Probability and Counting Lesson 1: Sample Spaces and Events

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To determine probabilities, it's important to keep in the sample space that we're wroking with. What do I mean by sample space?

I mean a set S (one can think of it as a universal set) that we call outcomes.

And the find the probability of something occuring we formalize this occurence as an event E, i.e., a subset $E \subset S$ of the sample space.

Determining the probability becomes about counting the outcomes in an event and counting the events of the sample space, and taking the ratio.

Sample Spaces

Let's start with an informal definition.

A **probability experiment** is a "random occurence", i.e., an action where a range of possible scenarios, known formally as **outcomes**, happens by chance.

Some Examples of probability experiments:

1. Flipping a coin, and the outcomes are the possible faces that it lands on.

2. Rolling a dice, and the outcomes are the possible numbers that it rolls into.

3. Drawing a hand of five cards from a standard deck of 52 cards, and the specific combination of five cards selected are the outcomes.

A **sample space** is the (universal) set S of all possible outcomes in a given probability experiment.

NOTE: The outcomes are elements of the sample space, though outcomes could be described as something more specific (see examples below)

<u>Example 1.1 (page 54):</u> Suppose that a six sided die is rolled once. What is the sample

space?
Answer: $S = \{1, \ldots, 6\}$

Another Example: Suppose that we toss a fair coin. What is the sample space?
Answer: $S = \{H, T\}$ (we write H and T as short for heads and tails)

Events

An **event** in a probability experiment/sample space is a subset $E \subset S$ of the sample space.

NOTE: An event is a collection of outcomes that contain some, all, or even none of the outcomes.

<u>Examples:</u>

For $S = \{1, \ldots, 6\}$ Then the set of even numbers $\{2, 4, 6\}$ is an event, so are the singletons $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$ and even the whole set S are events.

For $S = \{H, T\}$ all the possible events are all the possible subsets, which are $\varnothing, \{H\}, \{T\}, \{H, T\}$

Computing Sizes of Sample Spaces and Events

In practice, sample spaces are more sophisticated to compute than simply rolling a dice or flipping a coin; oftentimes, you flip a coin, say, 3 times, or roll a dice twice, or even flip a coin, then roll a dice. And in those scenarios, we need to remember the product set.

$$S \times T = \left\{(s, t) : s \in S \text{ and } t \in T\right\},$$

where S and T are themselves sets, and the product formula

$$n(S \times T) = n(S) \cdot n(T)$$

In general, we may have multiple products, like $A_1 \times \cdots \times A_n$

$$A_1 \times \cdots \times A_n = \{(a_1, \ldots a_n) : a_i \in A_i\},$$

where $A_1, \ldots A_n$ are sets, and we have a generalization of that formula (which call the

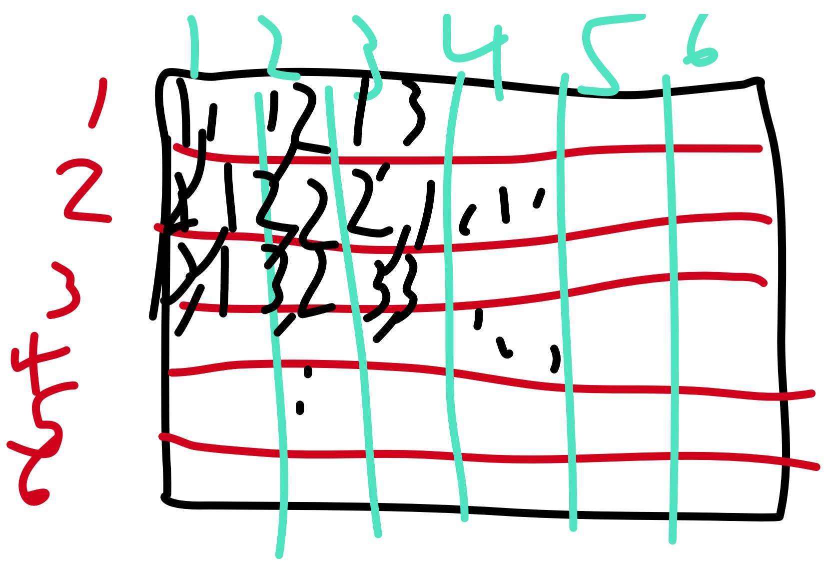
"Multiplication Principle"

$$n(A_1 \times \cdots \times A_n) = n(A_1) \cdot \cdots \cdot n(A_n).$$

Example 1.4 (page 57) We roll a dice twice, and we want to find its sample space, and number of elements in it. The sample space is

$$S = \{1, \ldots, 6\} \times \{1, \ldots, 6\} = \{1, \ldots, 6\}^2$$

NOTE: In general we define $\underbrace{S \times \cdots \times S}_{n \text{ times}} := S^n$



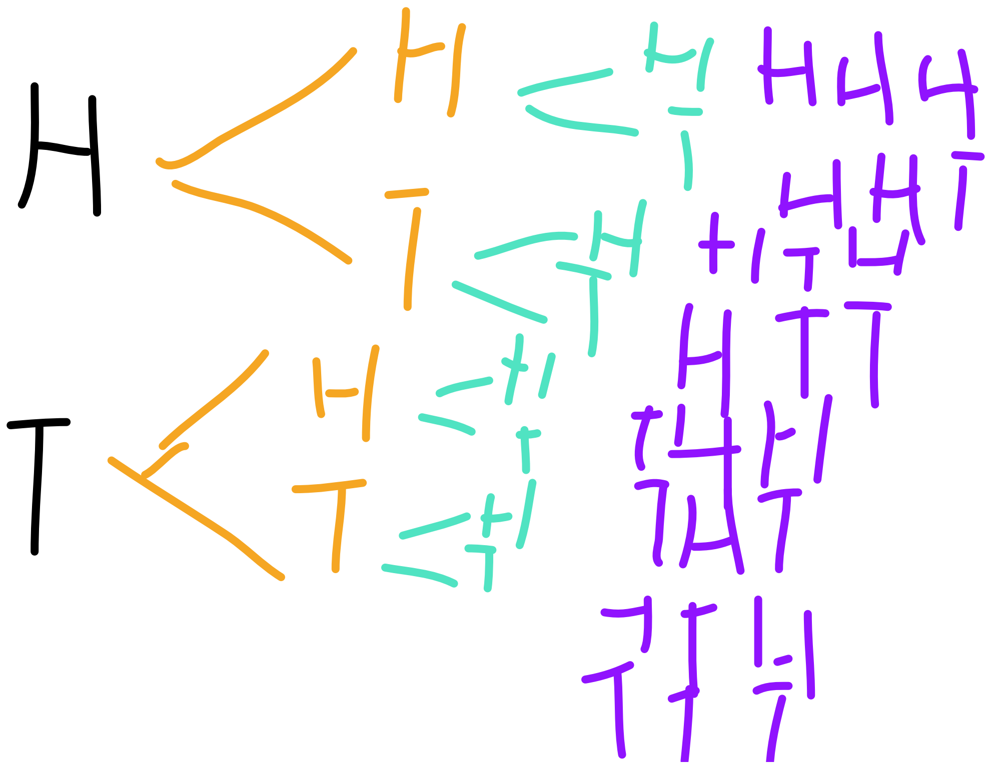
And we can use the product formula to determine the number of elements

$$n(S) = n(\{1, \ldots, 6\}) \cdot n(\{1, \ldots, 6\}) = 6 \cdot 6 = 36$$

Example 1.6 (page 56): We flip a coin three times. The sample space is

$$S = \{H, T\} \times \{H, T\} \times \{H, T\},$$

we can visualize multiple iterated products using an outcome tree as follows



and note that say, HHH is shorthand for (H, H, H)

So then we can write
$S = \{HHH, HHT, HTH, HTT, THH, THT,\ TTH, TTT\}$

$n(S) = n(\{H, T\})^3 = 2^3 = 8$

<u>Example 1.5. (Page 58)</u> A fair coin is tossed three times

Let's identify

E = event that exactly 2 heads come up
F = the event that heads come up on the second toss
G = the event that tails comes up at least once

So to find these events, we go through the outcomes of
$S = \{HHH, HHT, HTH, HTT, THH, THT,\ TTH, TTT\}$

and identify the events that fit the description

$E = \{HHT, HTH, THH\}$
$F = \{HHH, HHT, THH, THT\}$
$G = \{HHH\}' = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

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