Probability and Counting Lesson 1: Sample Spaces and Events

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Let's conclude this lecture by talking about how sets go hand-in hand with probability by showing how sets are utilized.

To determine probabilities, it's important to keep in mind the *sample space* that we're working with. What do I mean by a sample space?

I mean a set S (one can think of it as a universal set) of what we call outcomes.

And to find the probability of something occuring, we formalize this occurence as an *event,* i.e., a subset of the sample space.

Next Time: We'll go into greater detail of defining sample spaces and events more rigorously and then seeing how it's important for computing the probability of something happening.

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Last Time: Recapped Sets lesson 4 (about counting the amount of elements in a set) and we introduced Probability Spaces in terms of Sample Spaces

Now we're going to talk about sample spaces (and events) in more rigorous detail

Since we're going through the trouble of defining sample spaces and events (using sets), the question is why bother?

The reason it's not obvious otherwise how to make sense and compute probabilities.

The Boy or Girl "Paradox"

Let me pose the folliwng "trick question"

My friend Nik has two kids, and he told you he has at least one girl; thus what is the probability that the other kid is female?

What I would think the answer is: $1/2$
Actual Answer: $1/3$

What is happening?

What's happening is the sample space is not what we expect (we'll go into more detail as to why that is later), and the moral of the story is the specific sample space where you compute the probability MATTERS.

Sample Spaces

Let's start with this informal definition

A **probability experiment** is a "random occurence", i.e., an actin where a rangle of possible scenarios, known formally as **outcomes**, happens by chance.

Some Examples of probability experiments:

1. Flipping a coin is a probability experiment, and the possible faces that it lands on are the outcomes.

2. Rolling a dice is a probability experiement, and the possible numbers that it rolls into are the outcomes.

3. Drawing a hand of five cards from a standard 52 card deck is probability experiement, and the specific combination of five cards selected are the outcomes.

A **Sample Space** is the (universal) set of all possible outcomes in a probability experiement.

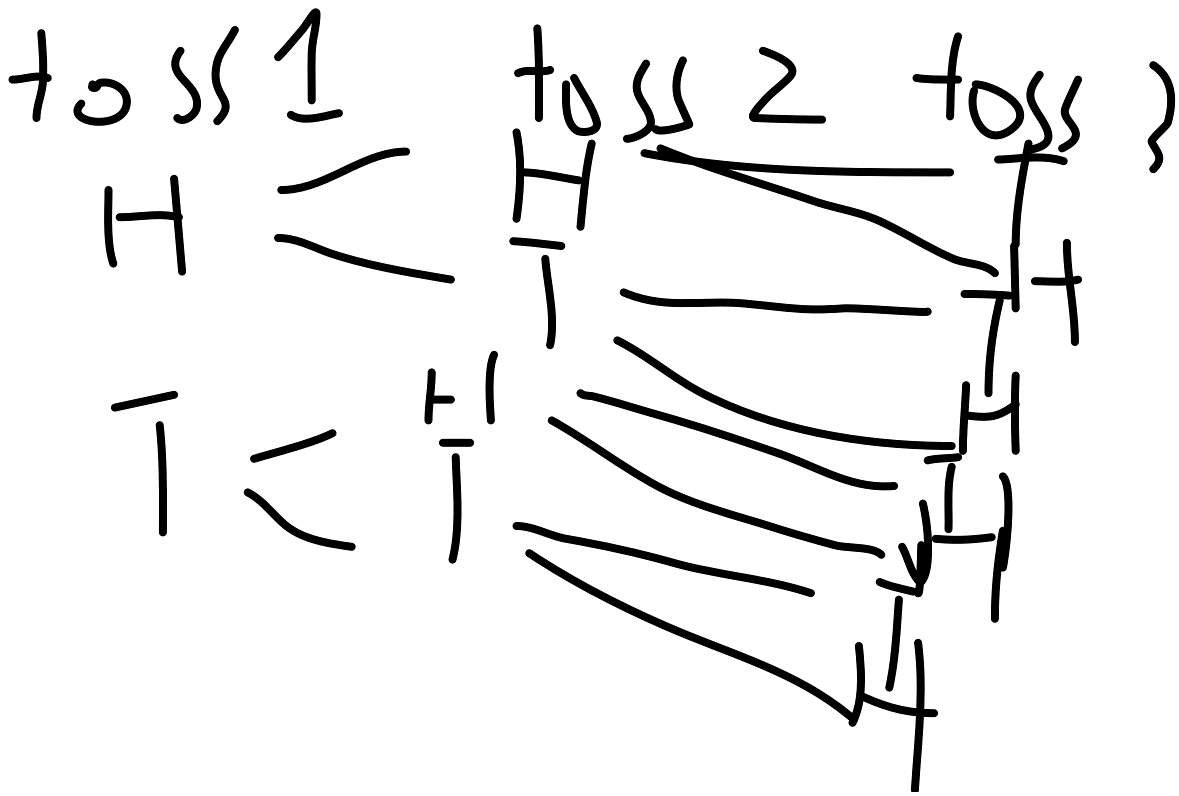
NOTE: The outcomes are <u>elements</u> of the sample space, though the outcomes could be described as something more specific (see the examples below)

Example 1.1 (page 54): Suppose that a six sided die is rolled once. What is the sample space?
Answer: $S = \{1, \ldots, 6\}$

Example 1.3 (page 55): A fair coin is tossed *three* times and the side showing up is recorded (in the order that the coin is flipped). What is the sample space of that?

Answer: The sequence of heads and tails that appear in the order that the coin is tossed. We often write such as, say HHT if two heads turn up in the first two tosses, then a tail

All such sequences make up the sample space. How do we get them all? We'll talk about "product sample spaces" in sec. 3, but for now, it helps to draw a tree.



We end up with eight such different sequences:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Events

An **event** in a Probability Experiment/Sample Space is a subset of the Sample Space.

NOTE: An event is a collection of outcomes that contain some, all, or even none of the outcomes.

Example 1.5 (page 58). A fair coin is tossed three times and the faces are recorded in a sequence (as in example 2.3). Here are some events

$E =$ The event that exactly 2 heads come up

F = the event that heads comes up on the second toss
G = the event that tails comes up at least once

more events in that example. Let's compute $E,\ F,$ and G as sets in list notation.
(Remember that $S = \{HHH,\ HHT,\ HTH,\ HTT,\ THH,\ THT,\ TTH,\ TTT\}$)

$E = \{HHT,\ HTH,\ THH\}$, because those outcomes are precisely the ones where exactly two heads appear.
$F = \{HHH,\ HHT, THH,\ THT\}$
G = everything except $HHH = \{HHH\}' = \{HHT,\ HTH,\ HTT,\ THH,\ THT,\ TTH,\ TTT\}$.

Probability Homework 1 Question:

(starts at page 63)

NOTE: "equally likely" means that each outcome has the same chance of happening. (I'll define it in greater detail in Lesson 2; the book defines it in greater in Ch. 2 section 2)

1.

a. The outcomes are
$1, 3, 5, 7, 2, 4, 6,\ 9$
Yes, it is equally likely, because each ball is selected at random and each ball has a distinct number.

b. The outcomes are
$red,\ blue$
Yes, it still is equally likely, because each color consists of an equivalent number of balls.

c. The outcomes are
$odd,\ even$

To find the answer, let's count the number of even balls.
$n(even\ balls) = 3$
$n(odd\ balls) = 5$
As a result, no it's not equally likely, because there are odd than even balls.

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More Probability Homework 1 Questions:

Problem 3. We'll do in class today
See the example in lesson 3 today.

9/11 UPDATE: For navigation purposes, I'll copy and paste the explanation here.

What is the sample space of the probability experiment where one rolls a dice then flips a coin.

So here, we have two-staged experiment where
$S_1 = \{1, \dots, 6\}$ (because we're rolling a dice)
$S_2 = \{H, T\}$ (because we're flipping a coin)

The outcomes of this experiment are the ordered pairs (s_1, s_2) where s_1 is a whole number between 1 and 6 and s_2 is either H or T
$S_1 \times S_2 = \{(1,\ H),\ (2,\ H),\ (3,\ H),\ \dots, (6,\ H),\ (1,\ T),\ \dots, (6,\ T)\}$