Exam 1 Practice Solutions

Problem 1.

1a. First we use distributive property and combine the like terms

$$x - 2(5 - 3x) = 8x - 14$$
$$x - 10 + 6x = 8x - 14,$$

and then we do algebra as follows:

$$\begin{aligned} 7x - 10 &= 8x - 14 \\ -7x \qquad &\quad -7x \\ -10 &= x - 14 \\ +14 \quad &\quad +14 \\ 4 &= x. \end{aligned}$$

1b. Multiply by a common denominator to get rid of the fractions. 12 is a common denominator

$$\begin{aligned} 1 + \frac{x}{3} - \frac{x}{4} &= x - \frac{5x}{6} \\ \times 12 \qquad &\quad \times 12 \\ 12 + 4x - 3x &= 12x - 10x, \end{aligned}$$

next we combine like-terms ($4x - 3x = x$ and $12x - 10x = 2x$)

$$12 + x = 2x.$$

Then we can do algebra as normal and get the solution

$$\begin{aligned} 12 + x &= 2x \\ -x \quad &\quad -x \\ 12 \quad &= x. \end{aligned}$$

1c. First we use distributive property and combine the like terms

$$\begin{aligned} 3(2 + n) + 6 &= 2(n + 3) + n \\ 6 + 3n + 6 &= 2n + 6 + n \\ 12 + 3n &= 3n + 6. \end{aligned}$$

and then we do algebra as follows:

$$\begin{aligned} 12 + 3n &= 3n + 6 \\ -3n \quad &\quad -3n \\ 12 \quad &= 6, \end{aligned}$$

but $12 \neq 6$ and we have a contradiction, so this problem has no solution.

Problem 2.

Using algebra, we have

$$
\begin{aligned}
ax - c &= bx + d \\
+c \quad &\quad + c \\
ax \quad &= bx + c + d \\
-bx \quad &\quad - bx \\
ax - bx &= c + d,
\end{aligned}
$$

then we get x be itself using the distributive property in reverse

$$
\begin{aligned}
ax - bx &= c + d \\
(a - b)x &= (c + d),
\end{aligned}
$$

and we divide the coefficient $(a - b)$ to get

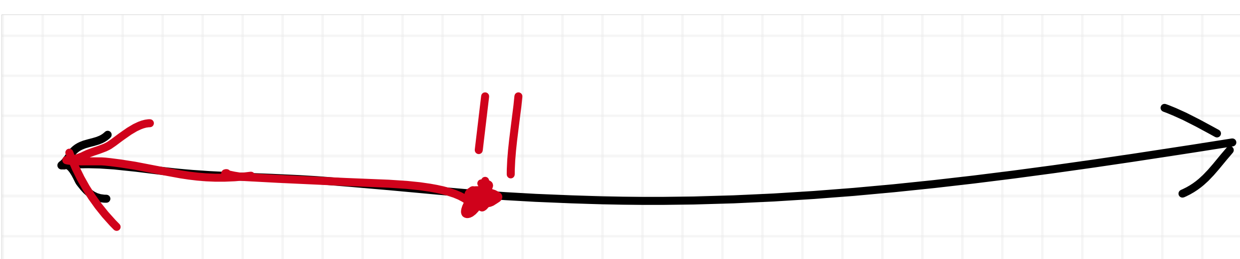
$$x = \frac{c + d}{a - b}.$$

Problem 3.

Using algebra on inequalities, we get

$$
\begin{aligned}
-4 \ &\leq \frac{1}{2}(3 - x) \\
-4 \ &\leq \frac{3}{2} - \frac{x}{2} \\
\times 2 \quad &\times 2 \\
-8 &\leq 3 - x \\
+x \quad &+ x \\
-8 + x &\leq 3 \\
+8 \quad &+ 8 \\
x \ &\leq 11
\end{aligned}
$$

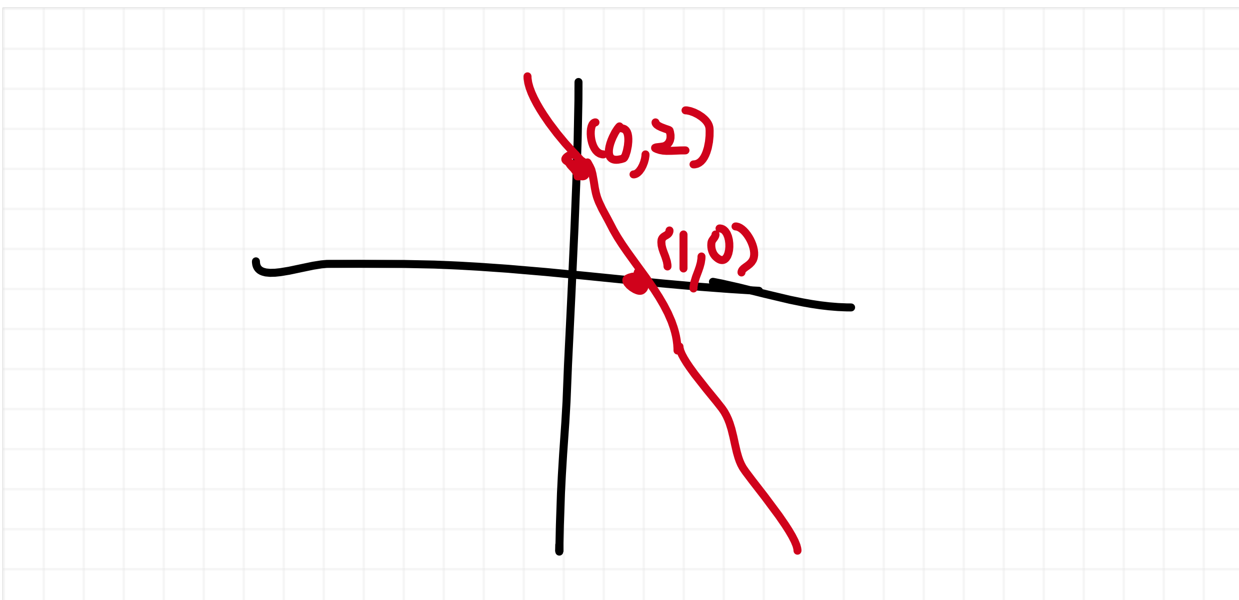
The inequality looks like the following on the number line



We then have the closed unbounded interval $\{x : x \leq 11\} = (-\infty,\ 11]$.

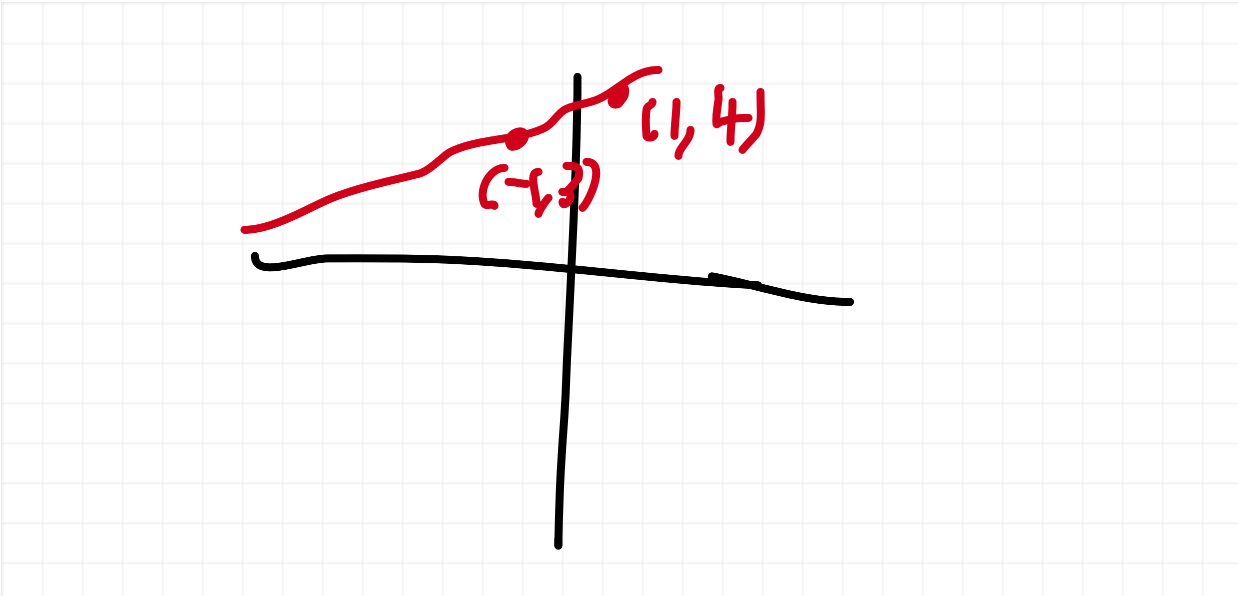
Problem 4.

4a. Plug in $x = 0$ and $y = 0$ to get the y and x intercepts $(0, 2)$ and $(1, 0)$ respectively, then plot the line through those two points:



4b. Using the point-slope formula, we have
$y - 3 = 1/2(x + 1)$,

so the line contains the point $(-1, 3)$ and $(1, 4)$ (found through knowing that the rise is $+1$ if the run is $+2$).

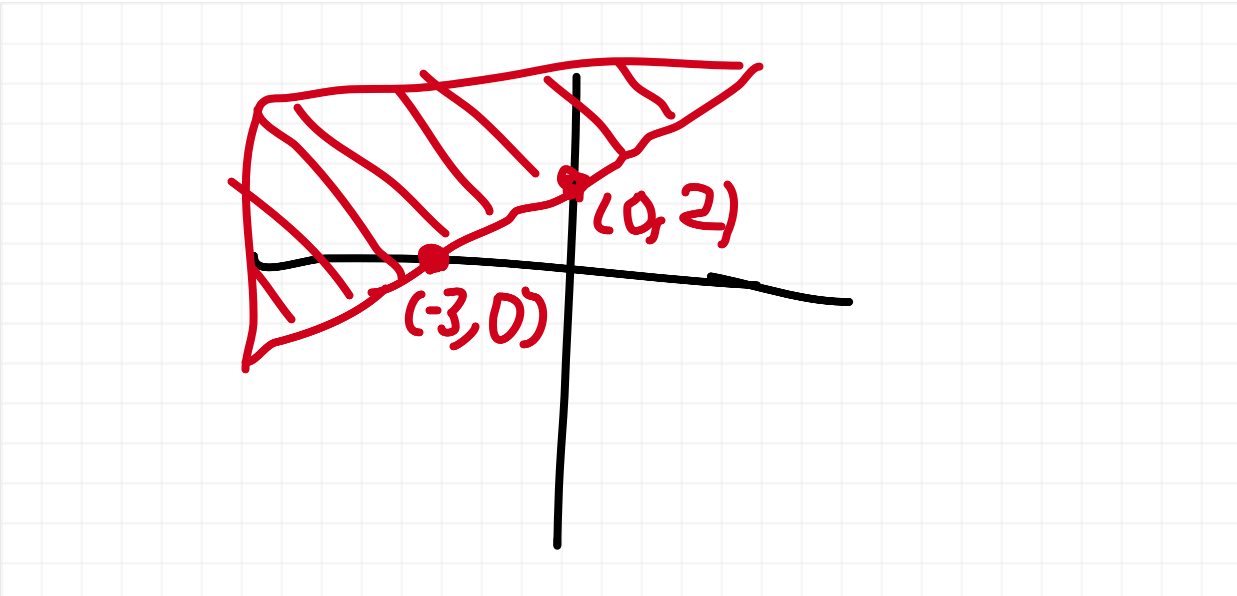


4c. First, we graph the line $3y - 2x = 6$ by connecting the x and y intercepts, which are $(-3, 0)$ and $(0, 2)$, respectively.

Next, we determine if we shade above or below the graph (and include the line by drawing the whole line, since the inequality ISN'T strict). We do this by putting the inequality in slope-intercept form as follows:

$$\begin{aligned} 3y - 2x &\geq 6 \\ +2x \quad &\quad +2x \\ 3y &\geq 2x + 6 \\ \div 3 \quad &\quad \div 3 \\ y &\geq \frac{2}{3}x + 2. \end{aligned}$$

We find since we find y is *greater than or equal* to the line, we shade *above* the line as done below.



Problem 5.

To find the slope, note that the general form is
$5x - 2y = -4$
with $A = 5$ and $B = -2$, and the slope is
$-\frac{A}{B} = -\frac{5}{-2} = \frac{5}{2}$
We can find the y intercept by plugging in $x = 0$ and getting
$-2y = -4$
$\div -2 \quad \div -2$
$y = 2,$
and we have y-intercept $(0, 2)$.

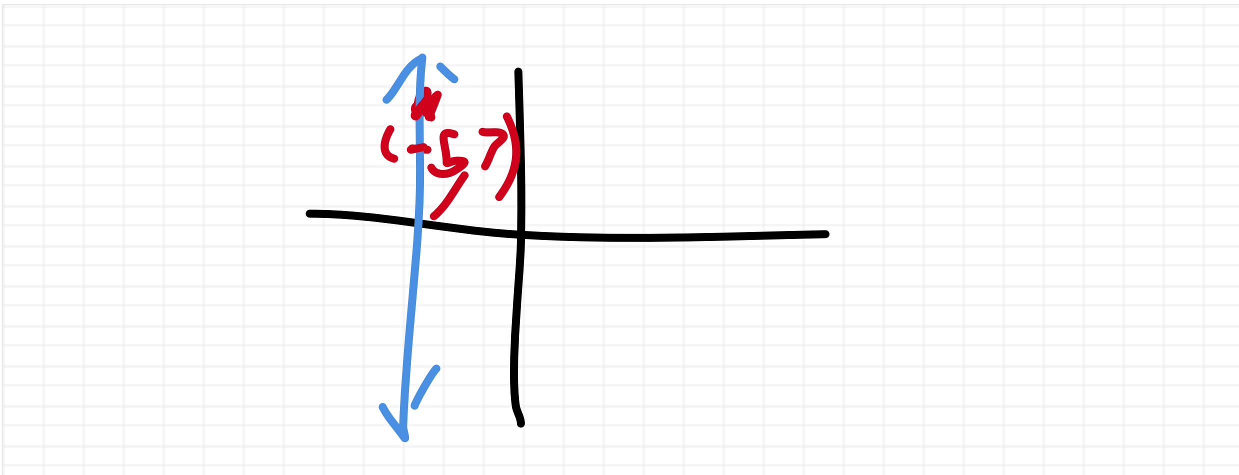
Another way to do it is find the slope-intercept formula
$5x - 2y = -4$
$-5x \qquad -5x$
$-2y = -5x - 4$
$\div -2 \quad \div -2$
$y = \frac{5}{2}x + 2$
and we have the slope $5/2$ and y-intercept $(0, 2)$

Problem 6.

6a. Using point slope form, we have

$y = -2x + 1/2$

6b. The equation for vertical line is always $x = a$ for some real number a. It must be that $a = -5$ since it's through the point $(-5, 7)$.



6c. Using point slope form, we have
$y - 0 = -3(x - 4)$

Problem 7.

We can solve using the substitution method as follows:

$$\begin{aligned} 7a - b &= 17 \\ -7a \quad & \quad -7a \\ -b &= 17 - 7a \\ \div -1 \quad & \quad \div -1 \\ b &= 7a - 17, \end{aligned}$$

Plugging in $b = 7a - 17$ into the other equation $3a - 2b = 1$, we get

$$\begin{aligned} 3a - 2(7a - 17) &= 1 \\ 3a - 14a + 34 &= 1 \\ -11a + 34 &= 1 \\ -34 \quad & \quad -34 \\ -11a &= -33 \\ \div -11 \quad & \quad \div -11 \\ a &= 3. \end{aligned}$$

Plugging in $a = 3$, we get
$b = 7(3) - 17 = 4,$

and we have the solution $(a,b)=(3,4)$.

Problem 8.

The equation
$x-3y+2z=0$
$-x+4y-3z=1$
$2x+y+z=3,$

has matrix form
$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -1 & 4 & -3 & 1 \\ 2 & 1 & 1 & 3 \end{array}\right)$$

Using row reduction, we get
$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -1 & 4 & -3 & 1 \\ 2 & 1 & 1 & 3 \end{array}\right)$$

row $2+$ row 1

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 2 & 1 & 1 & 3 \end{array}\right)$$

row $3-2\cdot$ row 1

$$\left(\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 7 & -3 & 3 \end{array}\right)$$

row $1+3\cdot$ row 2

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 7 & -3 & 3 \end{array}\right)$$

row $3-7\cdot$ row 2

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 4 & -4 \end{array}\right)$$

row $3 \div 4$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

row $1 +$ row 3

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

row $2 +$ row 3

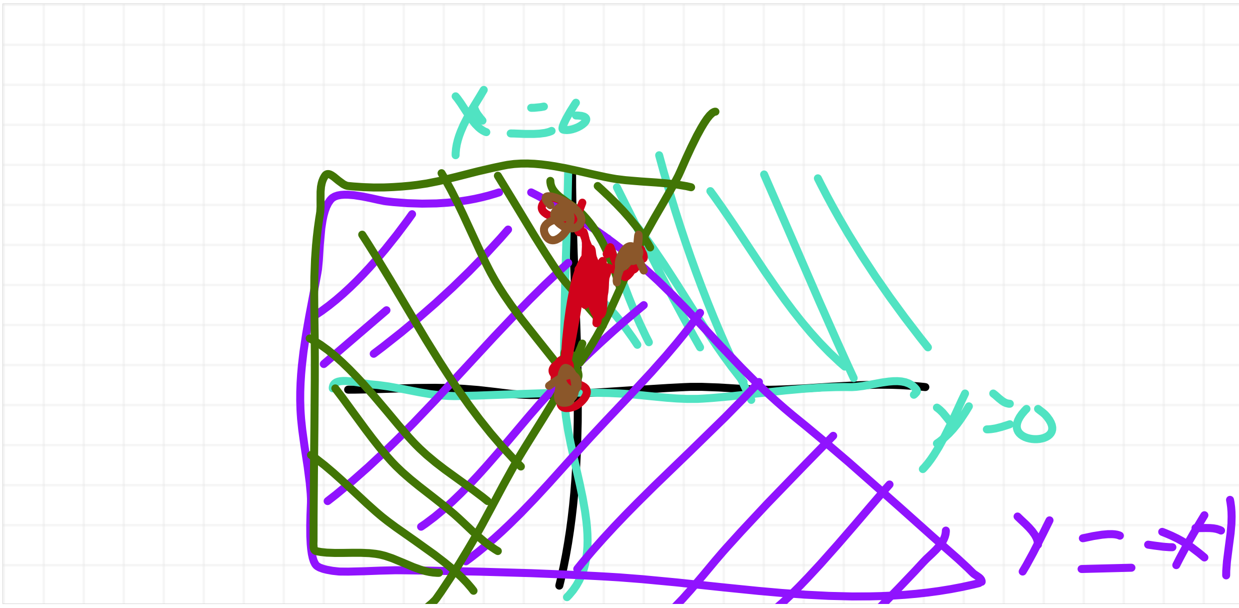
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right),$$

and we get the solution $(x, y, z) = (2, 0, -1)$.

Problem 9.

Note we have the following system of inequalities:
$x \geq 0,\ y \geq 0$
$x + y \leq 6 \quad y \leq -x + 6$
$y \geq 2x + 1.$

9a. The solution is the red-shaded region where the shaded areas in the diagram below intersect:



9b. To find the vertices, we find the corner points by identifying where the lines intersect in the boundary of the solution set (see the brown points in the diagram above). We solve the system of equations for each of the three intersections as follows:

$x = 0$
$y = 2x + 1$

$y = 2(0) + 1$
$y = 1$
$(x, y) = (0, 1)$,

$x = 0$
$y = -x + 6$

$(x, y) = (0, 6)$,

$y = 2x + 1$
$y = -x + 6$

$$
\begin{aligned}
-x + 6 &= 2x + 1 \\
+x \quad &\quad\ \ + x \\
6 &= 3x + 1 \\
-1 \quad &\quad\ \ - 1 \\
5 &= 3x \\
5/3 &= x
\end{aligned}
$$

$$y = 2\frac{5}{3} + 1 = \frac{10}{3} + 1 \cdot \frac{3}{3} = \frac{13}{3}$$

$$(x, y) = (5/3, 13/3).$$

So the vertices are the points $(0, 1)$, $(0, 6)$, $(5/3, 13/3)$.