2022-2023 James E Davis Trimester 1 Algebra 2 Week 6 Class Notes

10/31

Properties of Matrices (Cont.)

From previous weeks:

$$A^n = A \cdot A \cdot \underbrace{\cdots}_{n \text{ times}} \cdot A$$

Exponent Poperties

Addition Property of Exponents: $A^{m+n} = A^m A^n$

We don't always have $(AB)^n = A^n B^n$ (because we don't always commutative property)

Example 2.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 A^3 = A^5$$

$$A^4 = A^3 A = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 \cdot 1 + 4 \cdot 1 & 4 \cdot 1 + 4 \cdot 1 \\ 4 \cdot 1 + 4 \cdot 1 & 4 \cdot 1 + 4 \cdot 1 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$$

$$A^5 = A^4 A = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$$

$$A^2 A^3 = \begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix} = A^5$$

Example 3.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

We shall show that $(AB)^2 \neq A^2B^2$

We calculated that

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = A,\ B^2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix},\ AB = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$(AB)^2 = (AB)(AB) = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \cdot 0 + (-1) \cdot 0 & 0 \cdot (-1) + (-1) \cdot 0 \\ 0 \cdot 0 + 0 \cdot 0 & 0 \cdot (-1) + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2B^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 1 \cdot 0 & 1 \cdot (-2) + 1 \cdot 1 \\ 0 \cdot 1 + 0 \cdot 0 & 0 \cdot (-2) + 0 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = (AB)^2$$

Order of Operations for Matrix Arithmetic

The order is from first to last:

1. Parantheses
2. Matrix Expononentiation
3. Scalar Multipication/Matrix Multiplication
4. Matrix Addition

Example 4. Set $A = \begin{bmatrix} 5 & 7 \\ 0 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 7 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $E = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Compute

$$C^5 = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$$

$$\begin{aligned} E - (A + B)C^4D = E - \begin{bmatrix} 8 & 14 \\ 2 & 11 \end{bmatrix}C^4D &= E - \begin{bmatrix} 8 & 14 \\ 2 & 11 \end{bmatrix}\begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}D \\ &= E - \begin{bmatrix} 8 & 14 \\ 2 & 11 \end{bmatrix}\begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}D \\ &= E - \begin{bmatrix} 81 \cdot 8 & 81 \cdot 14 \\ 81 \cdot 2 & 81 \cdot 11 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 3 \end{bmatrix} - \begin{bmatrix} 1134 \\ 821 \end{bmatrix} \\ &= \begin{bmatrix} -1129 \\ -818 \end{bmatrix} \end{aligned}$$

11/1

Properties of Matrices (Cont.)

Inverse Matrices and Determinants

Properties of Inverse Matrices

Recall that an inverse matrix B of a matrix A is a matrix such that

$AB = BA = I_n,$

for some $n \geq 1.$

Here are some properties for inverse matrices:

Proterty 1: In order for matrices to (possibly) have inverses, the matrix must bew square.

Property 2: Not all (even nonzero) matrices have a multiplicitive inverse.

Property 3. If a matrix has an inverse, then it is unique.

Property 4. If A has an inverse, then so does A^{-1} and $\left(A^{-1}\right)^{-1} = A.$, and more generally, if A has inverse, then A^n has an inverse, and $\left(A^n\right)^{-1} = \left(A^{-1}\right)^n$, and we'll call $\left(A^{-1}\right)^n$ A^{-0}..

Determinants

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we define the determiinant det(A) to be the following number:

det(A) $= ad - bc$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \det A = ad -$$

Note: Determinants can be defined more generally for any square matrix, but for the sake simplicity, we only use the definition for the 2×2 matrix.

Example 1.

1.
$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, det$(A) = (1)(1) - (-1)(1) = 1 - (-1) = 2$

2.

$B = \begin{bmatrix} -2 & 0 \\ 0 & 7 \end{bmatrix}$, det$(B) = -14$

3.

$C = \begin{bmatrix} 1 & 3 \\ 5 & -7 \end{bmatrix}$ det$(C) = -22$

Inverse of a 2×2 Matrix

Let's say

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and det$(A) \neq 0$

Then A has an inverse and

$$A^{-1} = \frac{1}{\text{det}(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example 2. Find the multiplicative inverse of the following matrix

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \quad \text{det}(A) = (1)(3) - (-1)(2) = 5$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

We can verify that it's multiplicative inverse as follows

$$AA^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} (1)\left(\frac{3}{5}\right) + (-1)\left(-\frac{2}{5}\right) & (1)\left(\frac{1}{5}\right) + (-1)\left(\frac{1}{5}\right) \\ (2)\left(\frac{3}{5}\right) + (3)\left(-\frac{2}{5}\right) & (2)\left(\frac{1}{5}\right) + (3)\left(\frac{1}{5}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let's say we had

$$B = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

One can also find inverses for square matrices of any dimension n, but there is not an eloquent specific formula to do so, and finding the inverse for $n > 2$ is outside the scope of this class.

%MOVE BELOW SECTIONS TO NEXT TRIMESTER

10/20

Previously...

Using Matrices to Solve Systems of Equations