Linear Equations Lesson 2: Graphs of Linear Equations

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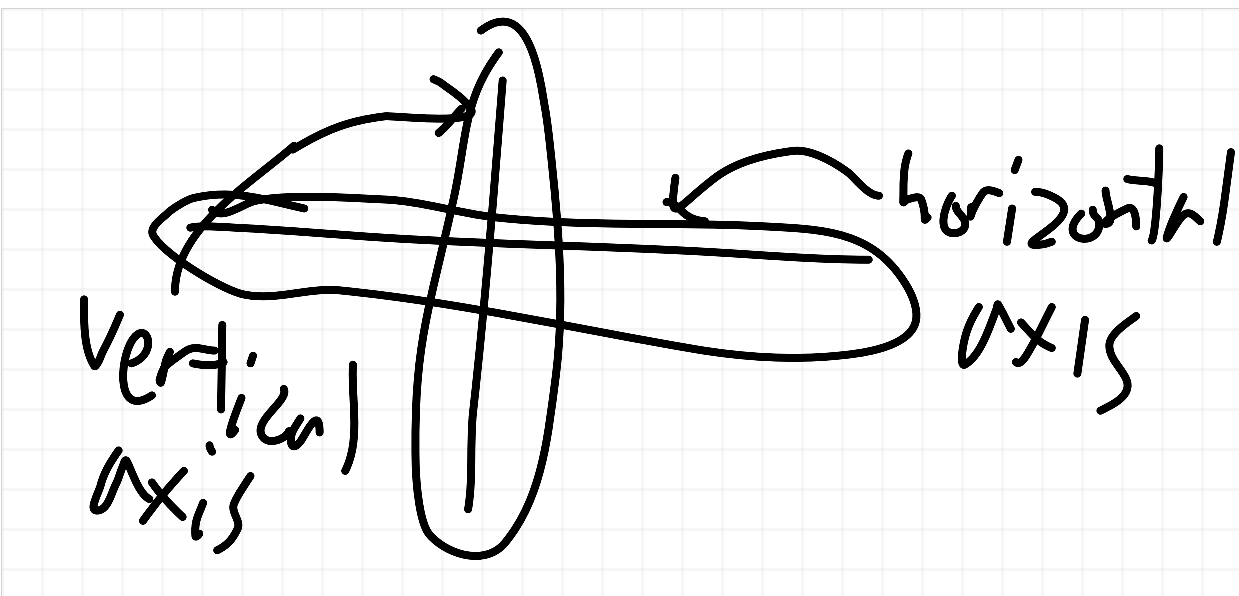
We'll cover 2.1-2.3 of *Gustafson and Frisk*. In this section, we'll focus on graphing lines and figuring out their equations.

Graphing Lines from Points

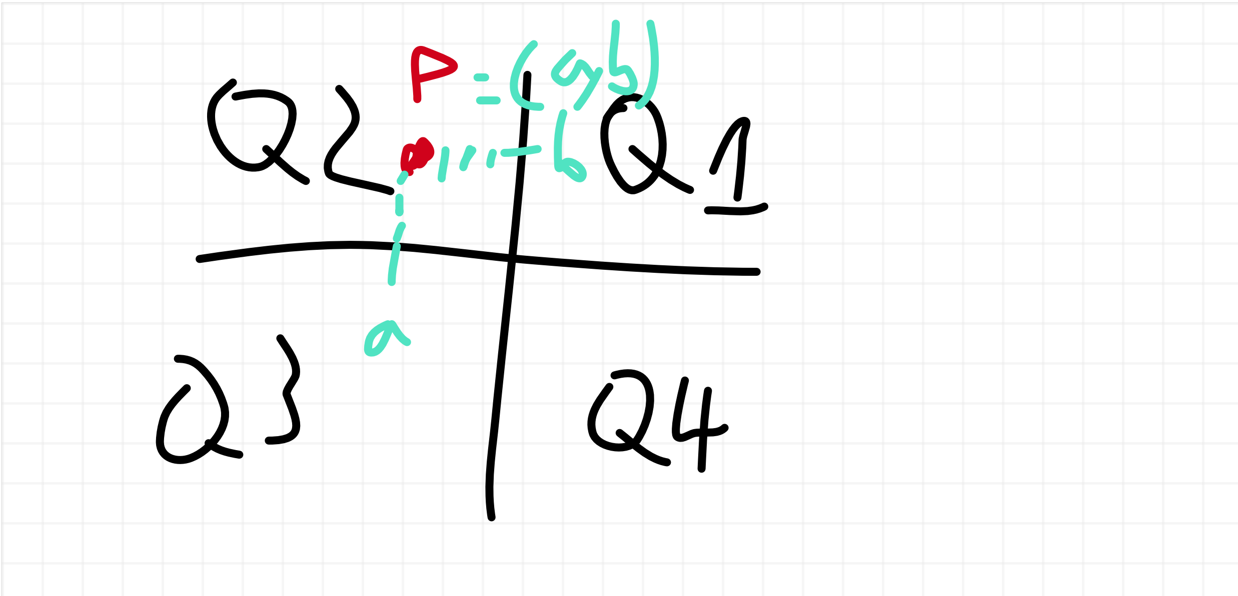
So in this course, we graph a lot of linea, and we do it in (2-dimensional) rectangular coordingates.

In a **rectangular coordinate system (the cartesian plane)**, we designate two lines, one vertical and horizontal that we look at as **axes**



and call the axes tht runs horizontally the **horizontal axis (x-axis)**, and we call the axis that runs vertically the **vertical axis (y-axis)**

We have in the cartesian plane four "quadrants" that are formed by the two axes of the plane



How do we talk about points (like P illustrated above)? We can do so by figuring out the horizontal and vertical values (x and y values) of P. Once we know these values, we can locate P, use "Coordinate System notation" to "locate" P.

Coordinate System Notation

We can locate the point in terms of what the horizontal and vertical value is. As an example, the point P has horizontal value a and vertical value b, so we write $P = (a, b)$ (as illustrated above) when we've figured the location of P.

Next Time: We'll complete our overview of the cartesian plane and get to graphing some lines.

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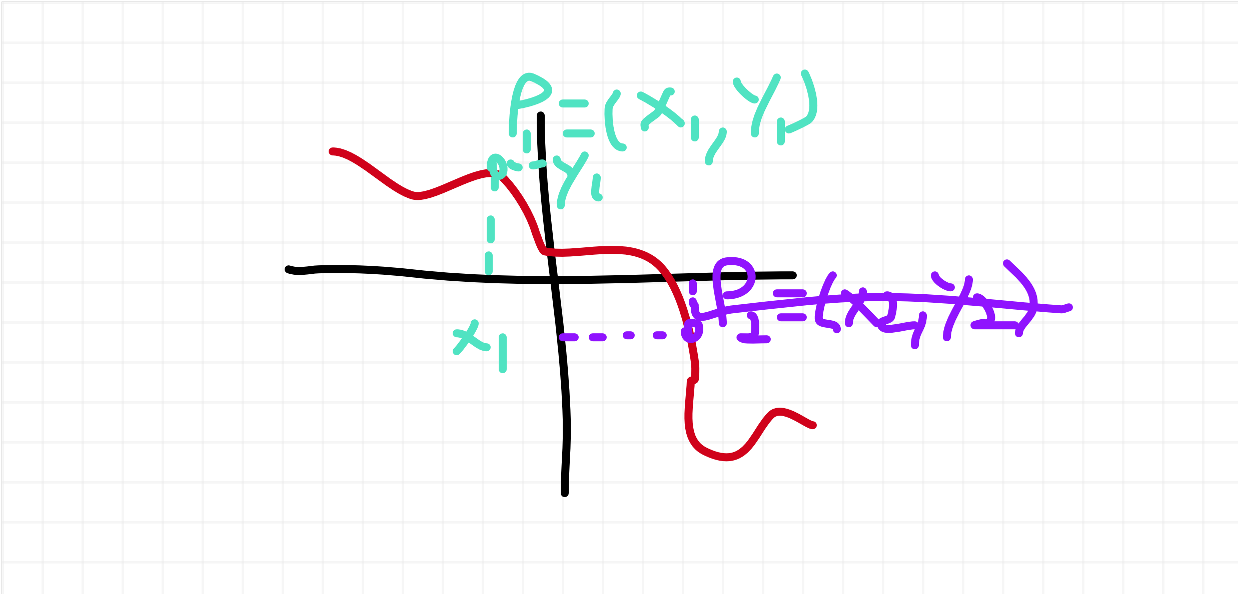
Last Time: We talked about identities, contradictions, and formulas in Section 1.5. Then we began Ch. 2 (more specifically section 2.1) by talking about the "Rectangular coordinate system" (I call it the "cartesian plane").

So in this chapter, we're going to go over graphing lines on the cartesian plane.

In general, we often take equations (of lines) and graph them. A **graph of an equation** is a drawing of all the points (x, y) that when plugged in satisfy the linear equation.



So let's say we have some equation $f(x) = y$ and it represents the red curve, then $f(x_1) = y_1$ for P_1 and $f(x_2) = y_2$ for P_2. So the general idea is to some equation $f(x) = y$ that represents all the points in the curve, and that conversely, we can represent curves with an equation $f(x) = y$.

Remember earlier (literally a day ago) when we talked about one-variable linear equations, which are of a form that simplifies to
$ax + b = 0,$
where a and b are arbitrary constants.

Now we graph **two-variable linear equations**, which are equations that are equivalent to equations of the form
$ax + by = c$ (this simplified form, as we'll learn soon is the *general form*)
$a, b,$ and c are arbitrary constants and x and y are variables.

Some Facts about lines:

1. The line connects any two points is the line in general
2. The "rate of change" (define that later) is constant

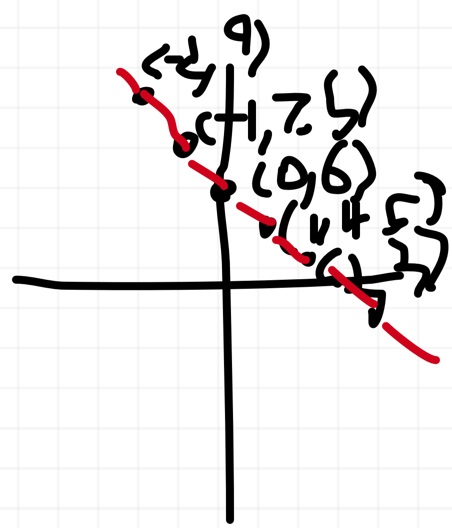
2.1 Example 3 (page 88-89) Graph $3x + 2y = 12$

We want to be able to find what y is given x (one variable in terms of the other). We want to solve for y in terms of x:

$$
\begin{aligned}
3x + 2y &= 12 \\
-3x \quad\quad &\quad -3x \\
2y &= 12 - 3x \\
\div 2 \quad &\quad \div 2 \\
y &= \frac{12 - 3x}{2}
\end{aligned}
$$

Now we can plug in values of x into the above equation and get output values of y.

x	y
-2	9
-1	7.5
0	6
1	4.5
2	3



In general, you only need two points to connect the dots; you don't need a lot of points. In some cases (if the slope/direction of the line), we'll only need one point.

A **vertical intercept (y-intercept)** of a line is the point of the line that intersects the vertical axis (y-axis). In coordinates, the point is of the form $(0, b)$.

A **horizontal intercept (x-intercept)** of a line is the point of the line that intersects the horizontal axis (x-axis). In coordinates, the point is of the form $(a, 0)$.

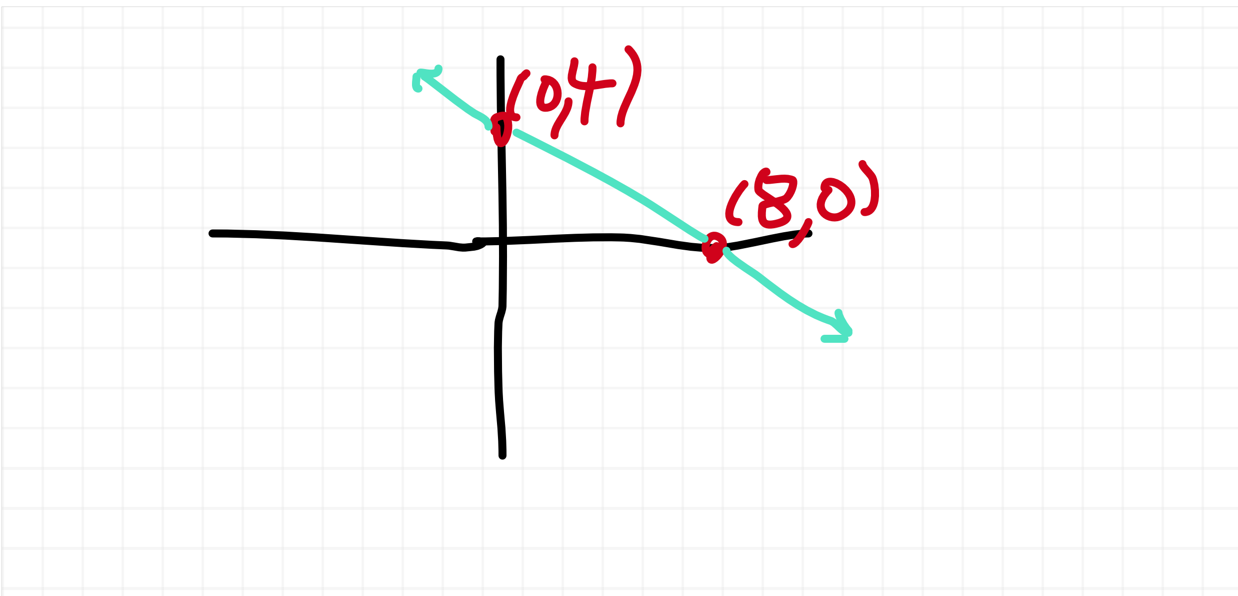
In geometry, we connect two points and we get the line between those two points, which satisfies the given equation completely, so to plot the entire line, we'll only need to connect the dots between two points, and the easiest to find are the horizontal and vertical intercepts.

<u>2.1 Example 2</u> (page 88). Graph $y = -1/2x + 4$.

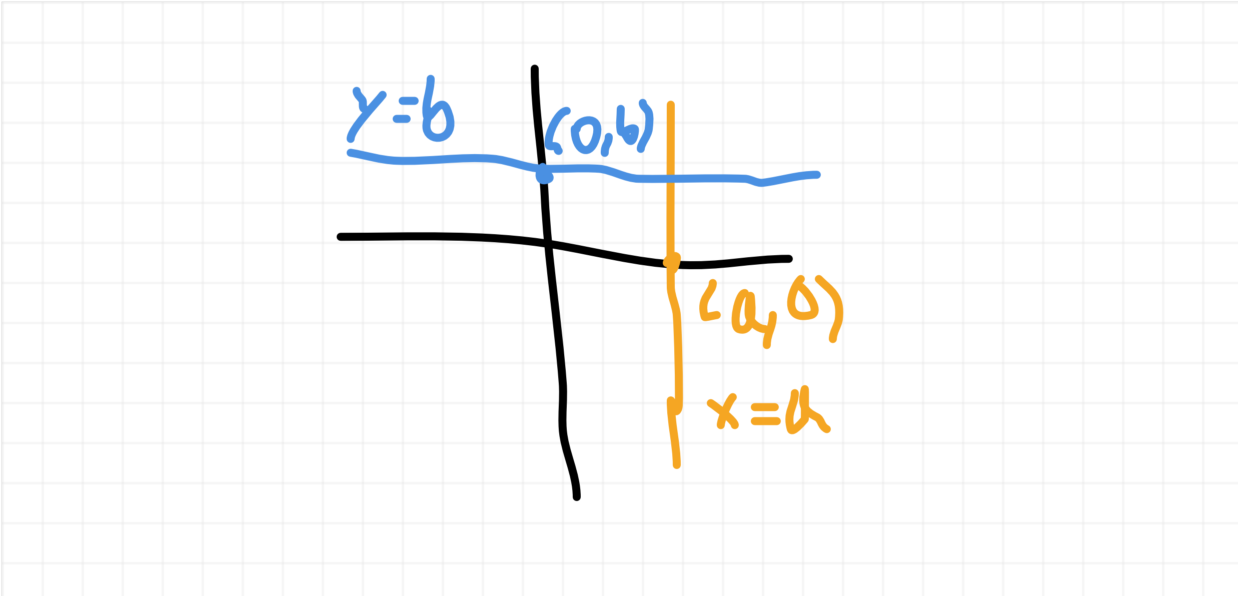
For the vertical intercept, we plug in $x = 0$ and get $y = -1/2(0) + 4 = 4$, so we have the point $(0, 4)$

For the horizontal intercept, we plug in $y = 0$, and get the equation $0 = -1/2x + 4$ and then we have to solve for x

$0 = -1/2x + 4$
$+1/2x \quad +1/2x$
$1/2x = 4$
$\times 2 \qquad \times 2$
$\quad x = 8$



Finding the vertical and horizontal intercepts works only when there are vertical and horizontal intercepts. There may not be one or the other when the line is horizontal or vertical



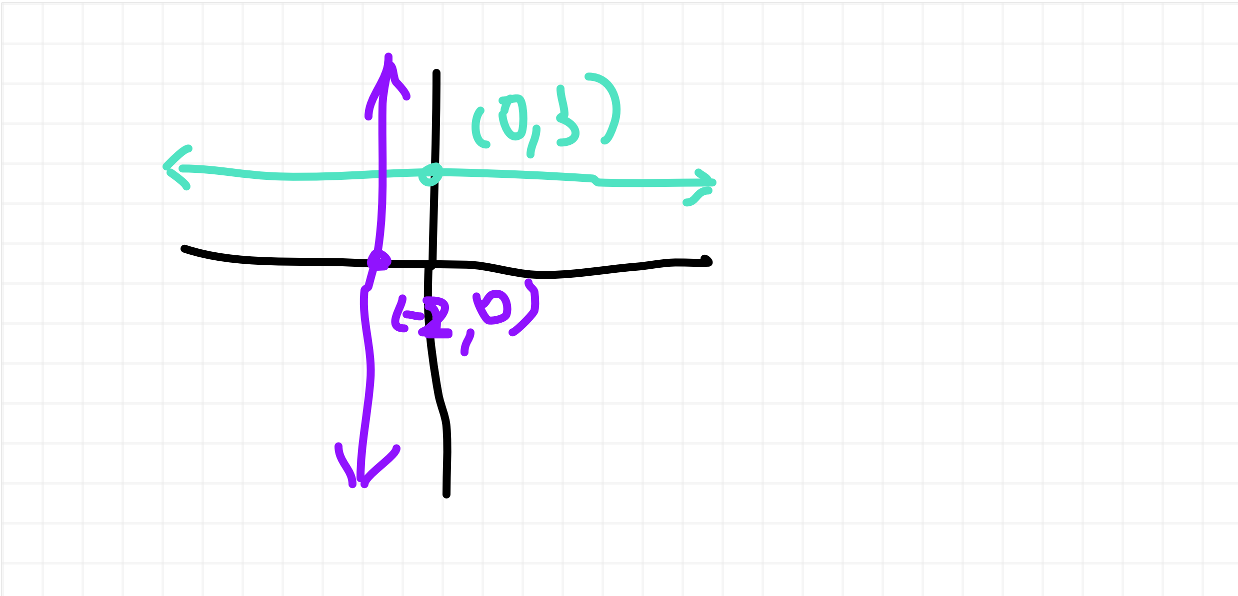
Vertical lines and horizontal can be drawn in other ways. In particular their equations are always of the following form:
$x = a$ for a **vertical line**
$y = b$ for a **horizontal line**

In general, as I've said before, connecting the dots between two points gives us the line, but for horzontal and vertical lines, we only need to know one such point (since we know the direction of the line, we can just plot point, and then plot the line going the vertical/horizontal direction)

2.1 Example 5 (page 90-91)

Graph $y = 3$ and $x = -2$

We know that $y = 3$ has vertical intercept $(0, 3)$ and we know that $x = -2$ has horizontal intercept $(-2, 0)$ so we just plot those points and then plot line going the appropriate horizontal/vertical direction



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Last time: We learned how to draw lines based on equations and based on points.

We end Section 2.1 by talking about midpoints between two points.

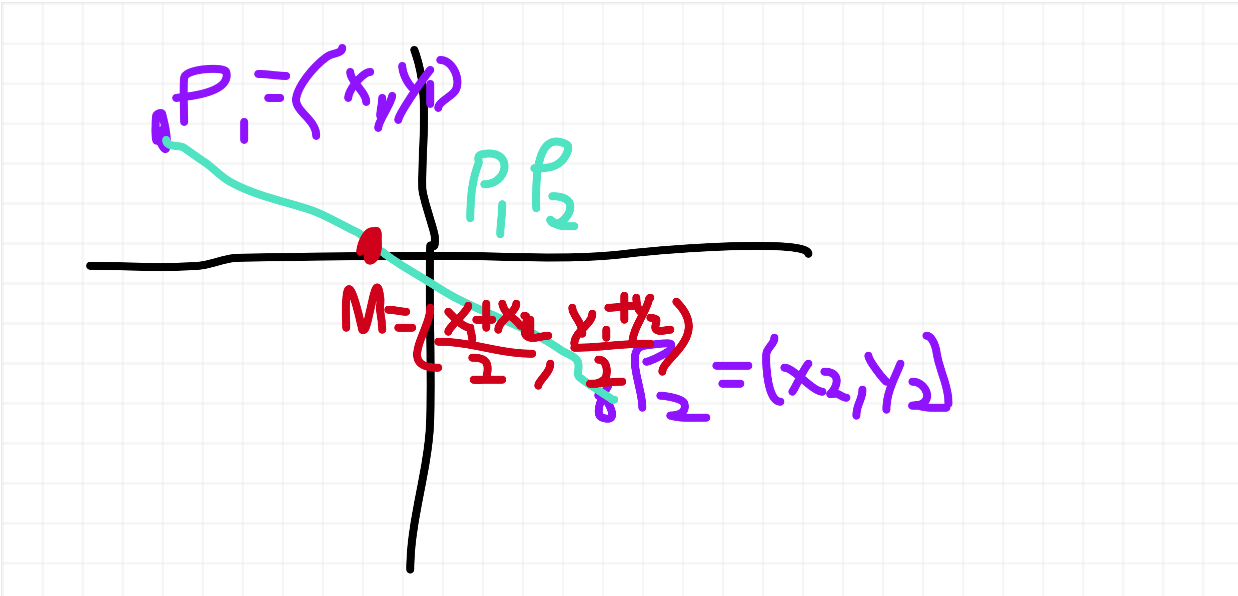
A **segment** between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is part of the line between those two points that is between P_1 as the starting point and P_2 as the endpoint.

We label the segment P_1P_2

A midpoint M of P_1P_2 is the point "in the middle" of the segement betwen P_1 and P_2

There is a midpoint formula to determine the value of the midpoint M, which is simply the average between the vertical and horizontal components:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



2.1 Example 8. (page 93) Find the midpoint of the line segment joining $P = (-2, 3)$ and $Q = (7, -5)$.

We plug in these numbers for $P = (x_1, y_1) = (-2, 3)$ and $Q = (x_2, y_2) = (7, -5)$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2+7}{2}, \frac{3+(-5)}{2}\right) = \left(\frac{5}{2}, \frac{-2}{2}\right) = (5/2, -2)$$

Questions on Homework 2

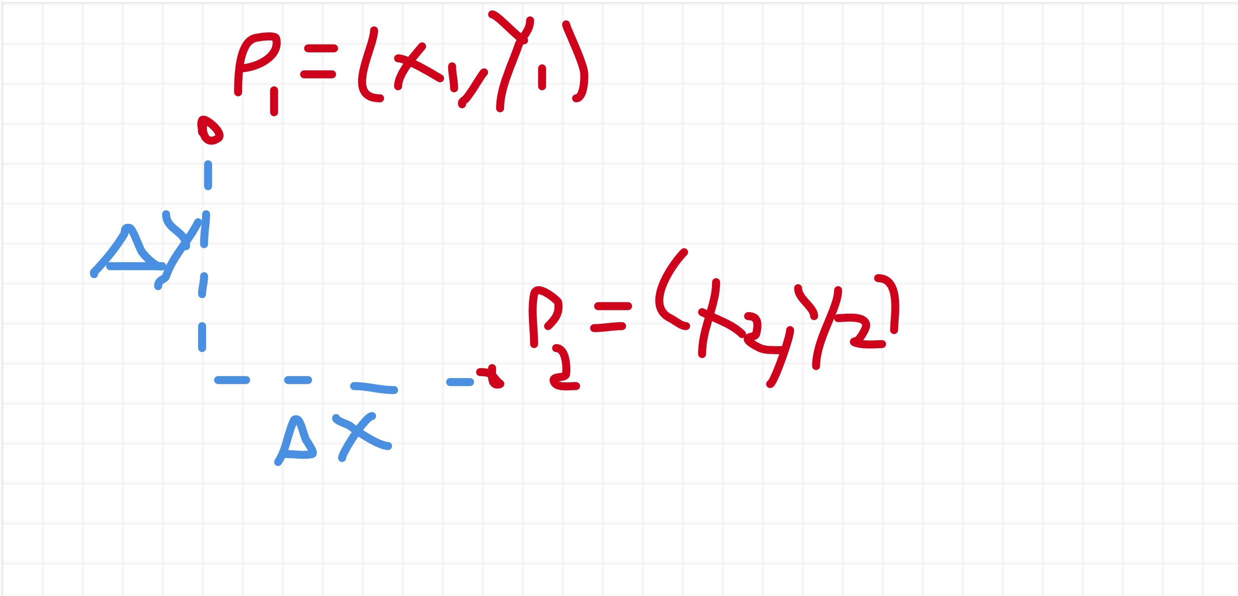
None.

Slopes of a Line

The **rate of change** from a point $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is ratioof the vertical change and the horizontal change

$$\text{rate of change from } P_1 \text{ to } P_2 \; = \frac{y_2 - y_1}{x_2 = x_1} = \frac{\text{"rise"}}{\text{"run"}} = \frac{\text{"change in y"}}{\text{"change in x"}}$$

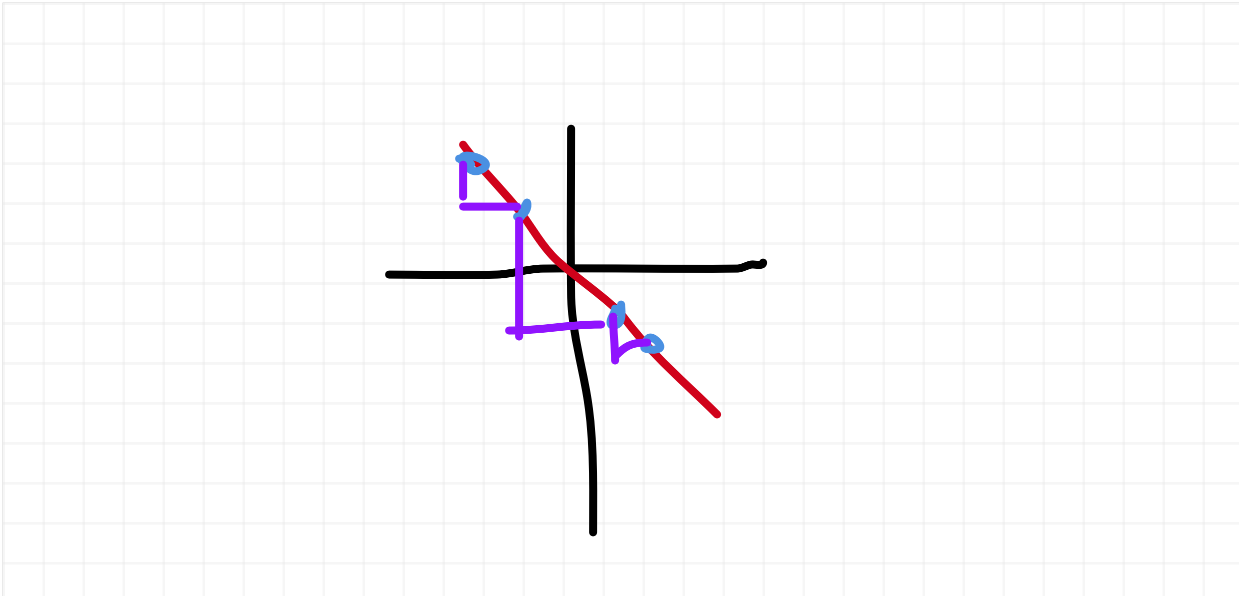
We often like to denote "change in y" quantity $\Delta y = y_2 - y_1$, similarly we like to denote the "change in x" quantity $\Delta x = x_2 - x_1$



Another way to denote the rate of change is "$\frac{\Delta y}{\Delta x}$".

NOTE: The "rate of change" does not always exist, and it doesn't exist when $x_2 - x_1$, i.e., when $\Delta x = x_2 - x_1 = 0$ (you can't divide by zero)

In a line the rate of change between any two points is constant when going from any point P_1 to P_2



The rates of change of each points in a line are the same since the triangle formed from each point moving to the next from the horizontal and vertical changes end up always being similar.

The constant rate of change quantity of any given line we call the **slope**.

We often use the letter "m" to refer to the slope. Since the slope is the "rate of change" the formula ends up being the rate of change between two points

$$m = \frac{y_2 - y_1}{x_2 = x_1} = \frac{\text{"rise"}}{\text{"run"}} = \frac{\text{"change in y"}}{\text{"change in x"}} = \frac{\Delta y}{\Delta x},$$

where $P_1 = (x_1, y_1),\ P_2 = (x_2, y_2)$ are *any* points (of your choice).

2.2 Example 2 (page 102).

Find the slope of the line determined by $3x - 4y = 12.$

We first find two points on the line. I'll choose the x and y intercepts.

The x intercept:
$y = 0$
$3x = 12$
$\div 3 \quad \div 3$
$x = 4 \qquad (4, 0)$

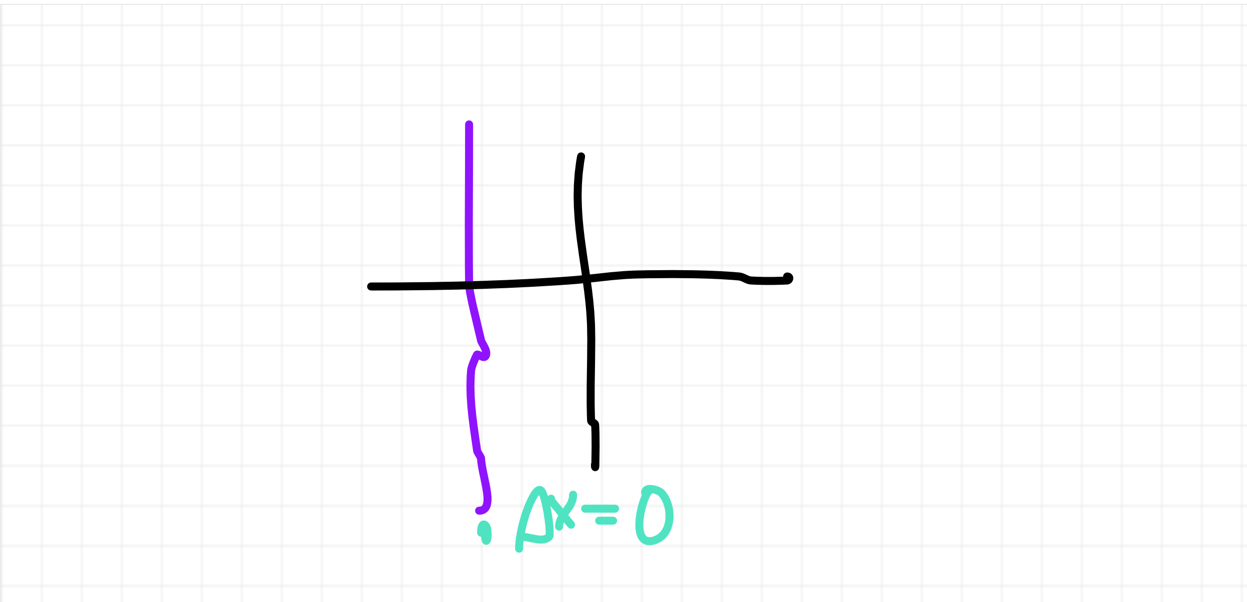
The y intercept:
$x = 0$
$-4y = 12$
$\div -4 \quad \div -4$
$y = -3 \qquad (0, -3)$

Set $P_1 = (4, 0),\ P_2 = (0, -3)$. Use the rate of change formula to get

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{0-4} = \frac{-3}{-4} = \frac{3}{4},$$

so we get $m = 3/4$.

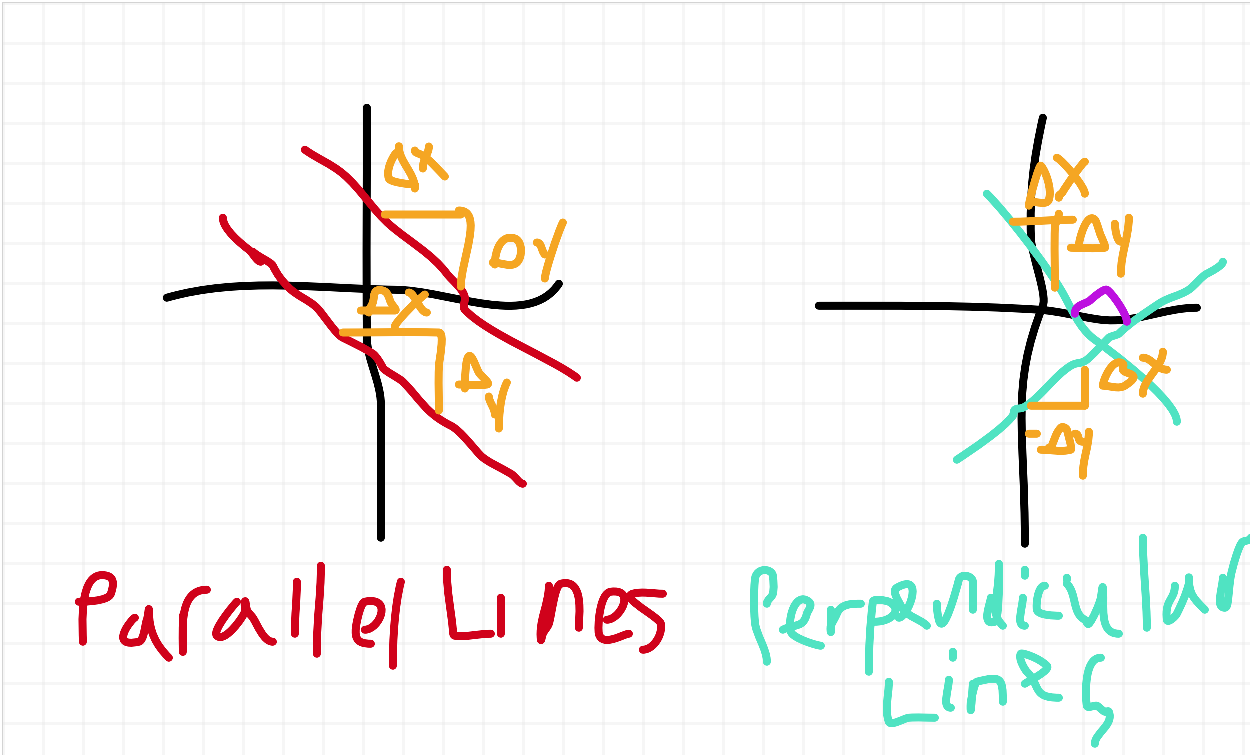
NOTE: Not all lines have slopes; in particular, vertal lines don't have a defined slope, because there is no change in x so we can't a "rate of change"



Every other kind of line has a slope (horizontal lines have slope 0).

Parallel and Perpendicular Lines

For those who need to recall basic geometry, parallel lines are lines that never intersect, and perpendicular lines are ones that intersect at a 90 degree angle



We are able to quantify parallel and perpendular lines using slopes as follows:

So parallel lines always have the same slope and vice versa:

$$m_2 = \frac{\Delta y}{\Delta x} = m_1$$

So for perpendicular lines, we always have the slope be the multiplicative inverse of the other:

$$m_2 = \frac{\Delta x}{-\Delta y} = -\frac{1}{\left(\frac{\Delta y}{\Delta x}\right)} = -\frac{1}{m_1}$$

Writing Equations of Lines

So in the previous sections (2.1,2.2), we went from an equation of a line to its graph, points, and slopes. Here (in section 2.3), we go from information about graphs, points, and slopes to equations of lines.

To write an equation of a line, the following information suffices:

1. point and the slope (the slope-intercept formula or the point-slope formula)
2. two points (the point-slope formula)
3. Coefficients A, B, C such such $Ax + By = C$ (the general form)

Next Time: We'll finish going over these formulas, and then we'll start with Ch. 3.

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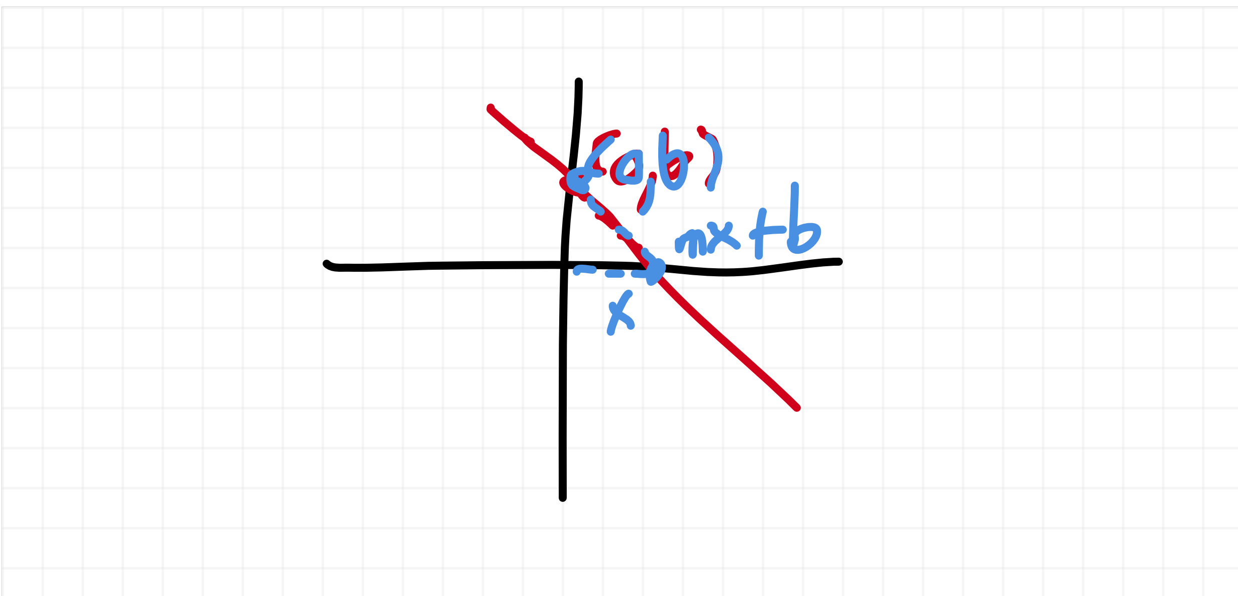
Last Time: We went over the midpoint formula and talked about slopes in section 2.2

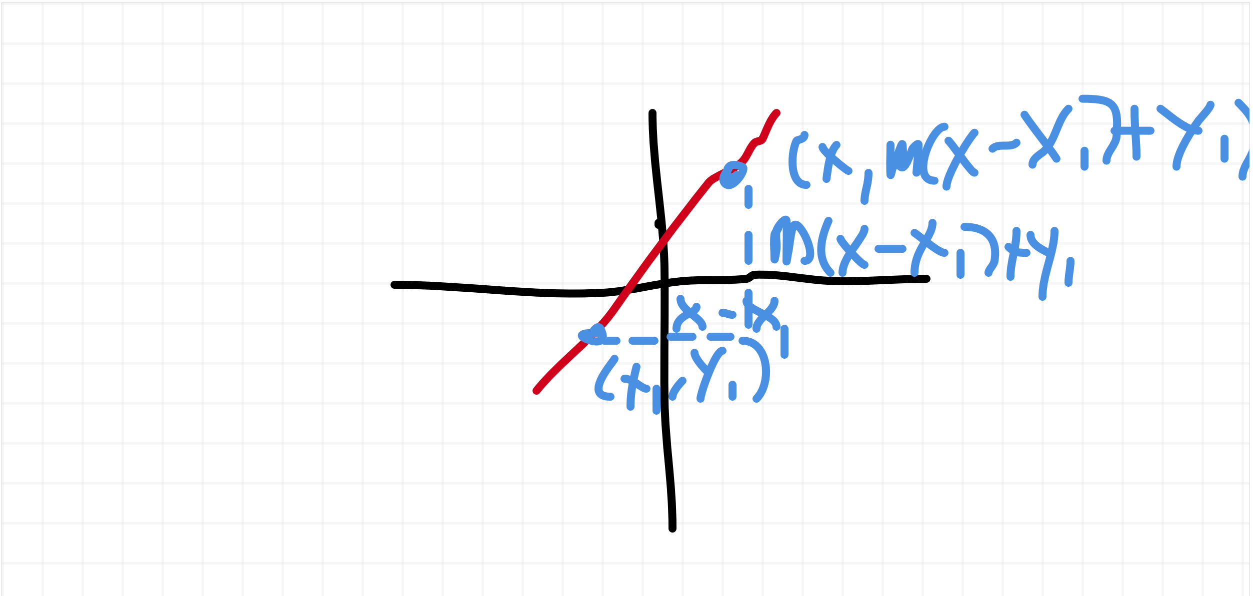
1. slope-intercept formula

If we have the y intercept $(0, b)$ and its slope m, then we have:
$y = mx + b$



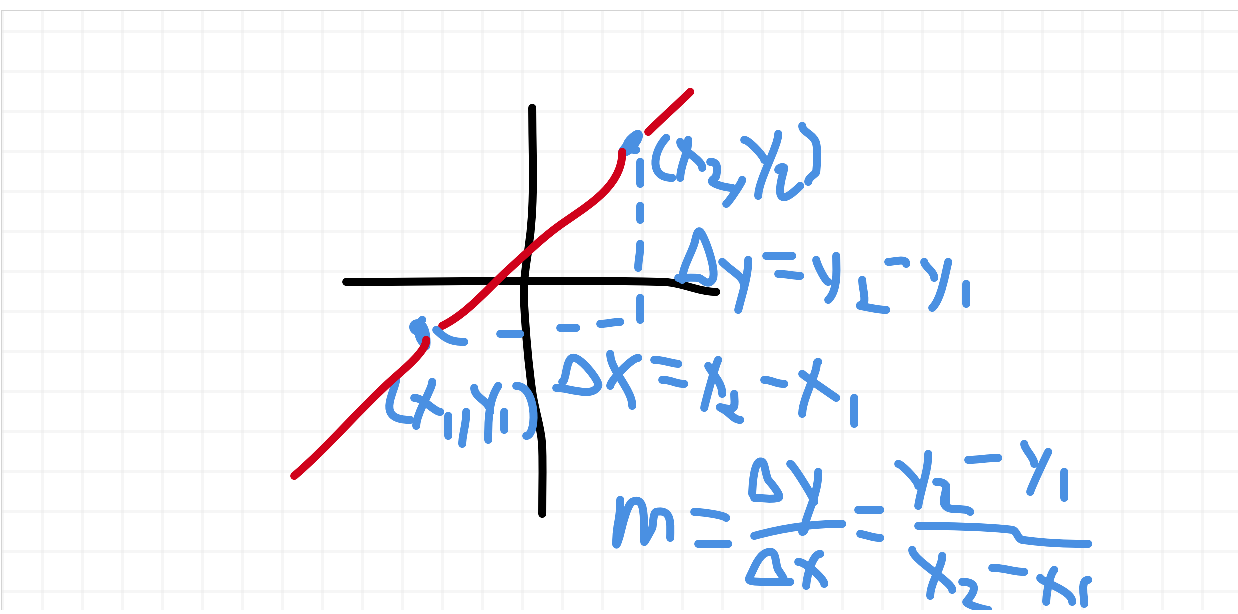
2. point-slope formula

If we have a point (x_1, y_1) and its slope m, then we have
$y - y_1 = m(x - x_1)$
$y = y_1 + m(x - x_1)$



If we have two points (x_1, y_1) and (x_2, y_2), then we have

$y - y_1 = m(x - x_1)$ and $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.



3. general form If we know that equation of the line is given in the form
$ax + by = c$,

for constants a, b, c such that $b \neq 0$, then we know that the slope $m = -\frac{a}{b}$ and the y-intercept is $(0, \frac{c}{b})$, because we get the slope-intercept formula of

$$\begin{aligned} ax + by &= c \\ -ax \quad & \quad -ax \\ by &= c - ax \\ \div b \quad & \div b \\ y &= -\frac{a}{b}x + \frac{c}{b} \end{aligned}$$

<u>2.3 Example 7.</u> Write the equation of the line passing through $(-2, 5)$ and parallel to the line $y = 8x - 3$.

We know from the line being parallel that the slope of $y = 8x - 3$ is the same slope as the other line. The slope of $y = 8x - 3$ is 8, since the equation is in slope-intercept form.

The slope of the equation of the parallel line containing $(-2, 5)$ is also 8. So we can write the equation of the line using point-slope form Plugging in $m = 8$ and $(x_1, y_1) = (-2, 5)$, we get $y - y_1 = m(x - x_1) = y - 5 = 8(x - (-2))$

NOTE: In general, not mandatory to simplify linear equations unless the exercise wants you to put it in a specific form

<u>2.3 Example 8.</u> Write the equation of the line passing through $(-2, 5)$ and perpendicular to the line $y = 8x - 3$

We know that from the line being perpendicular that the slope of $y = 8x - 3$ is the negative multiplicative reciprical of the other line. So since the slope of $y = 8x - 3$ is 8, the slope of the perpendicular line containing $(-2, 5)$ is $-1/8$.

The equation of the desired line we can find by plugging in $m = -1/8$ and $(x_1, y_1) = (-2, 5)$, to get

$$y - y_1 = m(x - x_1) = y - 5 = -\frac{1}{8}(x - (-2))$$

<u>2.3 Example 9.</u> Show that lines represented by $4x + 3y = 7$ and $3x - 4y = 12$ are perpendicular

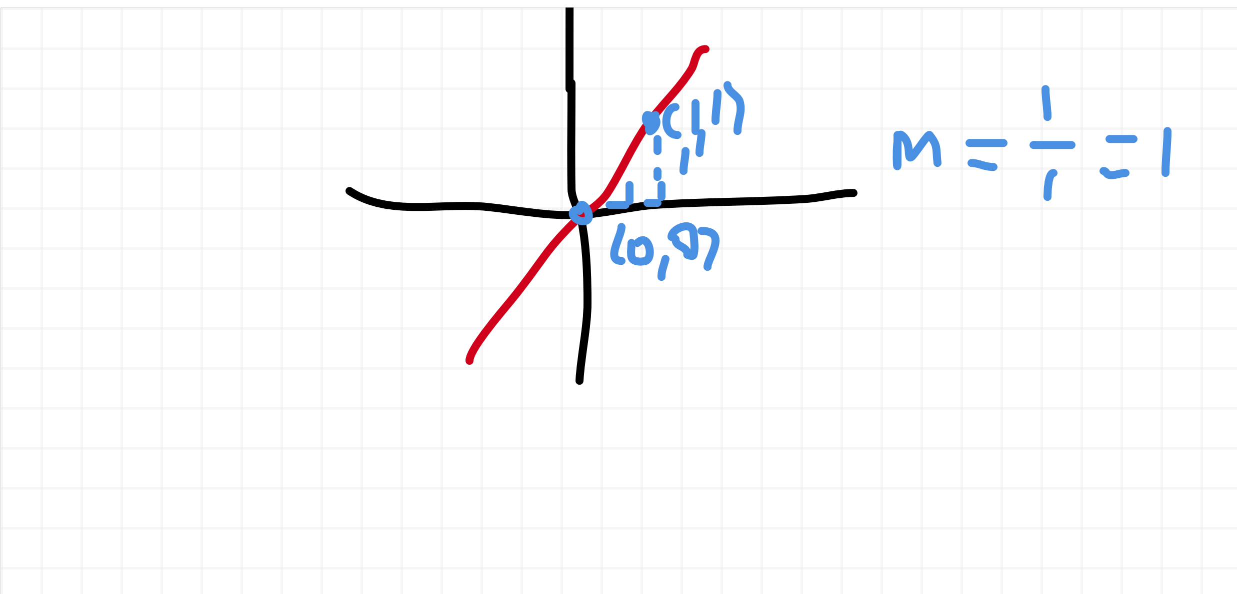
In any general form equation $ax + bx = c$, we know that the slope is always $-\frac{a}{b}$, so we know that the slope of $4x + 3y = 7$ is $-\frac{4}{3}$ and the slope of $3x - 4y = 12$ is $-\frac{3}{-4} = \frac{3}{4} = \frac{1}{4/3} = \frac{-1}{-1} \cdot \frac{1}{4/3} = -\frac{1}{-4/3}$.

Questions on Homework 3

exercise 34. find the slope of the line determined by $x = y$

Note that this equation is in slope-intercept form in the sense that $y = 1x + 0 = x$

so we know that the slope is 1 just from that



Questions on Homework 4