# Linear Equations and Inequalities Homework Explanations

## **Linear Equations Homework 1**

Question 6 (page 54).

$$\left(\frac{x^4y^3}{x^5y}\right)^3 = \left(\frac{y^2}{x^1}\right)^3 = \frac{y^6}{x^3}.$$

Question 47 (page 54).

$$3a - 22 = -2a - 7$$
  
 $+2a + 2a$   
 $5a - 22 = -7$   
 $+22 + 22$   
 $5a = 15$   
 $\div 5 \div 5$   
 $a = 3$ 

Question 70 (page 54).

$$\frac{x}{2} + \frac{x}{3} = 10$$

We have fractions, so we're going to need to actually do step 1.

Step 1. Find a common multiple of the denominators. 6 is a good common multiple (Note that we just need a common multiple, we don't need to worry about the least common multiple) because 2 and 3 both divide it.

NOTE: Don't forget to multiply divide, and add on both sides. Number one source of errors is forgeting to do so!

$$\frac{x}{2} + \frac{x}{3} = 10$$

$$\frac{x}{2} \cdot 6 + \frac{x}{3} \cdot 6 = 10 \cdot 6$$

$$\frac{x}{2} \cdot 6 = 6x/2 = 3x$$

$$\frac{x}{2} \cdot 6 = 6x/2 = 3x$$

$$\frac{x}{3} \cdot 6 = 6x/3 = 2x$$

$$3x + 2x = 60$$

Step 2. Deal with parenthesis and combine like terms. No paranethesis, but we do have liketerms, i.e. 3x and 2x (since they're both terms that is a coefficient times the variable)

$$5x = 60$$

We can then go straight to step 4, and divide

$$5x = 60$$

$$\div 5 \div 5$$

$$x = 12$$

When in doubt, check your answer. (step 5)

Question 81 (page 54).

$$4(2-3t)+6t = -6t+8$$

Step 1: no fractions

Step 2: Get rid of the parenthesis and combine like terms

$$4 \cdot (2-3t) = 8-12t$$

$$8 - 12t + 6t = -6t + 8$$

$$8 - 6t = -6t + 8$$

Note, we have the same thing on both sides, so we have an identity. We know that because plug in any number for t and it works.

Question 82 (page 54).

$$2x - 6 = -2x + 4(x - 2)$$

Step 1: no fractions

Step 2: We have parenthesis and like-terms

$$4 \cdot (x-2) = 4x - 8$$

$$2x - 6 = -2x + 4x - 8$$

$$2x - 6 = 2x - 8$$

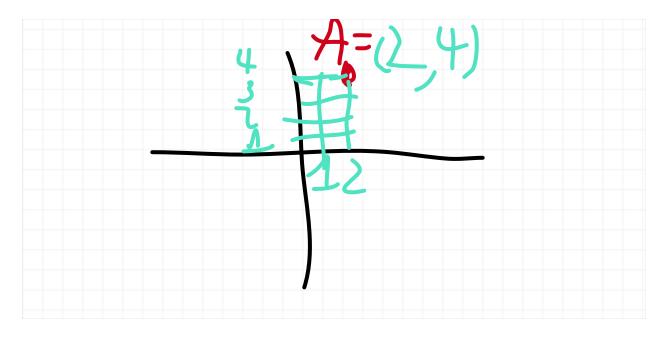
$$-2x - 2x$$

$$-6 = -8$$

We get a CONTRADICTION because  $-6 \neq -8$ .

# **Linear Equations Homework 2**

Question 27 (page 96).



Question 39 (page 96).

Plot some points (we only need 2)

$$y = -x + 4$$

$$y = 0$$

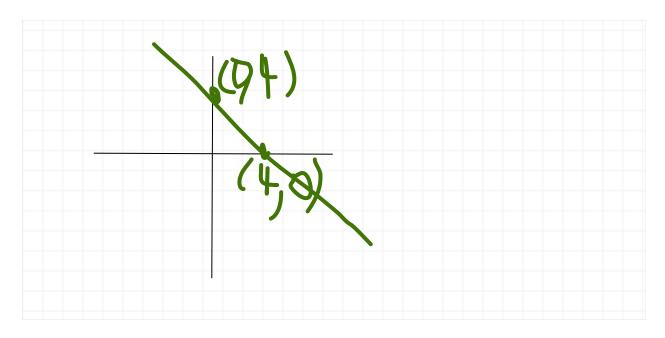
$$0 = -x + 4$$

$$+x + x$$

$$x = 4$$

$$x = 0$$

$$y = -(0) + 4 = 4$$



# Question 57 (page 97).

Want to find the midpoint of PQ

$$P = (x_1, y_1) = (0, 0), \ Q = (x_2, y_2) = (6, 8)$$

$$x_1 = 0, \ x_2 = 6$$

$$\frac{x_1 + x_2}{2} = \frac{0 + 6}{2} = 3$$

$$y_1 = 0, \ y_2 = 8$$

$$\frac{y_1 + y_2}{2} = \frac{0 + 8}{2} = \frac{8}{2} = 4$$

$$M = \left(\frac{x_1 + x_2}{2}, \ \frac{y_1 + y_2}{2}\right) = (3, 4)$$

#### Question 73 (page 97).

The table gives the amount  $\boldsymbol{y}$  (in dollars) that a student can earn for working  $\boldsymbol{x}$  hours.

Plot the ordered pairs and estimate the amount for 8 hours

Step 1. Let's rewrite the points on the table in terms of coordinates:

Step 2. Connect the dots, and drawing out the line that contains dots

Step 3. Find the point 8 and look at approximately what that point is.

#### **Linear Equations Homework 3**

Question 34 (page 106).

x = y; find the slope.

How we find the slope of a line is the rate of change between any two points (we can pick our favorite).

One of my favorite points is the origin (0,0), Let's plug in 1 for x to get another point (1, 1). Then use the rate of change formula

$$m = \frac{1-0}{1-0} = 1.$$

Question 38 (page 107).

$$x + y = \frac{2 - 3y}{3}$$

Three main ways of finding the slope

(i) Find two points of the line, and find their rate of change

$$x = 0$$

$$y = \frac{2 - 3y}{3}$$

$$\times 3 \times 3$$

$$3y = 2 - 3y$$

$$+3y + 3y$$

$$6y = 2$$

$$y = 0$$

$$x = \frac{2}{3} \left( \frac{2}{3}, 0 \right)$$

Plug that into the rate of change formula

$$\frac{\Delta y}{\Delta x} = \frac{0 - \frac{1}{3}}{\frac{2}{3} - 0} = \frac{-\frac{1}{3}}{\frac{2}{3}} = \frac{-1}{2}.$$

(ii). Convert the equation into one of the three formulas

the easiest would be the *slope-intercept* formula, since that entails solving for y

We started with

$$x + y = \frac{2 - 3y}{3}$$

$$x + y = \frac{2 - 3y}{3}$$

$$\times 3 \qquad \times 3$$

$$3(x + y) = 2 - 3y$$

$$3x + 3y = 2 - 3y$$

$$+3y \qquad +3y$$

$$3x + 6y = 2$$

$$-3x \qquad -3x$$

$$6y = 2 - 3x$$

$$\div 6 \qquad \div 6$$

$$y = \frac{1}{3} - \frac{1}{2}x$$

$$y = \frac{1}{3} + \left(-\frac{1}{2}\right)x$$

the slope is 
$$-\frac{1}{2}$$
.

Question 45 (page 107).

$$m_1 = 3, m_2 = -\frac{1}{3}$$

parallel or perpendicular or neither?

is it parallel? Are the slopes the same?

Obviously no, so they're not parallel.

perpendicular?

$$m_2 = -\frac{1}{m_1}$$

$$m_2 = -\frac{1}{3}$$

$$-\frac{1}{m_1} = -\frac{1}{3} = m_2$$

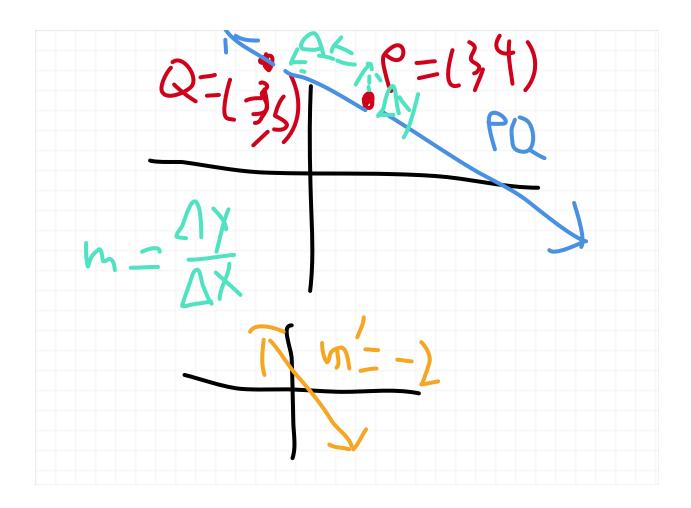
The lines are perpendicular.

Question 54 (page 107). To find if PQ is parallel or perpendicular (or neither) to a line with slope -2, we want to see if the slope of PQ meets the slope conditions of parallel or perpendicular lines.

Parallel lines:  $m_1 = m_2$ 

Perpendicular lines:  $m_2 = -\frac{1}{m_1}$ 

$$P(3,4), Q(-3,5)$$



$$m = \text{slope of } PQ = \frac{\Delta y}{\Delta x} = \frac{5-4}{-3-3} = \frac{1}{-6} = -\frac{1}{6}$$

Letting  $m^\prime$  be the slope of the other line, let's check to see if

$$m = m'$$
 or  $m = -\frac{1}{m'}$ 

$$m = m'$$
? No  $-2 \neq -\frac{1}{6}$ , not parallel.

$$m=-\frac{1}{m'}$$
? No

$$-\frac{1}{m'} = -\frac{1}{-2} = \frac{1}{2} \neq -\frac{1}{6} = m$$

The answer is that it's neither.

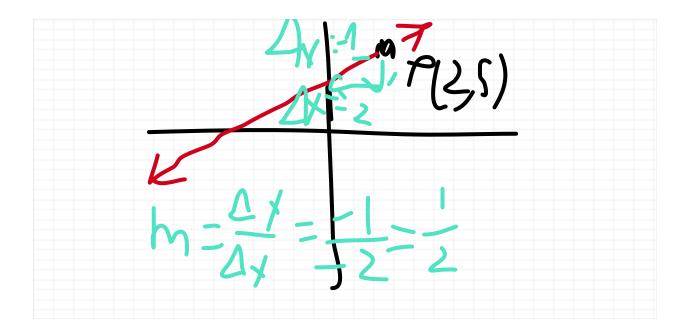
Question 64 (page 106).

$$x = 0$$

No slope.

## **Linear Equations Homework 4**

Question 17 (page 119).



So we have m=1/2 and  $(x_1,y_1)=(2,5)$ , so using point slope form, we plug in these values to get

$$y-y_1 = m(x-x_1)$$
  
 $y-5 = 1/2(x-2)$ 

Simplify to general form ax + by = c

$$y-5 = 1/2x-1$$
  
+5 +5  
 $y = 1/2x+4$   
 $-1/2x - 1/2x$ 

$$-\frac{1}{2}x + y = 4$$

# **Linear Equations Homework 5**

Question 34 (page 159).

$$2x = 5y - 11$$

$$3x = 2y$$

$$2x = 5y - 11$$

$$-5y - 5y$$

$$2x - 5y = -11$$

slope: 
$$-\frac{2}{-5} = \frac{2}{5}$$
 y-intercept:  $\left(0, \frac{11}{5}\right)$ 

$$3x = 2y$$

$$-2y - 2y$$

$$3x - 2y = 0$$

slope: 
$$-\frac{3}{-2} = \frac{3}{2}$$
 y-intercept:  $(0,0)$ 

NOTE: Make sure to draw the line very well (using graph paper)

## **Linear Equations Homework 6**

27.

$$2x + 3y = 8$$

$$3x - 2y = -1$$

$$2x + 3y = 8$$

$$\times 2 \times 2$$

$$4x + 6y = 16$$

$$3x - 2y = -1$$

$$9x - 6y = -3$$

$$4x + 6y = 16$$

$$+(9x - 6y) + (-3)$$

$$13x = 13$$

$$\div 13 \div 13$$

$$x = 1$$

$$2(1) + 3y = 8$$

$$2 + 3y = 8$$

$$-2 - 2$$

$$3y = 6$$

$$\div 3 \div 3$$

$$y = 2$$

$$(x,y)=(1,2)$$

#### Question 47 (page 172).

$$\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$$

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{6}$$

You're expected to solve for  $x' = \frac{1}{x}$  and  $y' = \frac{1}{y}$ 

$$x' + y' = \frac{5}{6}$$
$$x' - y' = \frac{1}{6}$$

Let's use the addition method

$$x' + y' = \frac{5}{6}$$

$$+(x' - y') + \frac{1}{6}$$

$$2x' = \frac{6}{6} = 1$$

$$\div 2 \div 2$$

$$x' = \frac{1}{2}$$
.

$$\frac{1}{2} + y' = \frac{5}{6}$$

$$\times 6 \times 6$$

$$3 + 6y' = 5$$

$$-3 \qquad -3$$
$$6y' = 2$$

$$6y' = 2$$

$$y'=\frac{1}{3}.$$

$$x = \frac{1}{x'} = 2,$$

$$y = \frac{1}{y'} = 3.$$

## **Linear Equations Homework 7**

Question 16 (page 192).

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 & 2 \\ & 1 & 5 \end{bmatrix}$$

How did we get from -2 to 1 and also from 2 to 5? It's clear that we didn't multiply row 2 by some constant because row 2 of the second matrix is not a factor of the first. So we know that we have the row operation

 $row 2 + t \cdot row 1$ 

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

 $row 2 + t \cdot row 2$ 

$$\begin{bmatrix} -1 & 3 & 2 \\ & 1 & 5 \end{bmatrix}$$

and we want to solve for t. So we solve the following equality

$$1 = -2 + t \cdot 3$$
$$+2 + 2$$

$$3 = t \cdot 3$$

$$1 = t$$

So t=1 and the row operation is row 2 + row 1, so we added row 2 by row 1 (let's check and the determine the blank)

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

row 2 + row 1

$$\begin{bmatrix} -1 & 3 & 2 \\ 0 & 1 & 5 \end{bmatrix}'$$

so the 2, 1 entry is 0.

Question 18 (page 192).

$$\begin{bmatrix} 2 & 1 & -3 \\ 2 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 3 & - \\ 2 & 6 & 1 \end{bmatrix}$$

$$6 = t2$$

$$3 = t$$

$$(6,3) = t(2,1)$$
?

$$(6,3) = 3(2,1)$$
?

$$(6,3) = (3 \cdot 2, 3 \cdot 1) = 3 \cdot (2,1)$$

$$\begin{bmatrix} 2 & 1 & -3 \\ 2 & 6 & 1 \end{bmatrix}$$

 $3 \times \text{row } 1$ 

$$\begin{bmatrix} 6 & 3 & -3 \cdot 3 \\ 2 & 6 & 1 \end{bmatrix}, \\ \begin{bmatrix} 6 & 3 & -9 \\ 2 & 6 & 1 \end{bmatrix}$$

Question 20 (page 192).

$$x + y = 3$$
$$x - y = -1$$

First we write the equation in matrix form

$$\left(\begin{array}{cc|c}
1 & 1 & 3 \\
1 & -1 & -1
\end{array}\right)$$

Next, we do reduced row operations to solve for x and y. First, observe that we already have the desired 1 coefficient in the 1,1 entry, so we can go straight to cancelling out row 2 with row 1 as follows:

$$\begin{pmatrix}
1 & 1 & 3 \\
1 & -1 & -1
\end{pmatrix}$$

row 2 - row 1

$$\begin{pmatrix}
1 & 1 & 3 \\
1-1 & -1-1 & -1-3
\end{pmatrix}, \\
\begin{pmatrix}
1 & 1 & 3 \\
0 & -2 & -4
\end{pmatrix}$$

Next, we want to get the 2, 2 entry to be 1. We do that by multiplying row 2 by -1/2 as follows:

$$\begin{pmatrix}
1 & 1 & 3 \\
0 & -2 & -4
\end{pmatrix}$$

row  $2 \div - 2$ 

$$\left(\begin{array}{cc|c}
1 & 1 & 3 \\
0 & 1 & 2
\end{array}\right)$$

Finally, we want to cancel the 1 on the 1,2 entry, because then the 1 in the 1,1 entry (i.e. the x variable is by itself)

$$\left(\begin{array}{cc|c}
1 & 1 & 3 \\
0 & 1 & 2
\end{array}\right)$$

row 1 - row 2

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$
,

so we now have the equation

$$x = 1$$
$$y = 2,$$

and our solution is (x, y) = (1, 2).

Question 29 (page 192).

$$2x + y + 3z = 3$$
$$-2x - y + z = 5$$
$$4x - 2y + 2z = 2$$

In matrix form, we have

$$\begin{pmatrix} 2 & 1 & 3 & 3 \\ -2 & -1 & 1 & 5 \\ 4 & -2 & 2 & 2 \end{pmatrix}$$

$$2x + y + 3z = 3$$

$$-2x - y + z = 5$$

$$4x - 2y + 2z = 2$$

row  $1 \div 2$  multiply by 1/2 on both sides on the first equation

$$\begin{pmatrix} 1 & 1/2 & 3/2 & 3/2 \\ -2 & -1 & 1 & 5 \\ 4 & -2 & 2 & 2 \end{pmatrix} \qquad x+1/2y+3/2z = 3/2$$

$$-2x-y+z=5$$

$$4x-2y+2z=2$$

$$row 2+2 \cdot row 1 \qquad -2x-y+z=5$$

$$+2(x+1/2y+3/2z) +2(3/2)$$

$$\begin{pmatrix}
1 & 1/2 & 3/2 & 3/2 \\
0 & 0 & 4 & 8 \\
4 & -2 & 2 & 2
\end{pmatrix}$$

row  $3 - 4 \cdot \text{row } 1$ 

$$\begin{pmatrix}
1 & 1/2 & 3/2 & 3/2 \\
0 & 0 & 4 & 8 \\
0 & -4 & -4 & -4
\end{pmatrix}$$

 $\mathsf{row}\ 2 \leftrightarrow \ \mathsf{row}\ 3$ 

$$\begin{pmatrix}
1 & 1/2 & 3/2 & 3/2 \\
0 & -4 & -4 & -4 \\
0 & 0 & 4 & 8
\end{pmatrix}$$

row  $2 \div -4$ 

$$\left(\begin{array}{ccc|c}
1 & 1/2 & 3/2 & 3/2 \\
0 & 1 & 1 & 1 \\
0 & 0 & 4 & 8
\end{array}\right)$$

row  $3 \div 4$ 

$$\left(\begin{array}{ccc|c}
1 & 1/2 & 3/2 & 3/2 \\
0 & 1 & 1 & 1 \\
0 & 0 & 1 & 2
\end{array}\right)$$

$$row 1 - \frac{1}{2} \cdot row 2$$

$$\left(\begin{array}{ccc|c}
1 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 \\
0 & 0 & 1 & 2
\end{array}\right)$$

row 2 - row 3

$$\left(\begin{array}{cc|cc|c}
1 & 0 & 1 & 1 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 2
\end{array}\right)$$

row 1 - row 3

$$\left(\begin{array}{ccc|c}
1 & 0 & 0 & -1 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 2
\end{array}\right)$$

$$x = -1$$

$$y = -1$$

$$z = 2$$

Question 32 (page 192).

$$2x + 3y - z = -8$$

$$x - y - z = -2$$

$$-4x + 3y + z = 6.$$

In matrix form, we have

$$\left(\begin{array}{ccc|c}
2 & 3 & -1 & -8 \\
1 & -1 & -1 & -2 \\
-4 & 3 & 1 & 6
\end{array}\right)$$

 $\mathsf{row}\ 1 \leftrightarrow \mathsf{row}\ 2$ 

$$\left(\begin{array}{ccc|c}
1 & -1 & -1 & -2 \\
2 & 3 & -1 & -8 \\
-4 & 3 & 1 & 6
\end{array}\right)$$

row  $2 - 2 \cdot \text{row } 1$ 

$$\begin{pmatrix}
1 & -1 & -1 & -2 \\
2-2 & 3-2(-1) & -1-2(-1) & -8-2(-2) \\
-4 & 3 & 1 & 6
\end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & -1 & -2 \\
0 & 5 & 1 & -4 \\
-4 & 3 & 1 & 6
\end{pmatrix}$$

row  $3 + 4 \cdot \text{row } 1$ 

$$\begin{pmatrix}
1 & -1 & -1 & | & -2 \\
0 & 5 & 1 & | & -4 \\
0 & -1 & -3 & | & -2
\end{pmatrix}$$

 $\mathsf{row}\ 2 \leftrightarrow \mathsf{row}\ 3$ 

$$\begin{pmatrix}
1 & -1 & -1 & | & -2 \\
0 & -1 & -3 & | & -2 \\
0 & 5 & 1 & | & -4
\end{pmatrix}$$

 $-1 \times \text{row } 2$ 

$$\begin{pmatrix}
1 & -1 & -1 & | & -2 \\
0 & 1 & 3 & | & 2 \\
0 & 5 & 1 & | & -4
\end{pmatrix}$$

row 1 + row 2

$$\begin{pmatrix}
1 & 0 & 2 & 0 \\
0 & 1 & 3 & 2 \\
0 & 5 & 1 & -4
\end{pmatrix}$$

row  $3-5 \cdot \text{row } 2$ 

$$\left(\begin{array}{cc|cc|c}
1 & 0 & 2 & 0 \\
0 & 1 & 3 & 2 \\
0 & 0 & -14 & -14
\end{array}\right)$$

row  $3 \div - 14$ 

$$\left(\begin{array}{ccc|c}
1 & 0 & 2 & 0 \\
0 & 1 & 3 & 2 \\
0 & 0 & 1 & 1
\end{array}\right)$$

row  $1 - 2 \cdot \text{row } 3$ 

$$\left(\begin{array}{ccc|c}
1 & 0 & 0 & -2 \\
0 & 1 & 3 & 2 \\
0 & 0 & 1 & 1
\end{array}\right)$$

row  $2 - 3 \cdot \text{row } 3$ 

$$\left(\begin{array}{ccc|c}
1 & 0 & 0 & -2 \\
0 & 1 & 0 & -1 \\
0 & 0 & 1 & 1
\end{array}\right)$$

We get the reduced equation of

$$x = -2$$

$$y = -1$$

$$z = 1$$

so we have the unique solution of (x, y, z) = (-2, -1, 1).

Question 33 (page 192)

$$x + y = 3$$

$$3x - y = 1$$

$$2x + y = 4$$

First we want to set up this equation as a matrix.

$$\begin{pmatrix}
1 & 1 & 3 \\
3 & -1 & 1 \\
2 & 1 & 4
\end{pmatrix}$$

row  $2 - 3 \cdot \text{row } 1$ 

$$\left(\begin{array}{cc|c}
1 & 1 & 3 \\
0 & -4 & -8 \\
2 & 1 & 4
\end{array}\right)$$

row  $3 - 2 \cdot \text{row } 1$ 

$$\begin{pmatrix}
1 & 1 & 3 \\
0 & -4 & -8 \\
0 & -1 & -2
\end{pmatrix}$$

 $\mathsf{row}\ 2 \div \!\!\!\! -4$ 

$$\begin{pmatrix}
1 & 1 & 3 \\
0 & 1 & 2 \\
0 & -1 & -2
\end{pmatrix}$$

row 1 - row 2

$$\begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 2 \\
0 & -1 & -2
\end{pmatrix}$$

row 3 + row 2

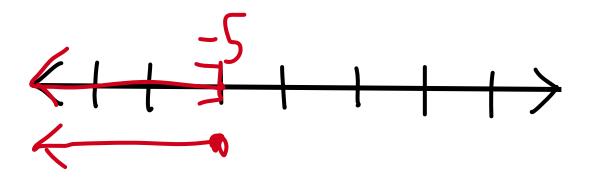
$$\left(\begin{array}{cc|c}
1 & 0 & 1 \\
0 & 1 & 2 \\
0 & 0 & 0
\end{array}\right).$$

# **Inequalities Homework 1**

Question 24 (page 218). Find the solution and graph

$$\begin{array}{ll} -2x+6 & \geq 16 \\ -6 & -6 \\ -2x & \geq 10 \\ \div -2 & \div -2 \ \left( -2 < 0 \ \text{dividing by a negative switches the inequality sign} \right) \\ x & \leq -5 \end{array}$$

The solution is the interval  $(-\infty, 5]$ , so note that 5 is included, so to draw the solution, we make sure to draw it with either a closed point (a fully drawn point) or a SQUARE bracket



## Question 28 (page 218). $-3 \le 3x < 12$

We do algebra for each of the inequalities individually, so we solve it for  $-3 \le 3x$  and 3x < 12 (and we could hypothetically do it simultaneously, but in general, doing it separate will work most generally)

$$-3 \le 3x$$
  

$$\div 3 \quad \div 3 \quad (3 > 0)$$
  

$$-1 \le x$$

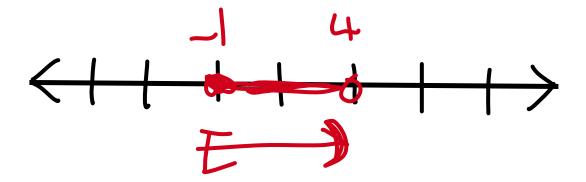
next we do 3x < 12

$$3x < 12$$

$$\div 3 \div 3$$

$$x < 4$$

So the solution is both of those inequalities put together  $-1 \le x < 4$ , which gives us the half-open interval [-1,4) (also can be viewed as the intersection of  $[-1,\infty)$  and  $(-\infty,4)$ )

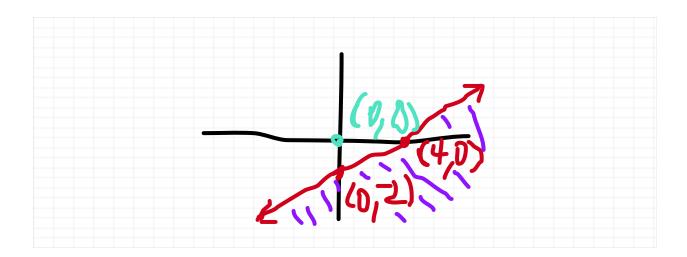


# **Inequalities Homework 2**

Question 25 (page 239).  $x - 2y \ge 4$ 

We find that the line x-2y=4 has x-intercept (4,0) and y-intercept (0,-2). We plug in the origin (0,0) into the inequality  $x-2y\geq 4$  and get

$$(0) - 2(0) = 0 \ge 4.$$



# **Inequalities Homework 3**

#### Question 11 (page 246)

$$3x + 2y > 6$$
$$x + 3y \le 2$$

$$3x + 2y > 6$$

$$-3x - 3x$$

$$2y > 6 - 3x$$

$$\div 2 \div 2 (2 > 0)$$

$$y > 3 - \frac{3}{2}x$$

$$x + 3y \le 2$$

$$-x - x$$

$$3y \le 2 - x$$

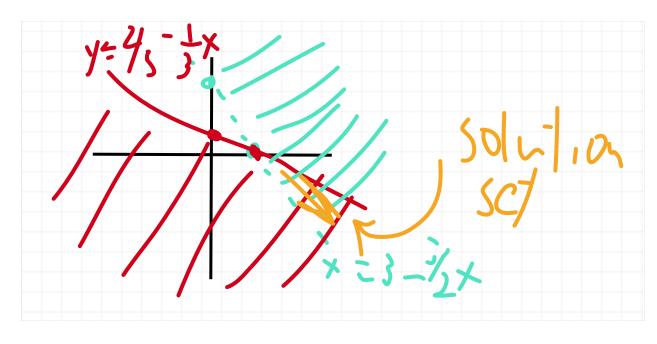
$$\div 3 \div 3 \quad (3 > 0)$$

$$y \le \frac{2}{3} - \frac{1}{3}x$$

We then have

$$y > 3 - \frac{3}{2}x$$
$$y \le \frac{2}{3} - \frac{1}{3}x.$$

We graph



#### Question 12 (page 246).

$$x + y < 2$$

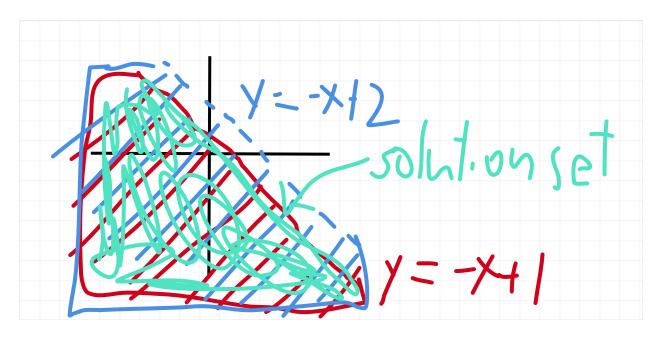
$$x + y \le 1$$

In slope-intercept form, the system of inequalities are

$$y < -x + 2$$

$$y \le -x + 1$$

#### We get the region

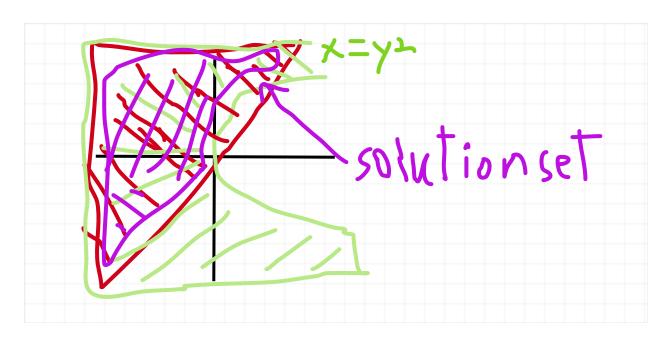


Question 16 (page 246): it involves a parabola, which is not linear.

$$x \le y^2$$

$$y \ge x$$

Note that graphically, we have a parabola above



## Question 20 (page 246).

 $2x + y \le 2$ 

 $y \ge x$ 

 $x \ge 0$ 

The first line in slope-intercept form is

$$y \leq -2x + 2$$

And so we draw the lines and get

