Linear Equations Lesson 1: Algebra Review

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NOTE: From now on, we'll begin using Gustafon and Frisk (7th ed. algebra textbook), so make sure that you have a copy by no later than next week (homework will be due this coming Monday and Tuesday).

This week: We'll go over Section 1.5, 2.1 (90% should be review)

Next week: We'll go over the first half chapter 2 (until I believe 2.4), and then we'll go over Chapter 3 (by the end of next week)

After Linear Equations: We'll go on to inequalities

So the next "algebra" unit has two modules (similar to the Sets and Probability unit) and these modules are Linear Equations and Inequalities. Inequalities feeds off of Linear Equations (kind of like how probability feeds off of sets).

Properties of Equality

Section 1.5 of Gustafon and Frisk

Totally review:

An **equation** is a statement of the form "$a = b$", where a and b are two **terms** (they could be terms with the $+$ and $\cdot$ symbol), so for exaple $2 \cdot 2 + 2 = 6$

As you probably remember, equality has the following preservation properties:

1. *(preservation of adding terms on the same side)*
If $a = b$, then $a + c = b + c$

2. *(preservation of multipying terms on each side)*
if $a = b$, then $a \cdot c = b \cdot c$ (note we often ac as shorthand for $a \cdot c$)

3. *(preservation of division)*
If $a = b$ and $c \neq 0$ (remember that we can't divide by zero), then $\frac{a}{c} = \frac{b}{c}$.

Next Time: We'll a single variabled Linear Equations and go over basic algebra for solving for x.

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NOTE: We're done with the Probability and Sets book, and now we move on to *Algebra for College Students 7th ed* by *Gustafon and Frisk* (in the canvas, I'll call it "Gustafon and Frisk" for short)

Last Time: We defined equations and reviewed preservation of equality.

Solving Linear Equations

Follow along at page 48 of Gustafon and Frisk:

A **linear equation** in *one variable* (usually we use the letter x to talk about the variable) is an equation that can be rewritten in the form $ax + c = 0$ (where a and c are real numbers and $a \neq 0$)

With a linear equation, we often like to find the **solution** to the linear equation, which is an equation that defines x, so it's of the form
$x = -c/a$ or just $x = b$

Example 2 (page 48): Solve $2x + 8 = 0$

$$\begin{array}{lll} 2x + 8 & = & 0 \\ \quad -8 & & -8 \\ 2x & = & -8 \\ \div 2 & & \div 2 \\ x & = & -4 \end{array}$$

Often a linear equation is not presented so nicely, sometimes we get something like
$2x + 7 = x + 5$,
$2x + 5x + 9 = 70$,
and so on, and they're linear equations, but they're not in the form we like of $ax = b$, or $ax + b = 0$. So we need to "combining like terms" so that we're able to proceed as we like.

First question: What exactly is a term? A **term** (i.e. an "algebraic term") either a number or a product of numbers and variables. It's a way of expressing numbers using variables (like x), constants (like $a,\ b,\ 4,\ 5,\ \pi$), and the $+,\ \cdot$ symbols (and maybe even the exponential symbol).

Terms with that common factors are called **like terms** or **similar terms**

A **coefficient** of a term of the form ax is the constant a that is multiplied by the variable.

Examples.
25 and 5 are like terms because 5 factors them both
$5x$ and $6x$ because x factors them both
$27x^2y^3$ and $-326x^2y^3$ are like terms, since x^2y^3 factor them both.

As a general rule, to solve a linear equation, we want to "combine like terms".

Example 4 (page 49). Solve $3(2x-1) = 2x+9$

$$\begin{aligned}
3(2x-1) &= 2x+9 \\
6x-3 &= 2x+9 \\
-2x \quad &\quad -2x \\
4x-3 &= 9 \\
+3 \quad &\quad +3 \\
4x \quad &= 12 \\
\div 4 \quad &\quad \div 4 \\
x \quad &= 3
\end{aligned}$$

(note that the first step is distributive property)

NOTE: If you're concerned that you made a mistake, there's a good way to check you answer, which is to plug the solution you found back in, so doing that, we get

$$\begin{aligned}
3(2(3)-1) &= 2(3)+9 \\
3(6-1) &= 6+9 \\
15 \quad &= 15
\end{aligned}$$

WE'RE GOOD!

Let's give a general step by step process to solve equations:

1. If the equation contins fractions, multiply both sides by a number that will eliminate the denominator.

2. Use the distributive property (as we did example 4) to remove all the parentheses and combine like terms.

3. Use addition and subtraction to get all the variables on one side of the equation, and all the numbers on the other side.

4. Use multiplication and division properties to make the coefficient of the variable equal to 1.

5. Check the result by plugging in the answer we got.

Example 5. Solve $\frac{5}{3}(x-3) = \frac{3}{2}(x-2)+2$.

We start off with the equation

$$\frac{5}{3}(x-3) = \frac{3}{2}(x-2)+2$$

Step 1, we multiply a common multiple of the denomenators to eliminate them

$$\frac{5}{3}(x-3) = \frac{3}{2}(x-2)+2$$

$$\times 6 \qquad \times 6$$
$$10(x-3) = 9(x-2)+12$$

NOTE 1: multiplying $5/3 \cdot 6 = 5 \cdot 2$, since the 3 and the 6 cancel.

NOTE 2: make sure to multiply everything being added up (don't forget to multiply a number being added)

Step 2, we then use distributive property to get rid of the parenthesis
$$10(x-3) = 10x-30$$
$$9(x-2)+12 = 9x-18+12$$

to give us
$$10x-30 = 9x-18+12$$

For Step 3, we combine like terms
$$10x-30 = 9x-6$$

Now we're in business to do steps 4 and add multiply on both sides to solve for x and we get
$$10x-30 = 9x-6$$
$$-9x \qquad -9x$$
$$x-30 = -6$$
$$+30 \qquad +30$$
$$x \qquad = 24$$

Now for the last step (step 5), we check our result:

$$\frac{5}{3}(24-3) = \frac{5}{3} \cdot 21 = 35$$
$$\frac{3}{2}(24-2)+2 = \frac{3}{2} \cdot 22+2 = 33+2 = 35$$

One more thing to note about linear equations is that the solutions don't always exist, and sometimes the solution doesn't exist the way we want them to. Sometimes equations are BAD EQUATIONS

Let me give two examples
$5x = 5x,$
$x = x+2,$

In the first example, any x is a solution, in other words, plug in any value of x and it works. Such a equation is called a **identity**

In the second example, any x doesn't work, since for any number a
$a \neq a+2$
Such an equation with no solution is called a **contradiction**

NOTE: For homework 1 (page 45), in problems 47-88, some of the problems have equations that are identities or contradictions, and if you run into those, you just write "identity" or "contradiction"

81.
$4(2-3t)+6t = -6t+8$ identity (that's the answer)

82.
$2x-6 = -2x+4(x-2)$ contradiction (that's the answer)

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Last time: We talked about what linear equations, and did some examples, and into the possible scenarios with finding a solution. In particular, we talked about the usual scenario where there's a single solution to an equation, and the other two scenarios where we have an "identity" and "contradiction".

Linear Formulas

Usually when we work with linear equations, we work with concrete numbers, so anything of the form

$ax + b = 0$ where a and b are concrete numbers, like $4, 5,\ 5/2,\ \pi$

Sometimes we do algebra on "formulas" where the numbers are letters and not concrete. We deal with this all the time in science.

$P = nRT$, $E = kx^2/2$, $a^2 + b^2 = c^2$

and note that the numbers are not concrete. Sometimes what we do in that instance is to designate one "letter" as our variable, and we "solve" for that variable in terms of the other letters.

Example 9 (page 53) Solve $A = 1/2bh$ for h

We want to have h by itself. So to do that, note that $1/2b$ we can treat as a number, and all we have to do is divide by that number and then we get h by itself.

$$1/2bh = A$$
$$\div 1/2b \quad \div 1/2b$$
$$h \qquad = \frac{A}{1/2b} = \frac{2A}{b}$$

To do this example more conventionally in a way that follows the steps, we can get rid of the fractions first.

$$1/2bh = A$$
$$\times 2 \qquad \times 2$$
$$bh = 2A$$
$$\div b \quad \div b$$
$$h = \frac{2A}{b}$$

Example 10 (page 53) Sove $A = p + prt$ for r

step 1. No fractions (check)

step 2. no parenthesis, and no like terms needed to combine (check) step 3. any addition or subtraction to get the coefficient by itsef?

yes we do

$A = p + prt$

$-p \quad -p$

$A - p = prt$

step 4. divide the coefficient to get 1 as the coefficient. What is the coefficient? The coefficient

is pt because $prt = (pt)r$. So we want to divide by pt to get

$A - p = (pt)r$

$\div pt \quad \div pt$

$\frac{A-p}{pt} = r$

NOTE: In step 5, we can check the answer (if we want to)

Questions on Homework 1

For 47-88, we find the unique solution if the solution exists, and mention that it's an identity or contradiction if either everything solution

What you want to do is proceed as normal with steps 1-5 to get the solution as if the equation was "normal" and had a solution, but then if you run into the following "hiccups", then you end up with an identity or a contradiction:

If you have something like $x = x$, $0 = 0$ when trying to solve the equation, you have an "identity"

And if you have a "literal contradiction" like $x = x + 2$ or $0 = 1$ then you have a "contradiction"

(page 55)

47.

$3a - 22 = -2a - 7$

$+2a \quad\quad + 2a$

$5a - 22 = -7$

$+22 \quad\quad + 22$

$5a = 15$

$\div 5 \quad \div 5$

$a = 3$

81.
$4(2-3t)+6t = -6t+8$

Step 1: no fractions

Step 2: Get rid of the parenthesis and combine like terms

$4 \cdot (2-3t) = 8-12t$

$8-12t+6t = -6t+8$
$8-6t = -6t+8$

Note, we have the same thing on both sides, so we have an identity. We know that because plug in any number for t and it works.

82.
$2x-6 = -2x+4(x-2)$

Step 1: no fractions

Step 2: We have parenthesis and like-terms

$4 \cdot (x-2) = 4x-8$
$2x-6 = -2x+4x-8$
$2x-6 = 2x-8$
$-2x \quad -2x$
$-6 \quad = \quad -8$

We get a CONTRADICTION because $-6 \neq -8$.

70.
$\frac{x}{2}+\frac{x}{3} = 10$

We have fractions, so we're going to need to actually do step 1.

Step 1. Find a common multiple of the denominators. 6 is a good common multiple (Note that we just need a common multiple, we don't need to worry about the least common multiple) because 2 and 3 both divide it.

NOTE: Don't forget to multiply divide,and add on both sides. Number one source of errors is forgeting to do so!

$\frac{x}{2} + \frac{x}{3} = 10$

$\times 6 \quad \times 6$

$\frac{x}{2} \ \cdot 6 + \frac{x}{3} \cdot 6 = 10 \ \cdot 6$

$\frac{x}{2} \cdot 6 = 6x/2 = 3x$
$\frac{x}{3} \ \cdot 6 = 6x/3 = 2x$

$3x + 2x = 60$

Step 2. Deal with parenthesis and combine like terms. No paranethesis, but we do have like-terms, i.e. $3x$ and $2x$ (since they're both terms that is a coefficient times the variable)

$5x = 60$

We can then go straight to step 4, and divide

$5x = 60$
$\div 5 \quad \div 5$
$x = 12$

When in doubt, check your answer. (step 5)