M106 Summer 2020 Recitation Lectures: Perspective

7/21 Lecture

Announcements:

NEW PERSPECTIVE UNIT!
3 homeworks uploaded (will find out if more are coming)
homework 1 due Thurs. 7/23
homework 2 due Fri. 7/24
homework 3 due Mon. 7/27
Exam 5 Wed. 7/29
Final Exam (all units) 7/31

My Goal: Try and get classes 1-10 all of this week, maybe finish up on Monday
Rest of Monday, and Tuesday I can focus on review
As much of next week as possible a complete review session

Attendence "grades" uploaded (on the local canvas!):
Remember:
-Attendence is 10% of your grade
-4 absences don't count negatively
-Each absence after 4 gives you .5% penalty of that 10% (so someone with 6 absences 9% out of 10%)
-Any student with over 8 *unexcused* automatically fails the course.

NOTE:

The current grade (on your GLOBAL canvas factors homeworks and exams, it doesn't factor in attendence)
The attendence grades (ON YOUR LOCAL CANVAS) how many times you haven't been here (disregard the numeric grade)

Class 1: Introduction

*This module 3 dimensional (3D) objects in 2 dimensions (2D)
*Being able to accurately represent 3D objects on a 2D plane

*This is actually very useful (arguably the MOST USEFUL module in M106!)

Real life applications:
-Animated movies and CGI movies
-Art
-Medical Imagery
-Building blueprints/architecural draings
-3D product designs
-Engineering
-Physics

We answer following question:
How do we "best" represent a 3D object on a 2D plane?

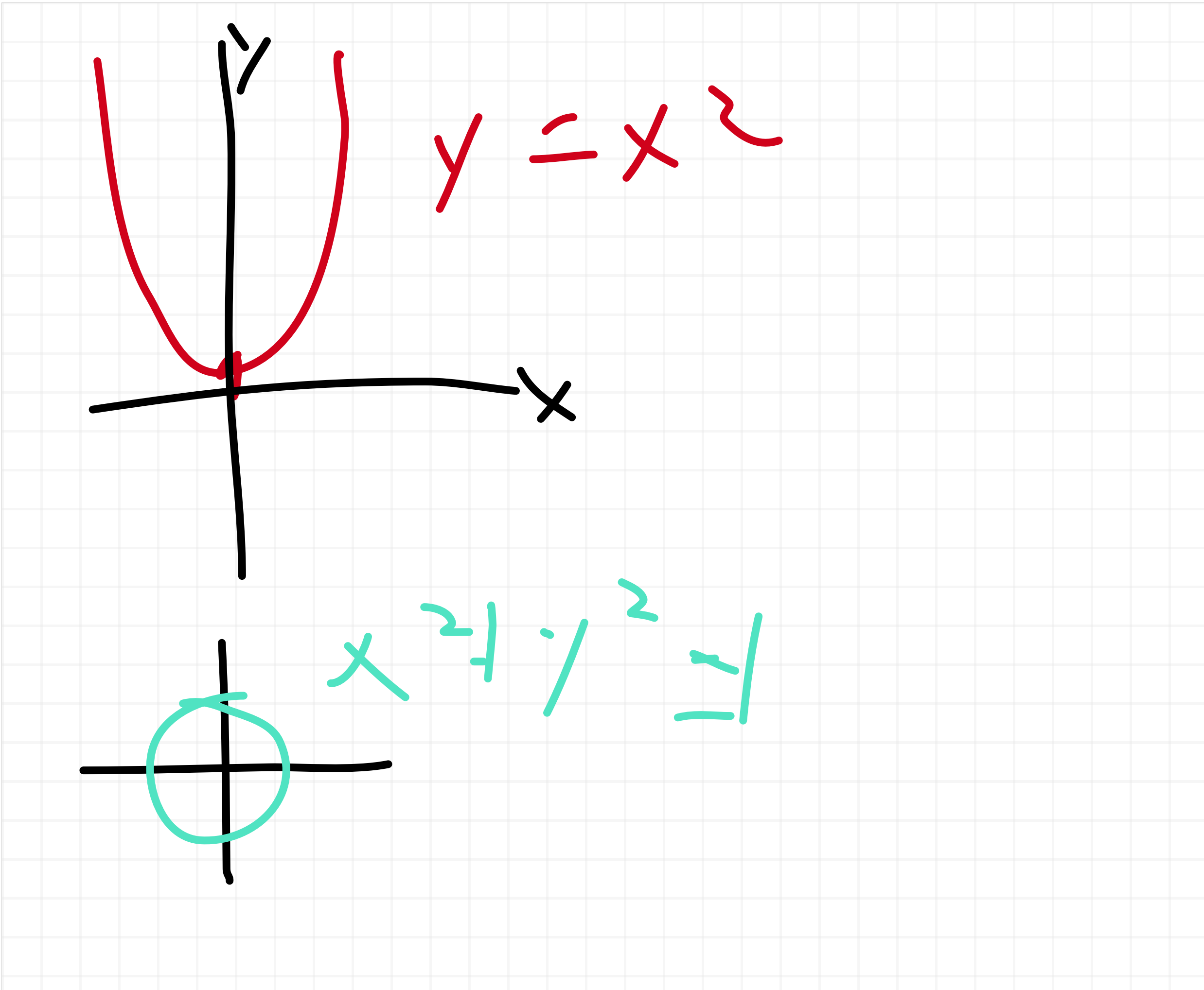
Parital/Naive answer: It depends, it's a matter of perspective (pun intended!)

Class 2: 2D Lines and Their Linear Equations

In the xy plane, different curves can be expressed in terms of different equations.

Example 2.1:

Equation:
$y = x^2$ (a parabola)
$x^2 + y^2 = 1$ (a circle)
$3y^5 - 2x^2y + x^{17} = xy + y - 5x^5$ (something that doesn't have a name)



We'll in this class on linear equations.

Example 2.2
The following equations are all linear equations
$y = x$
$y + 4 = 3x - 2$
$3x - 2y = 5$
$x = 2$

These equations are linear because they represent a line in the xy plane

A **line** is curve that is made up of a point and a (constant) slope.

A **linear equation** is a representation of a line on the xy plane in terms of an equation (using x and y as variables)

NOTE: Linear equations in the xy plane describe a line and every line line in the plane can be described by a linear equation. (Thm. 2.3)

Many lines can be described by slope/intercept form.

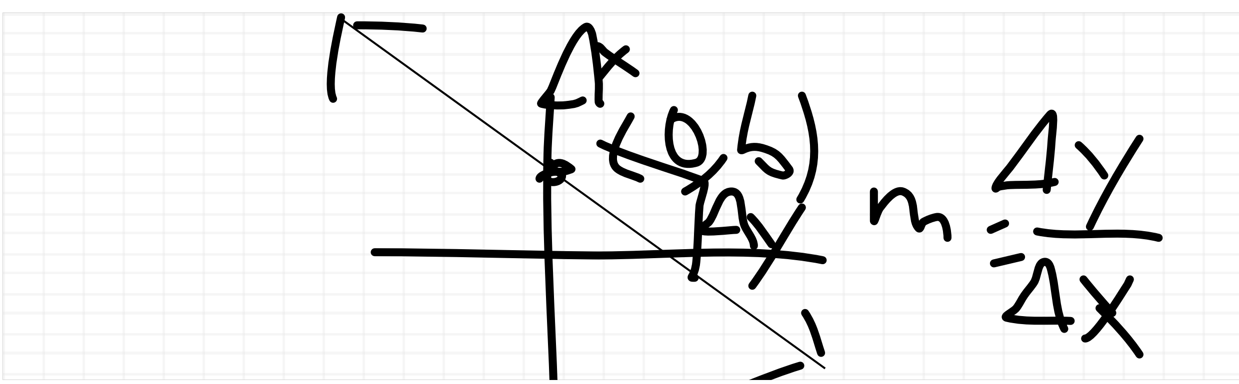
Slope/intercept form (Def. 2.6):

The **slope/intercept form** of a line is a equation of a line in the form $y = mx + b$

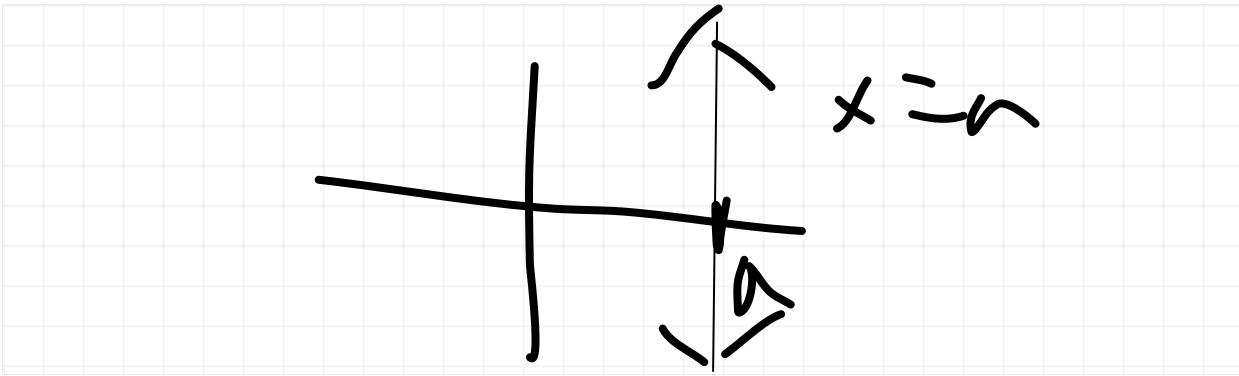
*The **slope** m is the rate of change in y per x "rise over run"
*The **(y-)intercept** b is the value at which the line crosses the y-axis.

NOTE:

1. This equation is very efficient when it comes to drawing the line. The equation gives you a point (on the y-intercept), i.e. $(0,\ b)$ and the slope m which you can draw the rest of the line after drawing the point with a straight edge.



2. NOT EVERY LINE has a slope/intercept form. Specifically vertical lines (of the form $x = a$) do not have a defined numeric slope (since there's infinite rise over no run).



To determine the point slope form of a line, you want *solve algebraically* for y in terms of x.

Example 2.7

$y = 2x + 1$ point/slope form $y = 2x + 1$

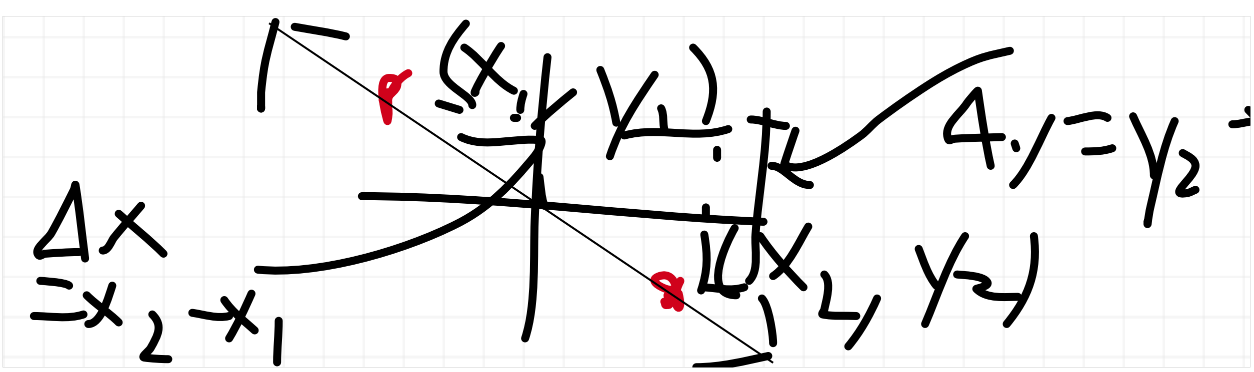
$y + 4 = 3x - 2$
$\quad -4 \qquad\quad -4$ point/slope form $y = 3x - 6$
$y \qquad = 3x - 6$

$3x - 2y = 5$
$-2y = -3x + 5$
$\div -2 \qquad \div -2$ point/slope form $y = 3/2x - 5/2$
$y = 3/2x - 5/2$

$y = 4$ (Note it's equivalent to $y = 0x + 4$) point/slope $y = 4$

$x = 7$ (Note it's a vertical line) point/slope form: Not possible

There's another important way to draw a line. One can determine a line purely through knowing two points.



The two point form:
For any two points on the xy plane $(x_1, y_1), (x_2, y_2)$, we get the following equation of a line (provided that $x_1 \neq x_2$)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

or

$y - y_1 = m(x - x_1)$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Note that we can reexpress this equation in point slope form as follows
$y = mx + (y_1 - mx_1)$ we find that $b = y_1 - mx_1$

Some terminology
If
$m > 0$, we call the slope "uphill"/positive
$m = 0$, we call the line horizontal
$m < 0$, we call the slope "downhill"/negative
"$m = \infty$" (i.e. $x_1 = x_2$) we call the slope vertical.

Parallel/Perpendicular lines (Def. 2.16)
-Lines with the same slope are parallel
-Lines that intersect at 90 degrees (i.e. "perpendicularly") are called perpendicular.

NOTE (Thm. 2.17): Using linear equations $y = m_1x + b_1$, $y = m_2x + b_2$, we get the following equivalent conditions of parallel and perpendicular lines

They are parallel if $m_1 = m_2$
They are perpendicular if $m_1 = -1/m_2$

7/22 Lecture

Announcements:

homework 1 due Thurs. 7/23
homework 2 due Fri. 7/24

homework 3 due Mon. 7/27
Exam 5 Wed. 7/29

Notes are 11 classes long

Final Exam Fri. 7/31

Homework 1 Questions:

Based on class 2

Previously:

We talked about the perspective module, how it's applied, and the main question it tries to answer: How do we "best" project 3D images into a 2D space? (Class 1)

In class 2, we talked about lines and how lines on the xy plane can be completely represented by equations.

We the **point/slope form**, i.e. "mx+b" equation of a line:
$y = mx + b$

We gave the **two point form** of a line: for two points (x_1, y_1), (x_2, y_2)
$y - y_1 = m(x - x_1)$
$y - y_2 = m(x - x_2)$
for $m = (y_2 - y_1) / (x_2 - x_1)$

Example 2.13:
Find the point/slope formula of the line that passes through $(0, 2)$ and $(3, 8)$:
$y - 2 = \frac{8-2}{3-0} \cdot (x - 0)$
$y - 2 = \frac{6}{3} \cdot x$
$y - 2 = 2x$
$\quad +2 \qquad +2$
$y = 2x + 2$

We talked about parallel and perpendicular lines and gave equivalent mathematical conditions for those qualities for $y = m_1 x + b_1,\ y = m_2 x + b_2$

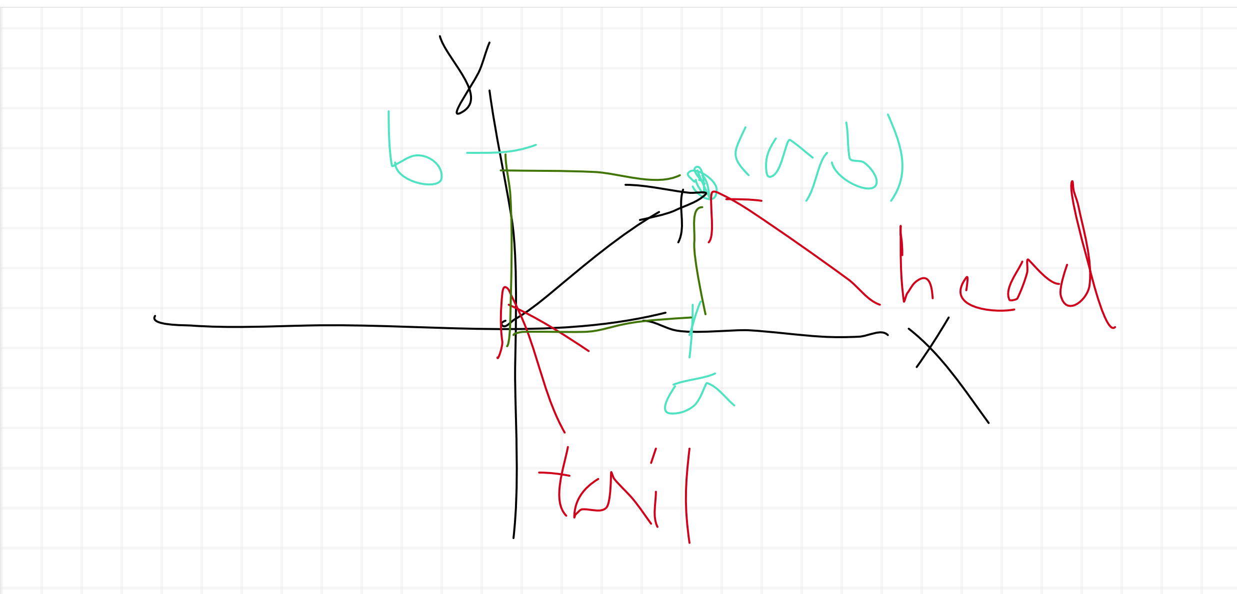
parallel: $m_1 = m_2$
perpendicular: $m_1 = -1/m_2$

Example 2.18:
1) The lines $y = 3x - 1$ and $y = 3x + 17$ are parallel because both of the slopes are 3.
2) The lines $y = -2x$ and $y = 1/2x + 4$ are perpendicular because $-2 = -1/(1/2)$.

Class 3-5 Overview

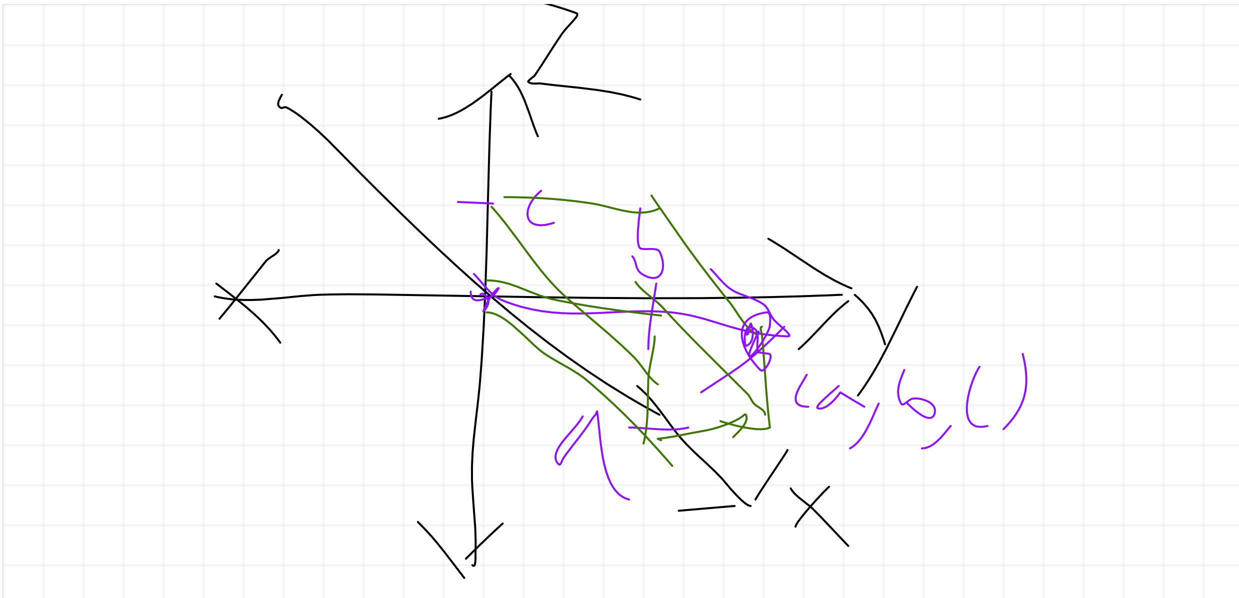
"vector" is just another name for "coordinate"

We interpret a 2D **vector** to be a straight arrow with its **tail** at the origin and its **head** at the point $(a,\ b)$:



a is the **x-component** and b is the **y-component**.

We can extend vectors to three dimensions (more generally an arbitrary number of dimensions) using a 3-tuple coordinate $(a,\ b,\ c)$:



Class 3, 5: Vectors

When talking about vectors abstractly, we simply talk about them as tuples
$\vec{v} = \boldsymbol{v} = (a,\ b)$

We often refer to "regular numbers" as scalars (and denote them using greek letters λ, μ and the letters s, t. Moreover, vectors and scalars lend themselves to their own "vector arithmetic" consisting of the following two operations: scalar multiplication (def. 3.1) and vector addition (def. 3.9)

Scalar Multiplication: We denote scalar multiplication by $\cdot$. For $\vec{v} = (a,\ b),\ \lambda$

$$\lambda \cdot \vec{v} = \lambda \cdot (a, b) = (\lambda a, \lambda b)$$

Vector Addition: We denote vector addition by $+$. For $\vec{v} = (a,\ b),\ \vec{w} = (c,\ d)$

$$\vec{v} + \vec{w} = (a, b) + (c, d) = (a + c,\ b + d)$$

NOTE: These operations generalize (as we do in class 5, def. 5.3) in 3 dimensions as follows: For $\vec{v} = (a,\ b,\ c),\ \vec{w} = (d, e, f),\ \lambda$

$$\lambda \cdot \vec{v} = \lambda \cdot (a, b,\ c) = (\lambda a, \lambda b,\ \lambda c)$$
$$\vec{v} + \vec{w} = (a, b, c) + (d, e, f) = (a + d,\ b + e,\ c + f)$$

Example 3.2:

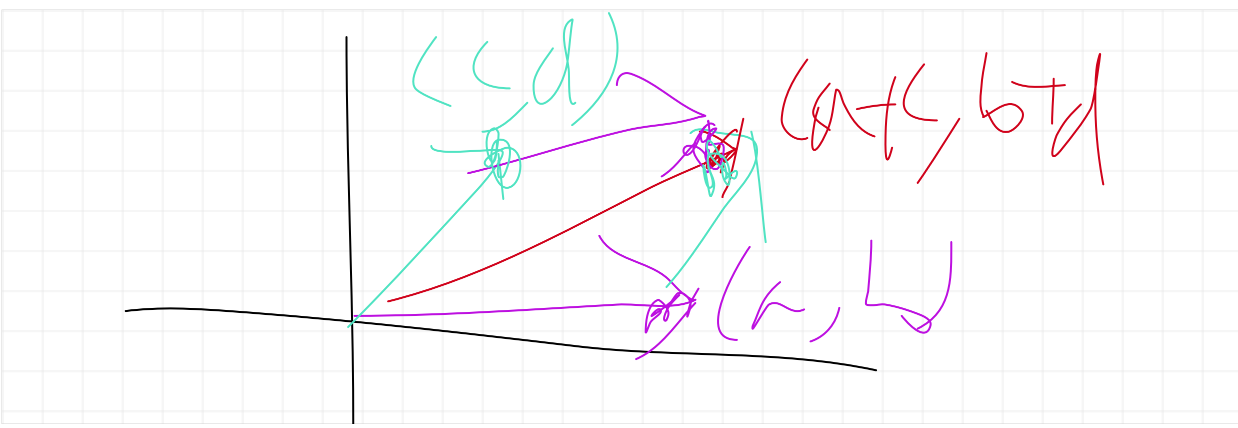
$3 \cdot (1,1) = (3,\ 3)$
$-2\ \cdot (2,\ -1) = (-4,\ 2)$

Example 3.10:

$(1,2) + (3,4) = (1+3, 2+4) = (4,6)$
$(-3,7) + (3,-2) = (0,5)$

NOTE: These operations can be illustrated geometrically

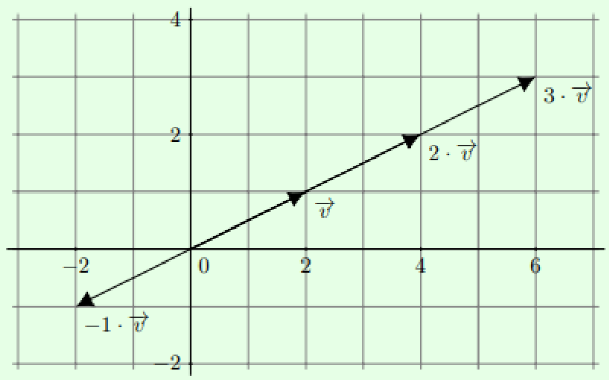
Vector Addition Geometrically



NOTE: Intiutively, we can see that $\vec{v} + \vec{w} = \vec{w} + \vec{v}$

Scalar Multiplication Geometrically

Example 3.4.
Let $\vec{v} = (2, 1)$.



Note the following (from Thm. 3.3)

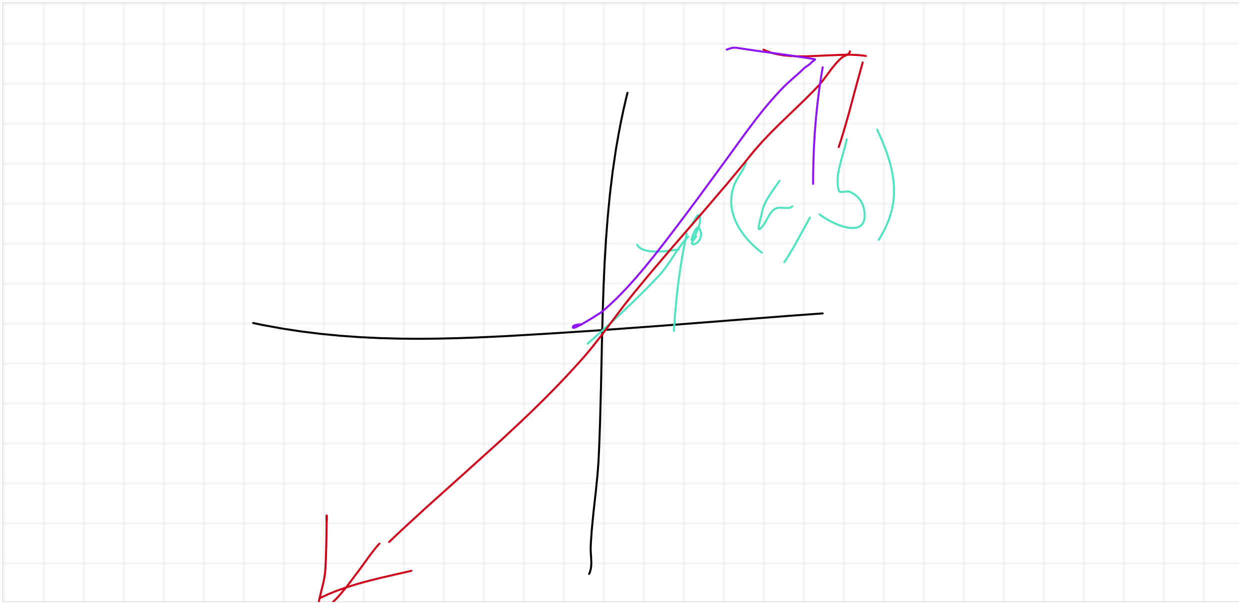
If $\lambda > 0$ (is positive), then the direction of $\lambda\vec{v}$ remains unchanged
If $\lambda < 0$ (is negative), then the direction of $\lambda\vec{v}$ is reversed

$\lambda > 1$ stretches the vector
$\lambda = 1$ does nothing
$0 < \lambda < 1$ shrinks the vector
$\lambda = 0$ "kills" the vector (i.e. $\lambda\vec{v} = (0,\ 0)$)
$-1 < \lambda < 0$ shrinks the vector and reverses it
$\lambda = -1$ reverses the direction
$\lambda < \ -1$ stretches the vector and reverses the direction

Lines and Rays Generated by Vectors: (Def. 3.5 and 3.7)

The **line generated by a vector** is the line obtained by taking scalar multiples of that vector

The **ray generated by a vector** is the half-line obtained by taking all *positive* scalar multiples of that vector.



Class 4: Linear Distortion

7/23 Lecture

Announcements:

Homework 1 due tonight and homework 2 due tomorrow night
Exam 4 grade ETA this afternoon to this evening

Remaining class plan:
Thurs. 7/23 Finish up vector arithmetic, linear distortion, parametric equations (class 3-6)
Fri. 7/24 Drawing projections (classes 7-10)
Mon. 7/27 Vanishing Points (class 11), review: symmetry, game theory
Teus. 7/28 review: graph theory, voting theory, and perspective
Wed. 7/29 Exam 5 (Perspective)
Thurs. 7/30 review: go over the perspective exam, go over the "hardest material" (based on survey)
Fri. 7/31 Final Exam

Office Hour schedule:
Fri. 7/24 2pm-3pm (make up for Monday)
Mon. 7/27 2pm-4pm
Teus. 7/28 2pm-4pm
Thurs. 7/29 2:30pm-4:30pm

NOTE: I can always schedule office hours by appointment

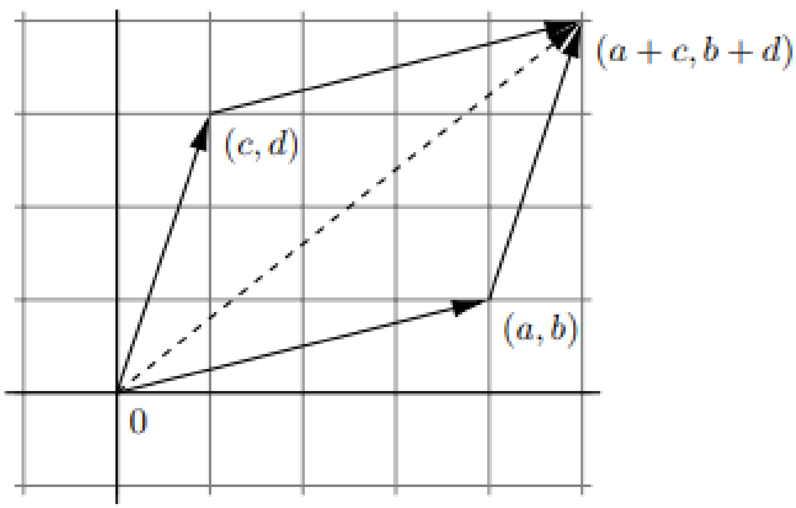
Homework 1-2 Questions:

Previously:

We talked about vectors and what they are
We talked about the vector operations of addition and scalar multiplication

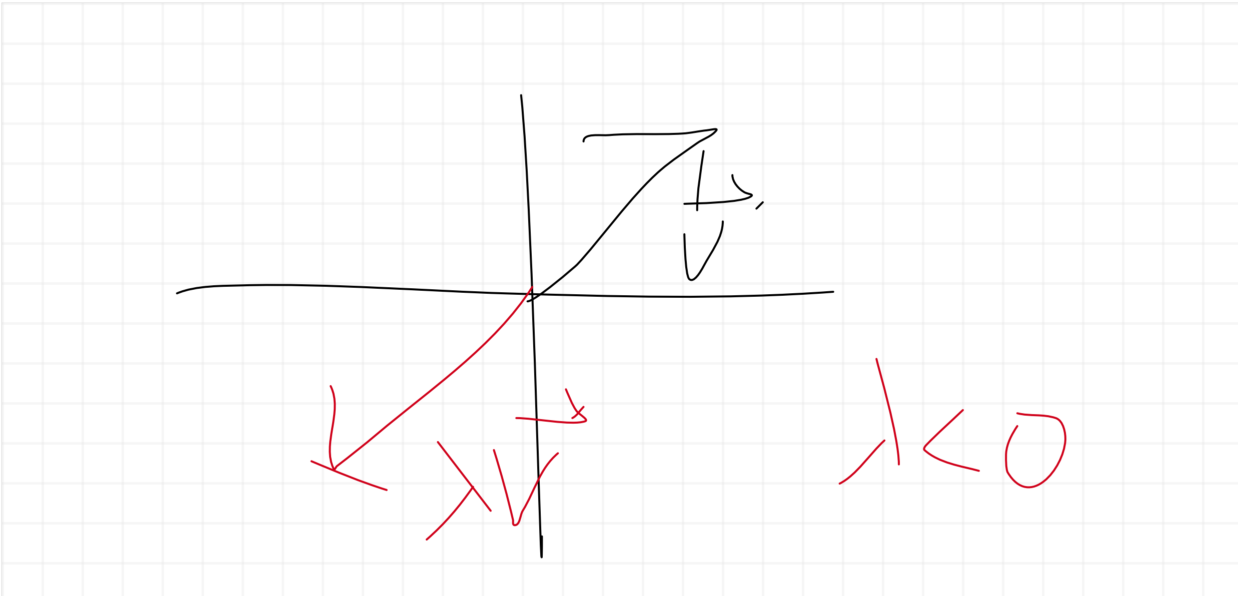
We also gave an intuitive picture on what these operations look like:

Vector Addition:

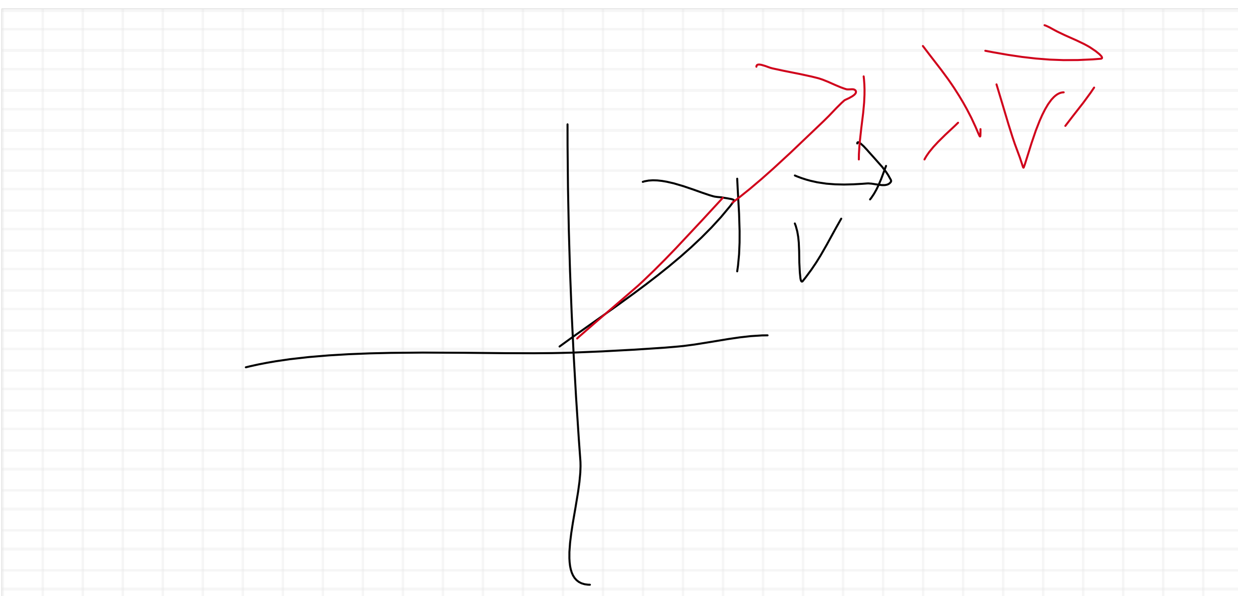


Scalar Multiplication:

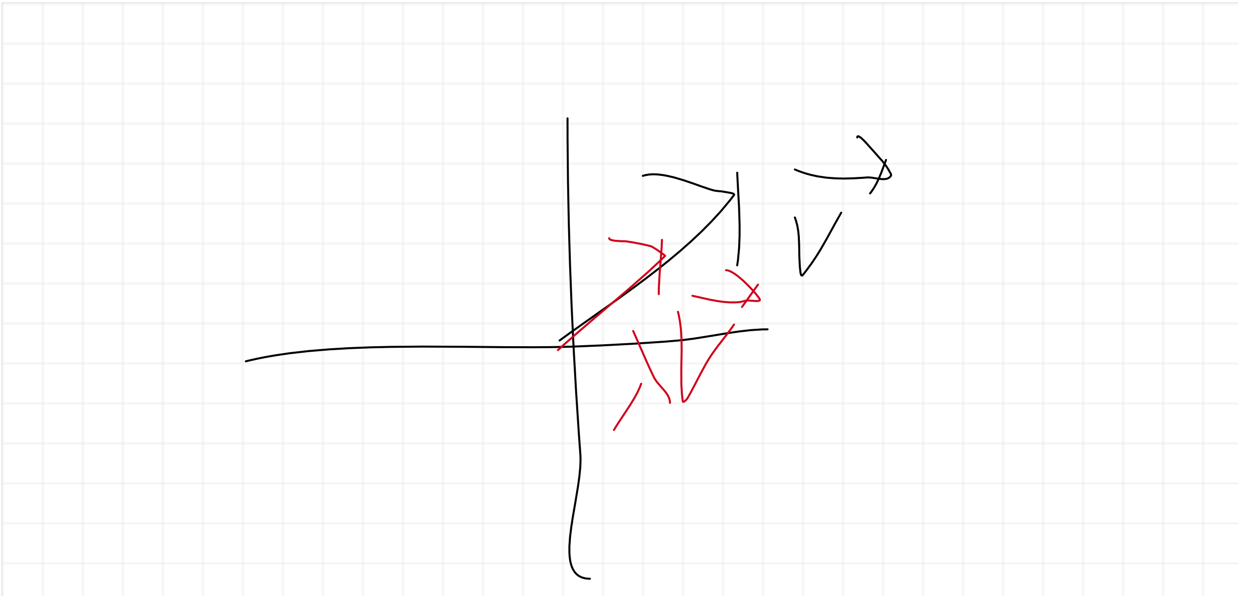
If $\lambda > 0$ (is positive), then the direction of $\lambda\vec{v}$ remains unchanged
If $\lambda < 0$ (is negative), then the direction of $\lambda\vec{v}$ is reversed



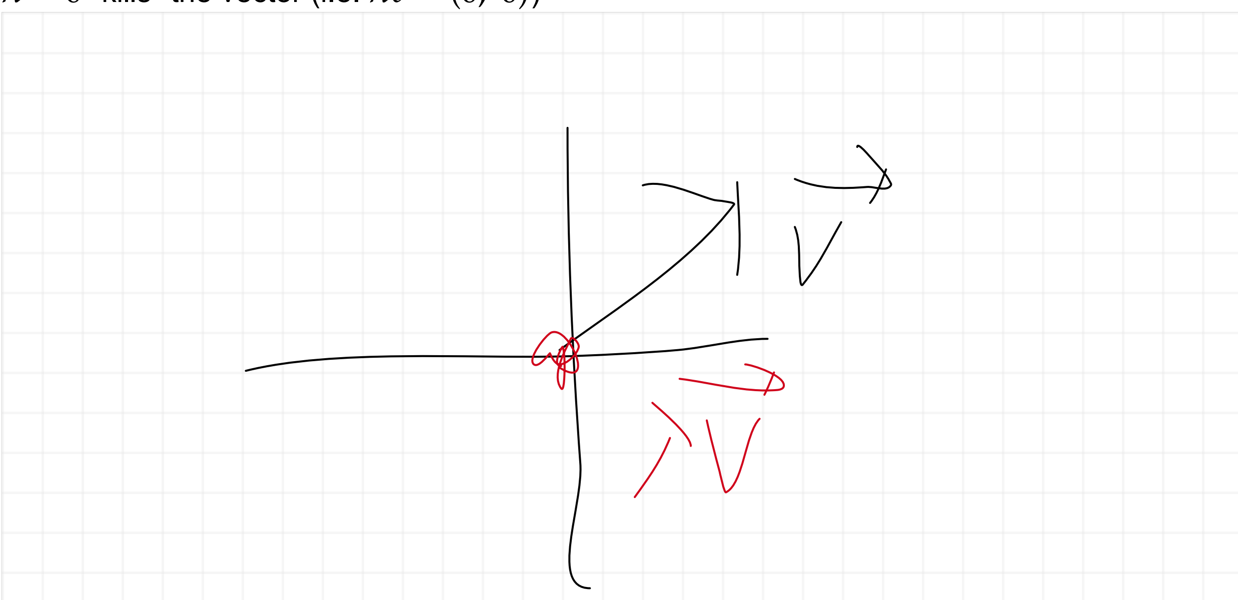
$\lambda > 1$ stretches the vector



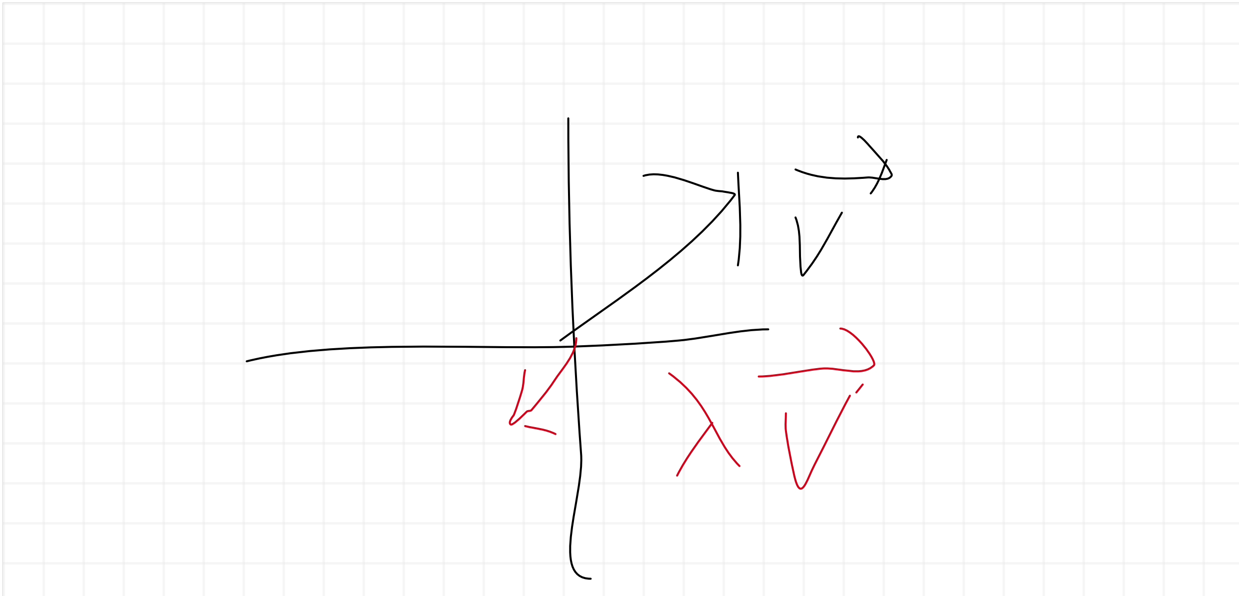
$\lambda = 1$ does nothing (i.e. $1 \cdot \vec{v} = \vec{v}$.)
$0 < \lambda < 1$ shrinks the vector



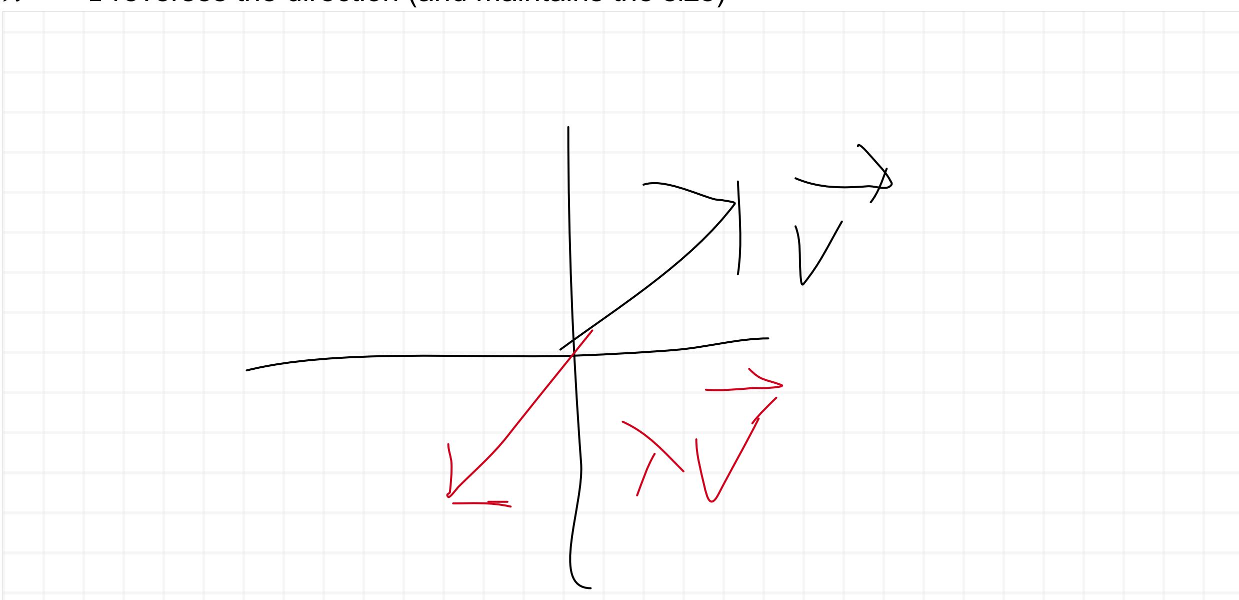
$\lambda = 0$ "kills" the vector (i.e. $\lambda\vec{v} = (0,\ 0)$)



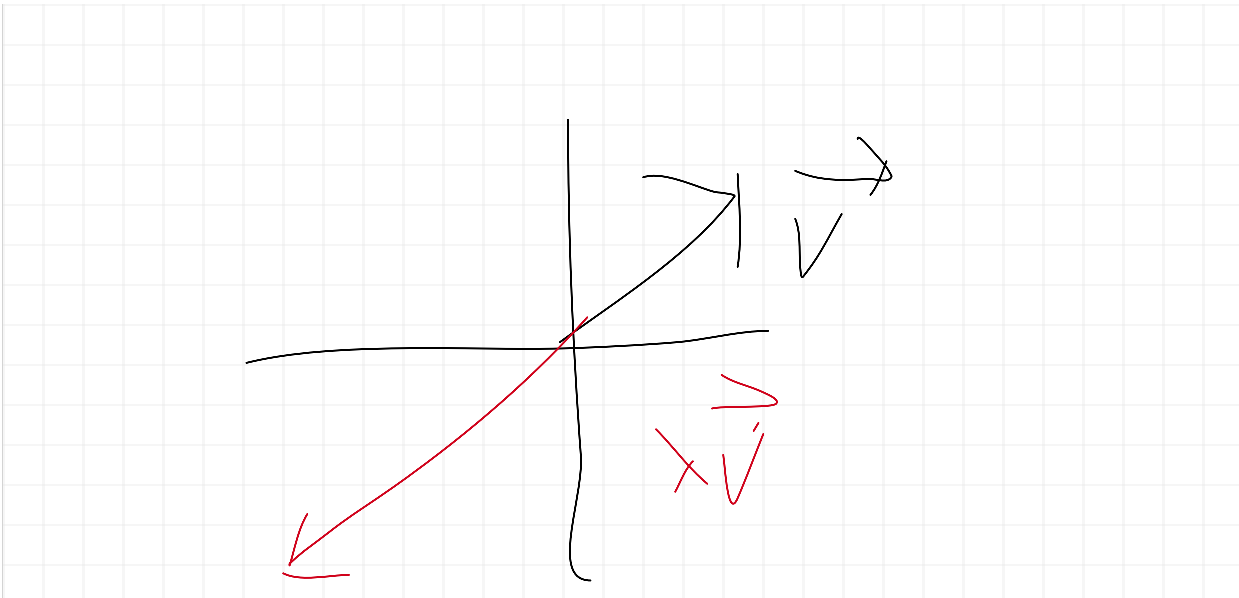
$-1 < \lambda < 0$ shrinks the vector and reverses it



$\lambda = -1$ reverses the direction (and maintains the size)



$\lambda < -1$ stretches the vector and reverses the direction



Class 4: Linear Distortion

Def. 4.1 and 4.5 define different linear distortions, which use scalars to "distort" image.

s-linear distortions: distortion of a figure is where you scalar multuply every point on a figure by the scalar s.

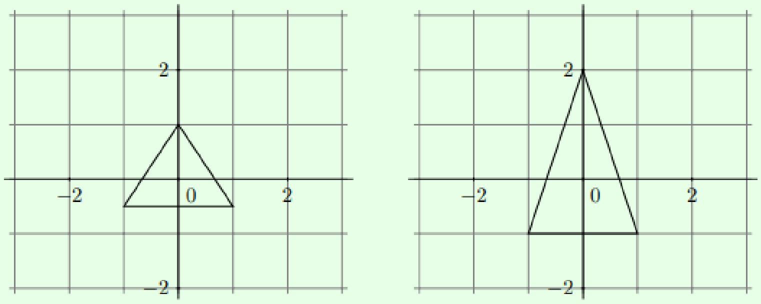
$$(x,\ y)\ \mapsto (sx,\ sy)$$

(s,t)-linear distortions: distortion of a figure where you multiply the x-coordinate by s, and the y-coordinate by t

$$(x,\ y)\ \mapsto (sx,\ ty)$$

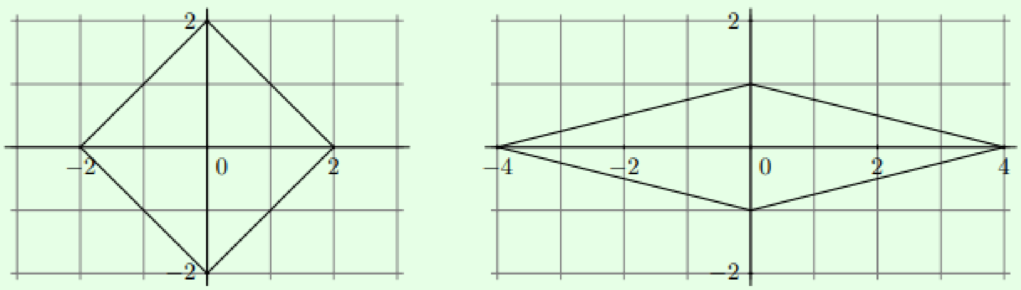
NOTE: s-linear distortions are the same thing as $(s,\ s)$-linear distortions

Example 4.7 ((s, t)-Distortion).
Sketch the triangle with vertices $(0, 1)$, $(-1, -\frac{1}{2})$, and $(1, -\frac{1}{2})$.
Now sketch the triangle after a $(1, 2)$- distortion:



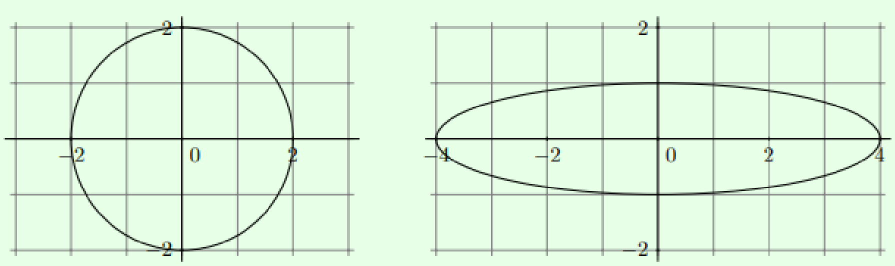
NOTE: (s,t)-distorted lines remain lines but with the slope adjusted.

Example 4.8 ((s, t)-Distortion).
Sketch the square with vertices $(2, 0)$, $(0, 2)$, $(-2, 0)$, and $(0, -2)$.
Now apply a $(2, \frac{1}{2})$- distortion and sketch the resulting figure:



NOTE: We can define ellipses pretty intuitively as (s, t) distortions of circles.

Example 4.10 (Ellipse).
Sketch the circle, center $(0, 0)$, radius 2.
Now apply a $(2, \frac{1}{2})$-distortion and sketch the resulting figure:



Class 3,5: Linear Combinations and Planes

As we can see, scalars can (and are) used to "distort" images of curves. But we can vectors and scalars to "draw" images as well.

Expressions of the form
$\lambda_1\overrightarrow{v_1} + \lambda_2\overrightarrow{v_2} + \cdots + \lambda_n\overrightarrow{v_n}$
are called **linear combinations**.

Thankfully you'll only have to deal with simpler expressions like $t\vec{v},\ \lambda\vec{v} + \mu\vec{w},\ \lambda\vec{v} + \mu\vec{w} + \vec{x}$.

Example 3.13:
$2 \cdot (3, 1) + \ -3\ \cdot (-2,\ 4) = (6,\ 2) + (6,\ -12) = (12,\ 10)$

NOTE: when you "subtract" a vector, that is the same as adding by the scalar multiple of -1 of that vector
$\vec{v} - \vec{w} = \vec{v} + (-1)\vec{w}$

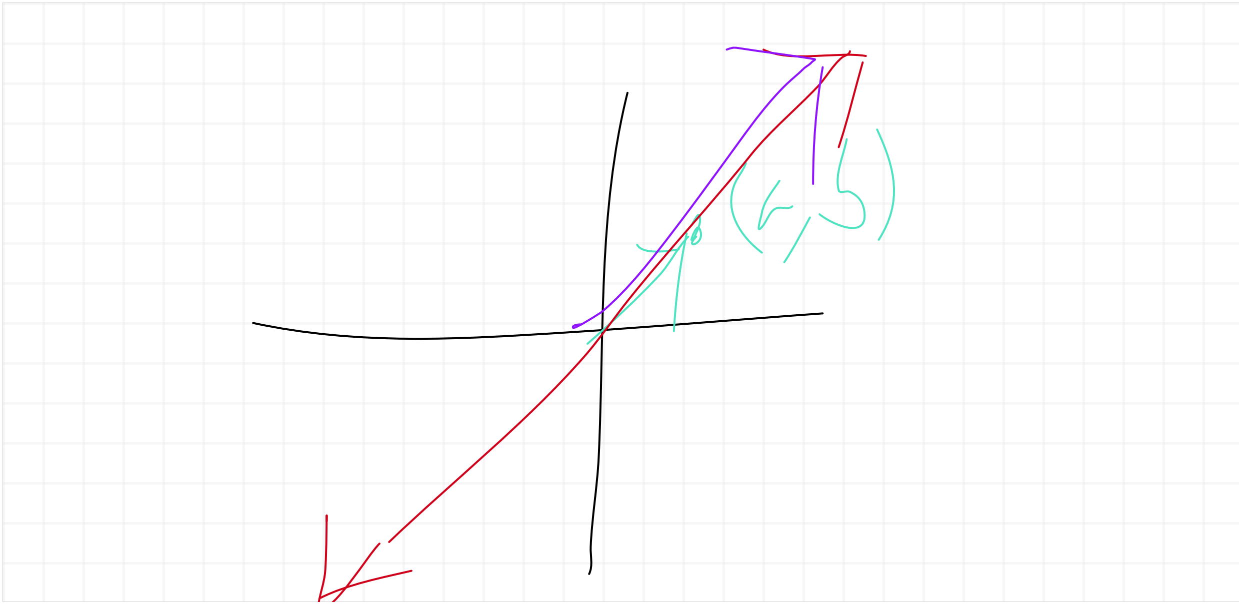
Example 5.4:

$$\begin{aligned}(1, 2, -3) + 2(0, -4, 1) - (2, 1, 0) &= (1, 2, -3) + (0,\ -8,\ 2) + (-2,\ -1,\ 0)\\ &= (1,\ -6,\ -1) + (-2,\ -1,\ 0)\\ &= (-1,\ -7, -1).\end{aligned}$$

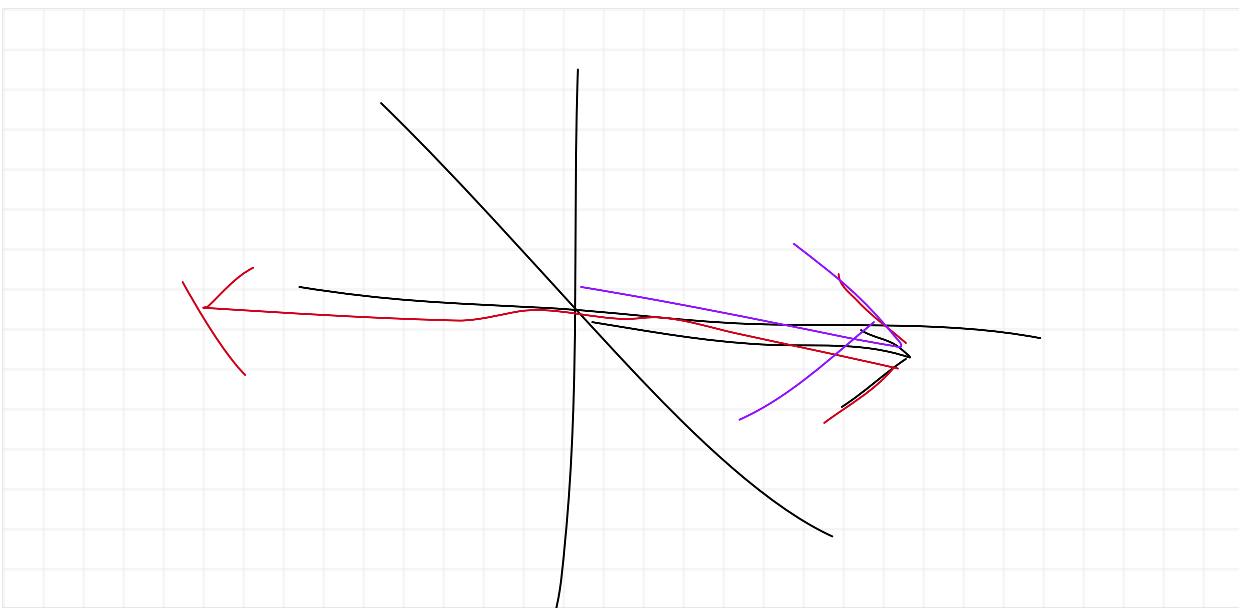
Using linear combinations, we can generate lines and rays:

An equivalent definition of **a line generated by** $\vec{v}$ is the set
$\{t\vec{v}\ :\ t\ \mathit{any\ real\ number}\}$

An equivalent definition of **a ray generated by** $\vec{v}$ is the set
$\{t\vec{v}\ :\ t\ \geq 0\}$



NOTE: We can generalize this idea in 3D

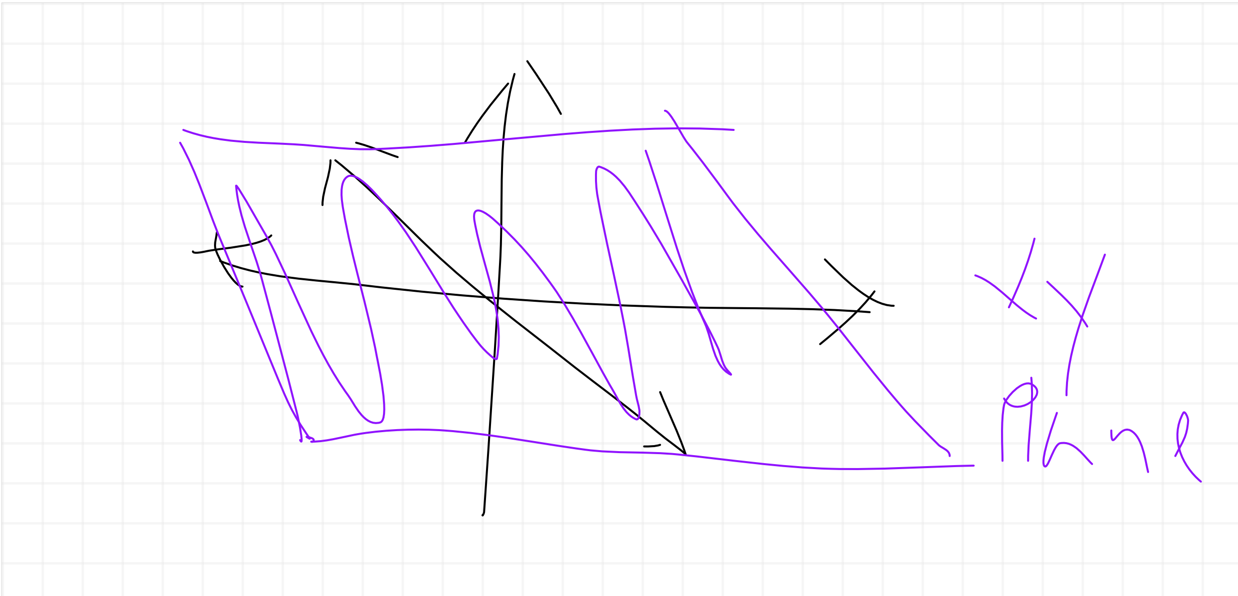


In 3D, we can generate all sorts of interesting images using linear combinations

For example, let's take $\vec{i} = (1,\ 0,\ 0)$, $\vec{j} = (0,\ 1,\ 0)$

$\{\lambda \vec{i} + \mu \vec{j}\ :\ \lambda,\ \mu\ any\ real\ number\}$

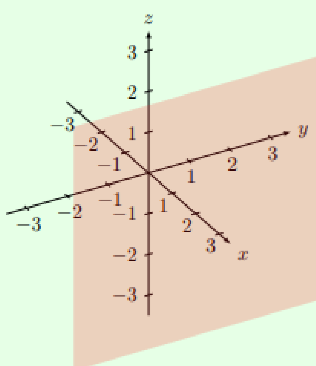
We get any point point of the form $(\lambda, 0, 0) + (0,\ \mu,\ 0) = (\lambda, \mu, 0)$



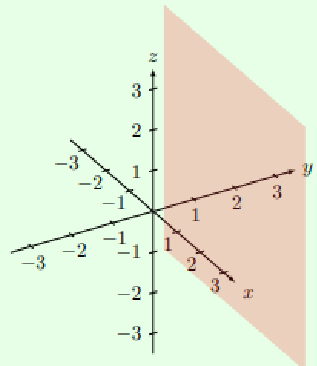
A **plane** in a 3D space is the set of linear combinations of two *linearly independent vector* (i.e., vectors that are not scalar multiples of each other)

Equivalent definition to the def. 5.7, i.e., a copy of the 2D plane sitting in a 3D space

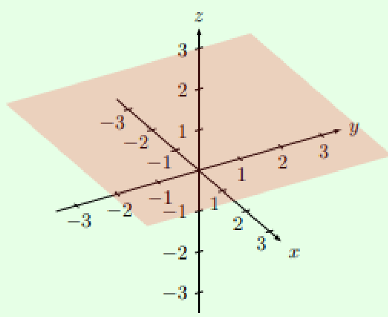
Example 5.9 (Planes).



The plane $x = 2$. Notice that the x-axis intersects the plane at the point $(2, 0, 0)$.



The plane $y = 2$. Notice that the y-axis intersects the plane at the point $(0, 2, 0)$.



The plane $z = 1$. Notice that the z-axis intersects the plane at the point $(0, 0, 1)$.

NOTE: We often give a plane in terms of a single linear equation, i.e. $x = a$.

Class 6: Parametric Equation of a Line in 2D and 3D

7/24 Lecture

Announcements:

Reminders for today: Homework 2 due tonight, office hours 2pm-3pm today
Exam 4 grades out
Homework 4 uploaded due Tues 7/28
Homework 5 is coming
Look out for BIG ANNOUNCEMENT as updated next week (includes make up opportunities)
Solutions to all the homeworks to be uploaded this weekend
Exam 5 this Wednesday the 7/29

Homework 2-3 Questions:

Poor wording on question 4 of homework

HINT:

Theorem 4.11: (s, t)-Linear distortions scale the area by a factor of $s \cdot t$.

Let's say that you have of area 1, and you aplly a (2, 1) distortion, now the rectangle has area of $2 \cdot 1 = 2$ times the original area, so it has twice the area, hence it has area 2.

Corollary (Thm. 4.4): s-Linear distortions ((s, s)-distortions) scale the area by a factor of s^2.

Previously:

Class 6: Parametric Equation of a Line in 2D and 3D

To recap what we talked about in the last few classes, we have 2 ways (in the 2D plane) of giving linear equations:

point/slope form, i.e. "y=mx+b" form:
form line from point (the y-intercept) and slope

2 point form:
forms line from 2 points

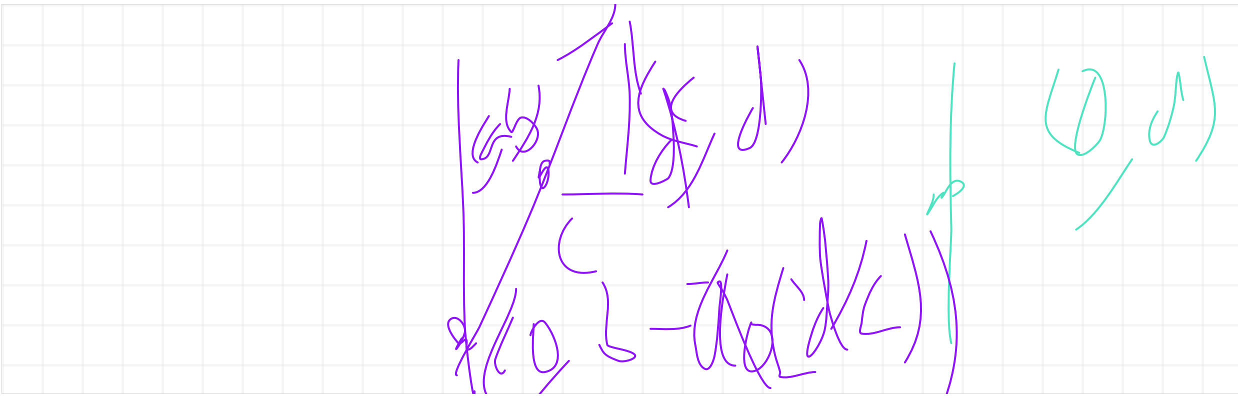
For this class, we have our "final form" for an equation of a line involving linear combinations

Introducing: parametric form
Uses a parametric equation

A **Parametric equation** forms a line from the set of linear combinations of the form
$\{\vec{u} + t\vec{v}: \ t \text{ is a real number}\}$

Can be done in 2D and 3D (note that these older versions, i.e. point/slope and the 2 point, can only be done in 2D)

Converting between parametric and point slope form:



Thm. 3.6 (more generalized): For the 2D parametric equation
$\{(a,\ b) + t(c,\ d):\ t\ real\ number\}$, we have

$$m = \begin{cases} d/c & if\ c \neq 0 \\ "\infty" & if\ c = 0 \end{cases}$$

If $c \neq 0$, then the y-intercept b' is determined by the following formula
$b' = b - a(d/c)$

Class 7, 9: Parallel Projections, Central Projections

Projections allow us to draw things 3 dimensions to 2 dimesnions, or even from 2 dimesnions to 1 dimension.

Remember that the big question of the perspective is how does one best 3D images in 2D.

And these types of projections are an answer to that question

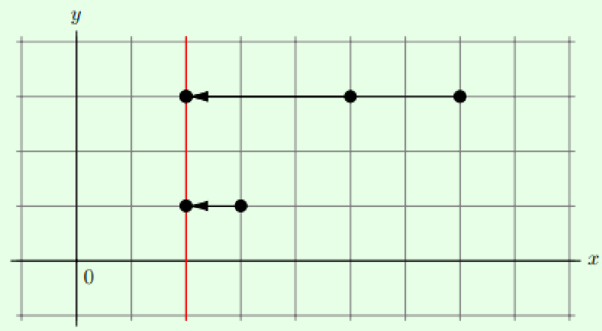
Parallel projection:

General idea: takes an image and projects it to a "parallel" area of smaller dimension.

2D parallel projection to a line (Def. 7.1):
Parallel projection to the line $x = a$: $(x,\ y) \mapsto (a,\ y)$
Parallel projection to the line $y = b$: $(x,\ y) \mapsto (x,\ b)$

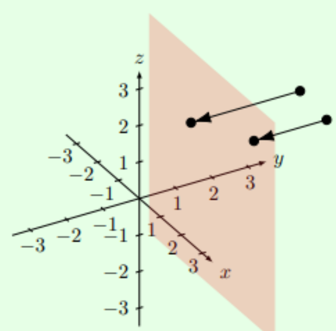
Example 7.3: Parallel project onto the line $x = 1$:
$(2,\ 3) \mapsto (1,\ 3),\ (5,\ 4) \mapsto (1,\ 4),\ (3,\ 4) \mapsto (1,\ 4)$

Example 7.2.
The following depicts parallel projection onto the line $x = 2$.



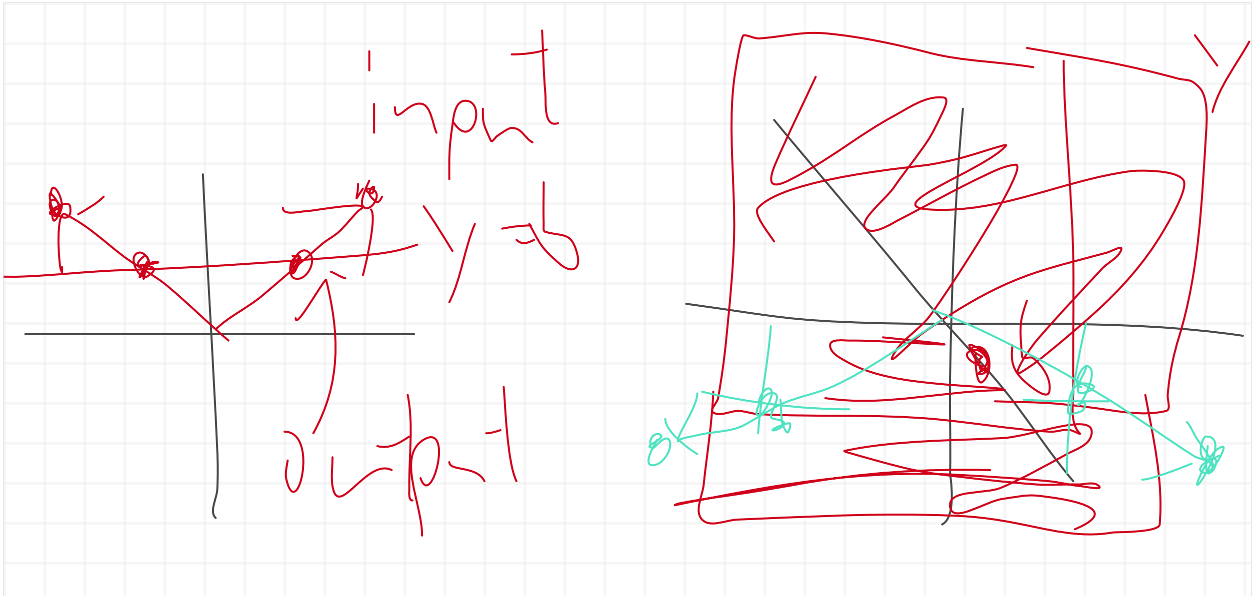
3D parallel projection (Def. 7.5) into a plane:
Parallel projection onto the plane $x = a$: $(x, y, z) \mapsto (a, y, z)$
Parallel projection onto the plane $y = b$: $(x, y, z) \mapsto (x, b, z)$
Parallel projection onto the plane $z = c$: $(x, y, z) \mapsto (x, y, c)$

Example 7.6.
The following depicts parallel projection onto the plane $y = 2$.



Central Projection:

General idea (Def. 9.1): Depict each object on the canvas at the spot "within the line of sight" (the "line of sight" being the origin), i.e. a ray from eye to the object.



NEXT TIME: Give an equivalent mathematical formula defintion, which will allow you to compute such a projection.

Projections of lines and vanishing points.

7/27 Lecture

Announcements:

Homework 1 grades out
Homework 2 grade ETA this afternoon
Office hours 2pm-4pm
homework 3 due tonight expect homework 3 grades tomorrow
homework 4-5 due tomorrow night (definitely can get homework 4 grades early)
expect upload of past solutions (all exams and past homework, plus perspective homework 1-2 this afternoon)

Homework 3-4 Questions:

Previously:

-Talked about the parametric form of lines (in both 2D and 3D)

-We defined parallel projections of lines/planes (in both 2D and 3D)

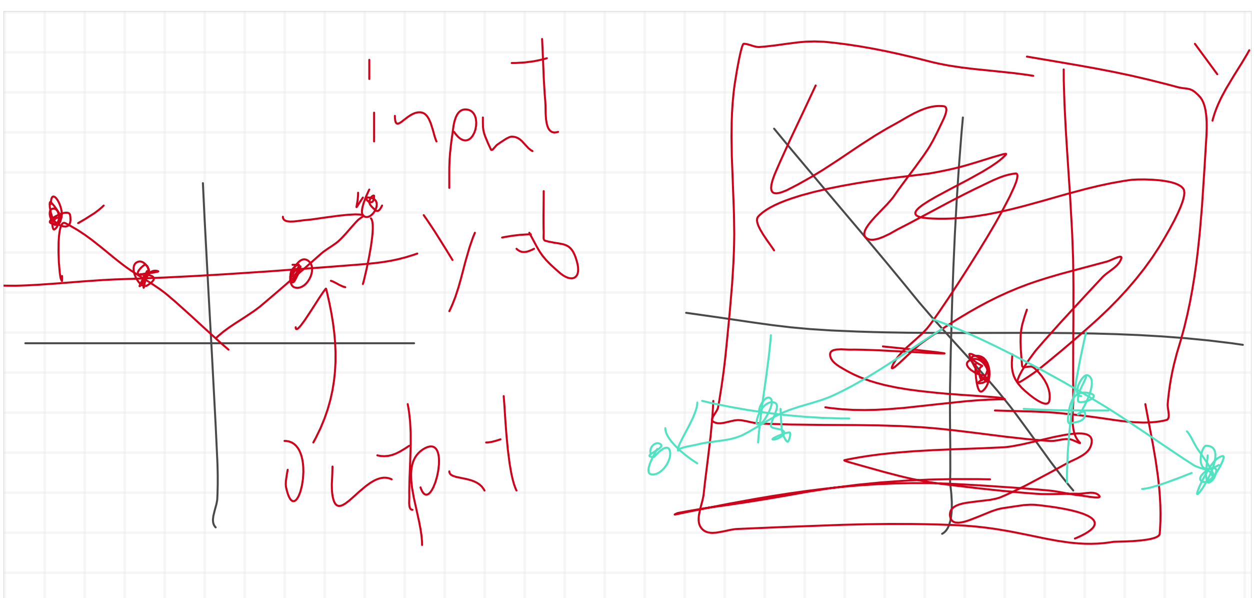
in 2D let's say that we parallel project to $x = a$
$(x, y) \mapsto (a, y)$
If $y = b$
$(x, y) \mapsto (x, b)$

in 3D if we parallel to $x = a$
$(x, y, z) \mapsto (a, y, z)$
$y = b$
$(x, y, z) \mapsto (x, b, z)$
$z = c$
$(x, y, z) \mapsto (x, y, c)$

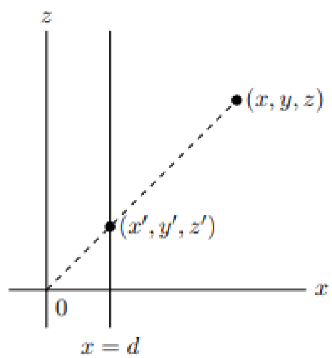
-We began the central projection, and that's where we left off

Class 9 (Cont.):

General idea (Def. 9.1): Depict each object on the canvas at the spot "within the line of sight" (the "line of sight" being the origin), i.e. a ray from eye to the object.



The following diagram demonstrates central projection of the point (x, y, z) onto the plane $x = d$, (with eye at the origin).



Central projection onto the plane $x = d$, maps $(x, y, z) \mapsto (x', y', z')$.

We will now try to calculate (x', y', z') in terms of x, y, z, and d.
Two facts:

1. (x', y', z') lies on the plane $x = d$.
2. (x', y', z') lies on the ray generated by (x, y, z).

From fact 1. we know that $(x', y', z') = (d, y, z)$.
From fact 2. we know that $(x', y', z') = \lambda(x, y, z)$.
Thus $(d, y, z) = (\lambda x, \lambda y, \lambda z)$, in particular $d = \lambda x$, so $\lambda = \frac{d}{x}$.
Thus $(x', y', z') = \frac{d}{x}(x, y, z) = d(1, \frac{y}{x}, \frac{z}{x})$.
We can do a similar calculation for central projection onto planes of the form $y = d$ or $z = d$.

Theorem 9.2: The central projection (with "central point" at the origin) maps the original to the following points:

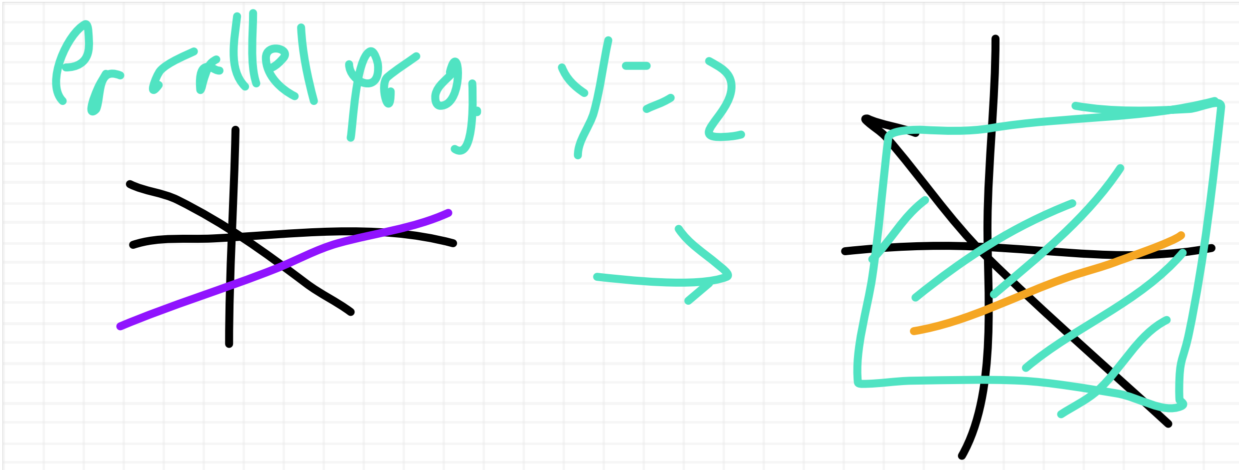
*For $x = d$: $(x,\ y,\ z) \mapsto d(1,\ y/x,\ z/x) = d/x(x,\ y,\ z)$
*For $y = d$: $(x,\ y,\ z) \mapsto d(x/y,\ 1,\ z/y) = d/y(x,\ y,\ z)$
*For $z = d$: $(x,\ y,\ z) \mapsto d(x/z,\ y/z,\ 1) = d/z(x,\ y,\ z)$

Example 9.3; The central projection of $(3,\ 5,\ 12)$ onto the plane $x = 2$ ($d = 2$) is
$(3, 5, 12) \mapsto 2(1,\ 5/3,\ 12/3) = (2,\ 10/3,\ 8)$

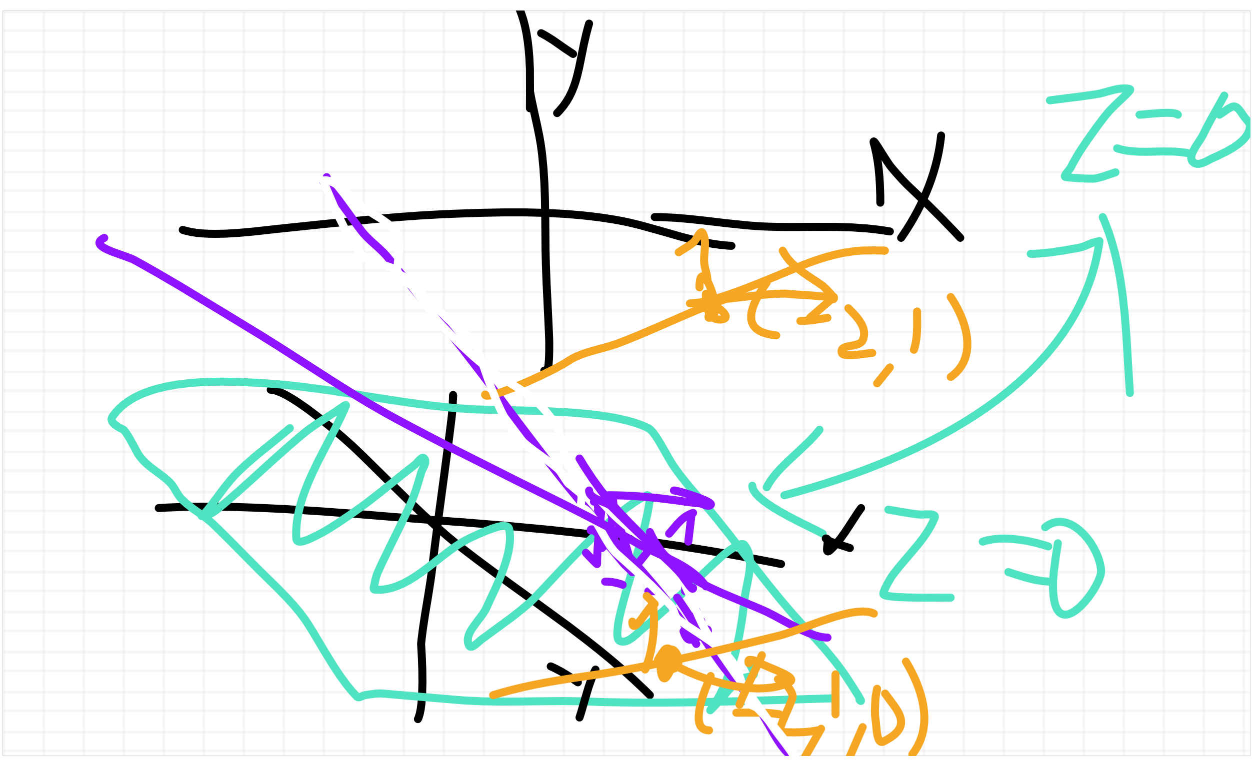
Class 8, 10-11: Parallel and Central Projections of Lines and Vanishing Points

Classes 8 and 10 talk about how a line/ray is parallel projected (class 8) or centrally projected (class 10)

Theorem 8.4: Parallel projections map line segments to either another line segment or a point

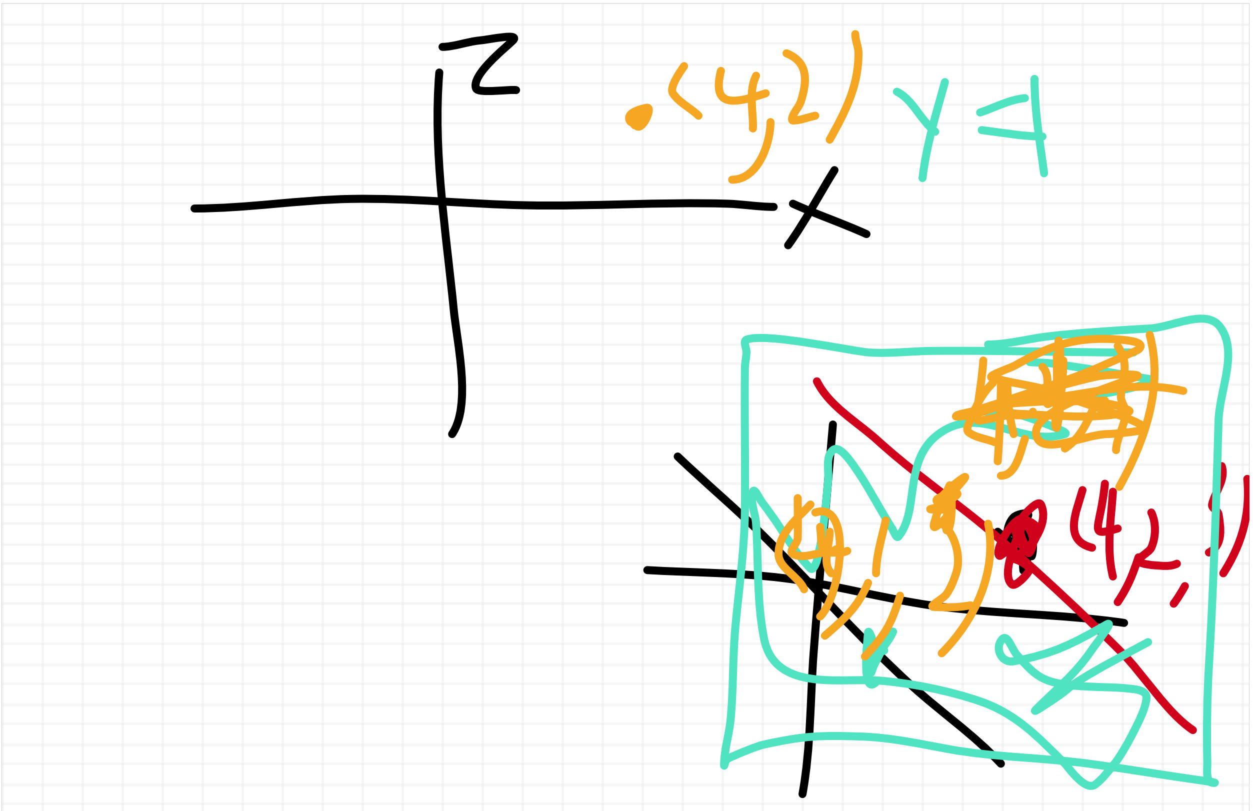


Example 8.2
Calculate the parallel projection of the line $(-2,\ 1,\ 3) + t(2,\ 1,\ -1)$ onto the plane $z = 0$ and graph
$(-2,\ 1,\ 3) + t(2,\ 1,\ -1) = (-2 + 2t,\ 1 + t,\ 3 - t)$
$(-2 + 2t,\ 1 + t,\ 3 - t)\ \mapsto (-2 + 2t,\ 1 + t,\ 0)$
$(-2 + 2t,\ 1 + t) = (-2,\ 1) + t(2,\ 1)$



Example 8.3
parallel project $(4,\ 2,\ 3) + t(0,\ 3,\ 0)$ onto $y = 1$

$(4,\ 2+3t, 3)\ \mapsto (4,\ 1, 2)$



NOTE: The line maps to a point precisely when it is perpendicular to the plane

Theorem 10.1

WRONG a ray doesn't necessarily map to another ray with a central projection (it may be curve that is different from a ray, though visually looking the "projection" as a 2D depiction of 3D, it may look like a segment)

CORRECTION: It is correct after all.

Example 11.1

Consider $(2,\ 1,\ 1) + t(1,\ 0,\ 0)$ and the central projection onto the plane $x = 1$

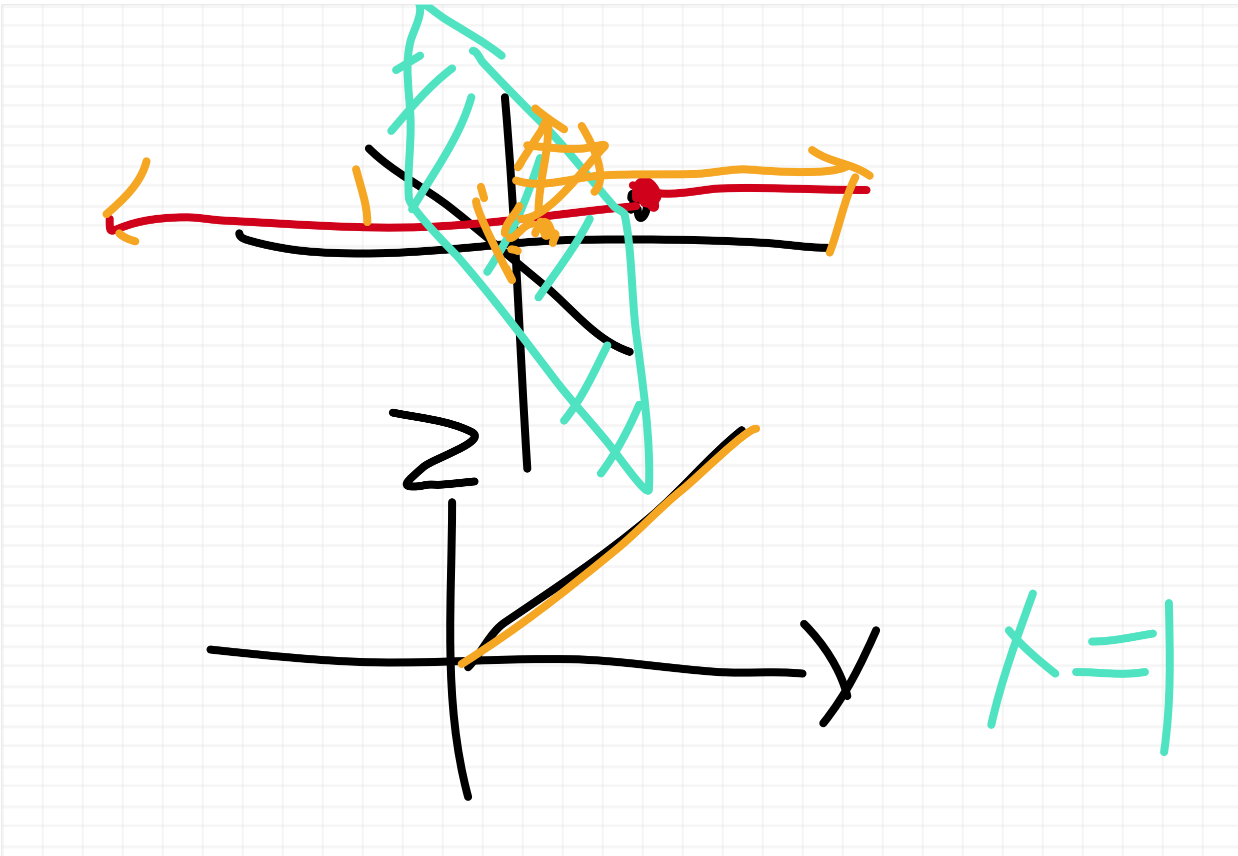
$$(2+t,\ 1,\ 1)\ \mapsto 1/(2+t)(2+t,\ 1,\ 1) = \left(1,\ \frac{1}{2+t},\ \frac{1}{2+t}\right)$$

$t = 0\ (2,\ 1,\ 1)\ \mapsto (1,\ 1/2, 1/2)$

$t = 1\ (3,\ 1,\ 1)\ \mapsto (1,\ 1/3,\ 1/3)$

$t = 10\ (12,\ 1,\ 1)\ \mapsto (1,\ 1/12,\ 1/12)$
$t = 100\ (102,\ 1,\ 1)\ \mapsto (1,\ 1/102,\ 1/102)$

NOTE: In this example, the line goes from infinitely closer in the xy plane to $(0,\ 0)$



NEXT TIME: Get into more detail about vanishing points.

7/28 Lecture

Announcements:

Homework 3 grade ETA this afternoon (solution uploaded then)
Homework 4 and 5 due tonight (early grades 7pm or earlier ETA tonight)
All past solutions, including last 4 exams, are uploaded (perspective homework 3 later)
Office hours this afternoon 2pm-4pm
Thursday will be review day (we'll review the hardest material of each unit)
PLEASE FILL OUT THE SURVEY (2 absences exponged)

Exam 5 tomorrow
Exam 5 grade/solution ETA (Thurs. afternoon/early evening)

Homework 30% grade:
Lowest 3 assignements will be deducted from your grade

Exam grade 60% your grade:

The final is as little as 10% of your grade, but as HIGH AS 20%
It becomes 20% when it's higher than your lowest exam grade

Attendence grade 10%:

They haven't been registered to your global grades

You can find out your attendence in the local canvas

Example: Let's say your local canvas tells your attendence 30/35
You've missed five days
Remember: 4 days don't negatively impact your grade at all
So a 31/35, that would mean you get all 10%
That means you lose .5%, that would have a 95/100 on your attendence (and recieve 9.5% out of 10%)
Caveat 1: Missing more than 8 days yields "automatic failure"
Caveat 2: Filling out the survey exponges 2 absences.

NOTE: Global grades reflect your homework and exams thus far (doesn't factor in attendence)

Homework 4-5 Questions:

Previously:

-Last Time: Projecting 3D lines in a 2D plane.

-Main takeaway: Parallel and central projections of 3D lines project line segments to other line segments and/or points (In particular, Theorem 10.1 is CORRECT, nvm what I said earlier)

-Gave a mathematical formula for central projection.

Theorem 9.2: The central projection (with "central point" at the origin) maps the original to the following points:

*For $x = d$: $(x,\ y,\ z)\ \mapsto d(1,\ y/x,\ z/x) = d/x(x,\ y,\ z)$
*For $y = d$: $(x,\ y,\ z)\ \mapsto d(x/y,\ 1,\ z/y) = d/y(x,\ y,\ z)$
*For $z = d$: $(x,\ y,\ z)\ \mapsto d(x/z,\ y/z,\ 1) = d/z(x,\ y,\ z)$

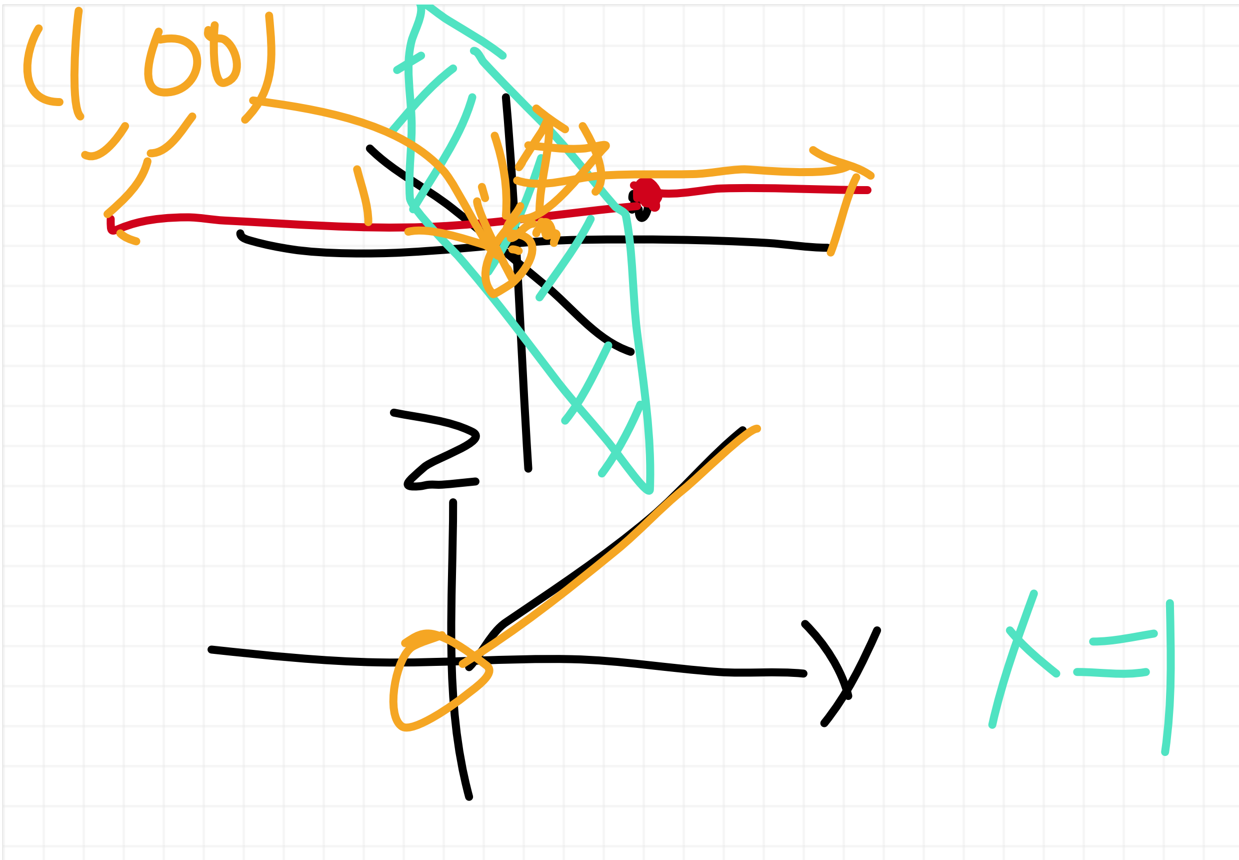
Left off at Example 11.1

Class 11: Vanishing Points

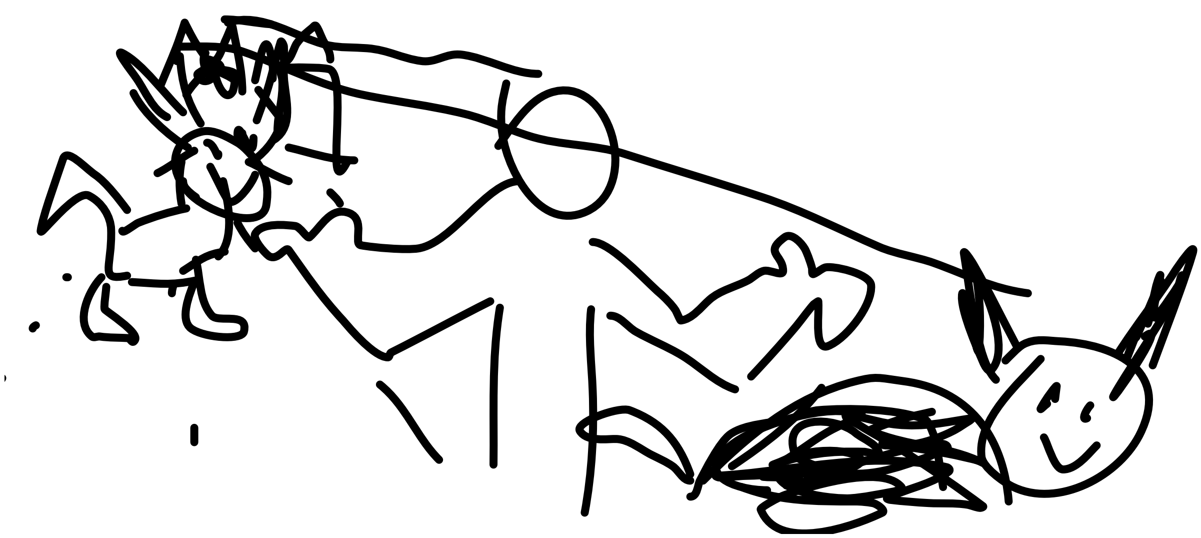
Example 11.1
Consider $(2,\ 1,\ 1) + t(1,\ 0,\ 0)$ and the central projection onto the plane $x = 1$

$(2+t,\ 1,\ 1)\ \mapsto 1/(2+t)(2+t,\ 1,\ 1) = \left(1,\ \frac{1}{2+t},\ \frac{1}{2+t}\right)$
$t = 0\ (2,\ 1,\ 1)\ \mapsto (1,\ 1/2, 1/2)$
$t = 1\ (3,\ 1,\ 1)\ \mapsto (1,\ 1/3,\ 1/3)$
$t = 10\ (12,\ 1,\ 1)\ \mapsto (1,\ 1/12,\ 1/12)$
$t = 100\ (102,\ 1,\ 1)\ \mapsto (1,\ 1/102,\ 1/102)$

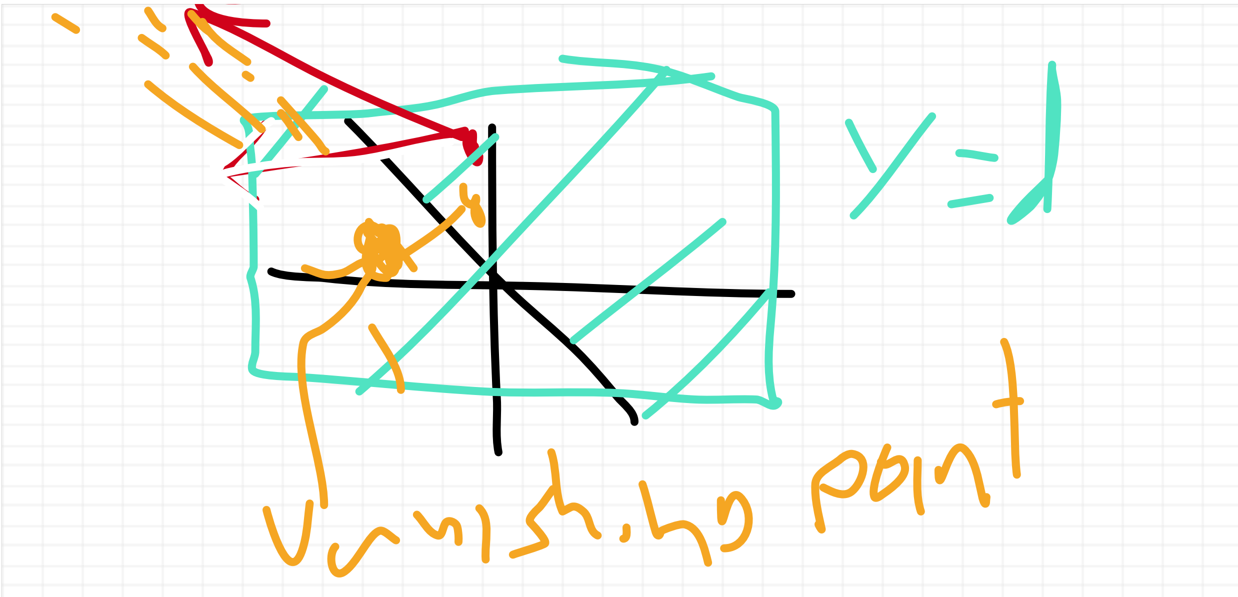
NOTE: In this example, the line goes infinitely closer as $t\ \to \infty$ in the plane $x = 1$ to $(0,\ 0)$:



General Idea: A vanishing point in general is the point at which a 2D canvas is picked to the point further away objects vanish from the viewer's perspective



What this artistic concept of a vanishing point does is to take a 3D image and project onto the canvas (which is viewed as a plane for $y = b$)



Example 11.4
Consider $(3,\ 0,\ -2) + t(2,\ 6,\ 4)$ projected onto $x = 2$
$x = d:\ (x,\ y,\ z)\ \mapsto d(1,\ y/x,\ z/x) = d/x(x,\ y,\ z)$

$$(3+2t,\ 0+6t,\ -2+4t)\ \mapsto 2/(3+2t)(3+2t,\ 6t,\ -2+4t) = \left(2,\ \frac{12t}{3+2t},\ \frac{-4+8t}{3+2t}\right)$$

$t = 0:\ (3,\ 0,\ -2)\ \mapsto (2,\ 0,\ -4/3)$
$t = 1:\ (5,\ 6,\ 2)\ \mapsto (2,\ 12/5,\ 4/5)$
$t = 10:\ (23,\ 60,\ 38)\ \mapsto\ (2,\ 120/23,\ 76/23)$
$t = 100:\ (203,\ 600,\ 398)\ \mapsto (2,\ 1200/203, 896/203)$

As $t\ \to \infty$, we get infinitely closer to the point $(2,\ 6,\ 4)$ When you look at the yz dimensions of the $x = 2$, we call the *vanishing point* $(6,\ 4)$

The formal definition of a **vanishing point** is a point that is approached in a 2D central projection of a 3D line as $t\ \to \infty$

Theorems 11.5 and 11.3 give us precise formulas for computing vanishing points

Theorem 11.5: For the plane $x = d$, the central projection of the line $(p, q, r) + t(a, b, c)$ maps to two rays with vanishing points $d(b/a,\ c/a)$.

Corollary (Thm 11.3): In the special case where the line is parallel to the plane $x = d$, the vanishing point is the orgin $(0,\ 0)$.

NOTE: You can generalize it with $y = d$ or $z = d$ as follows:

$y = d$: $d(a/b,\ c/b)$
$z = d$: $d(a/c,\ b/c)$