Sets Lesson 3: Venn Diagrams

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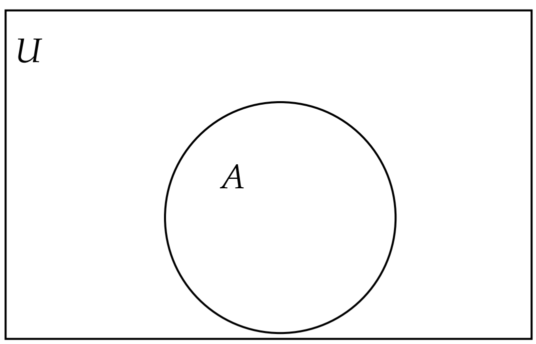
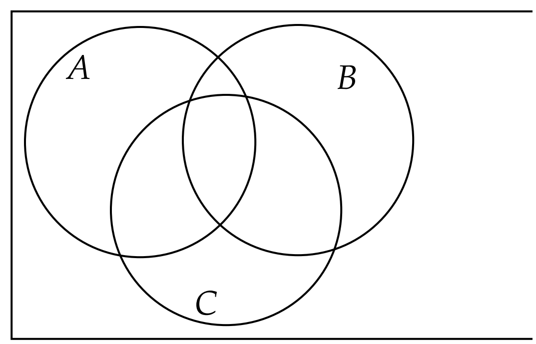
A Venn Diagram helps us picture an expression involving sets

NOTE: Typically circles represent sets, with the exception of the square that contains every circle, which is typically symbolizes the universal set U (we label the rectangle in the corner with U)



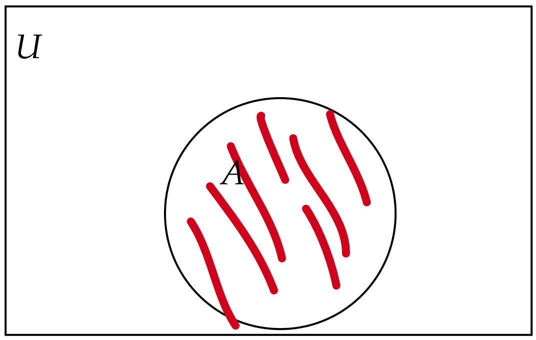


Venn diagrams are a great tool to solve problems related sets, because visually we can see the overlap that can help us visualize intersections, unions, and complements.

We can represent sets, particularly sets that are determined through set operations, by shading regions in the venn diagram.

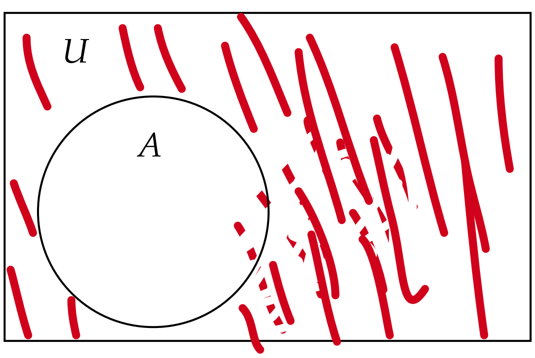
Representing Complements, Intersections, and Unions

Here are some conventions when it comes to figuring out how to shade regions based on which set operations are applied.



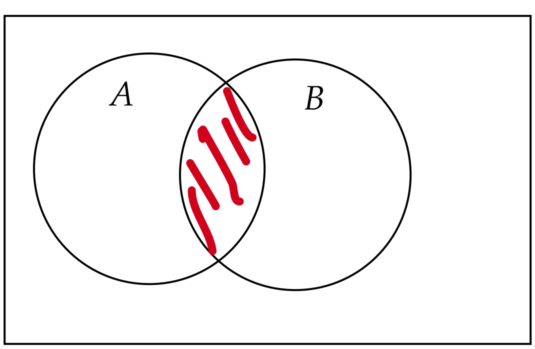
A Venn diagram representing the set A

To do that, we shade the entire region that a given set A occupies.



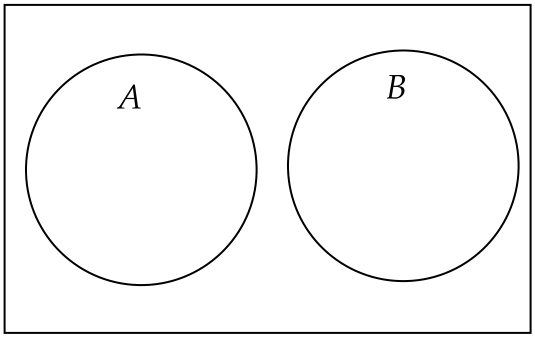
A Venn diagram representing the set A

the region that makes up the entire region outside the regior
of the original set



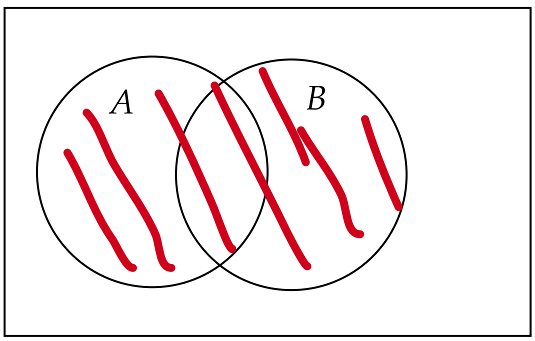
Venn diagram representing the set A

We shade any region of overlap (if any exists) that simultaneously occupy both A and B



NOTE: If A and B have no overlap, th
A and B are **disjoint**, i.e., $A \cap B = \varnothing$
and we shade nothing.

ANOTHER NOTE: Whenever we shade
empty set, we shade nothing.
"What does the empty set 'look like'?"
It looks like nothing.

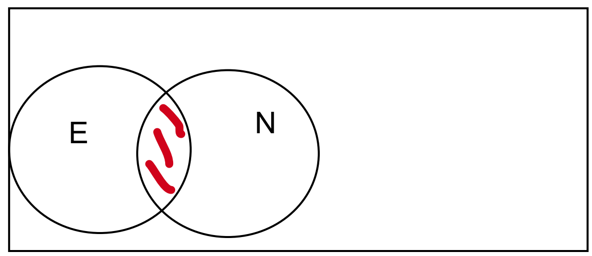


A Venn diagram representing $A \cup B$

We shade everything that is inside A OR
(OR both)

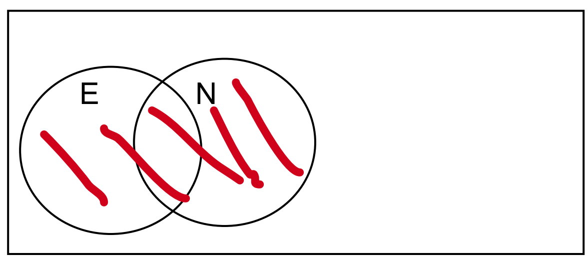
Example 3.2
Describe the shaded region using Set Builder Notation
U is the set of integers
$E = \{x | x \text{ is even}\}$
$N = \{x | x \text{ is negative}\}$

(a)



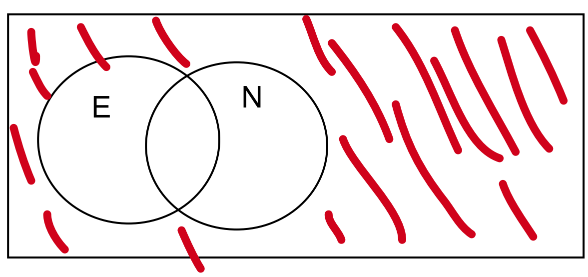
Answer: $\{x|x \text{ is even and negative}\}$

(b)



Answer: $\{x|x \text{ is even or negative}\}$

(c)



Answer:
$$\begin{aligned} &\{x|x \text{ is even or negative}\}' \\ &= \{x|x \text{ is neither even nor negat} \\ &= \{x|x \text{ is odd and nonnegative}\} \end{aligned}$$

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Last Time: We talked about Shading Venn Diagrams and what the shading represents when talking about set operations (intersection, union, and complement).

Representing Complex Set Expressions

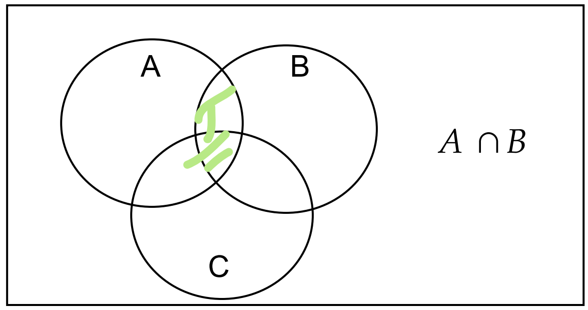
Question: How do we deal with multiple set operations (in particular, with more than two sets)?

Answer: As the order of operations of set operations (section 2) indicate, it is a multi-step drawing process.

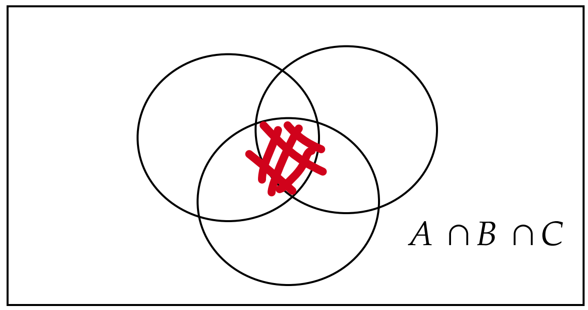
Examples.

$A \cap B \cap C$

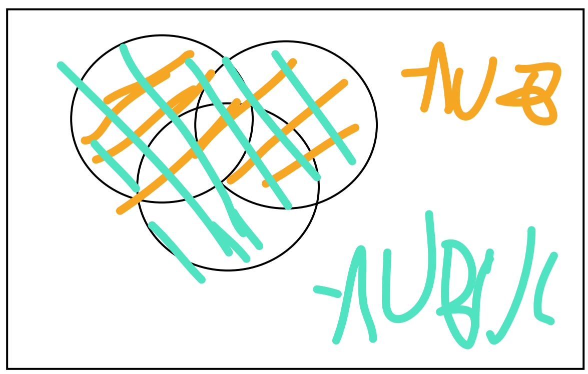
$A \cap B \cap C$:
the order of operations
does intersections and unions
from left to right
first we $A \cap B$



Next, we now that we found
the shaded region for $A \cap B$,
we intersect it with C



$A \cup B \cup C$

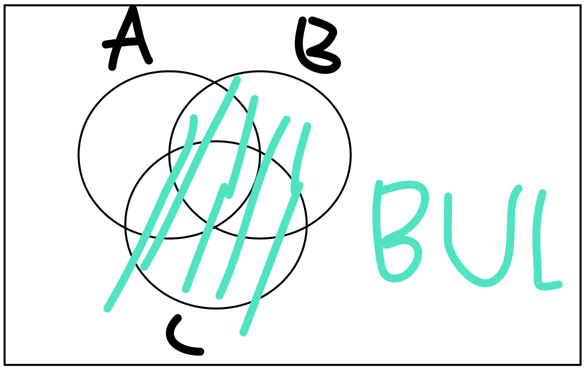


NOTE: You can do multiple shadings in the same Venn Diagram, but it may be clearer with a lot of steps to do multiple Venn Diagrams.

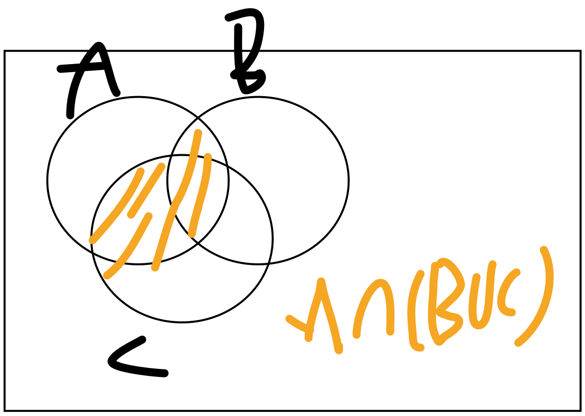
General Process:

1. Follow order of peration and find which operations go first
2. Shade the region in that given step, determined through any given region, or regions determined in any previous step.
3. If there's no additional left, then the final region is determined.

Example 3.4: Draw the Venn Diagram for $A \cap (B \cup C)$

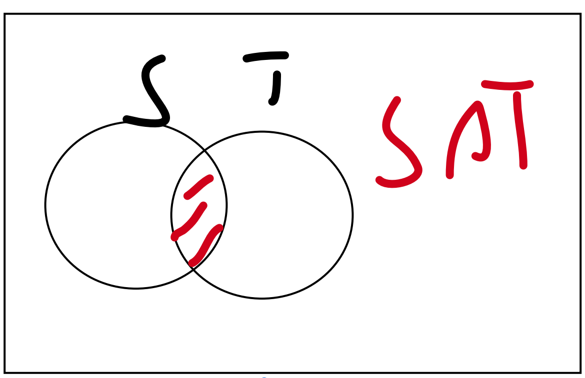


we did parenthesis $(B \cup C)$ first

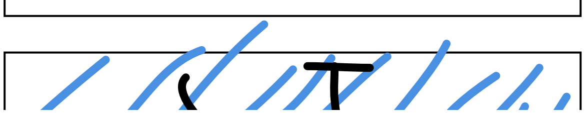


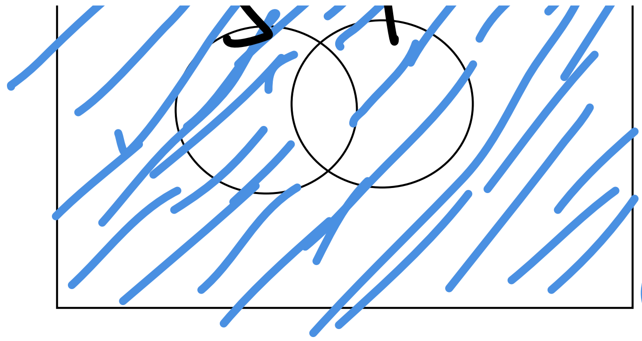
next, we intersect with A to get our <u>final answer</u>.

Example 3.5: $(S \cap T)'$



We do the intersection first, since ag
it's in the parentheses.





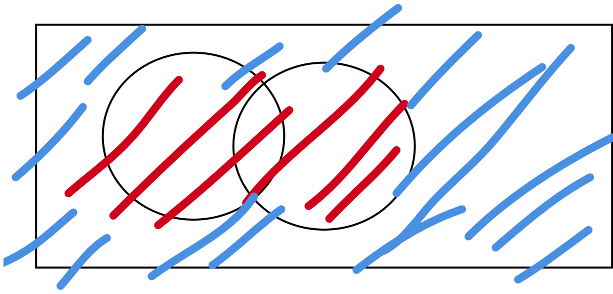
For the final step, we do the complement shading everything outside of what we sh earlier, and the blue is the final answer fo $(S \cap T)'$.

$(S \cap T)'$

Sets Homework 3 Questions

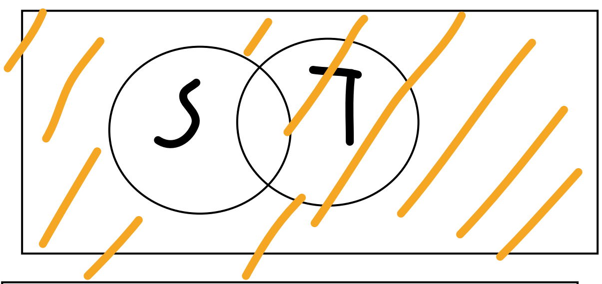
NOTE: On homework 3, if you know the steps in your head, you don't need to write all the Venn Diagrams to get to the final shading. But make sure you get the answer right.

8. Use Venn Diagrams to show $(S \cup T)' = S' \cap T'$ (one of DeMorgan's laws, and the other is in ex. 9 of hw 3)



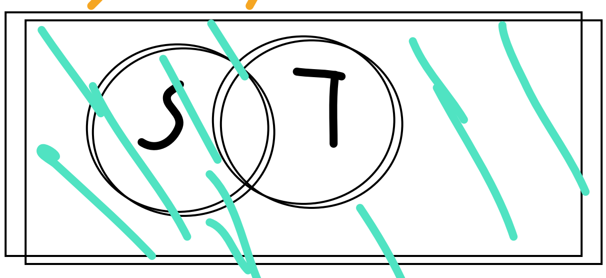
$(S \cup T)'$ 1. $S \cup T$ 2. $(S \cup T)'$

final answer is in blue

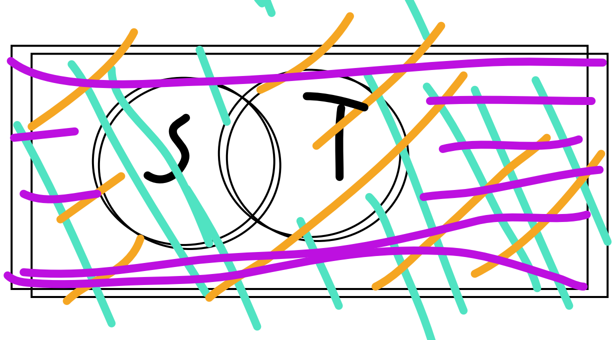


$S' \cap T'$

S'



T'

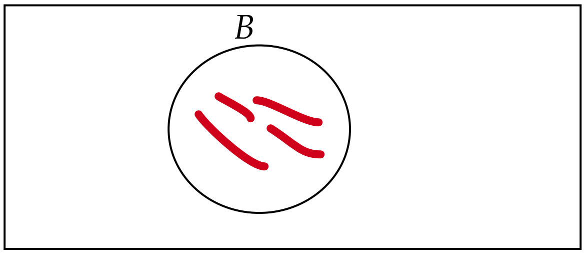


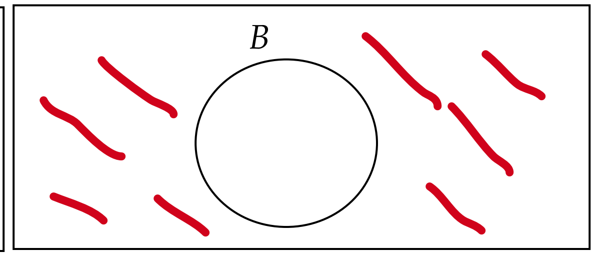
$S' \cap T'$

final answer is in purple, and notice
the final answer for $S' \cap T'$ matches $(S \cup T)'$ in blue abo

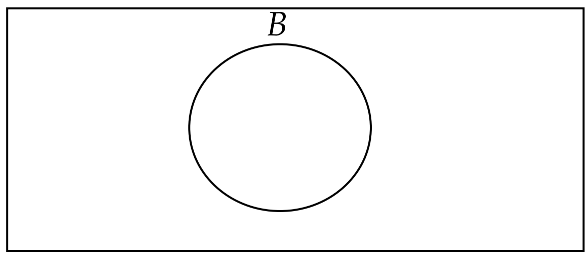
To start, we'll do 1 h. of Sets homework 3 (page 33)
h. $A \cup (B \cap B')$

4c) 3h) 1h)

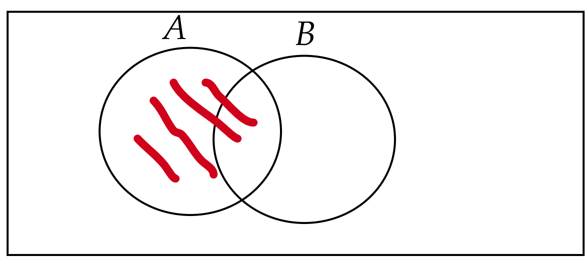




B B'



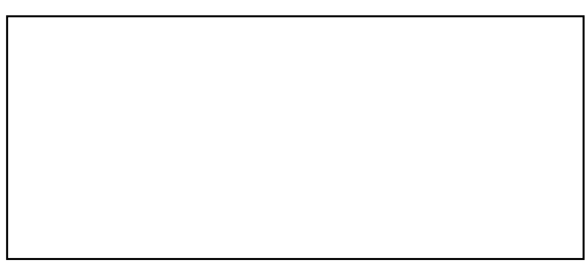
$B \cap B'$



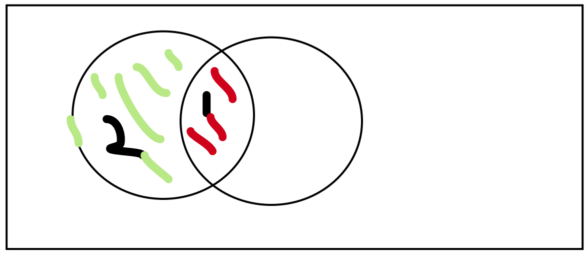
$A \cup (B \cap B')$

3h

$$(S' \cap S) \cup (S \cap T) = \varnothing \cup (S \cap T) = S \cap T$$



4c



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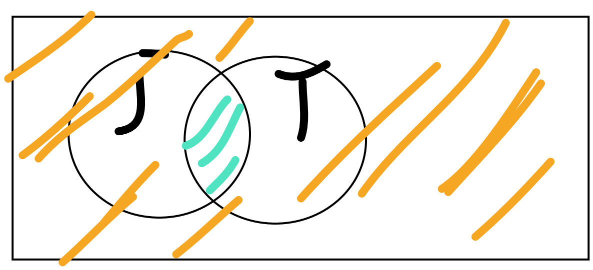
More Sets Homework 3 Questions

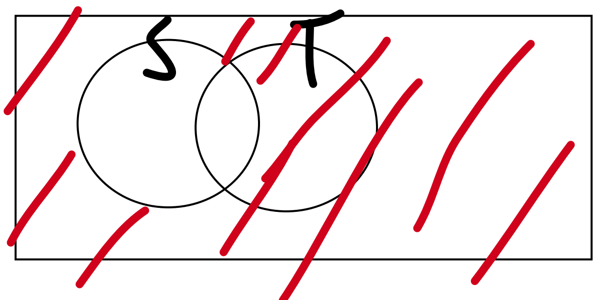
Question 9:
Prove using illustrations that $(S \cap T)' = S' \cup T'$
"neither S nor T"
"not S and not T"
"both not S and T"
Don't get it mixed up with Question 8 (which is about $(S \cup T)'$)

One thing to think about: $(S \cap T)'$ means
"not (S and T)" (different from "(not S) and T")
"not S or not T"

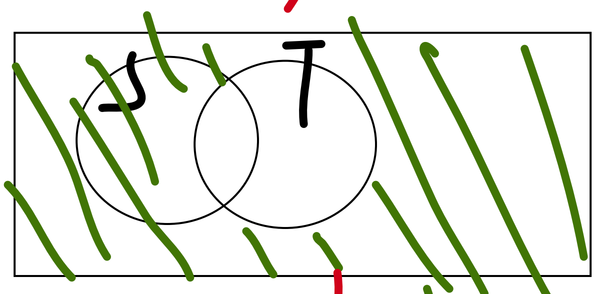


$S \cap T$

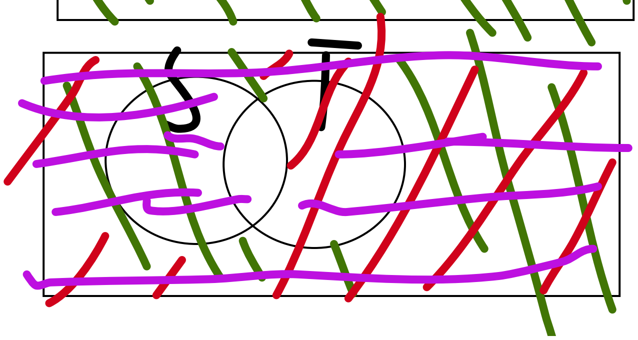
$(S \cap T)'$



S'



T'



$S' \cup T'$