Sets Lesson 4: Using Venn Diagrams

9/1

In practice, we use sets (and the logical reasoning of sets give us) to solve counting problems such as this one.

Example 4.1 (page 37)
Of 100 students enrolled in an algebra course, 65 were also taking English composition and 50 were also taking Psych. If 35 of the algebra students were enrolled in both English and Psych, how many were not enrolled in either English or Psych?

If you don't know how to do this problem, not to worry. This example highlights the usefulness of sets in our ability to count how many objects/people represent a given category.

Notation for Counting Elements

We use $n(S)$ notation for a given set S, which means the number of elements.

$n(S) =$ "the number of elements in S"

Example 4.3 Suppose $U = \{a, b, c, \ldots, g\}$
$A = \{a, d, e\},\ B = \{b, c, d, g\},\ C = \{b, c\}$

$n(A) = 3,\ n(B) = 4,\ n(C) = 2$

Next time: We'll go over counting more sophisticated sets such as $n(A \cup C)$ and $n(B')$, and we'll show how the diagrams help us, and go over some important formulas.

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Last Time: We finished lesson 3 and talked multistaged diagram drawings for complicated sets, and then we began talking about counting sets. We left off at example 4.3
$n(U) = 7$ because there is 7 different elements total in U
$A \cup C = \{a, b, c, d, g\}$
$n(A \cup C) = 5$ because there is five different elements total in $A \cup C$
$B' = \{a, e, f\}$
$n(B') = 3$ " "
$n(A \cap B)$
we find $A \cap B = \{d\}$
$n(A \cap B) = 1$

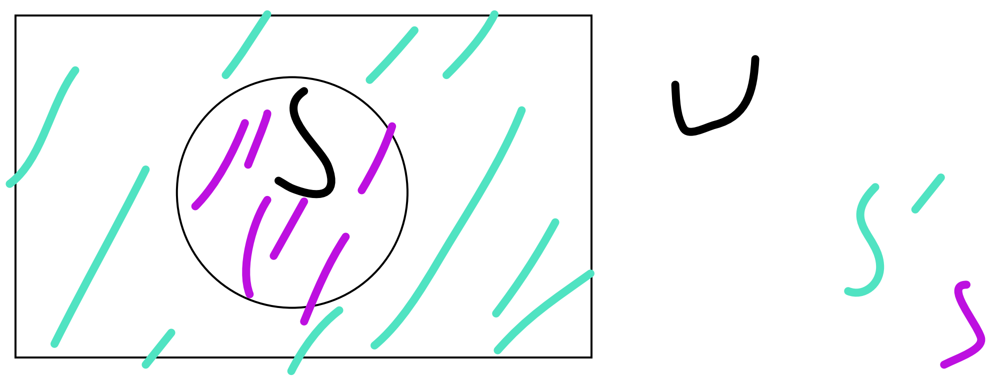
Note that $n(B) + n(B') = n(U)$ in this example?

Is that coincidence? I think not!

Formulas for Counting Sets

So there's ways compute sets other than counting their individual elements, which are useful when there's too many elements to count by hand.

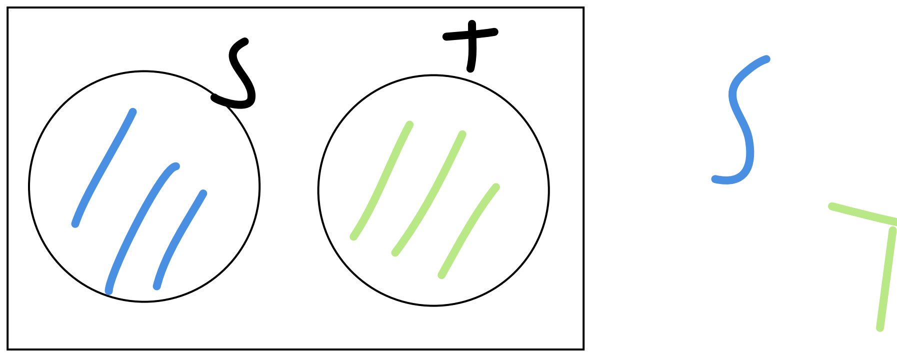
Formula 1: The Complement Formula
For any set S,
$n(S) + n(S') = n(U)$



Useful Variation:
$n(S') = n(U) - n(S)$

Formula 2: The Disjoint Union Addition Formula

If S and T are disjoint (if they don't have any elements in common, $S \cap T = \varnothing$), then
$n(S \cup T) = n(S) + n(T)$



NOTE:

In this situation, we don't worry about overlap, which is why they add together (this not true when the sets do overlap, ie., when S and T are not disjoint.

This is a generalization of formula 1 in the sense that $U = S \cup S'$ and S and S' are disjoint

This formula can be generalized even with multiple disjoint $S_1, \ \ldots, S_n$ for arbitrary n in the following way:

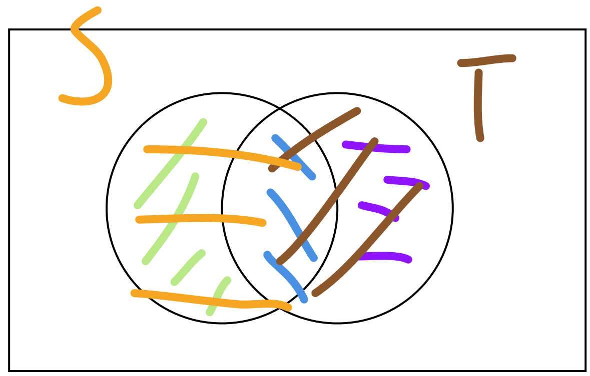
$n(S_1 \cup \cdots \cup S_n) = n(S_1) + \cdots + n(S_n)$

Formula 3: The Intersection-Union Formula

The most general one (each formula is generalized by the next)

$n(S \cup T) = n(S) + n(T) - n(S \cap T)$

In the general situation where the intersection is nonempty, the idea is to sum up the two sets, and trim the overlap to get the union



$S \cap T'$

$S \cap T$

$S' \cap T$

$n(S \cup T)$ is derived from the disjoint union addition rule, in the end.

Important Variant of formula 2:
$n(S \cap T) = n(S) + n(T) - n(S \cup T)$

Example 4.4 page 44: S has 20 elements (means $n(S) = 20$), T has 15 elements $\left(\text{means } n(T) = 15\right)$, and 6 elements "in common" (this means $n(S \cap T) = 6$). Find how many elements are in the union

To do that, we use formula 3 (the intersection-union formula)
$n(S \cup T) = n(S) + n(T) - n(S \cap T) = 20$ +15-6=29

Different Example (not in the book):
S has 30 elements, T has 10, and $S \cup T$ has 35 elements. What is the number of elements in $S \cap T$:
From this problem, we know:
$n(S) = 30,\ n(T) = 10,\ n(S \cup T) = 35$

To solve this, we use the variant:
$n(S \cap T) = n(S) + n(T) - n(S \cup T) \ = 30 + 10 - 35 = 5$.

Next Time: We'll go over more sophisticated examples of using these formulas, and the diagram (whenever you have three sets), and then we'll try to cover sample spaces.

9/3

Last Time: We talked counting elements of sets and some formulas to help us do that.

This lecture, we plan to count sets using a picture, which will help us solve problems involving three sets.

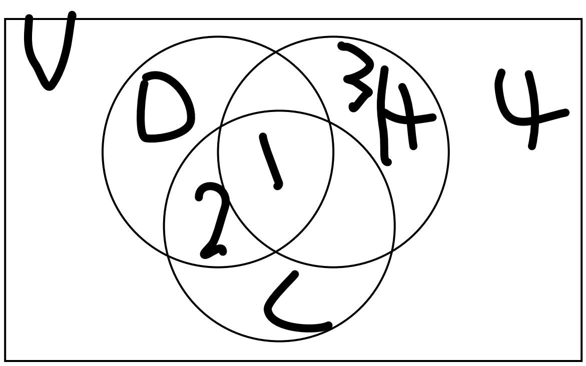
Solving Counting Problems Given in English

"It's a reading class, not a math class" (at least for the Sets and Probability unit!)

Example 3.3 (page 27)

$C =$ All respondents who drink Classic Coke
$D =$ all respondents who drink Diet Coke
$H =$ All respondents who drink Cherry Coke

Describe in English the respondents represented by each number.



Region 1: "People who drink all three types of coke" ($C \cap D \cap H$)
Region 2: "People who drink diet and classic but not cherry" ($C \cap D \cap H'$)
"People who only drink diet and classic"
NOTE: For three sets, make sure that the region includes information about what is NOT included in the region.
Note that there's a difference between region 2 and $D \cap C$ (which is region 1 and 2)
Region 3: "People who only drink cherry" ($D' \cap C' \cap H$)
"People who drink Cherry but not Classic and not Diet"
Region 4:
"People who don't any of the three cokes" ($(D \cup H \cup C)'$)
"People who drink neither diet, classic, nor cherry"

"People don't drink diet, don't drink classic, and don't drink cherry" ($D' \cap H' \cap C'$)
NOTE: In region 4, there's a more general DeMorgan's Law going on.

Example 4.1 (page 37)
Of 100 students enrolled in an algebra course, 65 were also taking English composition and 50 were also taking Psych. If 35 of the algebra students were enrolled in both English and Psych, how many were not enrolled in either English or Psych?

So to solve a problem like this, first we interpret the english.
$U =$ all students taking algebra
$E =$ those students who take english
$P =$ those students taking psych

$n(U) = 100$
$n(E) = 65$
$n(P) = 50$
$n(E \cap P) = 35$

not enrolled in either English <u>or </u>Psych=$(E \cup P)'$
The "or" signifies <u>union</u> and the "not enrolled in either" signifies <u>negation</u>.

Usually (look out for context) "and" signifies intersection (when it's part of a object consisting two things).

We can use the forulas, and we can visualize the diagram.

One is to use the formulas.

We can find $n(E \cup P)$ by using the union-intersection formula (since we have the info for that formula)

$n(E \cup P) = n(E) + n(P) - n(E \cap P) = 65 + 50 - 35 = 80$.
We have one more step, which is to use the complement formula to solve for what we want, which is $(E \cup P)'$

$n((E \cup P)') = n(U) - n(E \cup P) = 100 - 80 = 20$.

But there's another way to do that!

Using Diagrams to Solve Counting Problems

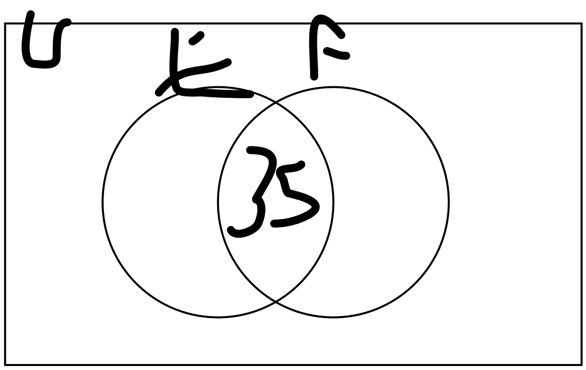
Formulas can get a bit tedious (esp. with three sets), so using diagrams is easier perhaps to keep up

With Two Sets:

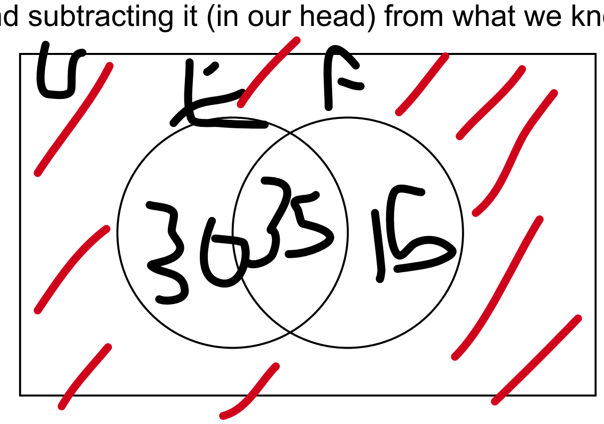
We'll compute compute example 4.1 in this way Recall that
$n(U) = 100$
$n(E) = 65$
$n(P) = 50$
$n(E \cap P) = 35$

We draw all the disjoint regions, using what we know about the intersections and using what we know about the compound regions of sets to find what each of disjoint components are

Observe how we know that the intersection is 35.



We can then determine $E \cap P'$ and $E' \cap P'$ by then taking the information about E and P and subtracting it (in our head) from what we know about the intersection. Doing this, we get



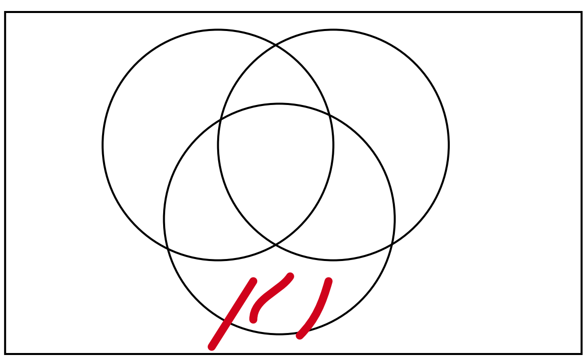
And to get the desired part (in red), we use the information about U and the information that we found about the disjoints components to get

$100 - (30 + 35 + 15) = 100 - 80 = 20$.

With Three Sets:
An advantage of solving problems this way is we can generalize easily to three sets

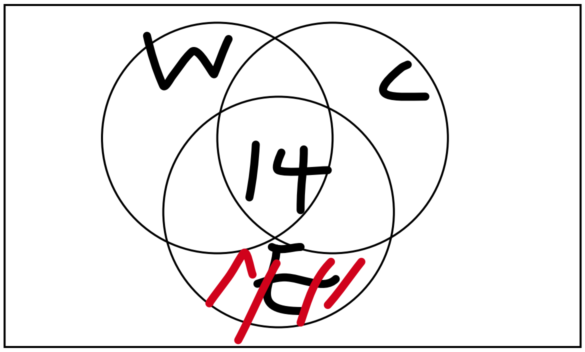
Example 4.7 (page 46)
There's email E, word processing W, and C for computerized instruction
63 used email (means $n(E) = 63$)
70 used word processing (means $n(W) = 70$)
44 used computerized instruction (means $n(C) = 44$)
20 had used email <u>and</u> computerized instruction ($n(E \cap C) = 20$)
34 had used word processing and computerized instruction ($n(W \cap C) = 34$)
38 had used word processing and email ($n(W \cap E) = 38$)
14 had used all three services ($n(W \cap E \cap C) = 14$)

a. How many of the students surveyed used email but not word preocessing?
$E \cap (W \cup C)'$
Let's identify the region we want:



Everything above is given

We start by taking what we know about the dead center disjoint region $W \cap E \cap C$, and writing that down



next, we want to take what we know about the intersections that overlap with $W \cap E \cap C$, and determine the other parts

We know $n(W \cap E) = 38$ $n(C \cap E) = 20$, and so we get

Now we're in a position to solve for the desired region, since we know $n(E) = 63$ and we take

$63 - (24 + 14 + 6) = 63 - 44 = 19$

Questions on Homework 4

None