Sets Lesson 1: Basic Definitions

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A **set** is a collection of objects
We refer to the "objects" as **elements**

Note that we give a lot of notation for sets, and we do that in order to be able to do math on sets.

So for this set unit, we're learning how to do math on sets.

How do we denote a set?
Often we use capital letters like A, B, C and often S.
When we talk about sets in letters, we often do that for *arbitrary* sets unless that set is specified.
(think of these capital letters as "variables")

We moreover have *specific ways* to denote sets so that we know all the elements of a given set
They are as follows:

Set Notation

1. A bracketed list:
Example:
$\{1, 2, 3\}, \{a, b, c\}, \{apple, orange, bananna\}$

NOTE 1: The capital letters come in when we write, say $A = \{1, 2, 3\}$, which means that when we mention A, we're referring to the set $\{1, 2, 3\}$

NOTE 2: Infinite sets are a thing and can be used by bracket notation; namely we can write a list and then put dots below it to signify a pattern
$\{1, 2, 3, \dots\}$

2. A bracket with a descriptor, i.e., what the Sets and Probability book calls "Set Builder notation"
Example:
$\left\{x | \ x \text{ is a positive whole number } \right\}$

$\left\{x \mid x \text{ is a fruit}\right\}$

$\left\{x \mid x \text{ is both at IU campus and not at IU campus}\right\}$

$\left\{x \mid x \text{ is an even number between 1 and 5}\right\} = \{2,\ 4\}$

Third example is a condtradictory statement and there are no elements, hence it's the "empty set", which we'll define in more detail soon.
The fourth example shows that set builder notation can be also used to denote lists of sets.

The takeaway: There's many different ways of talking about the same set

Elementhood and Equality of Sets, and Subsets

NOTATION: To signify that any object a is an element of a set A, we write $a \in A$.

What does it mean for a set to be equal?
It means generally speaking that they have the same element

Two sets A and B are said to be **equal** (we write $A = B$) if every $a \in A$ is $\in B$ and every $b \in B$ is $\in A$.

Examples:
$\{1, 2, 3\} = \{2, 3, 1\}$ (order of lists don't make a set different)
Because every element of $\{1,\ 2,\ 3\}$ is also in $\{2,\ 3,\ 1\}$ and vice versa.

$\{1,\ 2,\ 3\} = \{1,\ 1,\ 2,\ 2,\ 3\}$ (repetition of lists don't matter)
Because every element of $\{1,\ 2,\ 3\}$ is an element of $\{1, 1, 2, 2,\ 3\}$ and vice versa

$\left\{x :\ x \text{ is a natural number}\right\} = \{1, 2, 3,\ \ldots\}$

$\left\{x\ :\ x \text{ is an even number between 1 and 5}\right\} = \{2,\ 4\}$

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Last Time: We went over what a set is, and set notation, elementhood, and equality (all in Sets Probility Ch. 1 Sec. 1)

Let's continue by defining a subset

We say that A is a **subset** (" $\subset$ ")of B if every $a \in A$ is also $\in B$; in other words every element of A is also an element of B

RECALL: $a \in$ A means "a is an element of A".
We refer to the "objects" a set A as **elements**
For example with $\{1, 2, 3\}$ $1,\ 2,\ and\ 3$ are elements of $\{1, 2, 3\}$.

DISCLAIMER: " $\subseteq$ " and " $\subset$ " mean *exactly the same thing*. And note we use " $\subsetneq$ " to mean "is a subset of but not equal".

NOTATION: We mean $A\ \subset B$ by "A is a subset of B"

Examples:
$\{1,\ 2\}\ \subset \{1, 2, 3\}$ because 1 and 2 of $\{1, 2\}$ are both in $\{1, 2, 3\}$ as well
$\left\{x | x \text{ is a even positive number}\right\}\ \subset \left\{x |\ x \text{ is a positive integer}\right\}$, because every even positive number is a positive number.

NOTE: Two sets that are equal are both subsets of each other, but $A\ \subset B$ does not generally
imply $A = B$. $\{1, 2\}\ \subset \{1, 2, 3\}$, but $\{1, 2\}\ \neq \{1, 2, 3\}$.

Sets Homework 1 Questions

what does the symbol "prime", i.e. in S' mean?
Answer: It means the "complement of S", and I'll define a complement

For 6-10 in general, we want to convert list notation to set-builder notation (in the first five problems, we went from set builder notation to list notation)
What this problem wants is a description that describes everything and only everything in the list
6.
$\{1, 2, 3, 4,\ \dots, 50\} = \left\{x | x \text{ is between 1 and 50}\right\} = \left\{x | 1\ \leq x \leq 50 \text{ and x is a whole number}\right\}$
$\{1, 2, 3, 4,\ \dots, 50\} \neq \{x |\ x \text{ is between 0 and 50}\}$ because $0\ \notin \{1, 2, 3, 4,\ \dots, 50\}$
7.
$\{1, 2, 3, 4,\ \dots\} = \{x |\ x\ \geq 1\} = \left\{x | \text{x is a positive whole number}\right\}$
10. HINT:
$\{x | x \text{ is a whole number}\}$ contains more elements than $\{-5, -10,\ -15,\ \dots\}$

11. $A = \{a, x, y, z\}$, $B = \{x,\ z\}$, and $C = \{w, y, z\}$ fill in the blanks
how to do one of the parts:
a. y is one of the elements, B is one of the sets

So that narrows it down to $\in$ not $\notin$
answer: $y \notin B$

18. T' is the complement of T
$T' = \{3, 9, 15\}$
$(T')'$=the complement of T'
b. $(T')' = T$

Warm-Up

List the elements in each set (note: this is what problems 1-5 want you to do)
$\left\{x | x \text{ is between 2 and 4 or between 9 and 10}\right\} = \{2, 3, 4, 9, 10\}$
$\left\{x | x \text{ is a even integer between 3 and 11}\right\} = \{4, 6, 8, 10\}$

The Universal Set, the Empty Set, and Complements

A **universal set** is defined to be a "reference set" with all the elements.

NOTE: We use the letter U to denote a universal set, and U is an arbitrary choice.

The **complement** of a set A (denoted A') means everything in the universal set U that is *not* in A.
$A' = \left\{x | x \in U \text{ and } x \notin A\right\}$

Examples:
Let's do a bit of problem 13: $S = \{2, 3, 4\}$, find S' for each part
13. a.
$U = \{1, 2, 3, 4, 5\}$
S' is everything not in S
What are the elements of U that are not in S
anything that is not $\{2, 3, 4\}$
$S' = \{1, 5\}$
b.
$U = \{1, 2, 3, \dots, 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = \left\{x | x \text{ is a whole number between 1 and 10}\right\}$
$S' = \{1, 5, 6, 7, 8, 9, 10\}$
c. $U = S$, so $S' = \varnothing$

The **empty set** is a set without *any* element, and is denoted $\varnothing$
ANY object $a \in U$ is *not* in the empty set, so $a \notin \varnothing$.

NOTE: There are many ways to talk about the empty set
1. $\{\} = \varnothing$
2. Set builder notation with contradictory information:
$\left\{x|x \text{ is between 2 and 4 and between 9 and 10}\right\} = \varnothing$
$\left\{x|x \text{ is an IU student and Purdue Student (at the same time)}\right\} = \varnothing$

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Last Time: We talked about subsets, the universal set, complements, and the empty set (finished off Ch. 1 sec. 1 of Sets and Probability).

Warm-up

1. Let $A = \{5, 6, 9,\ 10\},\ B = \{5, 9\},\ C = \{8\}$

Tell me whether this is true or false
a. $A \subset B$ false
b. $A \subset C$ false
c. $B \subset A$ true
d. $C \subset A$ false

2.
$X = \{x|\ x\ is\ an\ integer\}$
$Y = \{x|x\ is\ divisible\ by\ 3\}$
$Z = \{1, 2, 3\}$
True or false
a. $Y \subset X$ true
b. $Z \subset X$ true
c. $Z \subset Y$ false
3. List all the subsets of:
$\{2, 3\}$
$\{2, 3\},\ \{2\},\ \{3\},\ \varnothing$

IMPORTANT NOTE: $\varnothing$ is a subset of everything, because "everything" that is in the emptyset is any the other set "vacuously" (because there's nothing in the empty set).

Sets Homework 1 Questions

23 $U = \{all\ students\}$
$E = \{x | x\ is\ enrolled\ in\ english\}$
$P = \{x | x\ is\ enrolled\ in\ Psych\ 101\}$

a. $E' = \varnothing$
$E' = all\ students\ not\ enrolled\ in\ english$
The empty set $\varnothing$ in general is the set with no elements, so to say that any set is equal to the empty set is to say (in english) that there's nobody that fits the description of that set.

Translation: "there are no students not enrolled in english"
"every student is enrolled in english"
$E = U$

c.
$P \subset E$
"everyone in psych is in English"

IN GENERAL: $A \subset B$ translates to "everything that has quality A also has quality B".

24. The experiment is chooseing two bills from the wallet and recording their total value
$S = \{x | x\ is\ a\ possible\ outcome\ of\ the\ experiment\}$

The problem asks to list all possible elements:

HINT: Every element is the sum of a given choice of two bills, so let's say two $\$1$ bills are chosen. Then
$\$1 + \$1 = \$2$
$\$1 + \$5 = \$6$
$\$1 + \$20 = \$21$
$and\ so\ on$

$\$2,\ \$6,\ and\ \$21 \in S$, and there more elements from the possible sums.
NOTE: I won't penalize you for writing the list in repitition (because it doesn't matter with sets).