Inequalities Lesson 2: Systems of Inequalities

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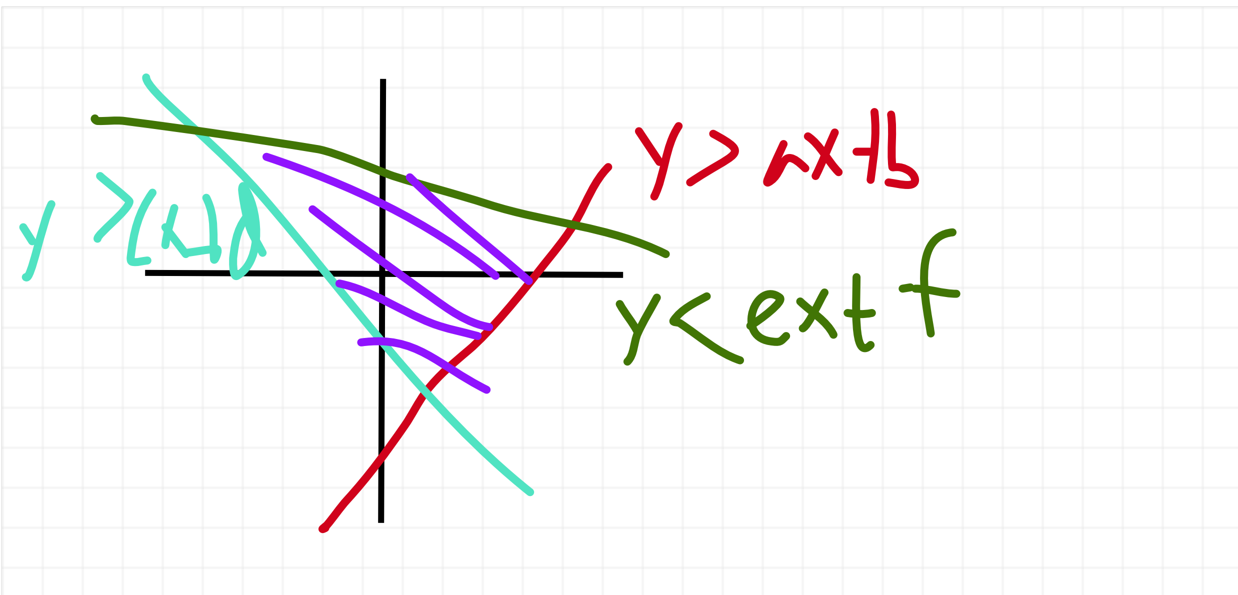
Graphing Systems of Linear Inequalities

A **system of linear inequalities** is a collection of linear inequalities (just as a system of equations is a collection of equations)

The solution set of a system of linear inequalities is all the points that satisfy all the conditions of the inequality (i.e. <u>the intersection</u> of the solution sets of each inequality).



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<u>4.4 Example 1.</u> Graph the solution set of

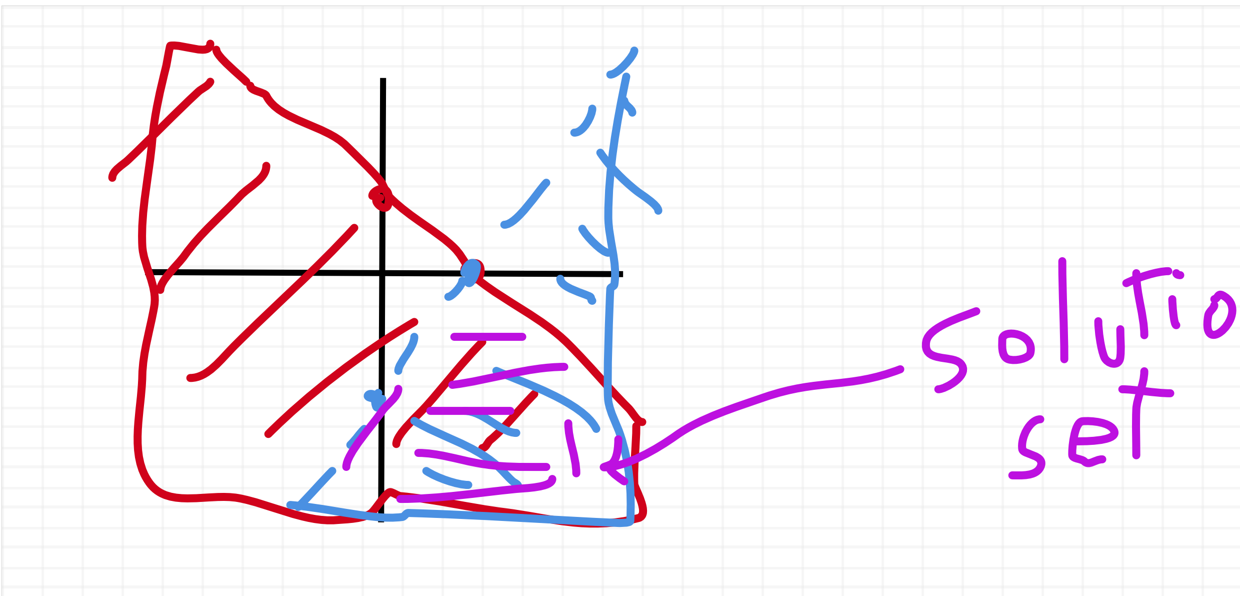
$x + y \le 1$
$2x - y > 2$

First we want to get y by itself (get the lines in slope-intercept form)

$$\begin{aligned} x + y &\le 1 \\ -x \quad &\quad -x \\ y &\le 1 - x \end{aligned}$$

$2x-y>2$
$-2x \qquad -2x$
$-y>2-2x$
$\div -1 \quad \div -1$ (note that $-1<0$)
$y<-2+2x$

We end up with
$y\le 1-x$
$y<-2+2x$



4.4 Example 3. (page 244)
$x\ge 1$
$y\ge x$
$4x+5y<20$

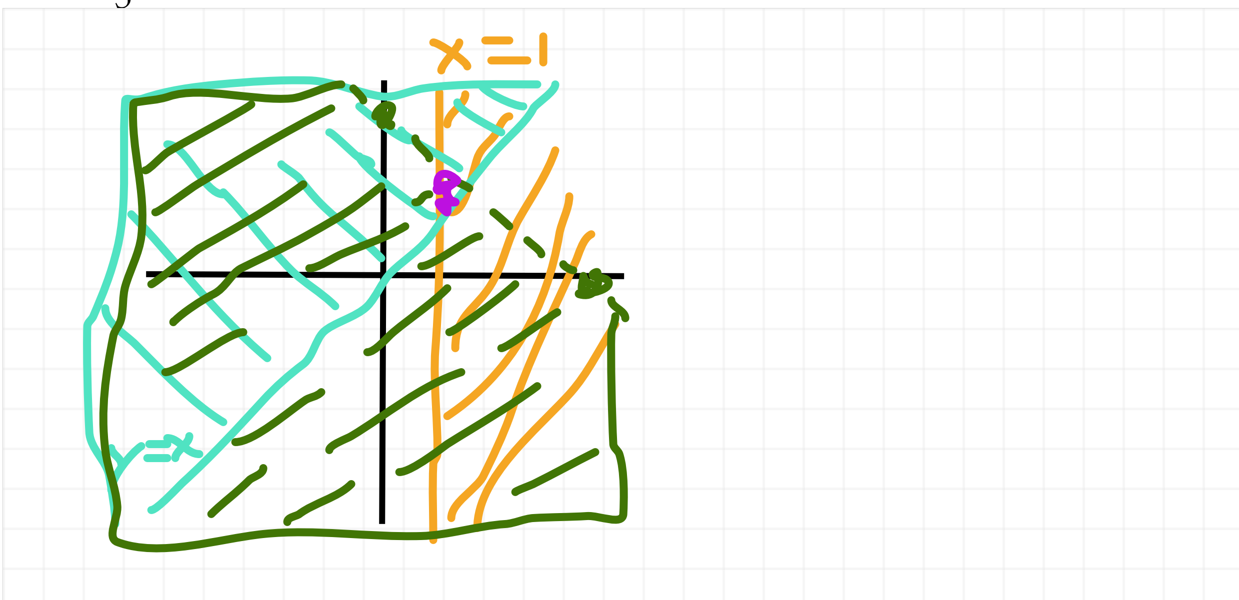
So we want get the third equation $4x+5y<20$ in slope intercept form.

$4x+5y<20$
$-4x \qquad -4x$
$5y<20-4x$
$\div 5 \quad \div 5$
$y<4-\frac{4}{5}x$

So we graph
$x\ge 1$

$y \geq x$

$y < 4 - \frac{4}{5}x$



It's always ideal to draw your lines to scale (unlike me)

Linear Programming

<u>4.5 Example 1 (page 250).</u> If $P(x, y) = 2x + 3y$, find the maximum value of P subject to the constraint:

$x + y \leq 4$
$2x + y \leq 6$
$x \geq 0$
$y \geq 0$

Find y by itself for $x + y \leq 4$ and $2x + y \leq 6$

$x + y \leq 4$
$-x \quad -x$
$y \leq 4 - x$

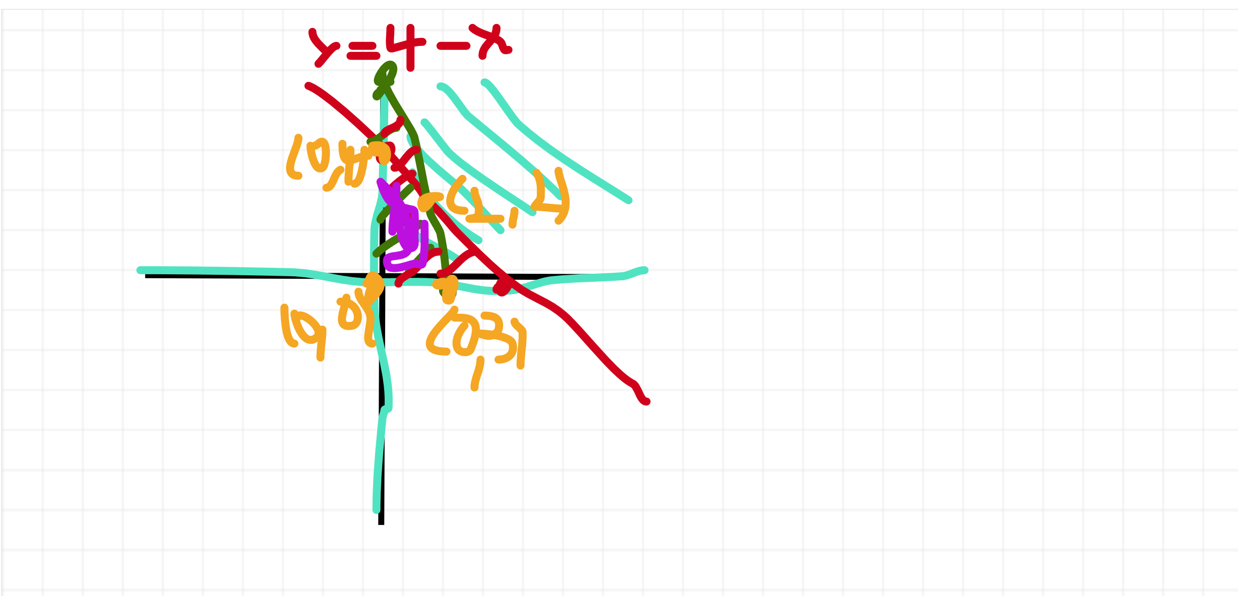
$2x + y \leq 6$
$-2x \quad -2x$
$y \leq 6 - 2x$

$y \leq 4 - x$
$y \leq 6 - 2x$
$x \geq 0$
$y \geq 0$

The intersection between $y = 4 - x$ and $y = 6 - 2x$ is at

$4 - x = 6 - 2x$
$\quad +x \qquad +x$
$4 \quad = \; 6 - x$
$-6 \qquad -6$
$-2 \quad = -x$
$\div -1 \quad \div -1$
$2 \quad = x$

The point is $(2, 2)$



Finally, we plug in all the possible corner points $(0, 0)$, $(0, 4)$, $(0, 3)$, $(2, 2)$

$P(x, y) = 2x + 3y$
$P(0, 0) = 0$
$P(0, 4) = 12$
$P(0, 3) = 6$
$P(2, 2) = 10$

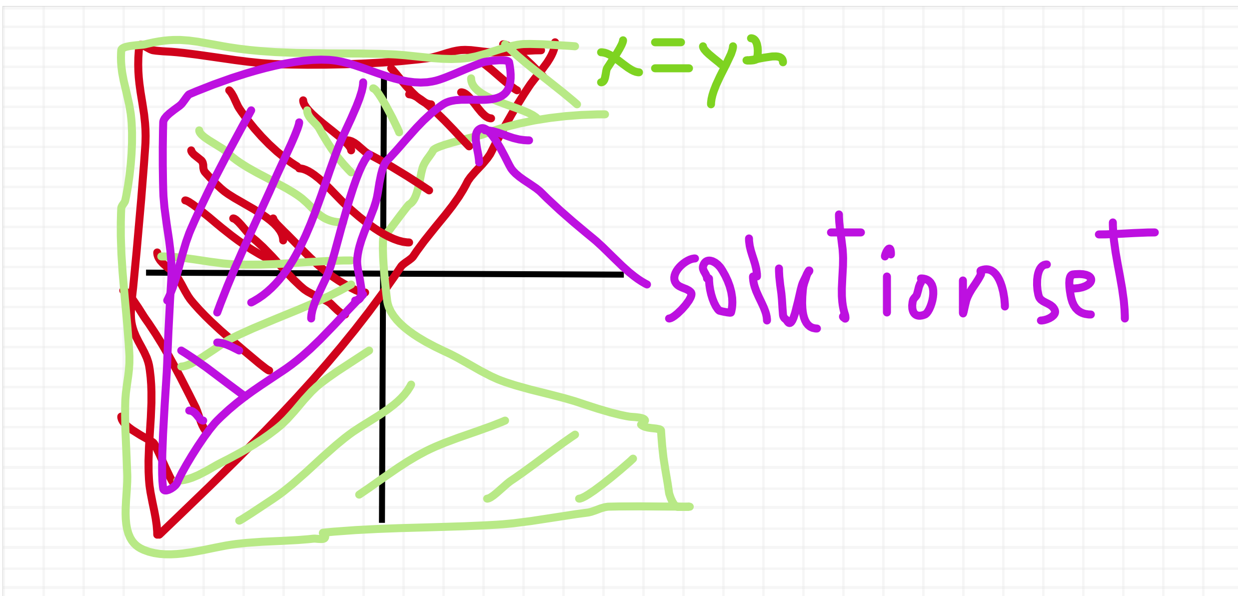
$(0, 4)$ is the point where the maximum is and 12 is the maximum.

Questions on Homework 3

<u>Question 16 of Homework 3 (page 246):</u> it involves a parabola, which is not linear.

$x \ \leq y^2$
$y \ \geq x$

Note that graphically, we have a parabola above



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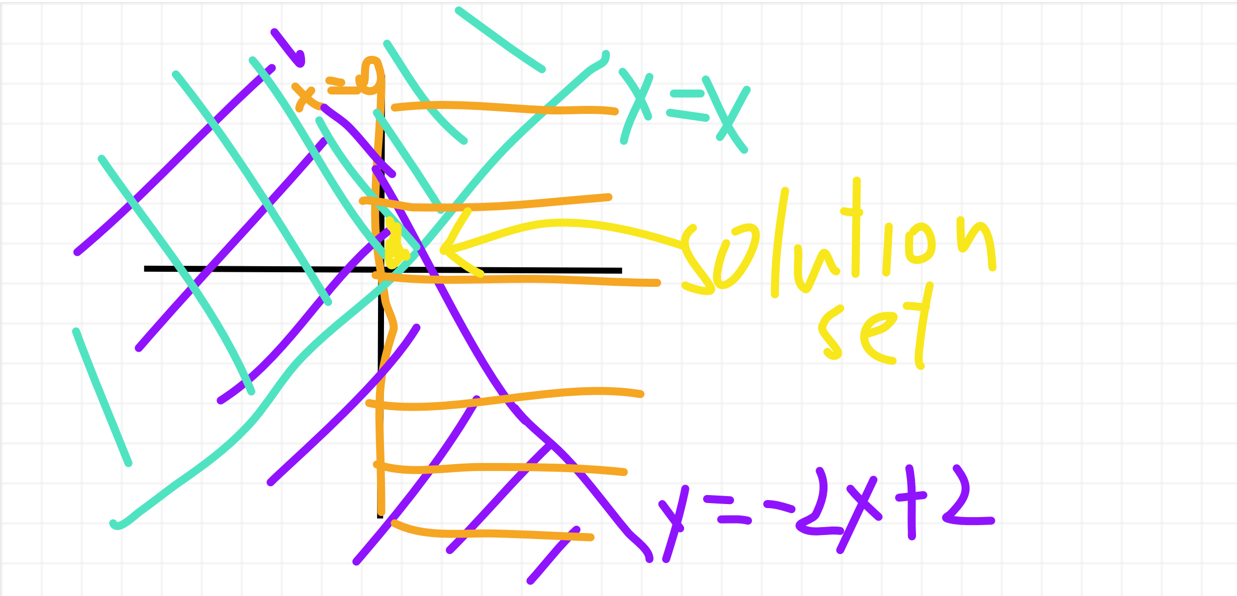
<u>Question 20 of Homework 3 (page 246).</u>
$2x + y \ \leq 2$
$y \geq x$
$x \geq 0$

The first line in slope-intercept form is
$y \ \leq -2x + 2$

And so we draw the lines and get



<u>Question 11 of Homework 3.</u>

$$3x + 2y > 6$$
$$x + 3y \le 2$$

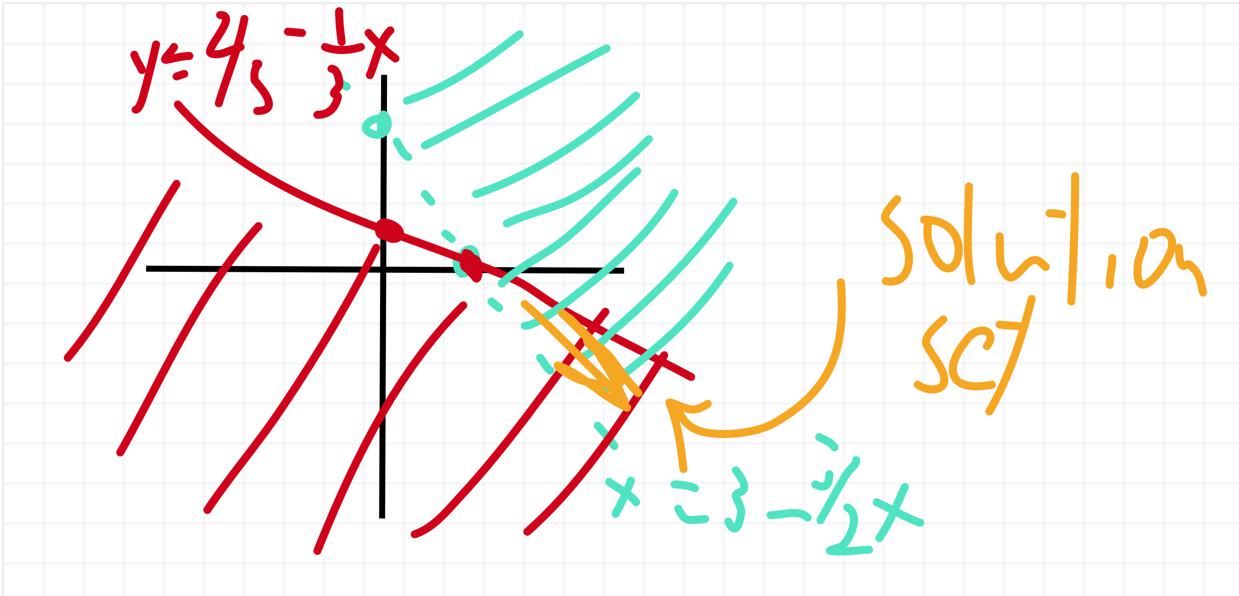
$$\begin{aligned} 3x + 2y &> 6 \\ -3x \qquad &\quad -3x \\ 2y &> 6 - 3x \\ \div 2 \qquad &\quad \div 2\ (2 > 0) \\ y &> 3 - \frac{3}{2}x \end{aligned}$$

$$\begin{aligned} x + 3y &\le 2 \\ -x \qquad &\quad -x \\ 3y &\le 2 - x \\ \div 3 \qquad &\quad \div 3\ \ (3 > 0) \\ y &\le \frac{2}{3} - \frac{1}{3}x \end{aligned}$$

We then have

$$y > 3 - \frac{3}{2}x$$
$$y \le \frac{2}{3} - \frac{1}{3}x.$$

We graph



Question 12 of Homework 3.

$x + y < 2$

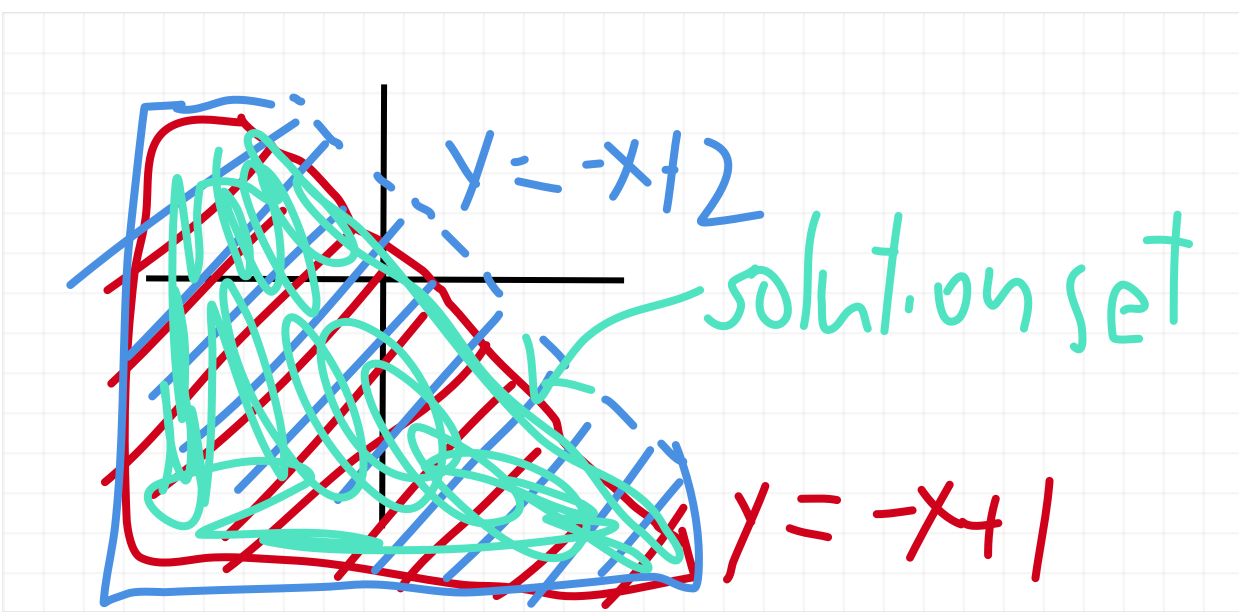
$x + y \leq 1$

In slope-intercept form, the system of inequalities are

$y < \; -x + 2$

$y \leq \; -x + 1$

We get the region



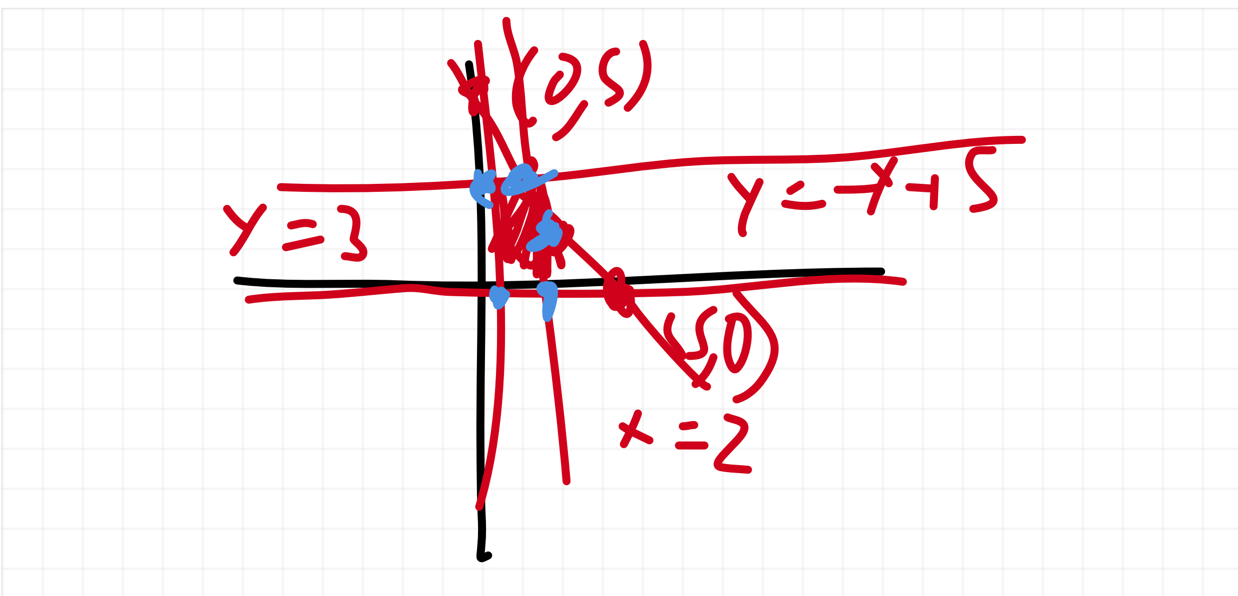
Questions on Homework 4

14. (page 256) Maximize the feasibility over $P = x - 2y$

$x + y \ \leq 5$
$y \leq 3$
$x \leq 2$
$x \geq 0$
$y \geq 0$

Note that $x + y \leq 5$ in slope intercept form is
$y \leq -x + 5$

Drawing the feasibility region, we get



The intersections are at $(0,3),\ (0,0),\ (2,0),\ (2,3)$

Plug in all the points
$P(x,y) = x - 2y$
$P(0,3) = \ -6$
$P(0,0) = 0$
$P(2,0) = 2$
$P(2,3) = \ -4$

The maximum $P(2,0) = 2$.

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