Fall II 2020 Exam 1A Solutions

Problem 1.

1a. Solve
$$\frac{1}{4}x - \frac{1}{5} = \frac{4}{5} + \frac{1}{5}x.$$

So to this, we should get rid of the fractions by multiplying by the common denomenator 20

$$\begin{aligned} &\frac{1}{4}x - \frac{1}{5} = \frac{4}{5} + \frac{1}{5}x \\ &\times 20 \qquad\quad \times 20 \\ &20\left(\frac{1}{4}x - \frac{1}{5}\right) = 20\left(\frac{4}{5} + \frac{1}{5}x\right). \end{aligned}$$

Now we use distributive property and cancel out the denomenators

$$\begin{aligned} 20\left(\frac{1}{4}x - \frac{1}{5}\right) &= \frac{20}{4}x - \frac{20}{5} = 5x - 4 \\ 20\left(\frac{4}{5} + \frac{1}{5}x\right) &= \frac{80}{5} + \frac{20}{5}x = 16 + 4x. \end{aligned}$$

Now, we have the equation

$$\begin{aligned} &5x - 4 = 16 + 4x \\ &-4x \qquad -4x \\ &x - 4 = 16 \\ &\;\; +4 \quad +4 \\ &x \qquad = 20, \end{aligned}$$

and we get $x = 20$.

1b. Solve
$$2(x-3) - 7 = 5(5-x) + 7(x+1).$$

So we have no fractions (thanks goodness!), but we have to use distributive property, as follows:
$$2(x-3) - 7 = 2x - 6 - 7 = 2x - 13$$

$$5(5-x)+7(x+1) = 25-5x+7x+7 = 32+2x$$

So now we have the equation

$$\begin{aligned} 2x-13 &= 32+2x \\ -2x \qquad & \qquad -2x \\ -13 &= 32 \\ +13 \quad & \quad +13 \\ 0 &= 45, \end{aligned}$$

which is a contradiction and *no solution exists.*

Problem 2. Solve $b(a+2) = 1$ for a.

So to do that, we have to use distributive property as follows:

$$b(a+2) = ba + b2 = ab + 2b$$

and then we get the following equation:

$$\begin{aligned} ab + 2b &= 1 \\ -2b \quad & \quad -2b \\ ab \qquad &= 1-2b \\ \div b \qquad & \quad \div b \\ a \qquad &= \frac{1-2b}{b} \end{aligned}$$

we get

$$a = \frac{1-2b}{b} = \frac{1}{b} - \frac{2b}{b} = \frac{1}{b} - 2.$$

3. Solve the following inequality and give the solution in **<u>interval notation</u>**:

$$5 - \frac{1}{3}x \leq 2$$

We start out by putting isolating the x coefficient using the addition property:

$$\begin{aligned} 5 - \frac{1}{3}x &\leq 2 \\ -5 \qquad & \quad -5 \\ -\frac{1}{3}x &\leq -3 \end{aligned}$$

Next, we cancel out the $-\frac{1}{3}$ coefficient by using the multiplication property:

$$-\frac{1}{3}x \leq -3$$
$$\times -3 \qquad \times -3 \quad \text{(note that } -3 \text{ is a negative number)}$$
$$x \geq 9,$$

in interval notation, this is expressed as $[9, \infty)$.

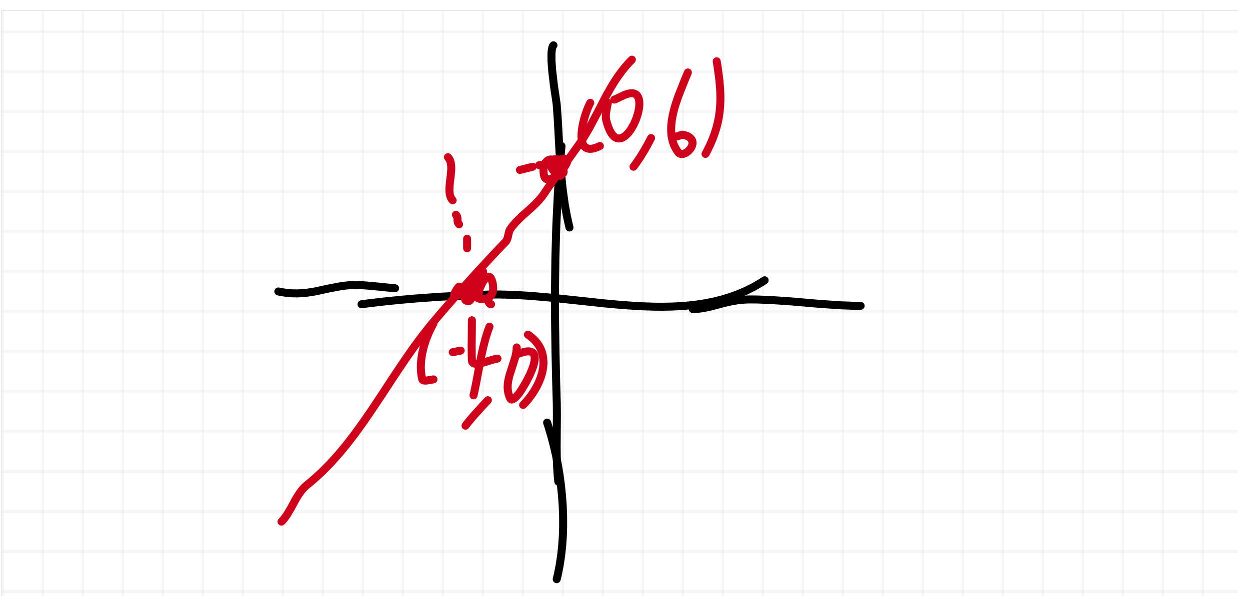
Problem 4. Sketch the graphs of the following equations

4a. $3x - 2y + 12 = 0$

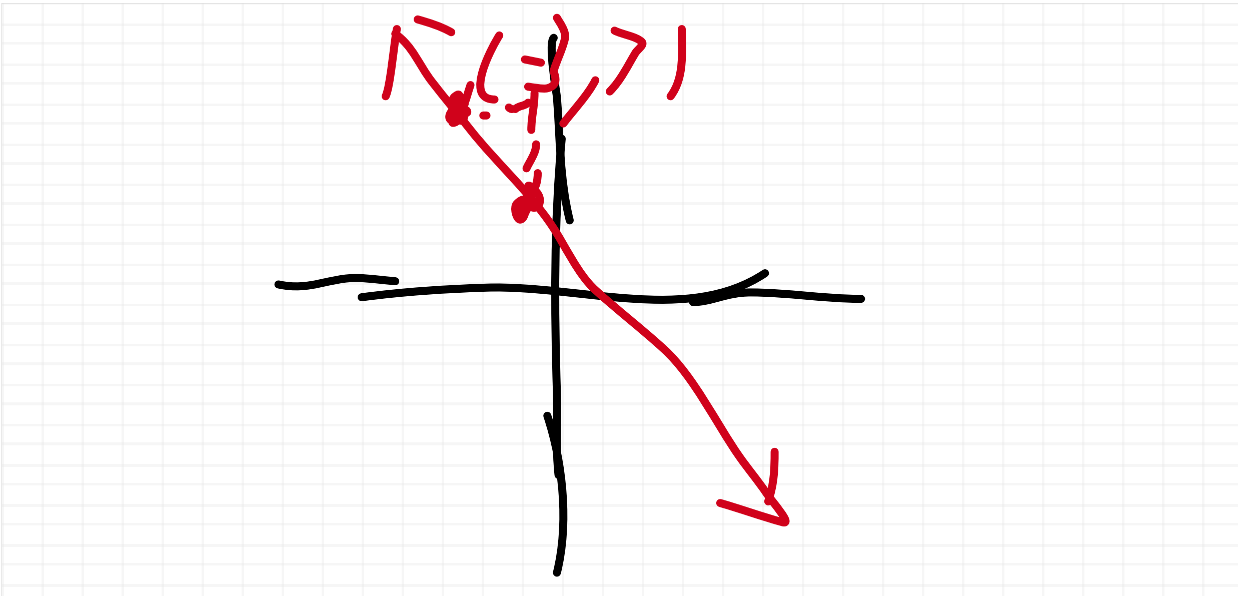
So we can rearrange in many different forms, but the general form takes the least work

$$3x - 2y + 12 = 0$$
$$-12 \qquad -12$$
$$3x - 2y = -12$$

Note that the general form has slope $-\frac{3}{-2} = \frac{3}{2}$ and the y-intercept is $\left(0, \frac{-12}{-2}\right) = (0, 6)$. So now we have all the information we need to draw the graph



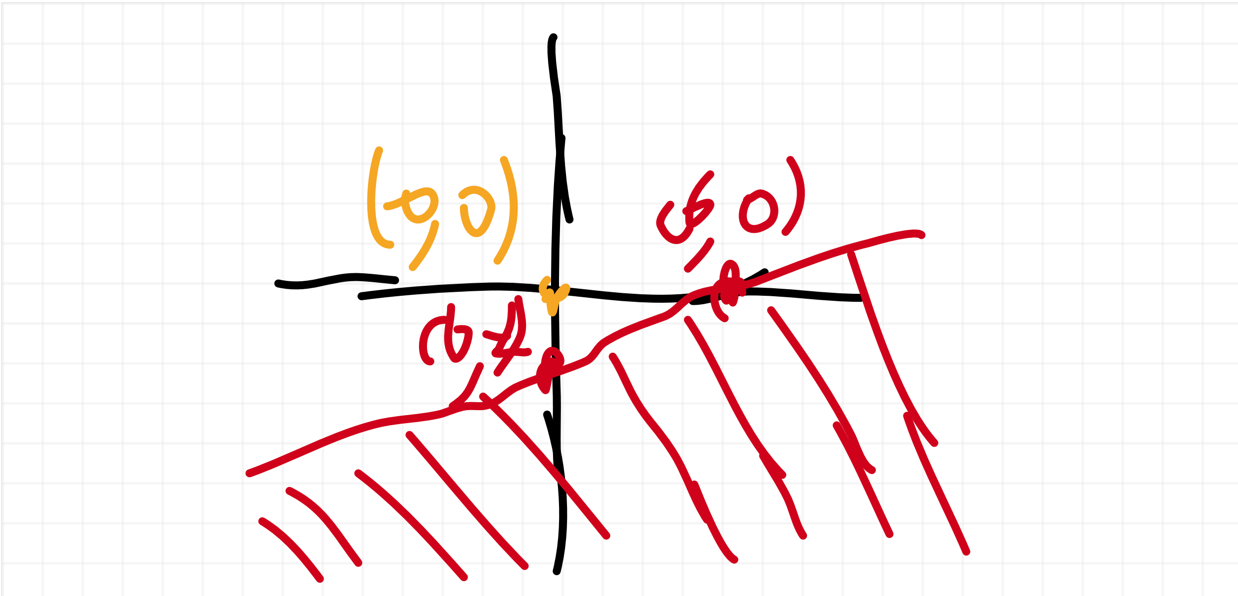
4b. The line with slope $-3/2$ and containing the point $(-3, 7)$



4c. $x - 3y \geq 6$. Note that the line $x - 3y = 6$ is in general form and has slope $-\dfrac{1}{-3} = \dfrac{1}{3}$ and with y-intercept $\left(0, \dfrac{6}{-3}\right) = (0, -2)$

We plug in the origin $(0, 0)$ and see that
$(0) - 3(0) = 0 \ngeq 6$,

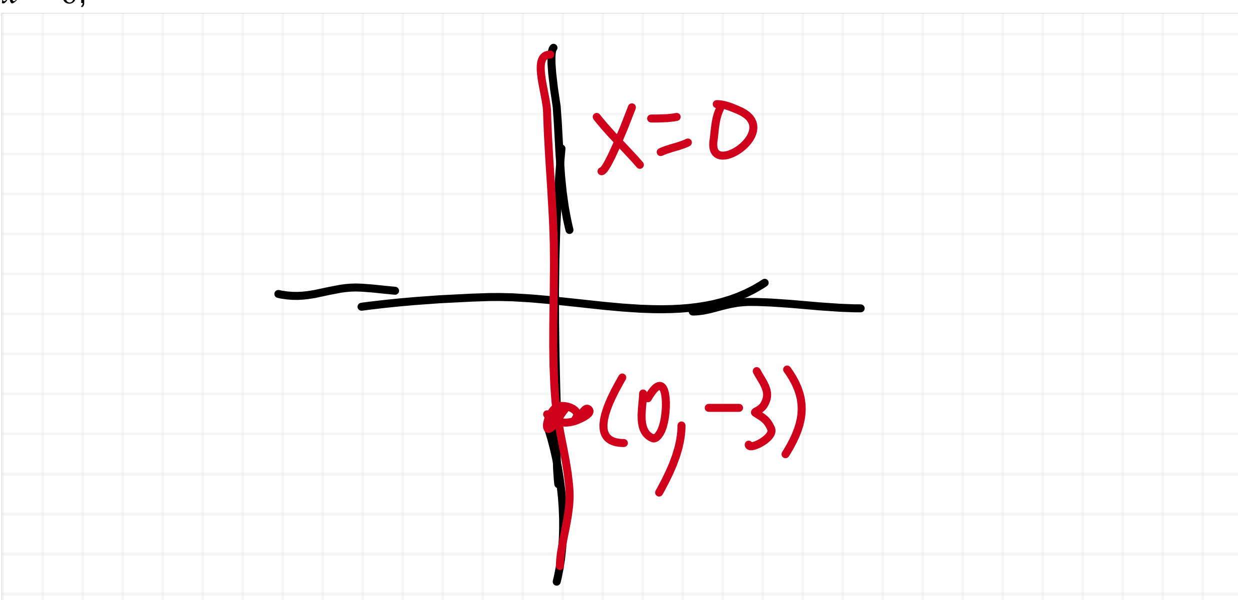
so the origin $(0, 0)$ is not included, and we draw below the line as a result.



Problem 5. Find the equation of

5a. The vertical line through $(0, -3)$

A vertical line has an equation of the form $x = a$, so there must a point in the line where $x = 0$,



so then the equation is $x = 0$.

5b. The line with slope -2 that contains the point $(0, 3)$.

To do that, we use the point-slope equation plugging in -2 for m and $(0, 3)$ for (x_0, y_0) as follows.

$$y - y_0 = m(x - x_0)$$
$$y - 3 = -2(x - 0)$$

So if you want to simplify the line, you can (no penalty if you don't).

$$y = -2x + 3$$

5c. The line containing $(-1, 4)$ and $(1, 2)$.

Here, we use the more elaborate point-slope equation where we determine the slope m using the formula

$$m = \frac{y_1 - y_0}{x_1 - x_0},$$

plugging in the points $(x_0, y_0) = (-1, 4)$ and $(x_1, y_1) = (1, 2)$ as follows:

$$m = \frac{2 - 4}{1 - (-1)} = \frac{-2}{2} = -1$$

and we get using the equation $y - y_0 = m(x - x_0)$:

$$y - 4 = -1(x + 1).$$

If you want, you can simplify this to get

$$y = -x + 3.$$

Problem 6. Solve the system:

$$5x - 3y = 18$$
$$2x + 7y = -1$$

To solve this system, we can use the substitution method as follows:

We take the top equation $5x - 3y = 18$ and solve for y in terms of x to get

$$5x - 3y = 18$$
$$-5x \qquad -5x$$
$$-3y = 18 - 5x$$
$$\div -3 \qquad \div -3$$
$$y = -6 + \frac{5}{3}x$$

So now we substitute $y = -6 + \frac{5}{3}x$ in the bottom equation $2x + 7y = -1$

$$2x + 7\left(-6 + \frac{5}{3}x\right) = -1$$

Using distributive property, we get

$$2x + 7\left(-6 + \frac{5}{3}x\right) = 2x - 42 + \frac{35}{3}x,$$

we end up with the equation

$$2x - 42 + \frac{35}{3}x = -1$$

To cancel out the fraction, we multiply by 3 to get:

$$2x - 42 + \frac{35}{3}x = -1$$
$$\times 3 \qquad \times 3$$
$$6x - 126 + 35x = -3$$

Next we combine like-terms

$$6x - 126 + 35x = -126 + 41x$$

Now we have the equation

$$-126 + 41x = -3$$
$$+126 \qquad +126$$
$$41x = 123$$
$$\div 41 \qquad \div 41$$

$x = 3$

Now we plug in $x = 3$ to $y = -6 + \frac{5}{3}x$ to get

$y = -6 + \frac{5}{3}(3) = -6 + 5 = -1,$

so we have the solution $(x, y) = (3, -1)$.

Problem 7. Sketch the solution to the system:
$x \leq 6,\ y \geq 0$
$y \leq x + 4$
$x + y \geq 4$

First, we plot the lines in each of the inequalities (note that the lines are drawn and not traced since the inequalities are not strict). The lines we plot are as follows:

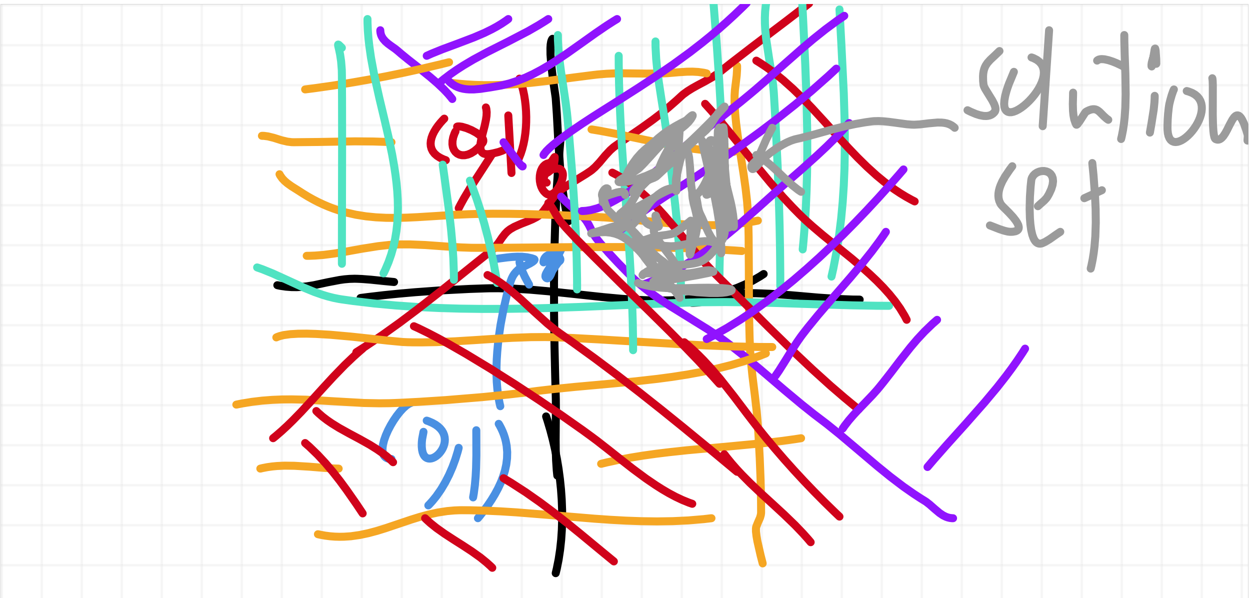
$x = 6,\ y = 0$

$y = x + 4$
(note that this equation is slope intercept form for the line with slope 1 with y-intercept $(0, 4)$)

$x + y = 4$
(note that this equation is the general form of the line with slope $-\frac{1}{1} = -1$ and y-intercept $\left(0, \frac{4}{1}\right) = (0, 4)$)

So to figure out the direction of each of the inequalities, we take the point $(0, 1)$ and plug it in to each of the equations and use it to figure out which regions are shaded in each individual inequality.

$x \leq 6$ $\quad (0) \leq 6?$ yes, so we shade to the right of the orange line
$y \geq 0$ $\quad (1) \geq (0)?$ yes, so we shade above the turquoise line
$y \leq x + 4$ $(1) \leq (0) + 4?$ yes, so we shade below the red line
$x + y \geq 4$ $(0) + (1) \geq 4?$ no, so we shade above the purple line



Problem 8. Use the matrix method to solve:

$x - y + z = 4$
$-x + 2y - z = 0$
$-x + y - 3z = -16$

NOTE: If you solve this system *any other way*, you will be given **no credit!**

First, we set up the problem as a matrix as follows

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ -1 & 2 & -1 & 0 \\ -1 & 1 & -3 & -16 \end{array}\right)$$

row 2 + row 1

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & 0 & 4 \\ -1 & 1 & -3 & -16 \end{array}\right)$$

row 3 + row 1

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -2 & -12 \end{array}\right)$$

row $1 +$ row 2

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -2 & -12 \end{array}\right)$$

row $3 \div -2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \end{array}\right)$$

row $1 -$ row 3

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \end{array}\right),$$

so we get solution $(x, y, z) = (2, 4, 6)$.