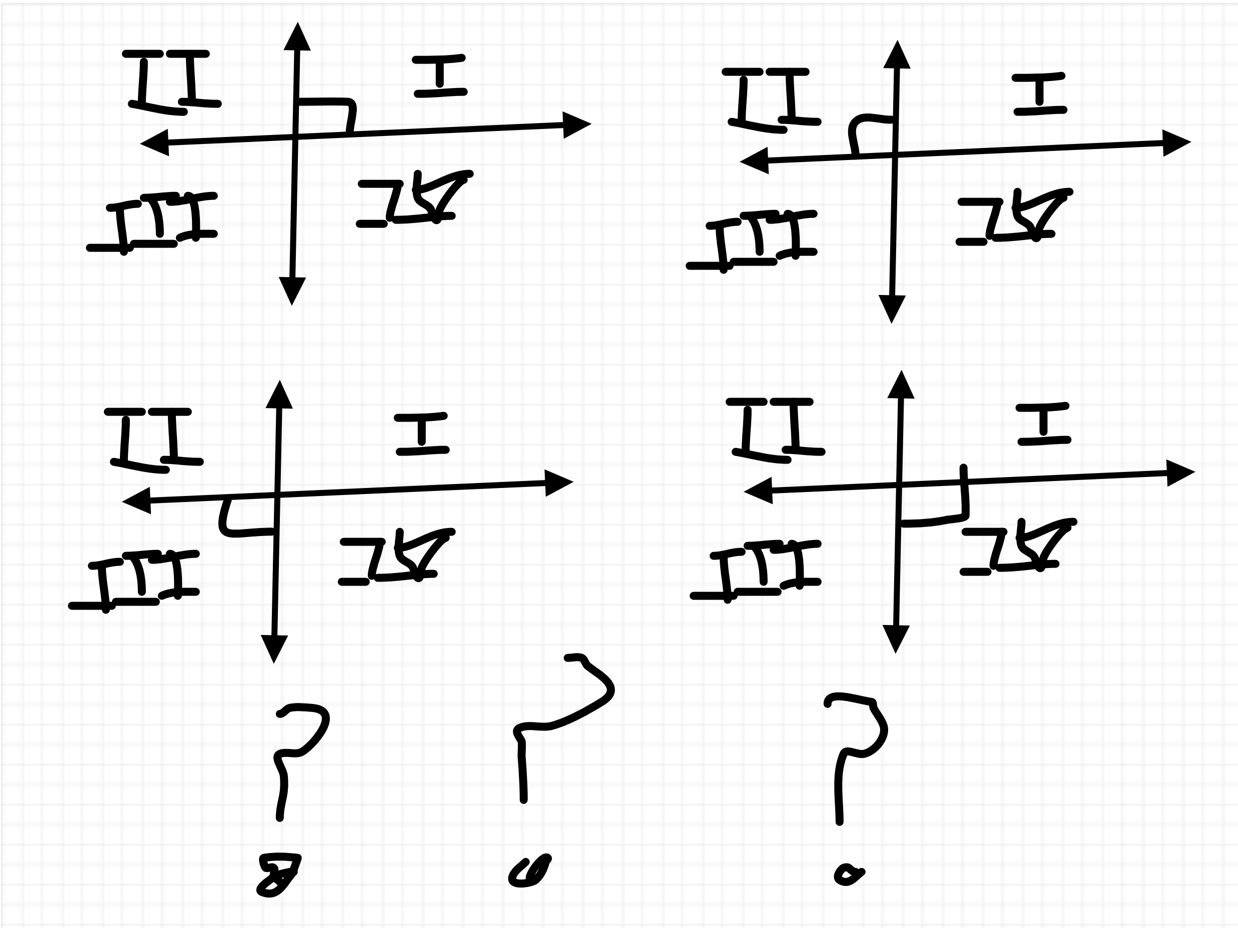
The (Woefully Incomplete) Definition of Perpendicular Lines

Definition 1. If two lines intersect to form a right angle, then they are **perpendicular lines**. We'll illustrate perpendicular lines as so (with a square on one of the angles)



Important Note: If we say that two lines are perpendicular, it's not clear (without additional theorems that we'll learn in section 3.6), we're NOT saying that all four angles of the intersection are 90 degrees, even though, yes, that is true.

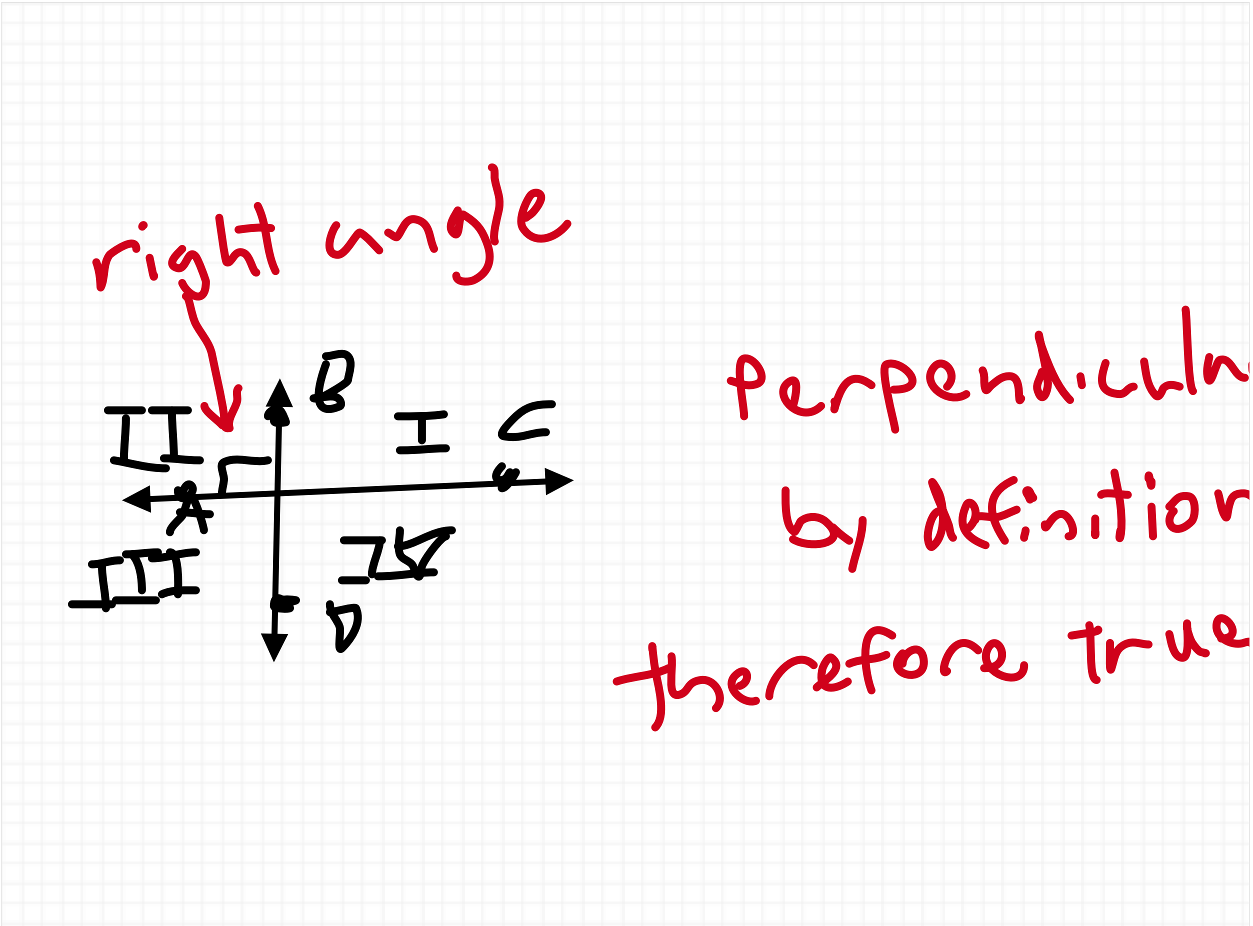
We're not even saying WHICH ANGLE is the 90 degree angle, so it could be that the third quadrant angle (or any quandrant) is the right angle!



Example 1. *(from Example 3 of Larson section 2.2, page 81)* Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $\overline{AC} \perp \overline{BD}$
b. $\angle AEB$ and $\angle CEB$ are a linear pair
c. $\overline{EA}$ and $\overline{EB}$ are opposite rays

Answer to a.



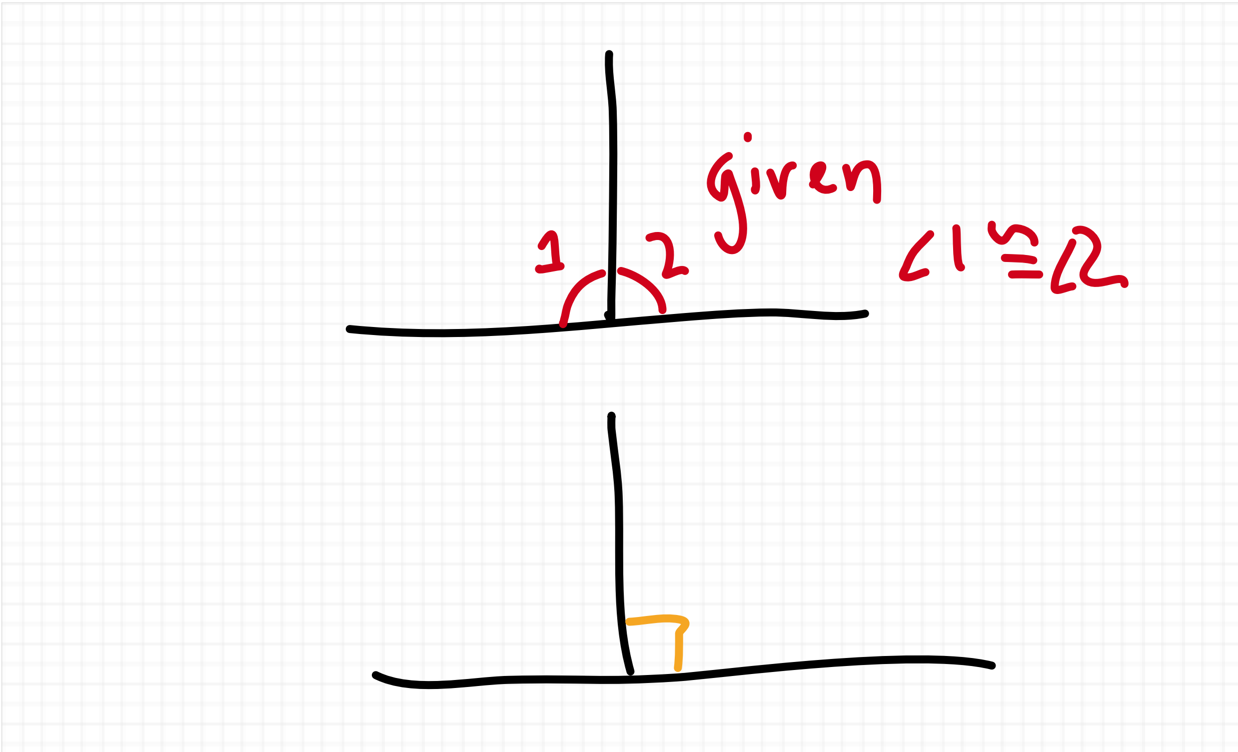
Answer to b. True

Answer to c. False

Proving Theorems About Perpendicular Lines

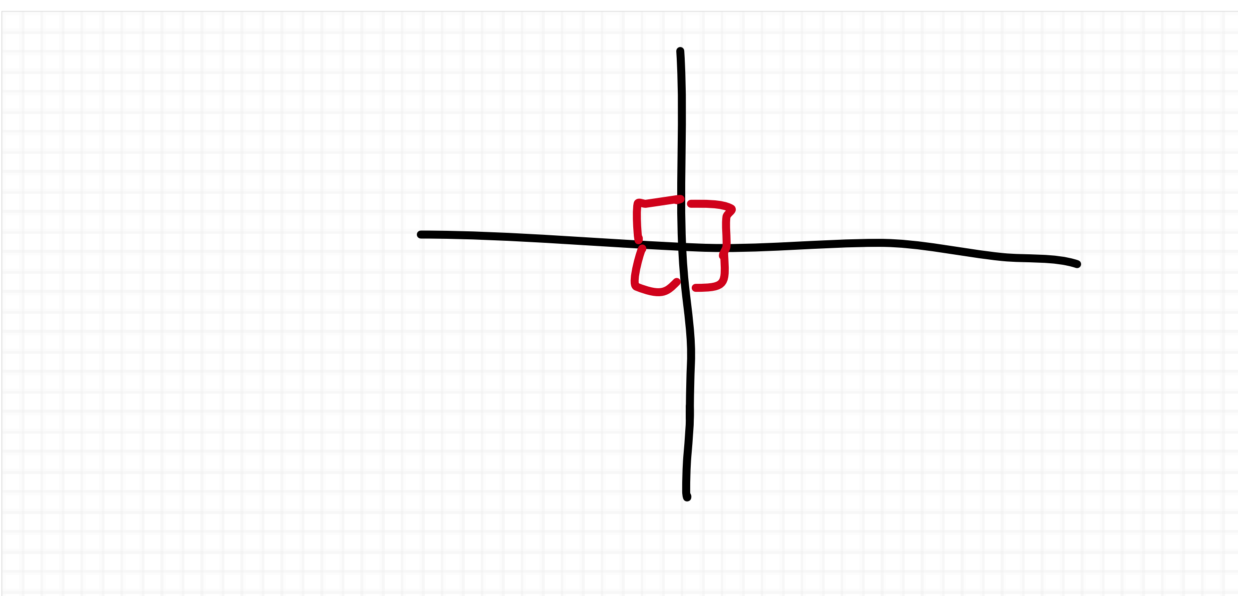
Here are some properties of perpendicular lines

Theorem 1. *Perpendicular Congruence Theorem I* If two lines intersect to form a linear pair of congruent angles, then the two lines are perpendicular.

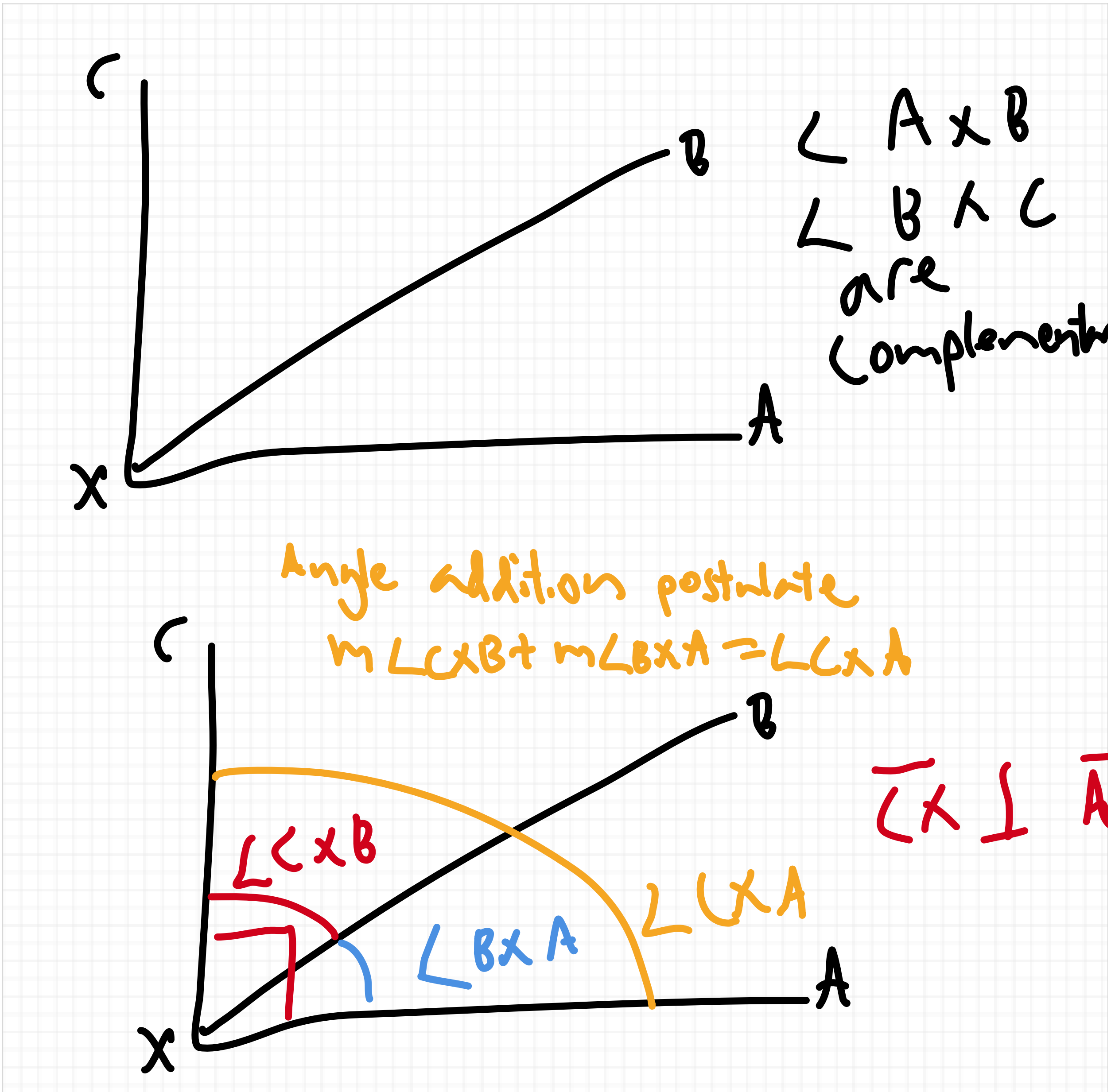


Theorem 2. *Perpendicular Congruence Theorem II* Two lines are perpendicular if and only if they form a linear pair of congruent angles.

Theorem 3 The BIG Perpendicular Theorem. If two lines are perpendicular, then they intersect to form four right angles. In other words, two lines are perpendicular if and only if all the angles between them are $90°$.



Theorem 4. *Perpendicular Complementary Angles Theorem* Two sides of two adjacent acute angles are perpendicular if and only if the angles are complementary.



This theorem is what I like to call a "well duh" theorem because the proof pretty much just follows from the definition.
EDIT: No it doesn't!

Proof.

First we show that if $\angle AXB$ and $\angle BXC$ are complementary, then $\overline{AX} \perp \overline{CX}$.

Statement	**Reason**
$\angle AXB$ and $\angle BXC$ are complementary	Given
$m\angle AXB + m\angle BXC = 90°$	Definition
$m\angle AXB + m\angle BXC = m\angle CXA$	Angle Addition Postulate
$m\angle CXA = 90°$	Substitution Property

Next, we show the converse, i.e., if $\overline{AX} \perp \overline{CX}$. then $\angle AXB$ and $\angle BXC$ are complementary

Statement	**Reason**
$\overline{AX} \perp \overline{CX}$.	Given
$m\angle CXA = 90°$	Theorem 4 (Theorem 3.8 of the book)
$m\angle AXB + m\angle BXC = m\angle CXA$	Angle Addition Postulate
$m\angle AXB + m\angle BXC = 90°$	Subsitution Property
$\angle AXB$ and $\angle BXC$ are complementary	Definition of complementary

QED

10/6

Previously...

We defined perpendicular lines, we lamented about how the definition of perpendicular is woafully incomplete, and then after Kia left, we started section 3.6 and mentioned the theorems that make the definition better.

"Reviewing" Properties of Equalities

Algebraic Properties of Equality

Addition Property
If $a = b$, then $a + c = b + c$

Subtraction Property
If $a = b$, then $a - c = b - c$

Multiplication Property
If $a = b$, then $ac = bc$

Division Property
If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

Substitution Property
If $a = b$, then a can be substituted for b in any equation or expression

Distributive Property
$a(b + c) = ab + ac$

With these properties, we can prove something is the solution to another thing by writing out each property when we solve the equation

Example 1. *(Example 1 from section 2.5)* Find and prove the solution to $2x + 5 = 20 - 3x$

Statement	**Reason**
$2x + 5 = 20 - 3x$	Given
$2x + 5 + 3x = 20 - 3x + 3x$	Addition Property
$5x + 5 = 20$	Combine like terms
$5x = 15$	Subtraction Property
$x = 3$	Division Property

Example 2. *(Example 2 from section 2.5)* Find and prove the solution to $-4(11x + 2) = 80$.

Statement	**Reason**
$-4(11x + 2) = 80$	Given
$-44x - 8 = 80$ $\quad +8 \quad +8$	Distributive Property
$-44x = 88$ $\div -44 \quad \div -44$	Addition Property
$x = -2$	Division Property

Example 3. *(from 2.5 exercise 12)* Solve and prove the solution to $4(5x - 9) = -2(x + 7)$

Statement	Reason
$4(5x-9) = -2(x+7)$	Given
$20x - 36 = -2x - 14$ $+2x \qquad\qquad +2x$	Distributive Property
$22x - 36 = -14$ $\quad +36 \qquad +36$	Addition Property
$22x \qquad = 22$ $\div 22 \qquad \div 22$	Addition Property
$x = 1$	Division Property

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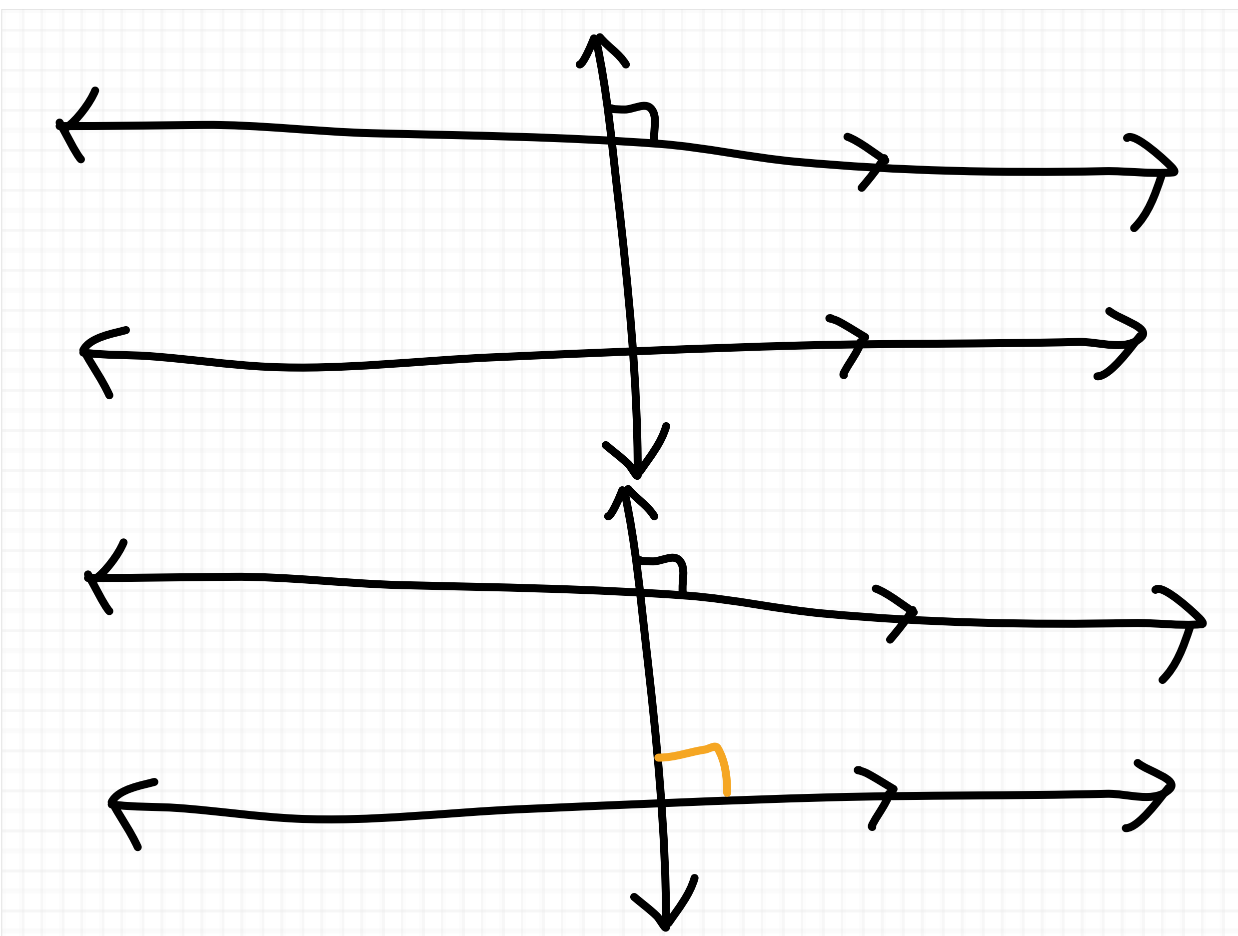
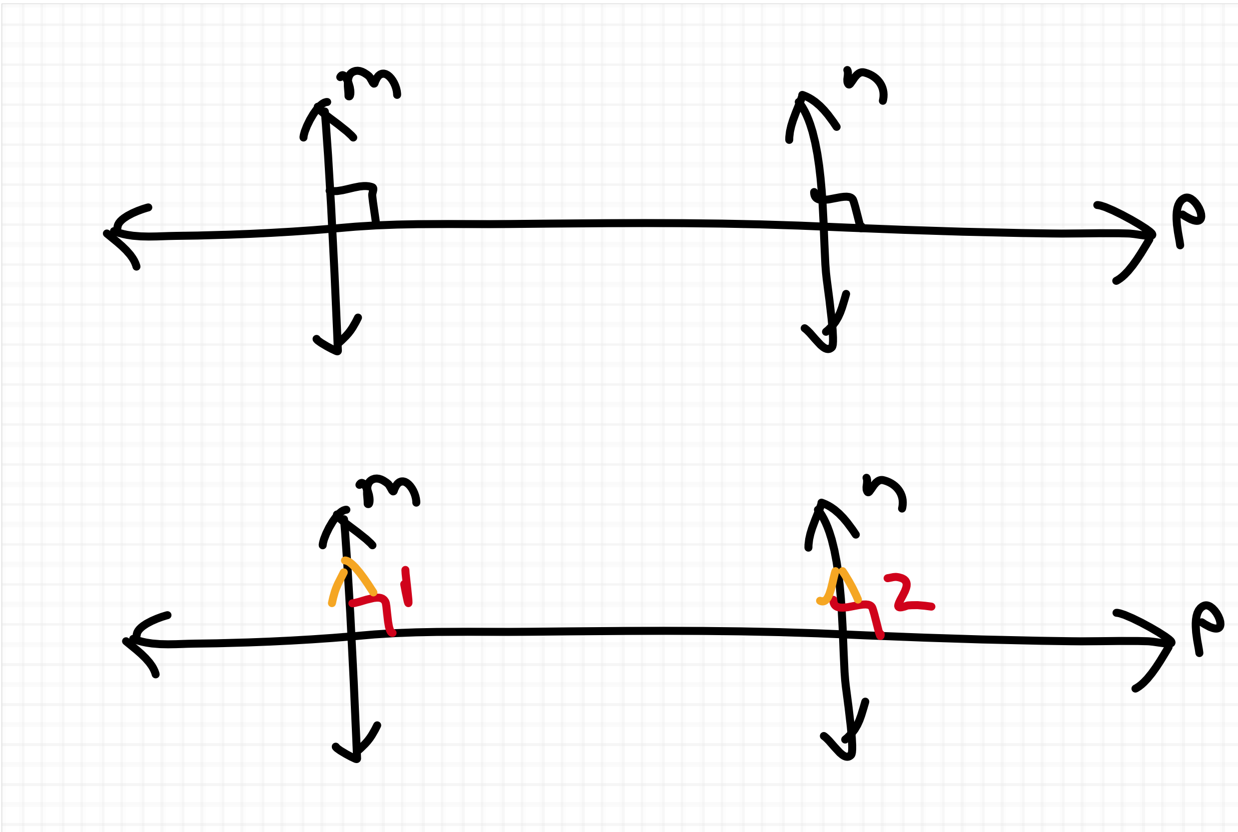
Perpendicular and Transversal Stuff

Theorem 1. *Perpendicular Transversal Theorem* If a transversal is perpendicular to two parallel lines, then it is perpendicular to the other.



Proof. We already did this as an exercise. QED

Theroem 2. *Lines Perpendicular to a Transversal Theorem* In a plane, if two line are perpendicular to the same line, then they are parallel to each other.



Proof.

Statement	**Reason**
$m \perp p,\ n \perp p$	Given
$m\angle 1 = 90°,\ m\angle 2 = 90°$	BIG perpendicular Theorem
$m\angle 1 = m\angle 2$	Substitution Property of Equality
$\angle 1 \cong \angle 2$	Definition of Congruence
$m \vert\vert n$	Corresponding angles converse postulate

QED