Fall II 2020 Exam 1A Solutions

Problem 1. Solve each equation:

1a.

$$\frac{2(x-3)}{5} - \frac{(x-2)}{10} = \frac{1}{5}$$

First, we need to multiply by the common denomenator 10 to get rid of the denomenators

$$\begin{array}{lll} \frac{2(x-3)}{5} - \frac{(x-2)}{10} & = & \frac{1}{5} \\ \times 10 & & \times 10 \\ 4(x-3)-(x-2) = 2 & & \end{array}$$

Next, we need to combine like-terms (need to use distributive property)

$$4(x-3)-(x-2) = 2$$
$$4x-12-x+2 = 2$$
$$3x-10 = 2$$

so then we have a nice linear equation setup and we can proceed to solve for x

$$\begin{array}{ll} 3x-10 & = 2 \\ \quad +10 & +10 \\ 3x & = 12 \\ \div 3 & \div 3 \\ x & = 4. \end{array}$$

1b.

$$6(3z+1)-5(2z-2) = 8(3+z)$$

First, we need to combine like-terms (need to use distributive property)

$$18z+6-10z+10 = 24+8z$$
$$8z+16 = 24+8z$$
$$\begin{array}{ll} -8z & -8z \\ \quad 16 = 24 & \end{array}$$

So we get $16 = 24$, but we know $16 \neq 24$, so the equation is a contradiction and there's no solution.

Problem 2. Solve $S = 2a^2 + 4ac$ for c.

First, we have get the term with c by itself.

$$\begin{aligned}
2a^2 + 4ac &= S \\
-2a^2 \qquad & \quad -2a^2 \\
4ac &= S - 2a^2 \\
\div 4a \quad & \quad \div 4a \\
c &= \frac{S - a^2}{4a}.
\end{aligned}$$

Problem 3. Solve the inequality and express the answer in ***in interval notation***:

$$2x - 5 \leq 3(x - 1) + 4$$

Just like with linear equalities, we combine-like terms and make the inequality in typical linear format

$$\begin{aligned}
2x - 5 &\leq 3x - 3 + 4 \\
2x - 5 &\leq 3x + 1 \\
-2x \quad & \quad -2x \\
-5 &\leq x + 1 \\
-1 \quad & \quad -1 \\
-6 &\leq x.
\end{aligned}$$

The interval is $\{x : x \geq -6\} = [-6, \infty)$.

Problem 4. Sketch the graph of each

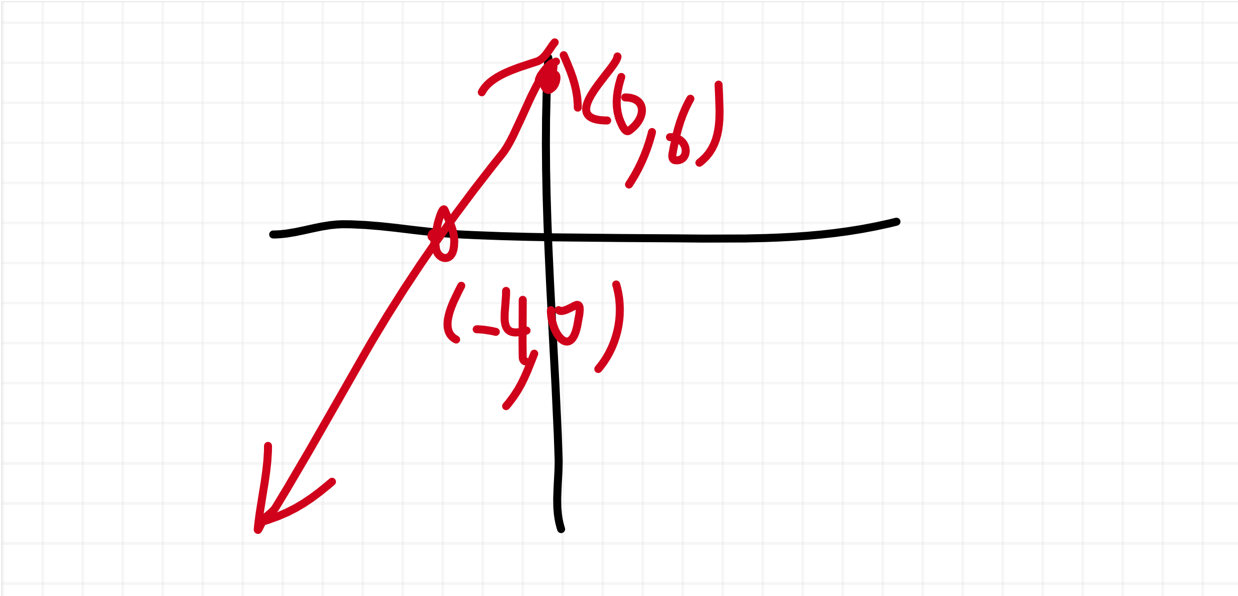
4a. $2y - 3x = 12$
We can find the line by finding two points (the x and y axes). To find the y-axis, plug in $x = 0$ to get

$$\begin{aligned}
2y - 3(0) &= 12 \\
2y &= 12 \\
\div 2 \quad & \div 2 \\
y &= 6.
\end{aligned}$$

Similarly, to find the x-axis, we plug in $y = 0$ to get

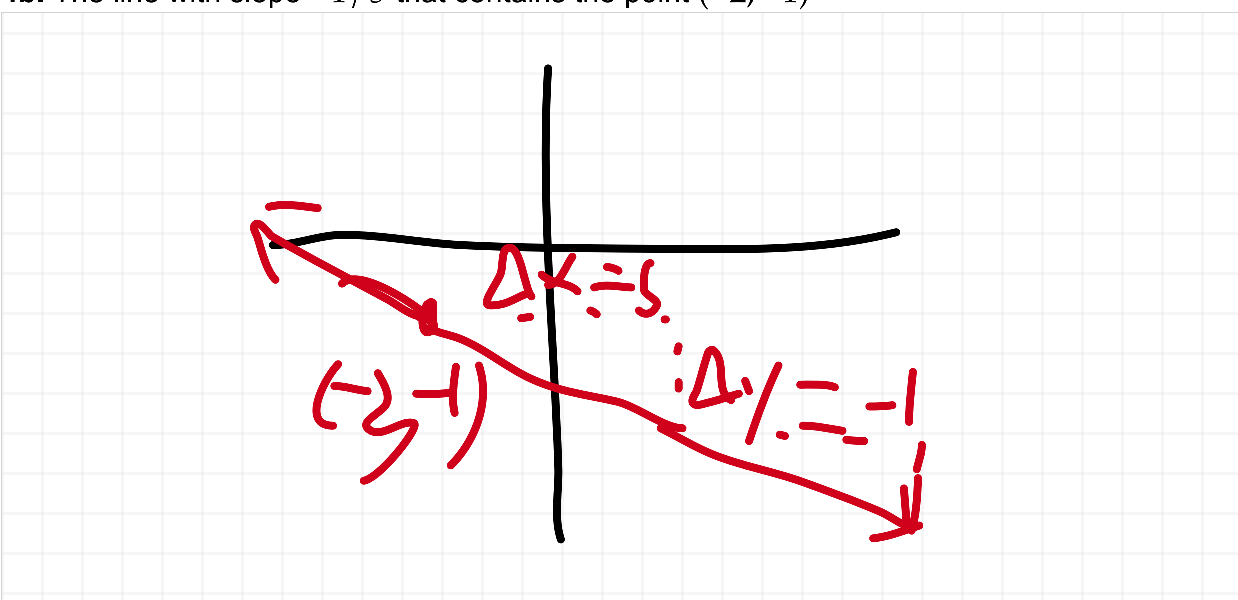
$$\begin{aligned}
2(0) - 3x &= 12 \\
-3x &= 12 \\
\div -3 \quad & \div -3 \\
x &= -4.
\end{aligned}$$

Those points are $(0, 6)$ and $(-4, 0)$



Note that we can also draw the line by finding one point and figuring out the slope (either through the general form formula $-A/B$ or through converting the equation to the slope-intercept formula.

4b. The line with slope $-1/5$ that contains the point $(-2, -1)$



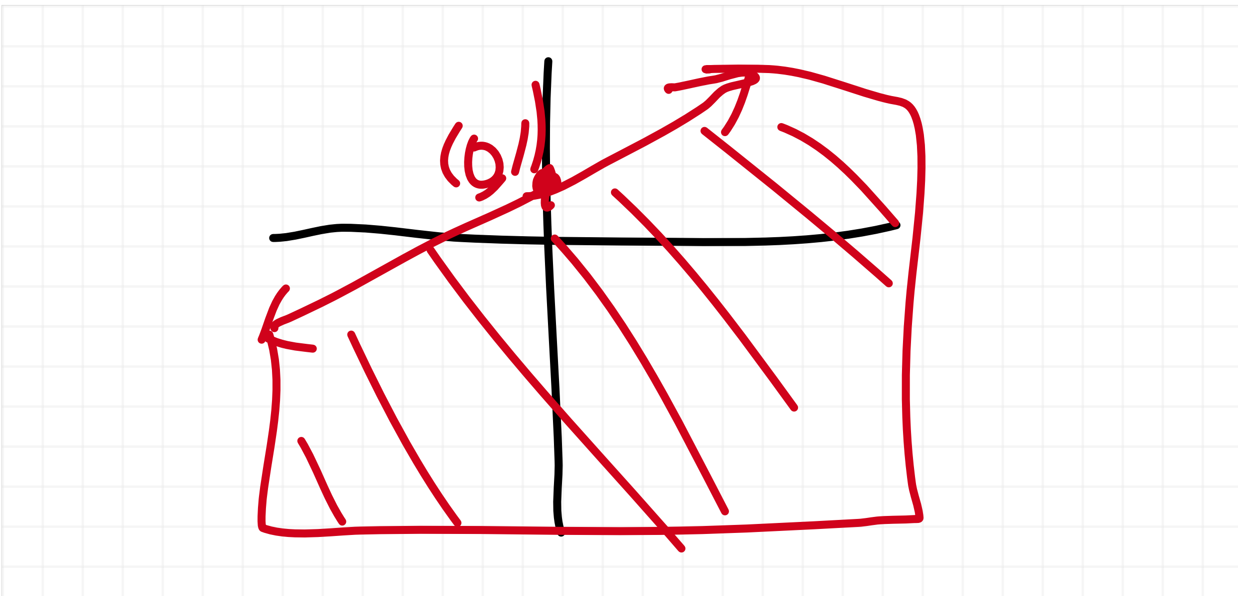
4c. The ***inequality*** $x \geq 2y - 2$

We can get the inequality in slope-intercept form as follows

$x \geq 2y - 2$

$$
\begin{aligned}
&+2 \qquad +2 \\
&x+2 \ \geq 2y \\
&\div 2 \qquad \div 2 \\
&\frac{1}{2}x+1 \geq y.
\end{aligned}
$$

Note that the inequality consists of all y BELOW the line



Problem 5. Find the equation of

5a. The line through $(3, 8)$ that is parallel to the $x-$ axis.

NOTE: The x-axis is a horizontal line (has slope 0)

So the desired is horizontal as well, since it's parallel, so it's of the form $y = a$, and it must be that $a = 8$ in order for the line to contain the point $(3, 8)$, so we have the equation

$y = 8$

5b. The line with slope -3 that crosses the y-axis at $(0, 7)$

Slope-intercept form $y = mx + b$ gives us the line right away, since we know $m = -3$ and $b = 7$:

$y = -3x + 7$

5c. The line containing $(-3,-4)$ and $(1,0)$

We use the point slope formula

$y - y_1 = m(x - x_1), \quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

for $P_1 = (1,0)$ and $P_2 = (-3,-4)$, we get

$m = \dfrac{-4-0}{-3-1} = \dfrac{-4}{-4} = 1.$

So plugging in m, y_1, x_1, have

$$
\begin{aligned}
y - 0 &= 1(x-1) \\
y &= x - 1.
\end{aligned}
$$

Problem 6. Solve the system:

$$
\begin{aligned}
3y &= 6 - 9x \\
6x + 5y - 1 &= 0
\end{aligned}
$$

Let's do the substitution method (note that we can also do it in the addition method):

$3y = 6 - 9x$

$\div 3 \quad \div 3$

$y = 2 - 3x,$

Plugging $y = 2 - 3x$ into the other equation, we get

$6x + 5(2-3x) - 1 = 0.$

First, we need to combine like-terms

$6x + 5(2-3x) - 1 = 0$

$6x + 10 - 15x - 1 = 0$

$-9x + 9 = 0$

$\quad -9 \quad -9$

$-9x \quad = -9$

$\div -9 \quad \div -9$

$x \quad = 1.$

So we need to also get the numeric value for y by plugging in $x = 1$ into $y = 2 - 3x$

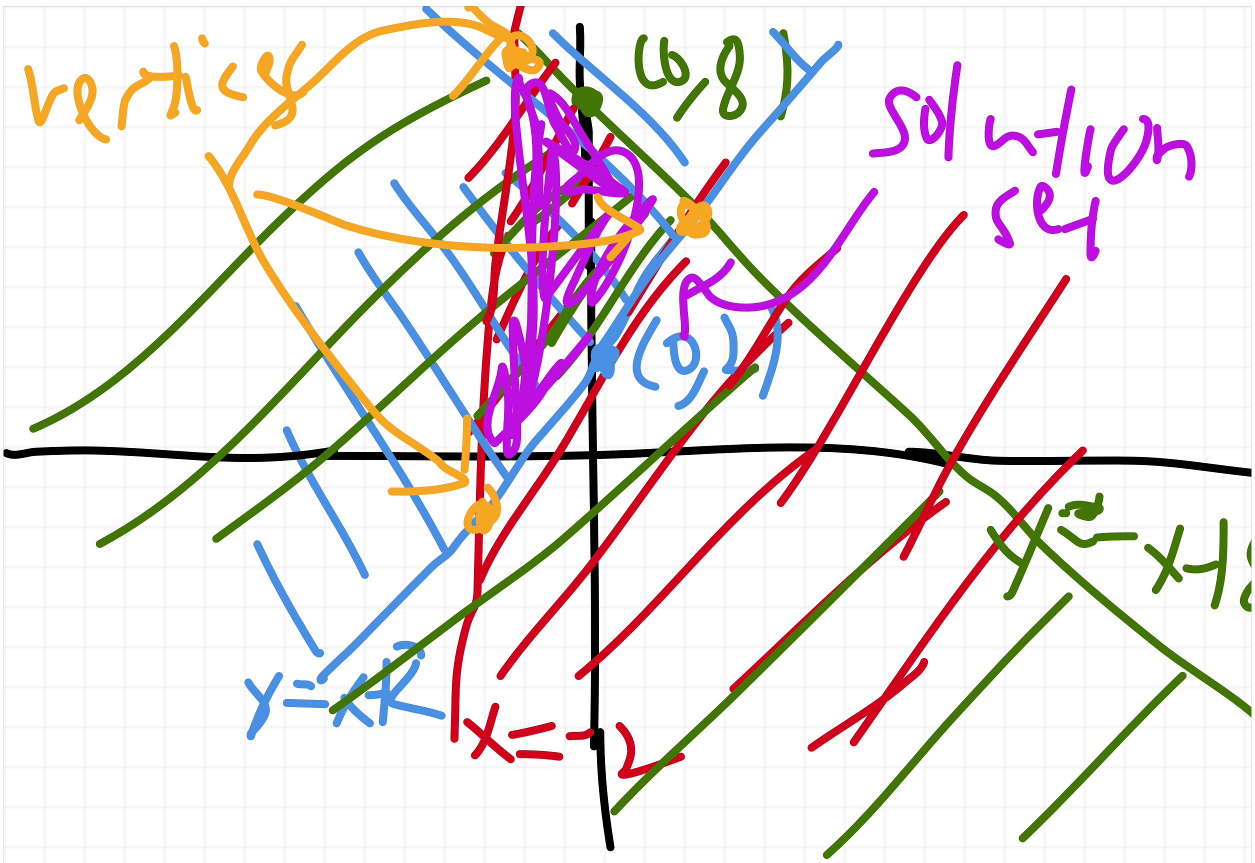
$y = 2 - 3(1) = -1.$

So we have the solution $(x, y) = (1, -1)$

Problem 7.

7a. Sketch the solution to the system

$x + y \leq 8 \quad y \leq -x + 8$
$y \geq x + 2$
$x \geq -2$



7b. Find the coordinates of each of the ***vertices*** (i.e., the corner points) of the solution

The first two vertices intersect with the vertical line $x = -2$ and the first two intersect with the following two lines

The bottom one is the following intersection

$x = -2$
$y = x + 2$

$y = (-2) + 2 = 0,$

which is the point $(-2, 0)$

The top one is the following intersection:
$x = -2$
$y = -x + 8$

$y = -(-2) + 8 = 10,$
which is the point $(-2, 10)$.

The middle one is the following intersection
$y = x + 2$
$y = -x + 8$

$$\begin{aligned}
x + 2 &= -x + 8 \\
+x \quad & \quad +x \\
2x + 2 &= 8 \\
-2 \quad & \quad -2 \\
2x \quad &= 6 \\
\div 2 \quad & \quad \div 2 \\
x \quad &= 3,
\end{aligned}$$

$y = (3) + 2 = 5,$
which is the point $(3, 5)$.

Problem 8. Use ***matrix methods*** to sove for the following system

$$\begin{aligned}
x + y - 2z &= 7 \\
2x + 3y + z &= 11 \\
2y - z &= 5
\end{aligned}$$

We have matrix form:

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 2 & 3 & 1 & 11 \\ 0 & 2 & -1 & 5 \end{array}\right)$$

row $2 - 2 \cdot$ row 1

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & 1 & 5 & -3 \\ 0 & 2 & -1 & 5 \end{array}\right)$$

row $1 -$ row 2

$$\left(\begin{array}{ccc|c} 1 & 0 & -7 & 10 \\ 0 & 1 & 5 & -3 \\ 0 & 2 & -1 & 5 \end{array}\right)$$

row $3 - 2 \cdot$ row 2

$$\left(\begin{array}{ccc|c} 1 & 0 & -7 & 10 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & -11 & 11 \end{array}\right)$$

row $3 \div - 11$

$$\left(\begin{array}{ccc|c} 1 & 0 & -7 & 10 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

row $1 + 7 \cdot$ row 3

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

row $2 - 5 \cdot$ row 3

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right),$$

so we have the solution $(x, y, z) = (3, 2, -1)$.