Linear Equations Lesson 1: Algebra Review

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This lesson will be entirely based on section 1.5 of *Gustafson and Frisk*. Totally review for high school.

Properties of Equality

A **term** is a way expressing a number in terms of $+, \ \cdot$ (sometimes we express $\cdot$ as $\times$), variables (letters) x, y, z and constants $1, 2, 3, 1.5, \frac{7}{3}, \pi, e$, etc. (sometimes we express constants using letters a, b, c when we want to talk about arbitrary such numbers)

<u>Examples.</u> Some terms include $1,\ 2+\pi,\ 3/2\cdot 3$, $a+2,\ x^2\cdot 5$

An **equation** is a statement of the form "$a = b$" where a and b are two terms**.**

As you probably remember, equality has the following preservation properties

1. *(preservation of adding terms on the same side)*

If $a = b$, then $a + c = b + c$

NOTE: It's also true that $a - c = b - c$ (since subtracting c is adding by it's additive inverse $-c$)

2. *(preservation of multiplying terms on the same side)*

If $a = b$, then $a \cdot c = b \cdot c$

We often write ac as shorthand for $a \cdot c$

3. *(preservation of division)*

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$ (remember that we can't divide by 0, so it must be that $c \neq 0$)

Solving Linear Equations

When we solve equations, we're going to focus SPECIFICALLY on linear equations.

A **linear equation** in *one variable* is an equation that can be rewritten (using preservation of inequality) in the form $ax + c = 0$ (where a and c are real numbers and x is a variable)

NOTE: When talking about linear equations, we use x to refer to the variable we want to solve for

With a linear equation, we often like to find the **solution** to the linear equation, which is the equation that efines x, so it's of the form
$x = -c/a$ for terms c and a if $a \neq 0$ or just $x = b$ for some term b

<u>1.5 Example 2</u> (page 48). Solve $2x + 8 = 0$. To solve for x, first we have the term $2x$ by itself, so we want to get that term by itself

$$\begin{aligned} 2x + 8 &= 0 \\ -8 \quad & \quad -8 \\ 2x \quad &= -8 \end{aligned}$$

Next, we want to divide by 2, which gives us x by itself.

$$\begin{aligned} 2x &= -8 \\ \div 2 \quad & \div 2 \\ x &= -4 \end{aligned}$$

Often a linear equation is not intially expressed so nicely; sometimes we get something like:
$2x + 7 = x + 5$
$2x + 5x + 9 = 70$
and so on, and they're linear equations, but they're not in the form $ax = b$, or $ax + b = 0$, so we need to "combine like terms" so we're able to proceed as we like.

Terms with common factors are called **like terms** or **similar terms**

A **coefficient** of the term ax is the term a that is multiplied by x.

For example, we $2x$ and $5x$ are like terms since they both factor into x. So in general, to solve for x in a linear equation, we want a single term that factors x and we do this in general by combining like terms.

<u>1.5 Example 4</u> (page 49). Solve $3(2x - 1) = 2x + 9$

$3(2x-1) = 2x+9$

we use distributive property to get
$3(2x-1) = 3 \cdot 2x - 3 \cdot 1 = 6x-3$

$$
\begin{aligned}
6x-3 &= 2x+9 \\
-2x \quad &\quad -2x \\
4x-3 &= 9 \\
+3 \quad &\quad +3 \\
4x \quad &= 12 \\
\div 4 \quad &\quad \div 4 \\
x \quad &= 3
\end{aligned}
$$

NOTE: If you're concerned that you made a mistake, there's a good way to check your answer, which is to plug in the solution you found back in (to see if it works). By doing that, we get

$3(2(3)-1) = 3(6-1) = 3 \cdot 5 = 15$
$2(3)+9 = 6+9 = 15$

So both sides of the equation equal 15, so the solution we found is correct.

We can give a general step-by-step process to solve equations, which is as follows:

1. If the equation contains fractions, multiply both sides by a common denominator (could be the least common denomenator, but it doesn't need to be) that will eliminate that denominator.

2. Use distributive property (as we did in example 4) to remove all the parentheses and combine like terms.

3. Use addition and subtraction (if necessary) to get all desired factor of the variable on one side of the equation, and all the numbers on the other side.

4. Use multiplication and division properties (if necessary) of equality to make the coefficient of the variable equal to 1, and here we have the solution for x.

5. *(optional)* Check the result by plugging in the solution we got.

1.5 Example 5 (page 50). Solve $\frac{5}{3}(x-3) = \frac{3}{2}(x-2)+2$

We start off with the equation

$$\frac{5}{3}(x-3) = \frac{3}{2}(x-2)+2$$

First we do step 1 and multiply by the common denominator 6 to eliminate the fractions

$$\frac{5}{3}(x-3) = \frac{3}{2}(x-2)+2$$

$$\times 6 \qquad\qquad \times 6$$

$$6\cdot\frac{5}{3}(x-3) = 6\cdot\left(\frac{3}{2}(x-2)+2\right)$$

Next, we do step 2 and use distributive property and combine like terms; we're doing to need to do ditributive property multiple times

$$6\cdot\frac{5}{3}(x-3) = 2\cdot 5(x-3) = 10(x-3) = 10x-30$$

$$\begin{aligned}6\cdot\left(\frac{3}{2}(x-2)+2\right) &= 6\cdot\frac{3}{2}(x-2)+6\cdot 2\\ &= 3\cdot 3(x-2)+6\cdot 2\\ &= 9(x-2)+12\\ &= 9x-18+12\\ &= 9x-6\end{aligned}$$

NOTE: In the arithmetic above, we cancelled out fractions as follows:

$$\overset{2}{\cancel{6}}\cdot\frac{5}{\cancel{3}} = 2\cdot 5 = 10$$

$$6 = 2\cdot 3$$

So now we have the simplified equation $10x - 30 = 9x - 6$. Now we do Step 3, which entails subtracting $9x$ and then adding 30

$$
\begin{aligned}
10x - 30 &= 9x - 6 \\
-9x \qquad &\quad - 9x \\
x - 30 &= -6 \\
+30 \quad &\quad + 30 \\
x \qquad &= 24
\end{aligned}
$$

Next, we do Step 4, which is to divide if necessary, and here we don't, since we already have the solution.

Now, for Step 5, we check the result

$$
\frac{5}{3}((24) - 3) = \frac{5}{3}(21) = 5 \cdot 7 = 35
$$
$$
\frac{3}{2}((24) - 2) + 2 = \frac{3}{2}(22) + 2 = 3 \cdot 11 + 2 = 33 + 2 = 35
$$

and they're both equal to 35, so we're good

One more thing to note about linear equations is that *solutions don't always exist*, and sometimes the solution doesn't exist in the way that we want them to (we may not have a single solution).

Let me give two examples:
$5x = 5x$
$x = x + 2$

In the first example, any x is a solution, in other words, plufg any value a in x and we have the equality $5a = 5a$. Such an equation with every number as the solution is called an **identity**.

In the second example, every possibility for x doesn't work, since for any number a we have $a \neq a + 2$
Such an equation is called a **contradiction**.

Next Time: We'll go into more detail about dealing with identities and contradictions, and then we'll go over linear formulas.

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Last Time: We went over how to solve single variable linear equations. We then left off at talking about so-called "contradictions" and "identities".

A question arises identities and contradictions, which is how do we determine whether a problem is solvable, or "not solvable" and either an "identity" or "contradiction".

To solve a problem generally (whether it's solvable or an identity or a contradiction), we want to do the steps to solve the equation as if there is a solution, and if we "can't" anymore because we up end simplifying the equation to "$0 = 0$" or "$a = 0$" for $a \neq 0$, and there's no "x" for us to solve, then we either have an identity or contradiction.

If we end up with "$0 = 0$" as our "final result", we have an *identity*.
If we end up with "$a = 0$" for $a \neq 0$ as our "final result", we have a *contradiction*.

<u>1.5 Example 7</u> (page 52) Solve: $2(x-1)+4 = 4(1+x)-(2x+2)$

First, don't need to do anything for step 1 since there are no fractions. Next, we do Step 2 and get for each side

$$
\begin{aligned}
2(x-1)+4 &= 2\cdot x - 2\cdot 1 + 4 = 2x-2+4 = 2x+2 \\
4(1+x)-(2x+2) &= 4\cdot 1 + 4\cdot x + (-1)\cdot 2x + (-1)\cdot 2 \\
&= 4+4x-2x-2 \\
&= 2+2x,
\end{aligned}
$$

and we end up with $2x+2 = 2+2x$, but we can do Step 3 and get

$$
\begin{array}{rcl}
2x+2 & = & 2+2x \\
-2x & & -2x \\
2 & = & 2 \\
-2 & & -2 \\
0 & = & 0,
\end{array}
$$

and we can't do Step 4 and we have $0 = 0$, so we have an identity.

<u>1.5 Example 7</u> (page 52) Sove: $\dfrac{x-1}{3} + 4x = \dfrac{3}{2} + \dfrac{13x-2}{3}$. We first do Step 1 and multiply by common denominator 6 to get

$$\begin{array}{l} \dfrac{x-1}{3} + 4x = \dfrac{3}{2} + \dfrac{13x-2}{3} \\ \times 6 \qquad\qquad \times 6 \\ 6 \cdot \left(\dfrac{x-1}{3} + 4x\right) = 6 \cdot \left(\dfrac{3}{2} + \dfrac{13x-2}{3}\right), \end{array}$$

then we do Step 2 and get

$$\begin{aligned} 6 \cdot \left(\frac{x-1}{3} + 4x\right) &= 6 \cdot \frac{x-1}{3} + 6 \cdot 4x \\ &= 2 \cdot (x-1) + 24x \\ &= 2 \cdot x - 2 \cdot 1 + 24x \\ &= 2x - 2 + 24x \\ &= 26x - 2 \end{aligned}$$

$$\begin{aligned} 6 \cdot \left(\frac{3}{2} + \frac{13x-2}{3}\right) &= 6 \cdot \frac{3}{2} + 6 \cdot \frac{13x-2}{3} \\ &= 3 \cdot 3 + 2 \cdot (13x-2) \\ &= 9 + 2 \cdot 13x - 2 \cdot 2 \\ &= 9 + 26x - 4 \\ &= 5 + 26x, \end{aligned}$$

so we have the simplified equation $26x - 2 = 5 + 26x$. We next do Step 3

$$\begin{array}{l} 26x - 2 = 5 + 26x \\ -26x \qquad\quad -26x \\ \quad -2 = 5 \\ \quad +2 \quad +2 \\ \quad\; 0 = 7, \end{array}$$

and we end up with a contradiction.

FINAL NOTE: No need to fully use the 5 Step process, and after this section, I'm personally going to shortcut through the single-variable 5 Step process (though there will be other multi-step processes that you may find helpful, or you can generally shortcut through those as well)

Linear Formulas

Sometimes, we have formulas given with letters used as arbitrary numbers. Examples of these include $a^2 + b^2 = c^2$, $p = vm$, $v = m/a$. In this class, you will be asked to solve for one letter as a "variable" in terms of the other letters in a "linear" equation in terms of that variable. Let's go through a few examples.

1.5 Example 9 (page 53) Solve for $A = \frac{1}{2}bh$ for h. We'll do it using the 5 step process.

Step 1. Multiply both side by 2

$$\begin{aligned} &A = \frac{1}{2}bh \\ &\times 2 \quad \times 2 \\ &2A = bh \end{aligned}$$

Step 2. All the like-terms are combined, so we can proceed to Step 3.

Step 3. We're good since we have a single term with h as a factor on one side

Step 4. We need to divide by b (assume $b \neq 0$) to get h on the other side

$$\begin{aligned} &2A = bh \\ &\div b \quad \div b \\ &\frac{2A}{b} = h. \end{aligned}$$

Step 5. We can check our solution and plug in $\frac{2A}{b}$ for h

$$\frac{1}{2}b\left(\frac{2A}{b}\right) = \frac{2}{2} \cdot \frac{b}{b} \cdot A = A,$$

which agrees with the right side.

1.5 Example 10. (page 53) Solve $A = p + prt$ for t

Step 1. No need, since no fractions

Step 2. No need, since like-terms are already combined

Step 3. We get the t factor to one side as follows:

$$
\begin{aligned}
&A = p + prt \\
&-p \quad -p \\
&A - p = prt
\end{aligned}
$$

Step 4. We divide by the coefficient pr to get t by itself:

$$
\begin{aligned}
&A - p = prt \\
&\div pr \qquad \div pr \\
&\frac{A-p}{pr} = t.
\end{aligned}
$$

Step 5. We plug in $\frac{A-p}{pr}$ for t to get

$$p + pr\left(\frac{A-p}{pr}\right) = p + (A-p) = p - p + A = A,$$

which agrees with the right side.

Questions on Homework 1

6. (page 54)

$$\left(\frac{x^4y^3}{x^5y}\right)^3 = \left(\frac{y^2}{x^1}\right)^3 = \frac{y^6}{x^3}.$$

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