2022-2023 James E Davis Trimester 1 Algebra 2 Week 1 Class Notes

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Previously...

We talked about two-varlable systems of equation, i.e., systems of equations (a group of equalities/inequalities) that involve at most two variables, and we talked about the possibilities for their solutions, in addition to going over a few possible ways of finding solutions

1. the graphing method-Graph the equation on the cartesian plane and find the points of intersection.
2. the subsitution method-Solve one of the equations the way you'd solve a one variable equation, i.e., we find one variable (any variable), and solve for that variable, in terns of the other variable.

The Substitution Method

Example 1.

$$-x + 5y = 22$$
$$7x - 2y = 19$$

Step 1. Solve for x with the equation $-x + 5y = 22$.

$$\begin{array}{ll} -x + 5y & = 22 \\ \quad -5y & \quad -5y \\ (-1) \cdot x & = 22 - 5y \\ \div -1 & \div -1 \\ x & = (-1)(22 - 5y) = -22 + 5y \end{array}$$

Step 2. Plug in the solution we found for step 1 ($x = -22 + 5y$) into the other equation $7x - 2y = 19$ and then solve for y

$$7(-22 + 5y) - 2y = 19$$
$$-154 + 35y - 2y = 19$$

$$
\begin{aligned}
-154 + 33y &= 19 \\
+154 \quad &\quad +154 \\
33y &= 173 \\
\div 33 \quad &\quad \div 33 \\
y &= 173/33
\end{aligned}
$$

Step 3. Plug in y into the previous solution for x (in terms of y), to get the value for the solution for x i.e., we plug in $y = 173/33$ into $x = -22 + 5y$ to get

$$x = -22 + 5(173/33) = \frac{-726}{33} + \frac{865}{33} = \frac{139}{33}$$

Step 4. The solution is

$$
\begin{aligned}
x &= 139/33 \\
y &= 173/33
\end{aligned}
$$

or the coordinate $(139/33, 173/33)$.

Example 2.

$$
\begin{aligned}
x + 2y &= 2 \\
5x - 3y &= -29
\end{aligned}
$$

We solve for x as follows.

$$
\begin{aligned}
x + 2y &= 2 \\
-2y \quad &\quad -2y \\
x \quad &= 2 - 2y
\end{aligned}
$$

Next, we plug in $x = 2 - 2y$ into the other equation

$$
\begin{aligned}
5(2 - 2y) - 3y &= -29 \\
10 - 10y - 3y &= -29 \\
10 - 13y &= -29 \\
-10 \quad &\quad -10 \\
-13y &= -39 \\
\div -13 \quad &\quad \div -13 \\
y &= 3
\end{aligned}
$$

Finally, we plug in $y = 3$ into the equation $x = 2 - 2y$, which gives us

$x = 2 - 2(3) = 2 - 6 = -4$

Our solution is then

$$x = -4$$
$$y = 3$$

or the coordinates $(-4, 3)$

Recall that sometimes a linear equation either has no solution or an entire line as a solution. This will happen when you plug in the solution in terms of the other variable of the first equation into the second, and try to solve for the first variable, only to get one of "$a = a$" or "$a = b$" for $a \neq b$. Recall that sometimes you solve a single variable equation, let's say

$$\begin{aligned} 2x - 7 &= 2x + 7 \\ -2x \quad &\quad - 2x \\ -7 &= 7 \end{aligned}$$

and get the above statement that $-7 = 7$, which is false, and we call in general,single variable equations that lead to the statement "$a = b$" for $a \neq b$ a **contradiction.** Other times, you may have

$$\begin{aligned} 3(x - 5) &= 3x - 15 \\ 3x - 15 &= 3x - 15 \\ -3x \quad &\quad - 3x \\ -15 &= -15 \end{aligned}$$

which is what we call an i**dentity**, i.e., we have a statement of the form "$a = a$" and our solution is all real numbers, so we write

$$\mathbb{R}, \ (-\infty, +\infty)$$

For two variable systems of equations, when we do the substitution, and get a contriction, we write

$$\varnothing.$$

Example 3.

$3x - 2y = 6$
$-9x + 6y = 9$

First, we'll solve for $3x - 2y = 6$

$$
\begin{aligned}
3x - 2y &= 6 \\
+2y \quad &\quad +2y \\
3x \quad &= 6 + 2y \\
\div 3 \quad &\quad \div 3 \\
x \quad &= 2 + \frac{2}{3}y
\end{aligned}
$$

Next, we plug in $x = 2 + \frac{2}{3}y$ into $-9x + 6y = 9$

$$
\begin{aligned}
-9\left(2 + \frac{2}{3}y\right) + 6y &= 9 \\
-18 - 6y + 6y &= 9 \\
-18 &= 9
\end{aligned}
$$

we get a contradiction, and we can write $\varnothing$, which is to say that there's no solution.

Example 3.

$3x - 2y = 6$
$-9x + 6y = -18$

We solve for x as before, so we get

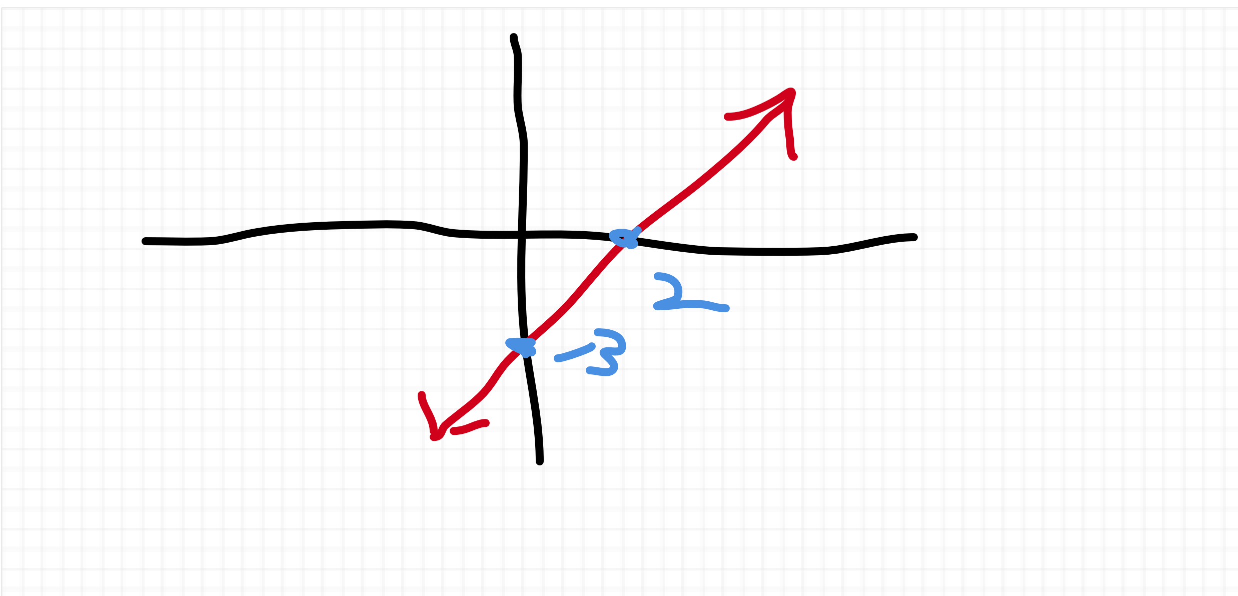
$$x = 2 + \frac{2}{3}y$$

We then plug in $x = 2 + \frac{2}{3}y$ as before into $-9x + 6y = -18$

$$
\begin{aligned}
-9\left(2+\frac{2}{3}y\right)+6y &= -18\\
-18-6y+6y &= -18\\
-18 &= -18
\end{aligned}
$$

giving us an identity, i.e., y can be any real number (which we can write as $\mathbb{R}$). The solution is then all ordered pairs (x, y) that satisfy the equation $x = 2 + \frac{2}{3}y$, so the solution set can be graphed as follows:

$$y = -3 + \frac{3}{2}x$$



Preview for Tomorrow

We'll do a few exercises involving the substitution method and then a few exercises as well with the graphing method, and we'll move on to building up the tools to do the next method, aka the determinant method.

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Previously...

We learned the substitution method and went over some examples.

The Warm-up

Using the substitution method, solve the following:

1. $\begin{aligned} x - 3y &= 13 \\ 5x + 3y &= 2 \end{aligned}$

2. $\begin{aligned} 7x - 6y &= -30 \\ x - 4y &= -20 \end{aligned}$

The Graphing Method

For this method, we simply graph the systems of equations and find the points of intersection, and find the solution that way.

To do this method, let's recall how to draw lines from a linear equation. There are a few ways to do this.

Way 1. *Find a point and the slope.* Immediately obvious what to do if the line is in point slope form, i.e.

$y = mx + b$

But lines are not always in point slope form. Now we can fix that by finding the equivalent linear equation that is in point-slope form by solving for y.

I don't recommend that, because 9 times out of 10 it takes work.

I recommend this other way

Way 2. *Find two points on the line.* Plug in what one value is based on the choice of another value. The easiest values to find are usually the x and y intercepts.

Example 1.

1. $x + 2y = 6$. We find the x and y intercepts as follows:

Set $x = 0$ and solve for y to get the y-intercept

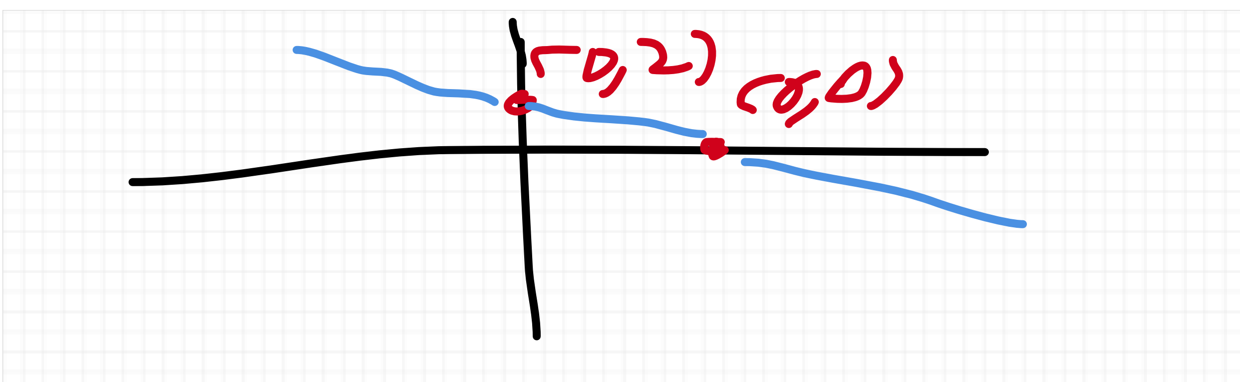
$$\begin{aligned} (0) + 2y &= 6 \\ 2y &= 6 \\ \div 3 \quad & \div 3 \\ y &= 2 \end{aligned}$$

This gives one of the two points $(0, 2)$

Next, set $y = 0$ and solve for x to get the x-intercept.

$x = 6$

This gives us the second point $(6, 0)$. Now we from those points we can graph it.



2. Now, we will graph $5x - y = 8$.

Set $x = 0$, and plug $x = 0$ in

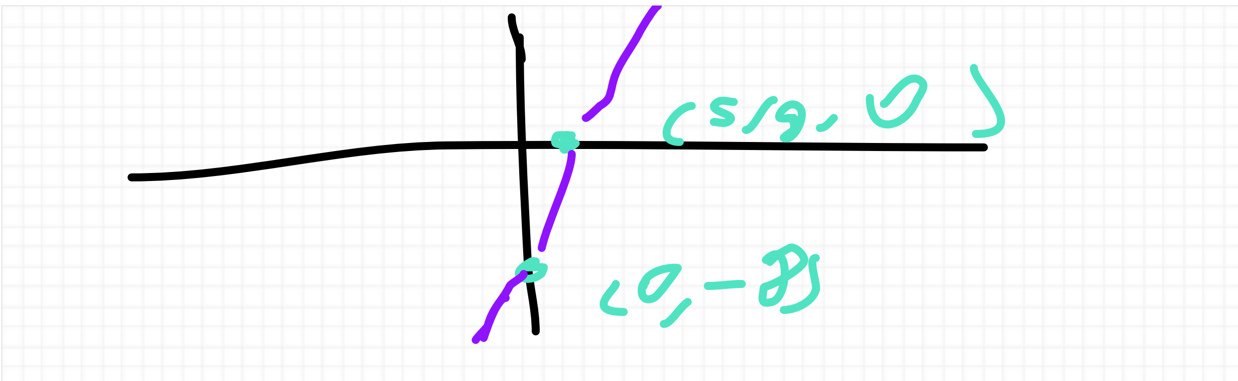
$$\begin{aligned} -y &= 8 \\ \div -1 \quad & \div -1 \\ y \quad &= -8 \end{aligned}$$

we have $(0, -8)$ for one point

For the other point, set $y = 0$, and plug $y = 0$ in

$$5x = 8$$
$$\div 5 \quad \div 5$$
$$x = \frac{5}{8}$$

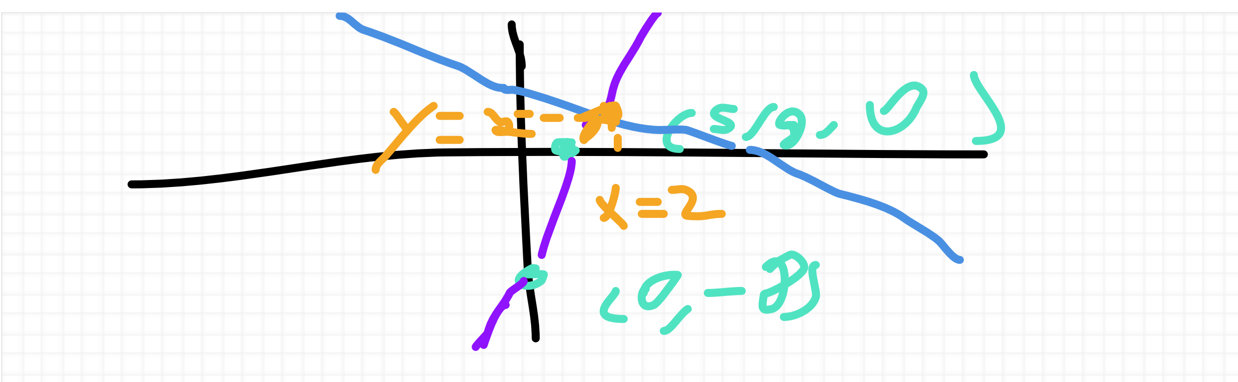
we have $\left(\frac{5}{8}, 0\right)$. We then have the line



Example 2. We find the solution to

$$x + 2y = 6$$
$$5x - y = 8$$

is determined by finding the intersection of the lines determined by the previous example

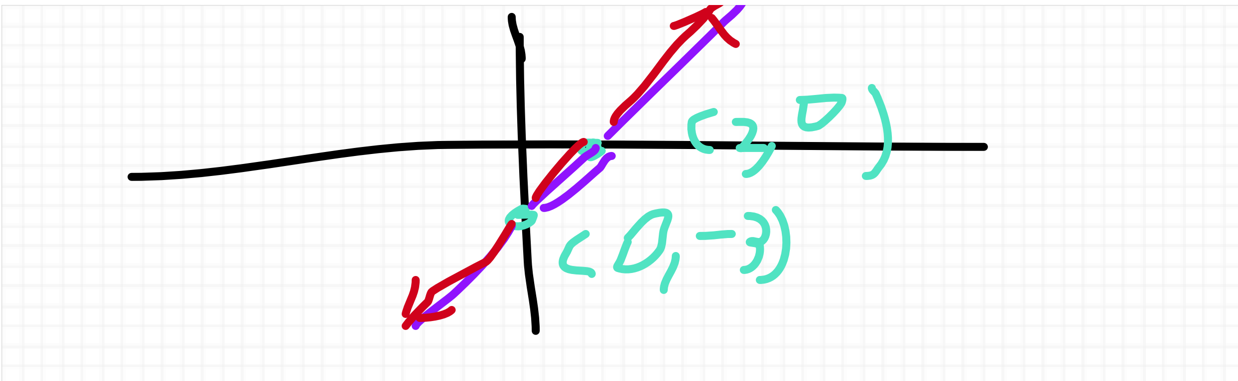


we find that the solution is $(2, 2)$, i.e., $x = 2$ and $y = 2$.

Example 3. Find the solution to

$$3x - 2y = 6$$
$$-9x + 6y = -18$$

We get that the equation of the line is the same, hence the solution is just all coordinates (x, y) of real numbers that satisfy the linear equation $3x - 2y = 6$.



Preview for Tomorrow

N/A

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Previously...

Last time we talked about the graphing method, and did a few examples in solving for equations with the graphing method.

The Warm-up

Use the graphing method to solve for the follwing systems of equations:

1. $\begin{aligned} x + 2y &= 5 \\ 3x - y &= 1 \end{aligned}$

2. $\begin{aligned} 3x - 2y &= 6 \\ -9x + 6y &= 9 \end{aligned}$

The Graphing Method (Cont.)

Example 4. Find the solution using the graphing method

$x + 2y = 5$
$3x - y = 1$

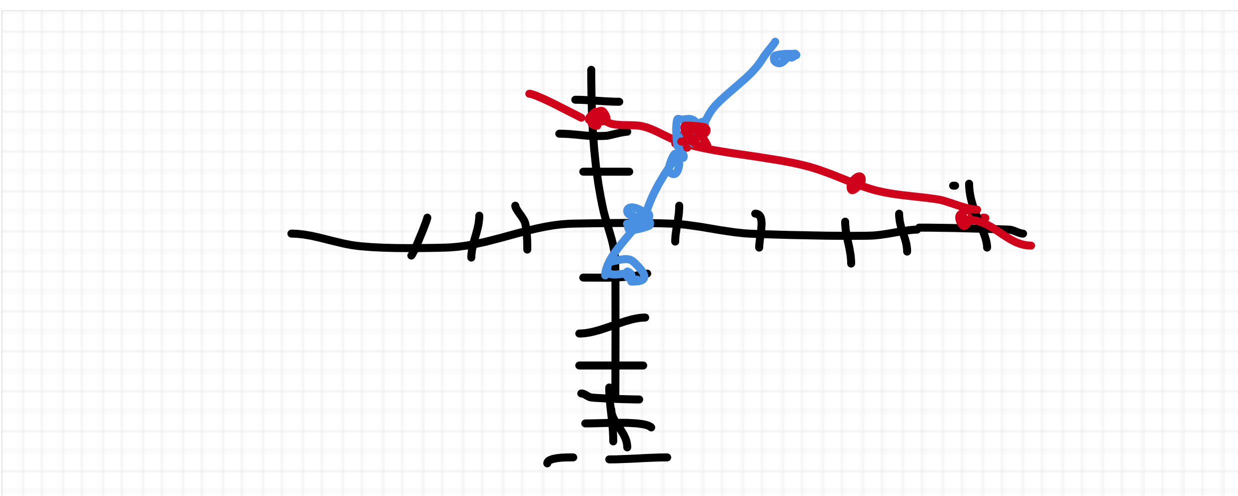
Once we find the two points, we can determine the slope as follows

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y \text{ values}}{\text{change in } x \text{ values}}$$

To make the graph more exact, you want to put points on every integer-valued coordinate that we find by finding the integer valued rise over an integer valued run.

On a test/quiz setting, the solution for graphing method equations will bew "nice", which is to say integer valued, so finding the integer valued coordinates of the graph and connecting the dots when drawing the line wiil get you the solution.

So for this example, the interteger-value coordinates between those two lines agree at $x = 1,\ y = 2$, or $(1, 2)$.



Example 5. Next, we find the solution using the graphing method of

$3x - 2y = 6$
$-9x + 6y = 9$

slople of $3x - 2y = 6$ is

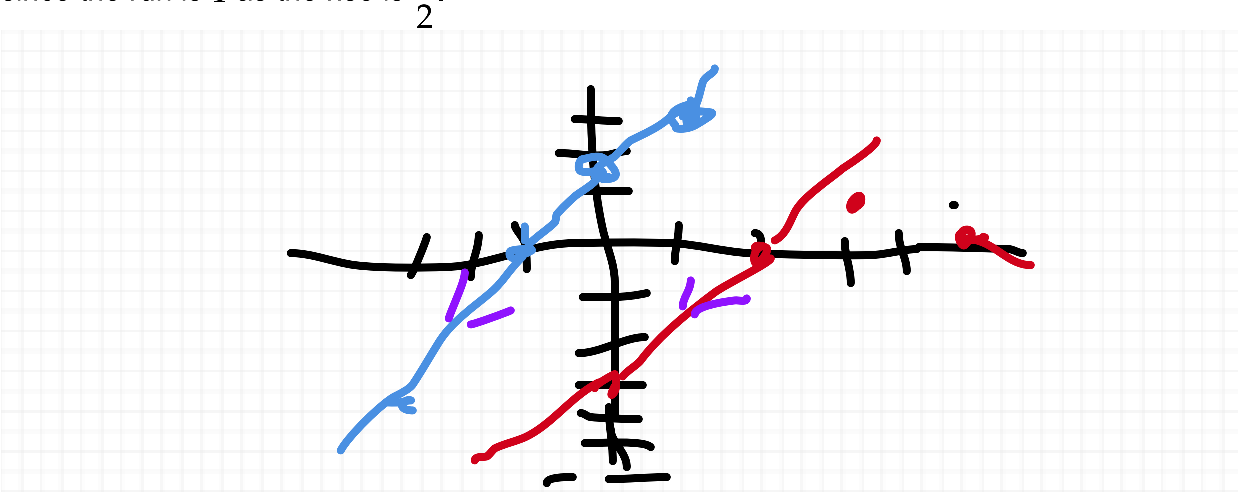
$$\frac{3}{2}$$

since the run is 2 as the rise is 3.

slope of $-9x + 6y = 9$ is also

$$\frac{3}{2}$$

since the run is 1 as the rise is $\frac{3}{2}$.



Since the lines have no intersection, there is no solution.

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The takeaway: When using the graph method to draw the lines very well and precise, or else you may not find the points of intersection.

Preview for What's Ahead

Before, we talked about some methods of finding solutions for two-variable systems of equations, one being the graph method, and the other being the substitution method. But there's some other methods of finding solution that we also want to cover. In particular, we have the determinant method and the matrix method.

The determinant method is the last way of finding a solution that you'll do by hand, but we'll also go over the inverse matrix method, and possibly use it as a way of finding a system of equation with three variables.

Both of these methods of solving systems of equations, involve matrices.

The general idea. Let's say we have a system of equations of the form

$$\begin{aligned} ax + by &= c \\ dx + ey &= f \end{aligned}$$

Then we can convert thie system of equation into the array, or matrix equation

$$\left[\begin{array}{cc|c} a & b & e \\ d & e & f \end{array}\right]$$

and then we can do one of two things. One, is that we let

$$A = \begin{bmatrix} a & b \\ d & e \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ D = \begin{bmatrix} e \\ f \end{bmatrix}$$

and we convert the above system of equations into matrix equation

$$AX = D$$

and if A is invertible, i.e., there exists A^{-1} such that

$$A^{-1}AX = X$$

and then we can multiply A^{-1} on both sides of the matrix equation as so

$$\begin{aligned} AX &= D \\ A^{-1}AX &= A^{-1}D \\ X &= A^{-1}D \end{aligned}$$

hence x and y are just the corresponding entries of $A^{-1}D$, since $X = \begin{bmatrix} x \\ y \end{bmatrix}$. This method is called the **inverse matrix method**. The other method, which is related to the inverse matrix method and also utilizes matrices, is called **the determinant method**, which involves finding the determinant of A (as defined above), which has the following formula

$$\det A = \det\begin{bmatrix} a & b \\ d & e \end{bmatrix} = ae - bd.$$

What are Matrices?

Matrices are two dimensional arrays of numbers, i.e., something that may look like this

$$\underset{2\times 2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}, \underset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}}, \underset{1\times 2}{\begin{bmatrix} 1 & 2 \end{bmatrix}}, \underset{2\times 4}{\begin{bmatrix} 5 & 6 & 7 & 8 \\ e & \pi & 7.34 & d \end{bmatrix}}$$

Each matrix has a number of rows r and a number of columns c. And to specify the kind of matrix with the kind of rows and columns that it has, we call it a $r \times c$ matrix

# of rows $\times$ # of columns

What is it that we can mathematically do with matrices?

1. Add/Subract Matrices
2. Multiply a "scalar" number by a matrix
2. Multiply two matrices with appropriate dimensions
3. Multtiplicatively Invert Matrices "Divide Matrices" (sometimes)

Adding/Subtracting Matrices

Any matrix that has the same row and column dimension, we can add/subtract. When we do that, we add the matrices entry by entry

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} a+w & b+x \\ c+y & d+z \end{bmatrix}$$

Example 1. Add the following matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix} = \begin{bmatrix} 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 5 & e \\ 7.5 & h \end{bmatrix} + \begin{bmatrix} a & z \\ \pi & 3 \end{bmatrix} = \begin{bmatrix} 5+a & e+z \\ 7.5+\pi & h+3 \end{bmatrix}$$

One important thing to note is if the matrices have either different amount of rows or different amount of columns, you cannot add them up

Example 2. We find that

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$

does not exist.

If A is a 5×7 matrix and B is a 7×7 matrix, then $A + B$ doesn't exist.

Preview for Tomorrow

We'll next learn about matrix multiplication, which is a bit more sophisticated than matrix addition. For instance, when we multiply $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$, we get

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

How we got that is through multiplying the rows of the first matrix, by the columns of the second.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

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We took the QUEST, and then watched Thor.