2022-2023 James E Davis Trimester 1 Algebra 1 Week 5 Class Notes

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Lines and Slopes (Cont.)

A vertical intercept (y-intercept) of a line is the point of the line that intersects the vertical axis (y-axis). In coordinates, the point is of the form $(0, b)$ ($x = 0,\ y = b$).

A horizontal intercept (x-intercept) of a line is the point of the line that intersects the horizontal axis (x-axis). In coordinates, the point is of the form $(a, 0)$ ($x = a,\ y = 0$).

In geometry, we connect two points and we get the line between those two points, which satisfies the equation completely, so to plot the entire line, we only need to connect the dots between two points, and the easiest to find (oftentimes) are the horizontal and vetical intercepts.

Example 2. Graph $y = -\frac{1}{2}x + 4$.

To graph this line, we're going to find the horizontal intercept and the vertical intercept.

To find the horizontal intercept, we set $y = 0$ and solve for x

$$
\begin{aligned}
y &= 0 \\
(0) &= -\frac{1}{2}x + 4 \\
-4 & \qquad -4 \\
-4 &= -\frac{1}{2}x \\
\times -2 & \quad \times -2 \\
8 &= x
\end{aligned}
$$

The horizontal intercept is $(8, 0)$

To find the vertical intercept, we set $x = 0$ and solve for y

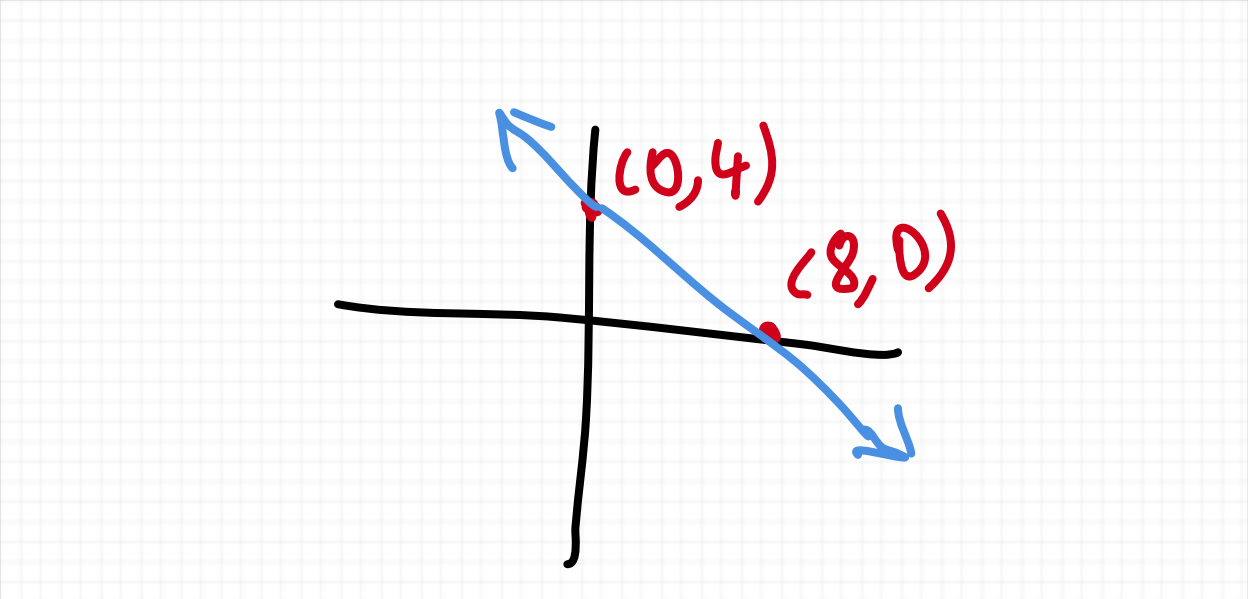
$x = 0$

$y = -\frac{1}{2}(0) + 4$

$y = 4$

The vertical intercept is $(0, 4)$

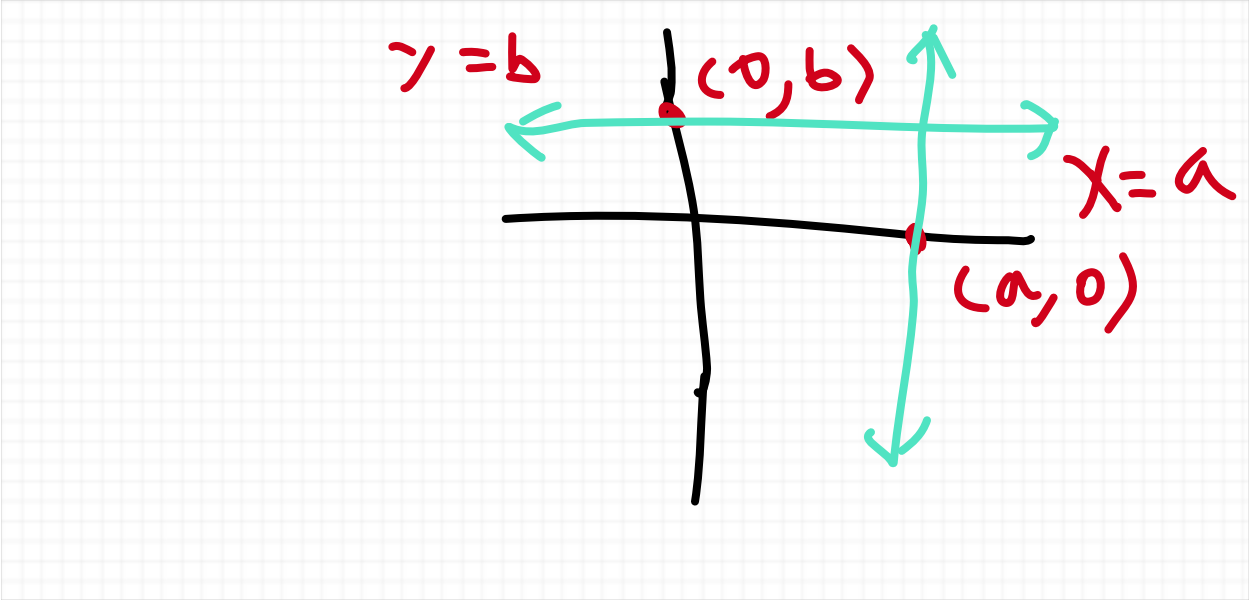
To draw the line, we shall plot the two points and connect the dots



Vertical and horizontal lines can be drawn in other ways. In particular, their equations are always of the following form:

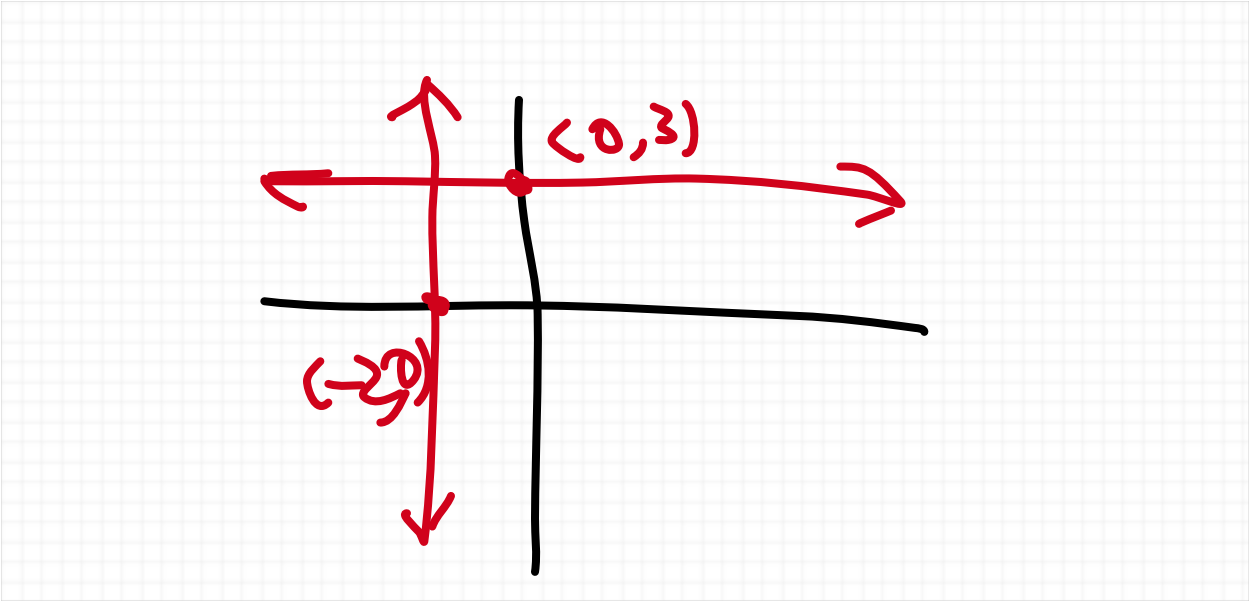
$x = a$ for **vertical lines**

$y = b$ for **horizontal lines**



With horizontal and vertical lines, all we need is one point.

Example 3. Graph the line $y = 3$ and the line $x = -2$



Next Time...

We'll practice plotting lines and move onward to understanding equations of lines.

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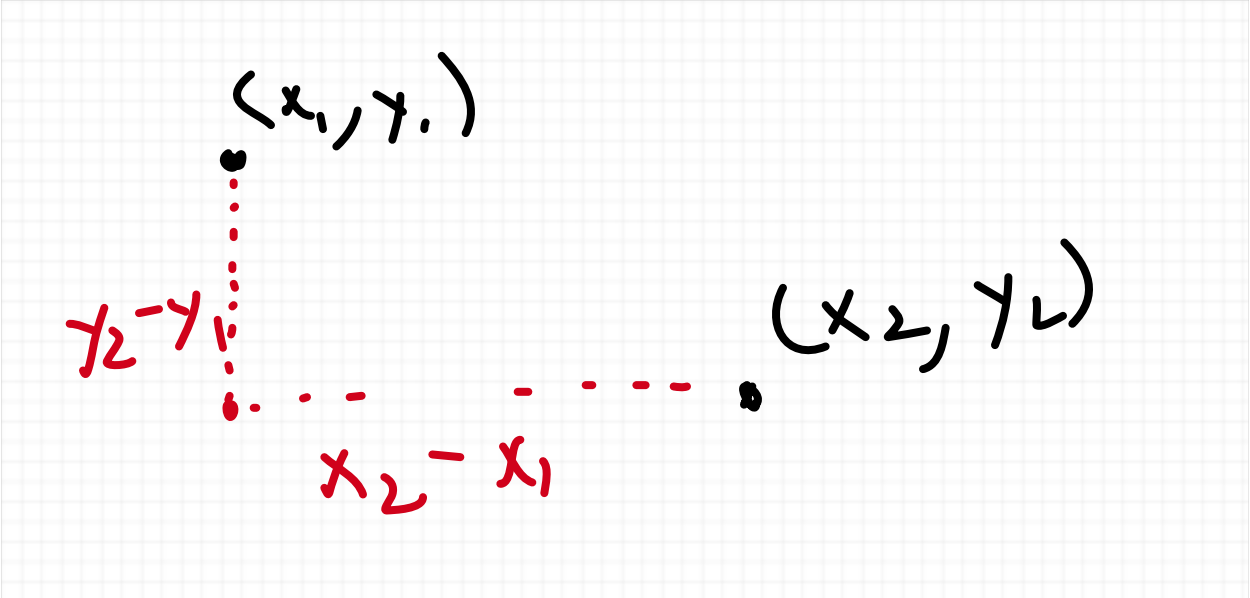
Last Time...

We talked about finding the intercepts and connecting points together to draw lines.

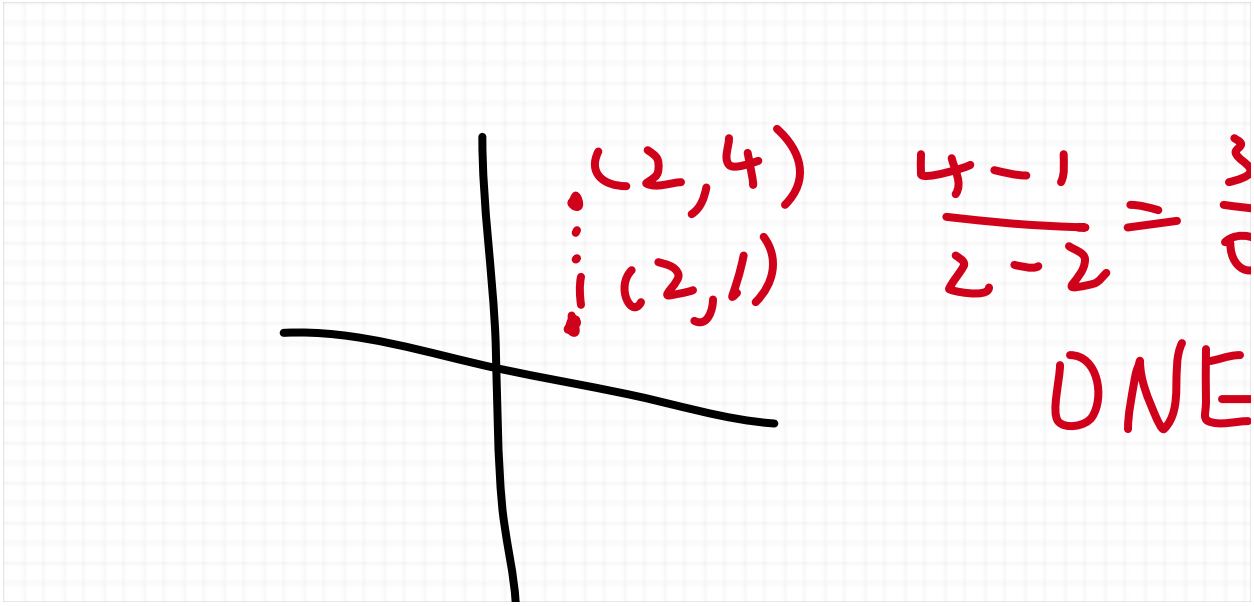
Lines and Slopes (Cont.)

The rate of change from an initial point (x_1, y_1) to the final point (x_2, y_2) is the ratio of the vertical over the horizontal change

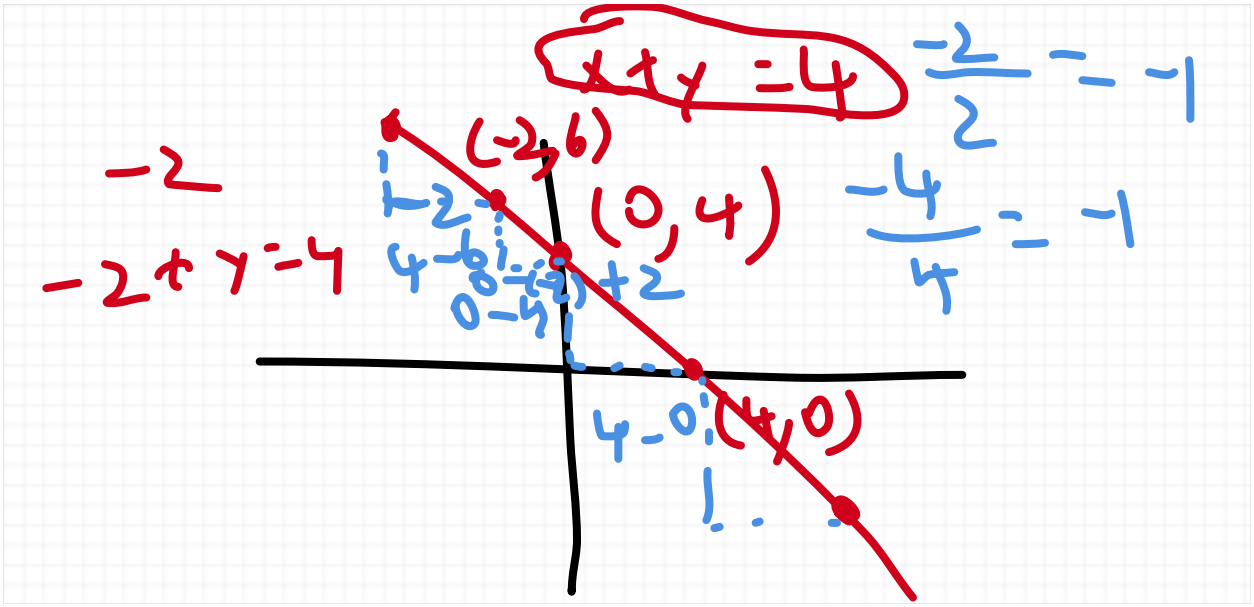
rate of change from (x_1, y_1) to $(x_2, y_2) = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{"rise"}}{\text{"run"}} = \dfrac{\text{"change in } y\text{"}}{\text{"change in } x\text{"}}$



IMPORTANT NOTE: The rate *does not always exist* between two points, when horizontally there's no change at all, then the rate of change doesn't exist since we can't divide by zero



With a line, the rate of change between any two points is always the same



The constant rate of change of any given line we call the **slope**.

We often call the slope m:

$m = \frac{y_2 - y_1}{x_2 - x_1}$ for ANY two points (x_1, y_1) (x_2, y_2)

Example 4. Find the slope of the line determined by $3x - 4y = 12$

We pick two points on the line; let's choose the x and y intecepts

x-intercept
$y = 0$
$3x - 4(0) = 12$
$3x = 12$
$\div 3 \quad \div 3$
$x \quad = 4 \qquad (4, 0)$

y-intercept
$x = 0$
$3(0) - 4y = 12$
$-4y = 12$
$\div -4 \quad \div -4$
$y = -3 \quad (0, -3)$

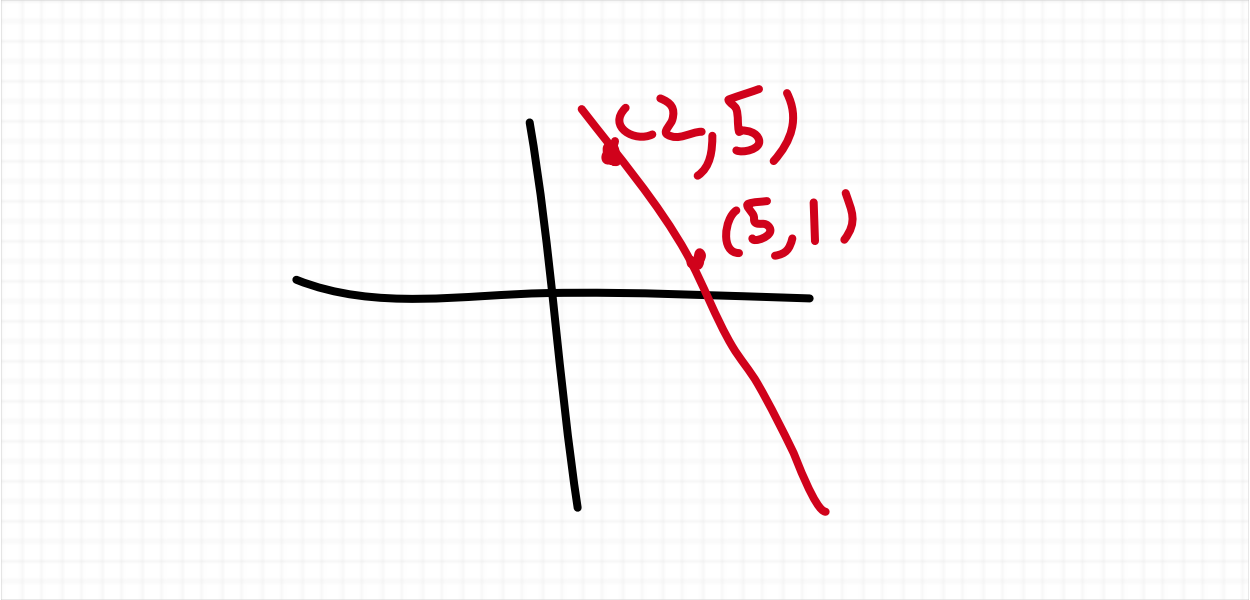
To calculate the slope, we use the rate of change formula on the points $(4, 0)$, $(0, -3)$

$$m = \frac{-3 - 0}{0 - 4} = \frac{-3}{-4} = \frac{3}{4}$$

Example 5. Let's say that we had a line illustrated as so

Find the slope:

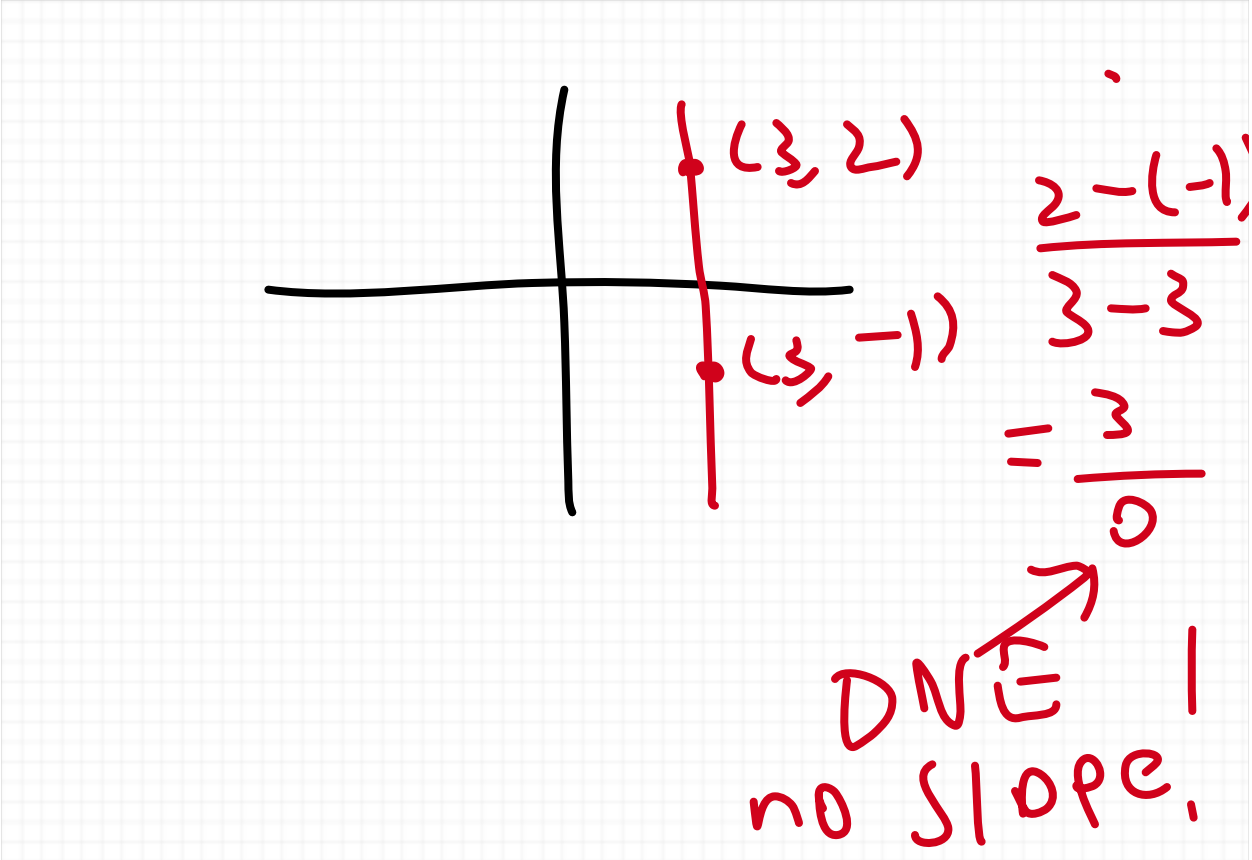
$$\frac{5 - 1}{2 - 5} = \frac{4}{-3} = -\frac{4}{3}$$



Example 6. Find the slope of the line containing two points $(1, 7)$ and $(3, 13)$

$$m = \frac{7 - 13}{1 - 3} = \frac{-6}{-2} = 3$$

Some lines, particularly vertical lines, have no slope, since they're vertical and have no horizontal change. Take the line $x = 3$. When finding the rate of change of two points, we run into a situation where we divide by zero, which we can't do.



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Last Time...

Lines and Slopes (Cont.)

Entrance Pass

1. For the line with the equation $x + 2y = 10$, do the following:

a. Find the vertical and horizontal intercept. Plot the points on a graph
Hint: Set $x = 0$, solve for y for the vertical intercept, set $y = 0$ solve for x for the horizontal intercept.

b. Use those points plotted in **part a.** to plot the entire line on a graph
c. Find the slope of the line.
Hint: The slope is found using the "rise" over "run" formula

Writing Equations of Lines

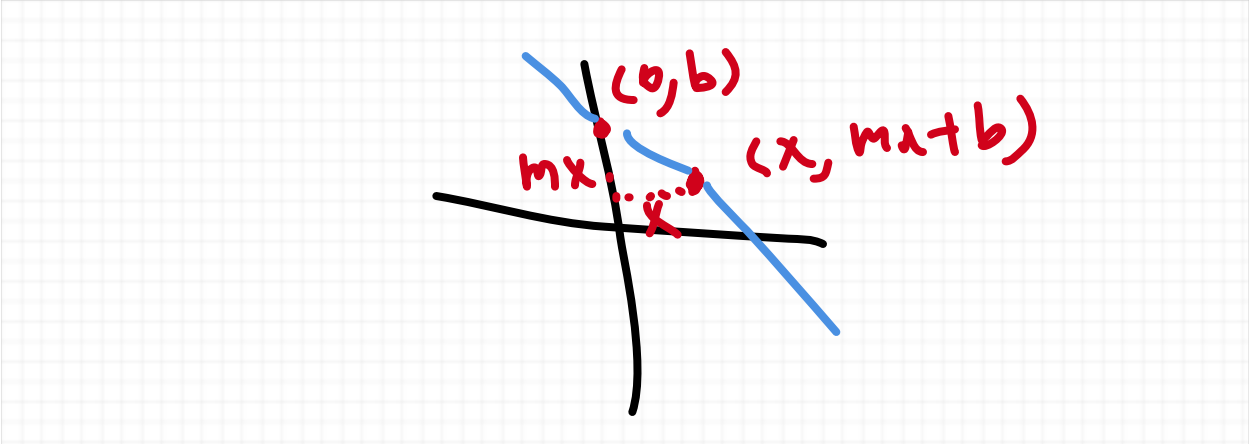
To write an equation of a line, the following information suffices:

1. Knowing the vertical intercept $(0, b)$ and the slope m (the slope-intercept formula)
2. Knowing either a point (x_1, y_1) and slope m or two points $(x_1, y_1), (x_2, y_2)$ (the point-slope formula)
3. Coefficients a, b, c such that $ax + by = c$ (the standard formula)

Slope-Intercept Formula

If we have the vertical intercept $(0, b)$ and its slope m, then we have

$$y = mx + b$$

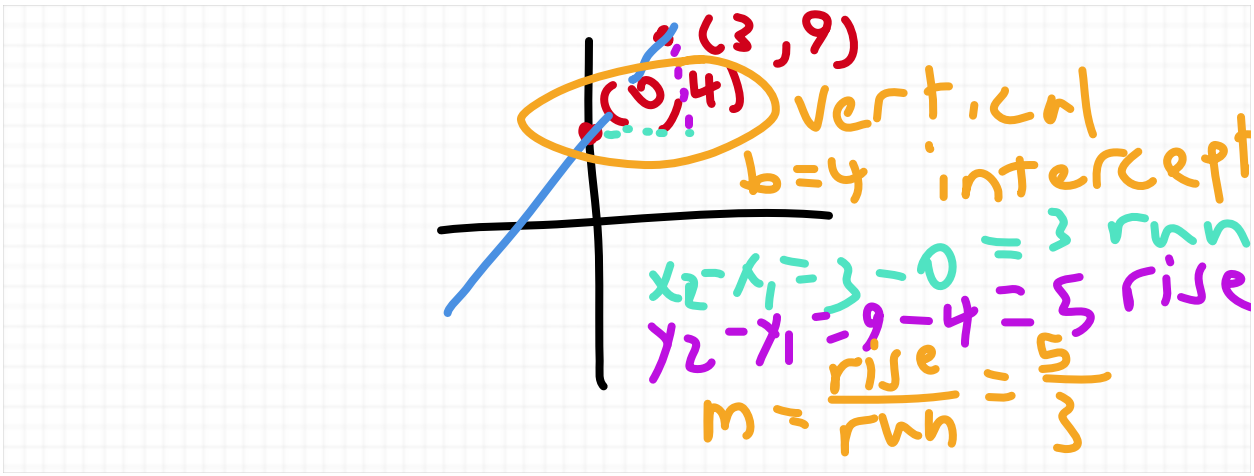


Example 1. Write an equation of the line given slope m and vertical intercept b

$$m = 3, b = 2$$

$$y = 3x + 2$$

Example 2. Find the equation of a line with points $(0, 4)$ and $(3, 9)$.

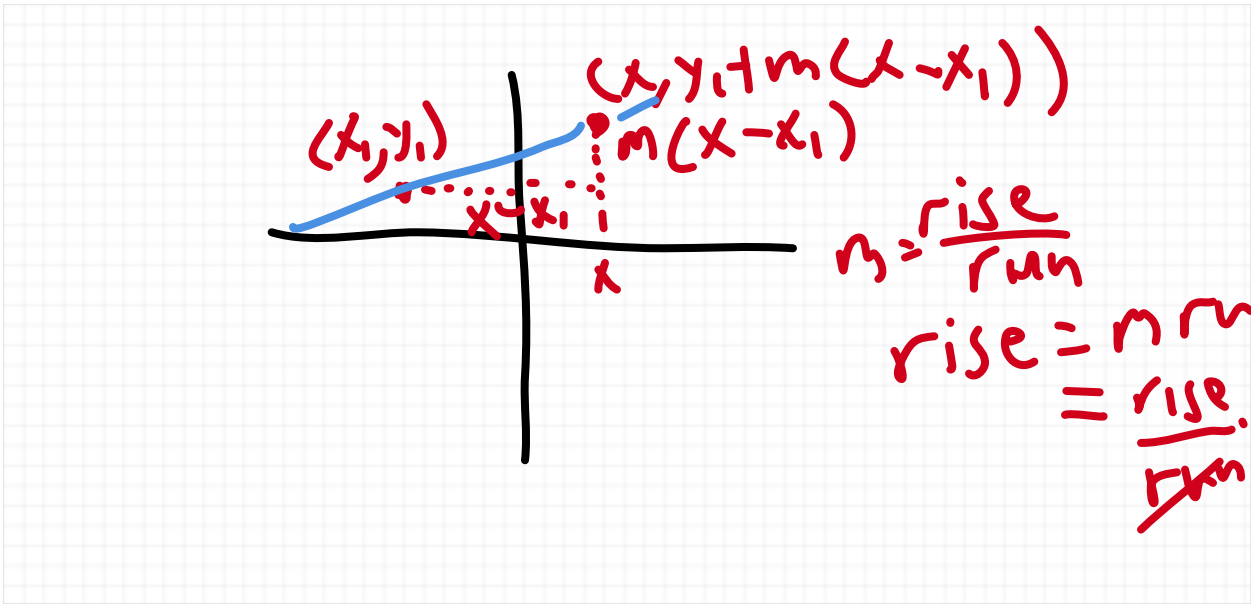


$$y = \frac{5}{3}x + 4$$

Point Slope Formula

If we have (x_1, y_1) and its slope m, then we have

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ +y_1 & \qquad +y_1 \\ y & = m(x - x_1) + y_1 \end{aligned}$$



Example 3. We have a point $(5, 4)$, $m = 4$. We plug in the point values and the slope into the

point-slope formula

$x_1 = 5, y_1 = 4,\ m = 4$
$y - y_1 = m(x - x_1)$
$y - 4 = 4(x - 5)$

Note: Whenever I have you write an equation of a line, I usually don't ask you to simplify it at all, unless I ask you to change it into a different form.

Let's say instead of a slope, we had two points $(x_1, y_1), (x_2, y_2)$. Then we can use the rise over run formula to compute the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

and then use the point-slope formula

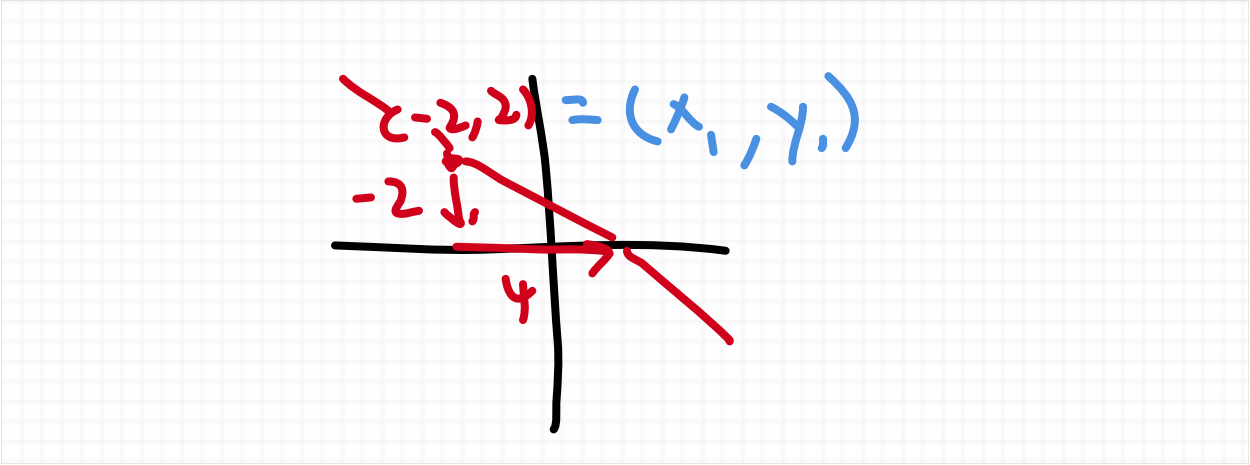
$y - y_1 = m(x - x_1)$
$y - y_2 = m(x - x_2)$

Example 4. Find the equation of a line containing $(1, 7)$ and $(3, 13)$

$$m = \frac{7 - 13}{1 - 3} = \frac{-6}{-2} = 3$$

$x_1 = 1,\ y_1 = 7,\ m = 3$
$y - y_1 = m(x - x_1)$
$y - 7 = m(x - 1)$

Example 5. Find the equation of the following line:



First, we need to find the slope m using what we know about the rise and run

$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{4} = -\frac{1}{2}$$

$$x_1 = -2,\ y_1 = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{1}{2}(x - (-2))$$
$$y - 2 = -\frac{1}{2}(x + 2)$$

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Last Time...

Writing Equations of Lines (Cont.)

10/18 Entrance Pass Solutions

1. For the line with the equation $x + 2y = 10$, do the following:

a. Find the vertical and horizontal intercept. Plot the points on a graph
Hint: Set $x = 0$, solve for y for the vertical intercept, set $y = 0$ solve for x for the horizontal intercept.

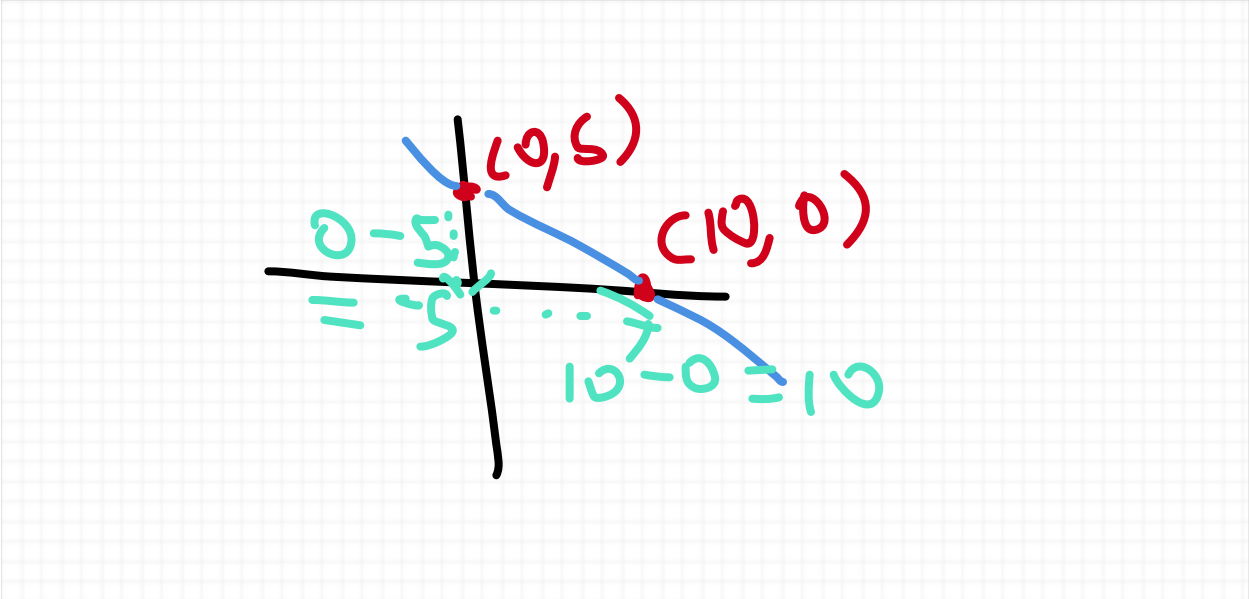
vertical intercept: Set $x = 0$, solve for y:

$$x = 0$$
$$(0) + 2y = 10$$
$$2y = 10$$
$$y = 5 \quad (0, 5)$$

horizontal intercept: Set $y = 0$, solve for x:

$$y = 0$$
$$x + 2(0) = 10$$
$$x = 10 \quad (10, 0)$$

b. Use those points plotted in **part a.** to plot the entire line on a graph



c. Find the slope of the line.
Hint: The slope is found using the "rise" over "run" formula

Use the rise over run formula on the intercepts $(0, 5)$ and then $(10, 0)$

$$m = \frac{0-5}{10-0} = \frac{-5}{10} = -\frac{1}{2}$$

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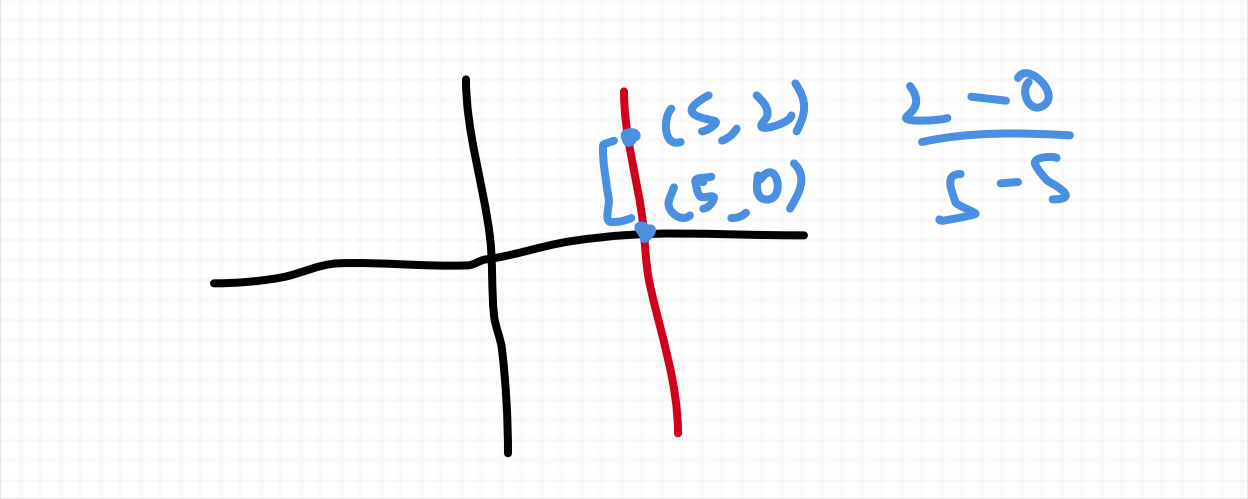
Last Time...

Warm up

Find the equation of the line:

1. Has y-intercept 5 and slope 2
2. Has x-intercept 5 and no slope
3. Has y-intercept 5 and x intercept -2

"NO SLOPE" is code for "vertical line"



Writing Equations of Lines (Cont.)