2022-2023 James E Davis Trimester 1 Algebra 1 Week 3 Class Notes

10/3

Previously...

We talked about the five step process for inequalities.

We shall modify the five step process (one last time, I promise!) as follows:

We elaborated algebra on equalitiues with absolute value to five steps as so:

$95 - |2x| < 81,\ 95 - |2x| \geq 81$

Step 1. Check if the absolute value on one side or not. If it's on one side, then we proceed to step 3. If not, we have to set absolute value to a and proceed to the next step.

$a = |2x|$

Step 2. Solve for a as an inequality

$$\begin{array}{ll}
95 - a < 81 & 95 - a \geq 81 \\
-95 \quad\ -95 & 95 \qquad\ -95 \\
-a < -14 & -a \geq -14 \\
\div -1 \ \ \div -1 & \div -1 \ \ \div -1 \\
a > 14 & a \leq 4
\end{array}$$

Step 3. Check if the absolute value has a positive or a negative on the other side. If it has a positive, carry on as normal. If it has a negative then we have two cases:

Case 1. If we have $a < b$ or $a \leq b$, for a the absolute value and b negative, then no solution exists (since we know $a \geq 0$) and we shade nothing.

Case 2. If we have $a > b$ or $a \geq b$, for a the absolute value and b negative, then every value of x is a solution since we know $a \geq 0$ automatically, regardless of what x is.

Step 4. Split the problem into cases (and then join it with an "**OR**" statement.

$|2x| > 14$ $\quad\quad\quad\quad$ $|2x| \leq 14$

$2x > 14$ or $2x < -14$ $\quad\quad$ $0 \leq 2x \leq -14$ or $0 > 2x \geq -14$

Step 5. Solve for x in both cases, and each case will be a solution (so the answer will usually have two solutions).

$2x > 14$ or $2x < -14$ $\quad\quad$ $0 \leq 2x \leq 14$ or $0 > 2x \geq -14$

$\div 2 \quad \div 2 \quad \div 2 \quad\quad \div 2 \quad\quad \div 2 \quad \div 2 \quad\quad \div 2 \quad \div 2$

$x > 7 \quad\quad x < -7 \quad\quad\quad 0 \leq x \leq 7 \quad\quad 0 > x \geq -7$

Inequalities With Absolute Values (Cont.)

Now we shall do some more examples for solving equations with absolute values.

Example 1. We shall graph the following inequality.

$|2x + 5| - 3 \leq -5$

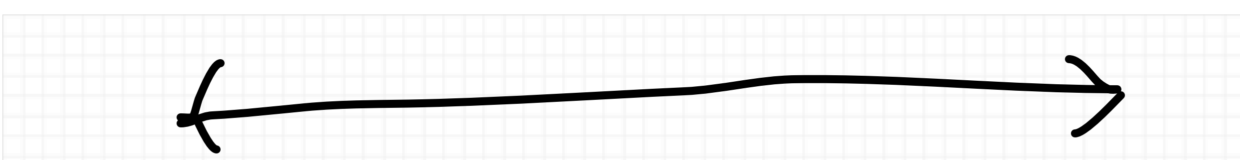
$a = |2x + 5|$

$a - 3 \leq -5$

$\;\; +3 \quad\; +3$

$a \quad\quad \leq -2$

Now we do step 3, and find that there is a negative on the other side of a, and recall that $a \geq 0$ since it's an absolute value, and we have a contradiction and no solution exists, so the graph is not shaded at all!



Example 2. We shall graph the following inequality.

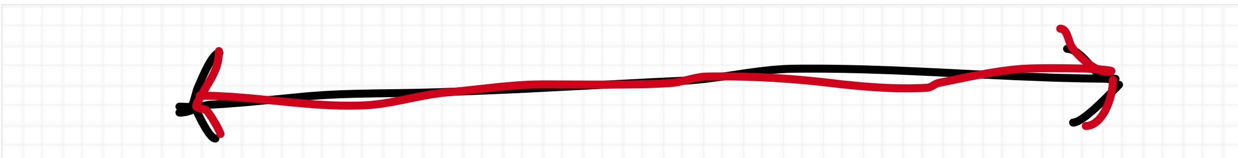
$|2x + 5| - 3 > -5$

$a - 3 > -5$

$\;\; +3 \quad +3$

$a \qquad > -2$

Now we do step 3, and find that a is greater a negative value. We already know (again from a being an absolute value) that $a \geq 0$. As a result, the value of x can be literally anything and we shade the entire graph



10/4

ASVAB Testing, no class

10/5

Going over 9/30 Exit Pass

9/30 Exit Pass Solutions

1. $|x+7| \leq 5$

The absolute value is on one side, so we split it into cases

Case 1: Case 2:

$$\begin{array}{lll} 0 \leq x+7 \leq 5 & \qquad & 0 > x+7 \geq -5 \\ -7 \quad -7 \quad -7 & & -7 \quad -7 \quad\quad -7 \\ -7 \leq x \qquad \leq -2 & & -7 > x \geq -12 \end{array}$$

2. $|6x-3| > 33$

Case 1: Case 2:

$$\begin{array}{lll} 6x-3 > 33 & \qquad & 6x-3 < -33 \\ \quad +3 \qquad +3 & & \qquad +3 \quad +3 \\ 6x \qquad > 36 & & 6x \qquad < -30 \\ \div 6 \qquad \div 6 & & \div 6 \qquad \div 6 \\ x \qquad > 6 & & x \qquad < -5 \end{array}$$

3. $7 + |x| \geq 85$

Before splitting into cases, we set $a = |x|$ and solve for a

$7 + a \geq 85$
$-7 \qquad -7$
$a \qquad \geq 78$

Now we split it into cases

Case 1: Case 2:
$x \geq 78$ $x \leq -78$

10/6

Interval Notation

Interval notation is another way of expressing inequalities.

There are two general types of intervals

1. **Bounded intervals**-Always have two endpoints, i.e. the smallest/largest points that

2. **Unbounded Intervals**-Have either one, sometimes zero endpoints

First we'll look at intervals visually

Interval notation expresses the following inequalities as so

(a, b) is called an **open bounded interval** and is notation for the expression $a < x < b$

$[a, b)$ and $(a, b]$ are called **half-open bounded intervals** and each are notation for the expressions $a \leq x < b$ and $a < x \leq b$, respectively

$[a, b]$ is called a **closed bounded interval** and is notation for the expression $a \leq x \leq b$

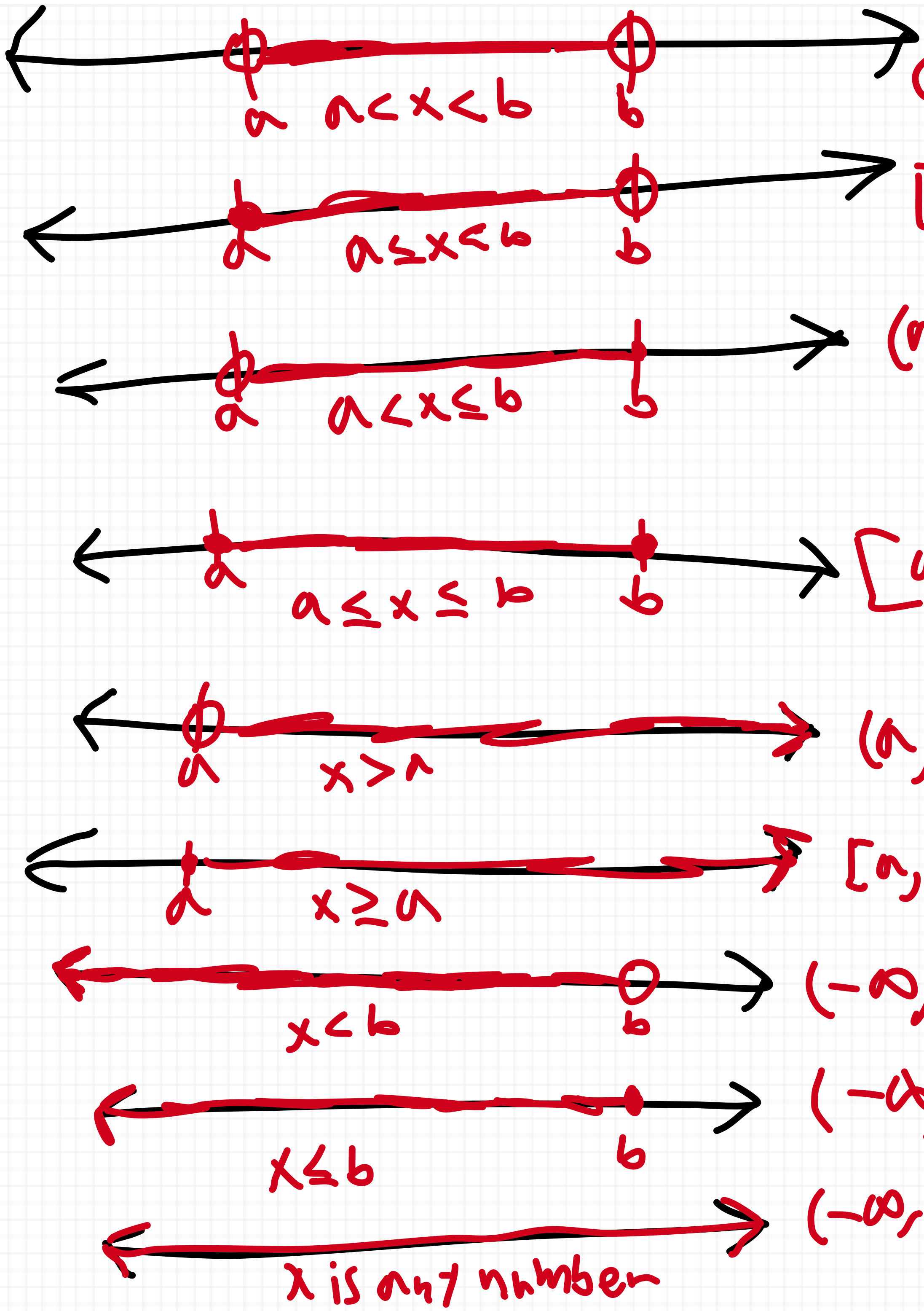
(a, ∞), representing $x > a$, and $(-\infty, b)$, representing $x < b$ are called **open unbounded intervals**

$[a, \infty)$, representing $x \geq a$ and $(-\infty, b]$, representing $x \leq b$.are called **closed unbounded**

intervals

The interval representing the entire number line represented by $(-\infty, \infty)$ or $\mathbb{R}$

An open parenthese $(,\ )$ represents an open endpoint (the open circle on the number line illustration), a closed parenthese $[,\ ]$ represents a closed endpoint (the closed circle on the number line illustration). As a result, each kind of interval corresponds to the following diagram:



$a < x < b$ (a, b)

$a \le x < b$ $[a, b)$

$a < x \le b$ $(a, b]$

$a \le x \le b$ $[a, b]$

$x > a$ (a, ∞)

$x \ge a$ $[a, \infty)$

$x < b$ $(-\infty, b)$

$x \le b$ $(-\infty, b]$

x is any number $(-\infty, \infty)$

Next Time...

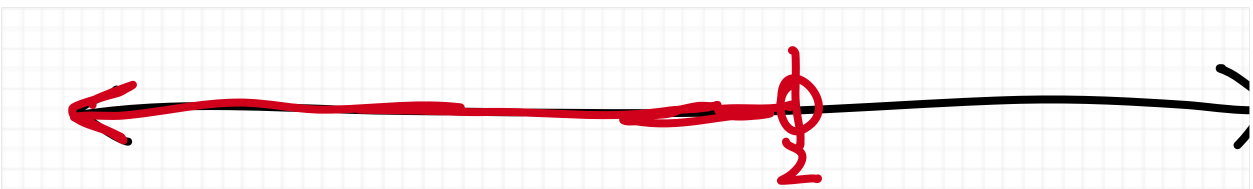
We will talk about intersections and unions applied to intervals.

Exit Pass...

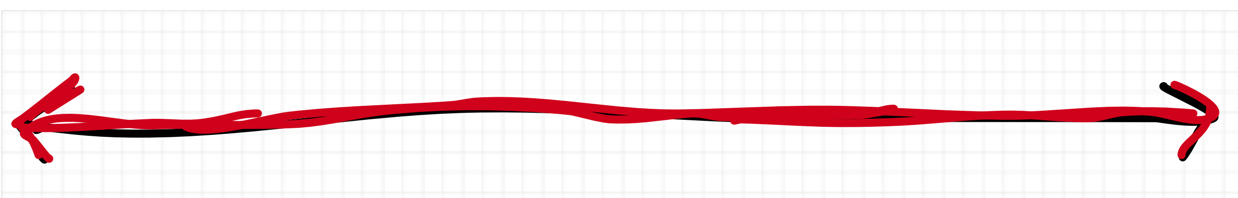
Find the correct interval notation for the following:

1. $1 \leq x \leq 5$

2.



3.



4. $x > 1$

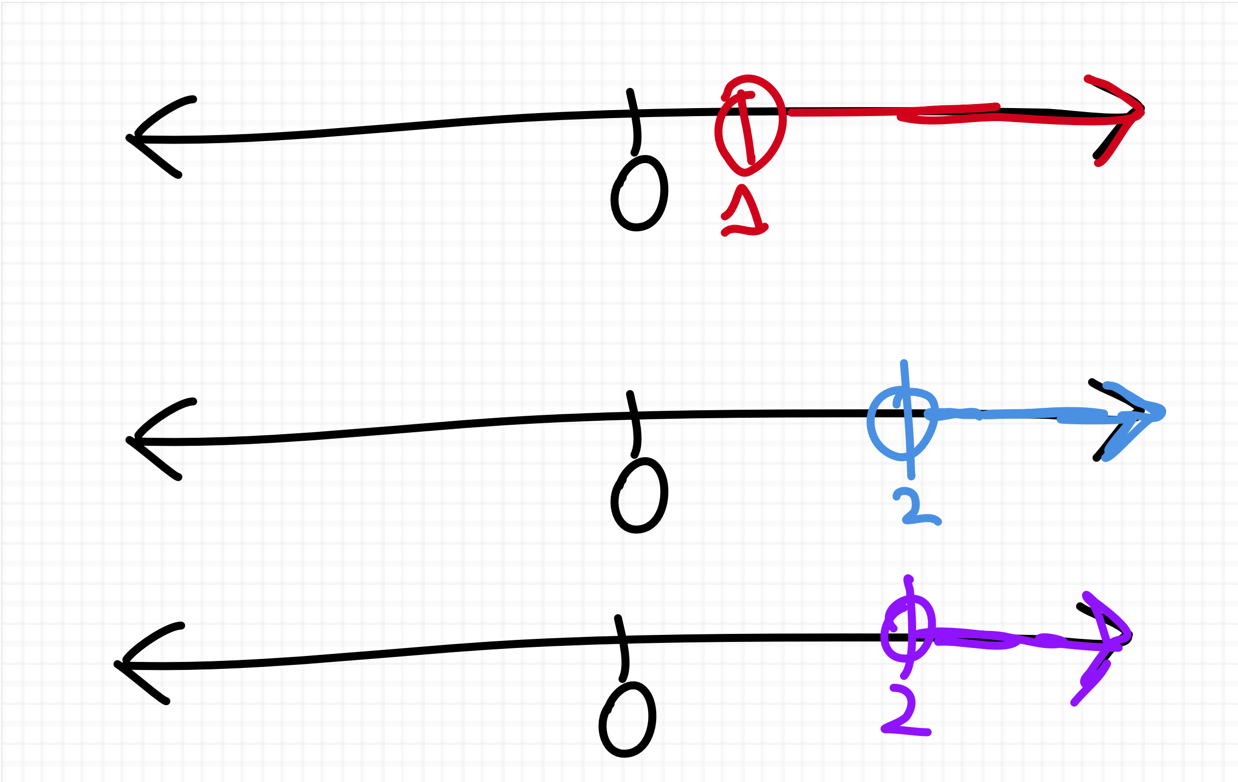
10/7

Previously...

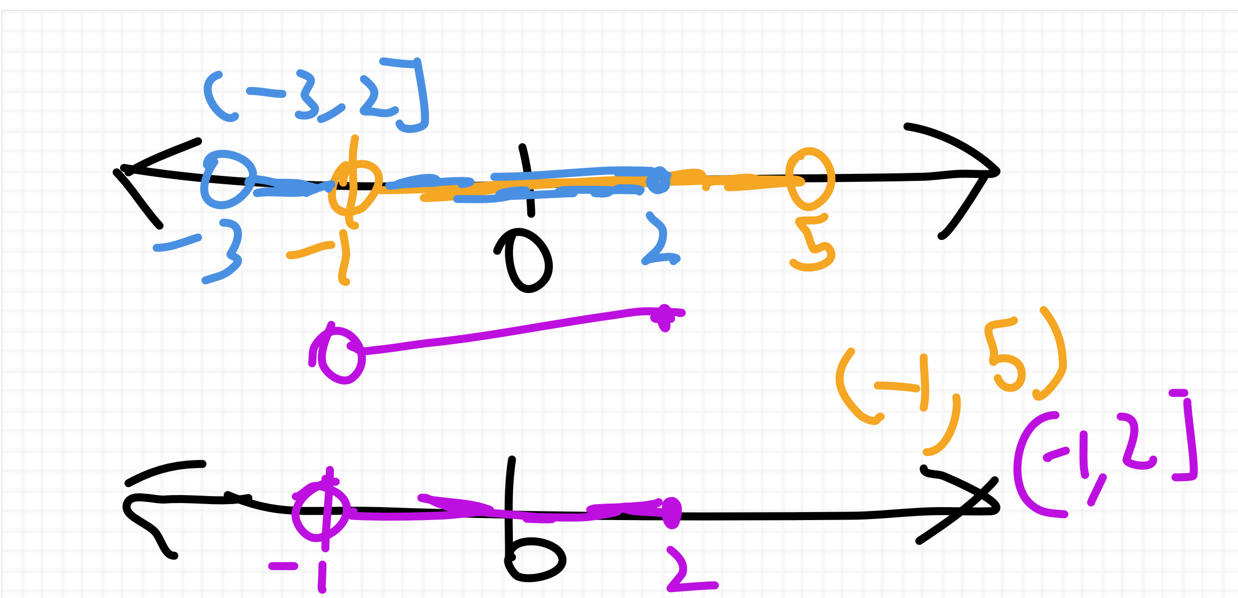
Unions and Intersections of Intervals

One can think of intervals as sets of numbers, and as a result, we apply unions and intersections of intervals.

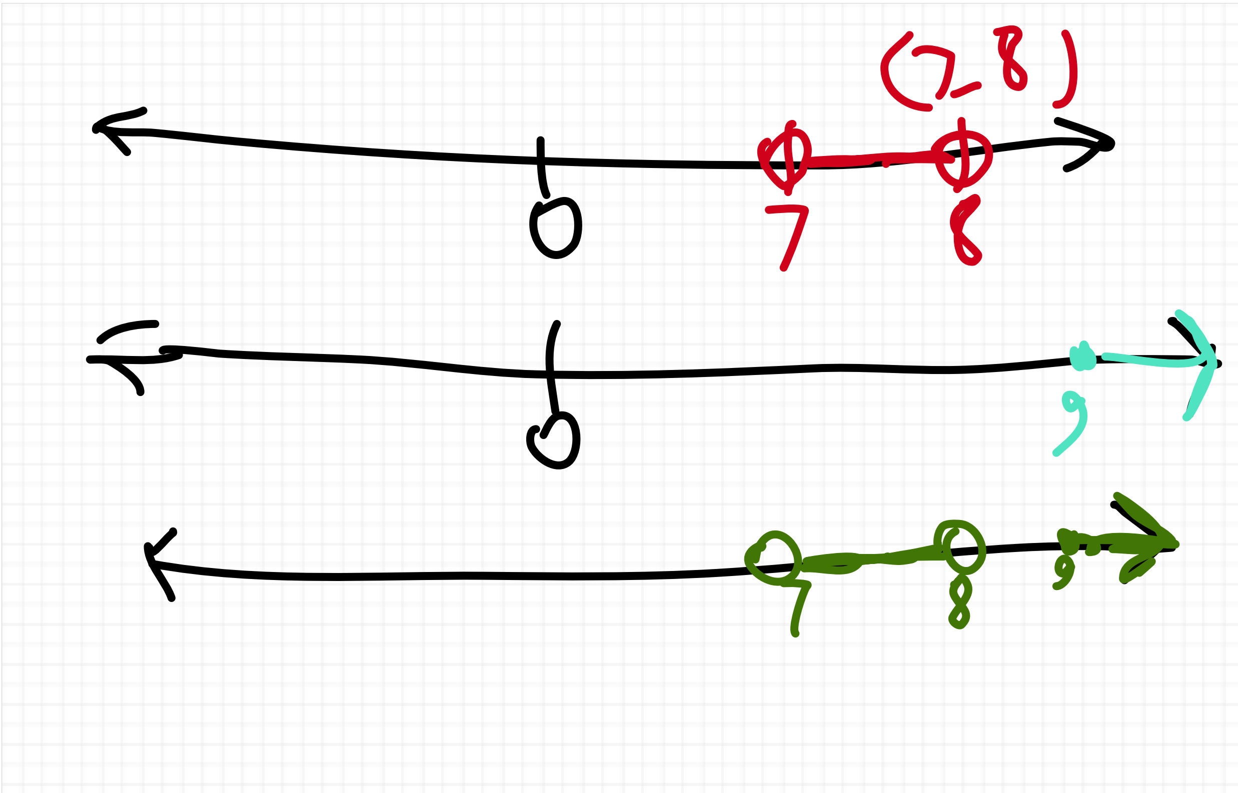
Example 1. Find the interval (in inteval notation) for $(1, \infty) \cap (2, \infty)$.



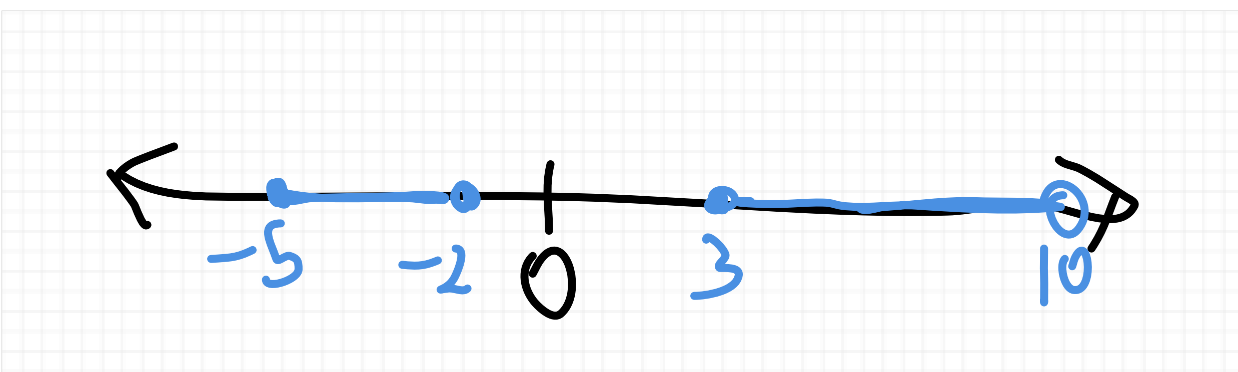
Example 2. Find the interval (in interval notation) of $(-1, 5) \cap (-3, 2]$.



Example 3. Illustrate $(7, 8) \cup [9, \infty)$.



Example 4. How can we express the illustration below in interval notation?



We express it as $[-5, -2] \cup [3, 10)$.

Exit Pass.

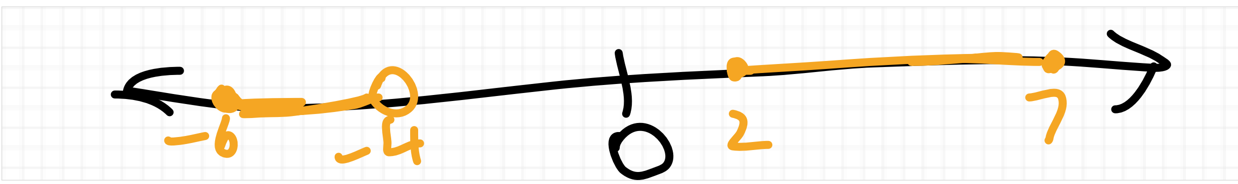
For **1.** and **2.**, illustrate/graph the following.

1. $(-\infty, 2) \cap [-5, 7]$

2. $(-\infty, -3) \cup [3, \infty)$

For **3.** and **4.**, write the following expressions of inequalities in interval notation

3.



4. $x < 3$ or $x > 11$

10/10

10/6 and 10/7 Exit Passes

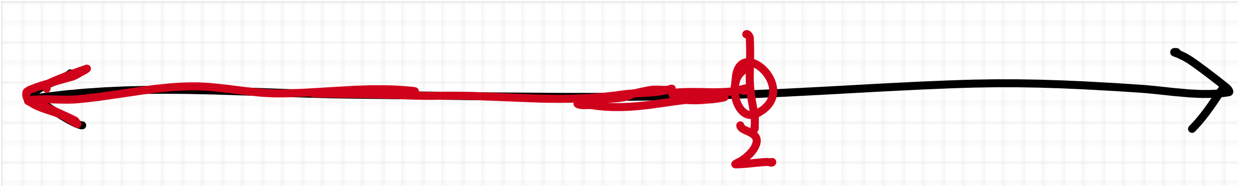
10/6 Exit Pass Solutions

Find the correct interval notation for the following:

1. $1 \leq x \leq 5$

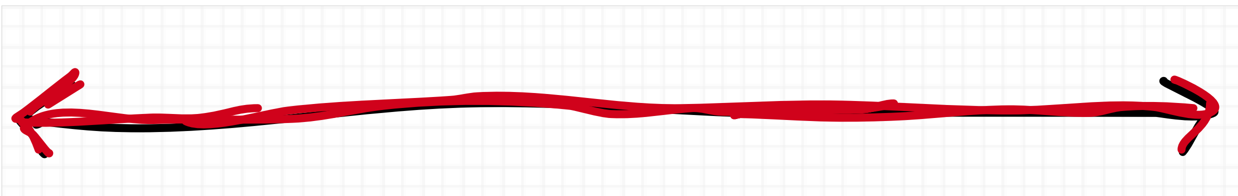
$[1, 5]$

2.



$(-\infty, 2)$

3.



$(-\infty, \infty)$ or $\mathbb{R}$

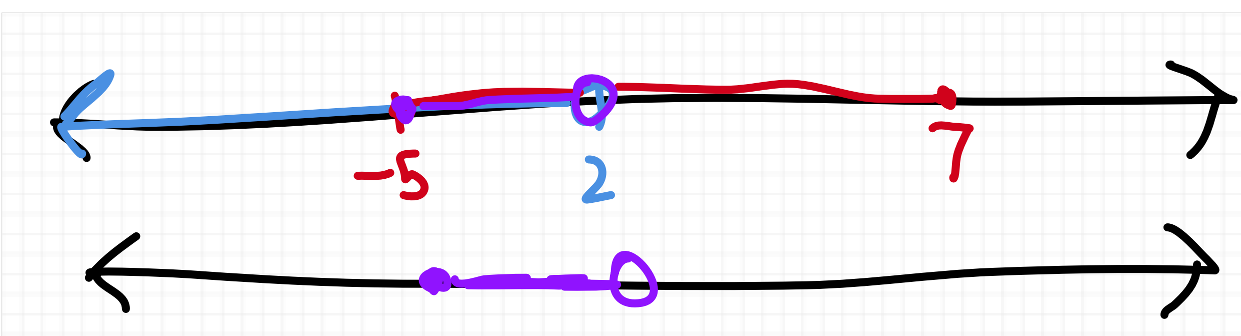
4. $x > 1$

$(1, \infty)$

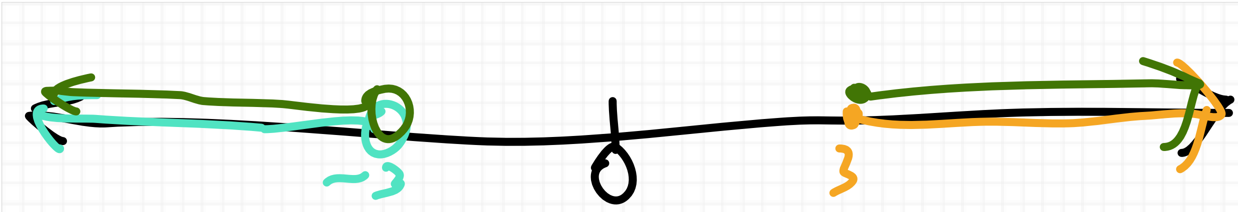
10/7 Exit Pass Solutions

For **1.** and **2.**, illustrate/graph the following.

1. $(-\infty, 2) \cap [-5, 7]$

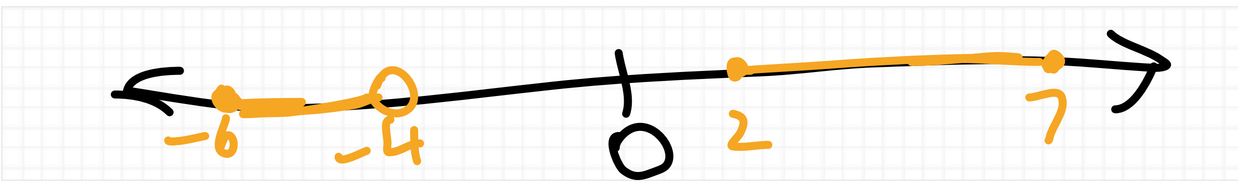


2. $(-\infty, -3) \cup [3, \infty)$



For 3. and 4., write the following expressions of inequalities in interval notation

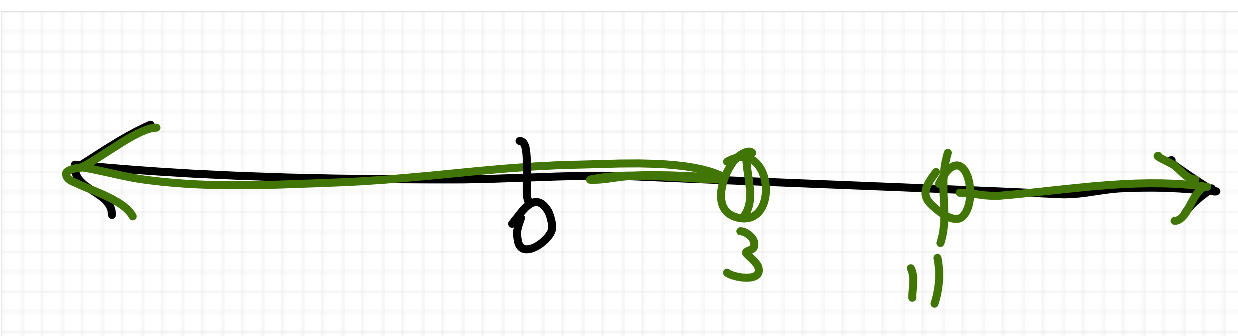
3.



$[-6, -4) \cup [2, 7]$

$[-4, -6) \cup [2, 7]$ is WRONG (because the order you place the brackets matter) and from this point forward, I'll take half a point off if you do that.

4. $x < 3$ or $x > 11$



$(-\infty, 3) \cup (11, \infty)$