M106 Summer 2020 Recitation Lectures: Voting Theory

7/9 Lecture

Announcements:

New module today! (Voting theory!) Yay! :)
Homework 1 is uploaded, due Tues July 14
More homework to come
Exam 4 is Monday July 20

Class 1: Introduction

Please go vote! In general

What is voting theory? It's a branch "social choice theory"

It takes the concept of voting, and mathematically models it. (mathematics of the decision voting)

In the game theory module, we modeled interactions where one makes simultaneous decisions involving two people (we specifically dealt with two player games)

With "voting theory", we model the decision to vote simultaneously (or sequentially) with an arbitrary amount of people.

So voting theory does a similar sort of thing with two games

Social Choice Theory (Def. 1.1) is the analysis of collective decision procedures, so the study of collective preferences, rankings of preferences, and the calculus to reach such a decision.

Social Choice Theory is often applied to economics
It has maybe some bearing on sociology
It applies to political science

Social choice theory is very general, and there's many subbranches of social choice theory

Fair divisions
Matching
Competition rankings
-Basketball brackets (bracketology!)
-Climbing Olympic scoring
-Sabremetrics (baseball, football, and even the game of basketball)

Finally of course voting theory (what our module covers)
In the abstract, here are some questions that voting theory seeks to address
1. How to choose a winner given everyone's preferences?
2. What are the most common voting systems?
3. Most important question of all (in your instructors), how "fair" are the different systems?

Class 2: Basic Definitions

Def. 2.1:
We call the members of an electorate the **voters** (or **individuals**)
-We often denote the set of all voters V
We call the set of possible choices the voters have the **alternatives**,
-We often denote the set of all possible alternatives by $\mathcal{A}$
-We often use the term **candidate** when the alternatives are specifically people.

In general, we use the following standard notation for V and $\mathcal{A}$:

We denote the i-th voter by v_i i.e. $V = \{v_1, v_2, \ldots, v_n\}$
Denote the alternatives/candidates by upper case letters, i.e. $\mathcal{A} = \{A, B, C, D, \ldots\}$
(note that sometimes "A" is used as an alternative and not the set $\mathcal{A}$)

Preference notation (Def. 2.3): Let $\succ_i$ denote the (strict) **preference relation** for voter v_i

i.e., we write $A \succ_i B$ if voter v_i strictly prefers alternative A to alternative B
(similar to strict dominance in game theory with action A vs. action B, except that preference is predetermined, whereas game theory, you had to calculate that preference based on the payoff structure)

A **voter preference** (Def. 2.4) is a ranking of all the alternatives in $\mathcal{A}$.

A **voter profile** (Def. 2.6) is a listing of all possible voter preferences, together with the

number of voters with those voter preferences.

NOTE: When zero voters vote for a given preference, we often omit it from the list. (we do that because there often is A LOT of different possible arrangements of voter preferences)

Thank back to Democrat primary, when you had Bernie Sanders, Elizabeth Warren, Joe Biden, Kamela Harris, Pete Buttigeig, Tulsi Gabbard, Amy Klobachar, Michael Bloomberg, and the list goes on, and we wouldn't to list ALL voter precences of all those candidates in a list, so we only go with the preferences that specific voters have.

Example 2.7 (Voter Profile - Highway Referendum).
Consider the highway referendum (Example 1.3).
The voter profile is:

Votes	28	9	2	30	0	31
Preferences	I	I	S	S	N	N
	S	N	I	N	I	S
	N	S	N	I	S	I

There are 28 voters with preference ranking $I \succ S \succ N$
There are 9 voters with preference ranking $I \succ N \succ S$
There are 2 voters with preference ranking $S \succ N \succ S$
And so on

In this case, there are six possible preferences, and it does give a voter preference arrangement that zero voters have, but if $N \succ I \succ S$ wasn't on the chart, we can assume that there is zero voters with that preference

Any voter preference with B on top of the list (i.e. most preferred) isn't there either, so we

can infer from the profile that NO VOTER has B as the top preference

Now we're going to talk about some properties of preference relations (Def. 2.9-2.10)

The preference relation $\succ$ has the following possible properties:

-**Complete**-for any two alternatives $A,\ B\ \in \mathcal{A}$ (" $\in$ " means is "is inside the set"), either have $A \succ B$ or $B\ \succ A$ or neither

NOTE: When neither $A\ \succ B$ nor $B\ \succ A$, we write $A\ \sim B$, we write " $\succ_i$" as " $\succ$ " (without the index either the context is clear or when we don't need the context)

-**Strict-**If for any two *distinct* alternatives, $A,\ B$ in $\mathcal{A}$ (in other words, $A \neq B$) then $A\ \succ B$ or $B \succ A$ but not both.

-**Transitive**-If $A \succ B$ and $B \succ C$, then $A \succ C$.

-**A strict total order** is an order that is complete, strict, and transitive.

NOTE: We ALWAYS make the assumption that a voter is rational, which both allows to formalize voting theory and really analyze it effectively (to the point some really powerful theorems we'll mention later about the "fairness"), but do note that there's a lot irrational about human decision making in practice. So a lot of voting theory is very limited in modeling actual voting in practice, but sometimes helpful to consider.

-More specifically, a voter is **rational** (and we call a voter a **rational voter**) if their voter preference relation $\succ_i$ is a strict total order.

The main consequences of the above are as follows:
*A rational voter can mathematically compare any two alternatives
*A rational voter NEVER likes two alternatives equally (since they're strict), no indifference by the voters
*A rational voter never has preference cycles $A \succ B \succ C \succ A$ (it also can't happen that $A \succ A$).

This highlights some limitations with voting theory, though voting theory is still quite useful.

7/10 Lecture

Announcements:

Homework 1 due Tues. 7/14
Expect Homework 2, 3 upload ETA next week
Expect video 508 to be uploaded next week (running a bit behind)
Exam 3 grade ETA next week

Previously:

Give an informal on Voting Theory (Class 1)
For class 2, we gave basic definitions, notation, and convention of a collection $\mathcal{V}$ of **voters,** the set of **alternatives/candidates** $\mathcal{A}$, and the preference relation $\succ_i$ for voter i, $v_i \in \mathcal{V}$.
We also talked about a **voter profile** (refer to illustration to refresh yourself)

Example 2.7 (Voter Profile - Highway Referendum).
Consider the highway referendum (Example 1.3).
The voter profile is:

Votes	28	9	2	30	0	31
Preferences	I	I	S	S	N	N
	S	N	I	N	I	S
	N	S	N	I	S	I

We make the assumption throughout this module that voting preferences are ALWAYS **rational,** i.e. their preference relation $\succ$ is a **strict total order**, which has the following properties:

Complete: Either $A \succ B$ or $B \succ A$. (no indifference happens)

Strict: Both $A \succ B$ and $B \succ A$ can't happen. Another way think about is the "tricotomy law", i.e., either $A \succ B$ or $B \succ A$ or $A \sim B$.

Transitive: If $A \succ B$ and $B \succ C$ then $A \succ C$.

Class 3-4: Voting Systems

Classes 3-5 (maybe 6) talk about the voting systems from simplest to most complicated.

A **voting system** is an algorithm that inputs a voter profile and outputs a candidate $A \in \mathcal{A}$, that is the **winner**.

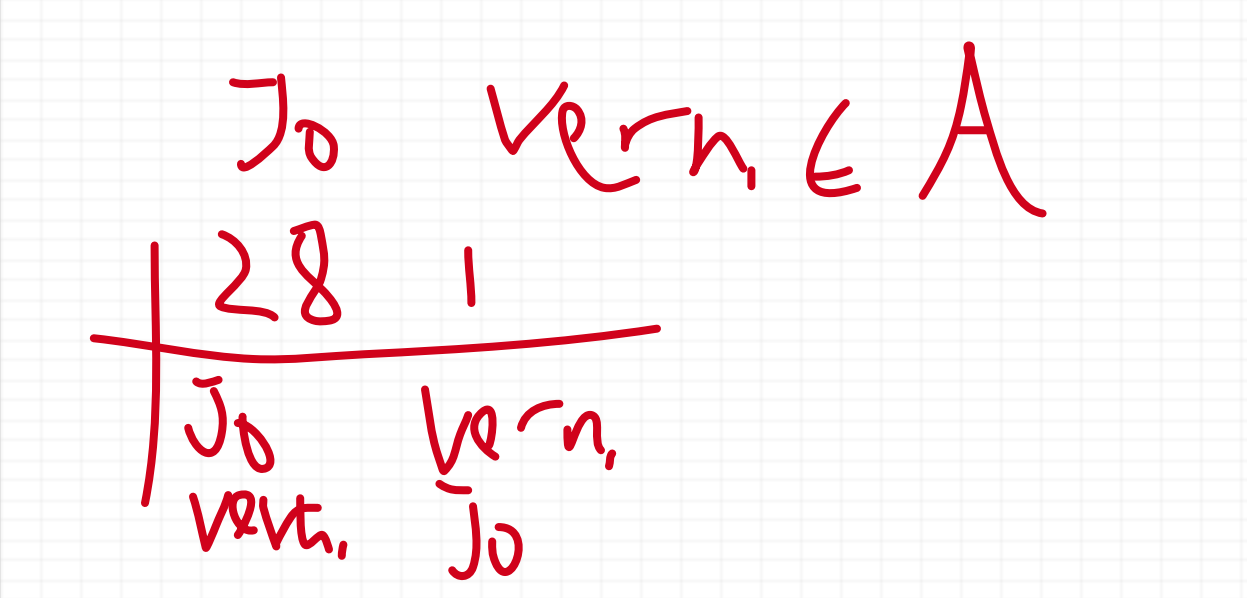
A voting system more generally speaking computes a **social ranking**, which is a preference relation (also denoted by $\succ$) determined by the voting system.

For the winner A, we have $A \succ B$, for every other $B \in \mathcal{A}$.

NOTE: The voting systems we use, a social ranking $\succ$ is generally-speaking in all the voting systems we cover are *strict* and *transitive*, but not necessarily complete. (indifference between alternatives may exist)

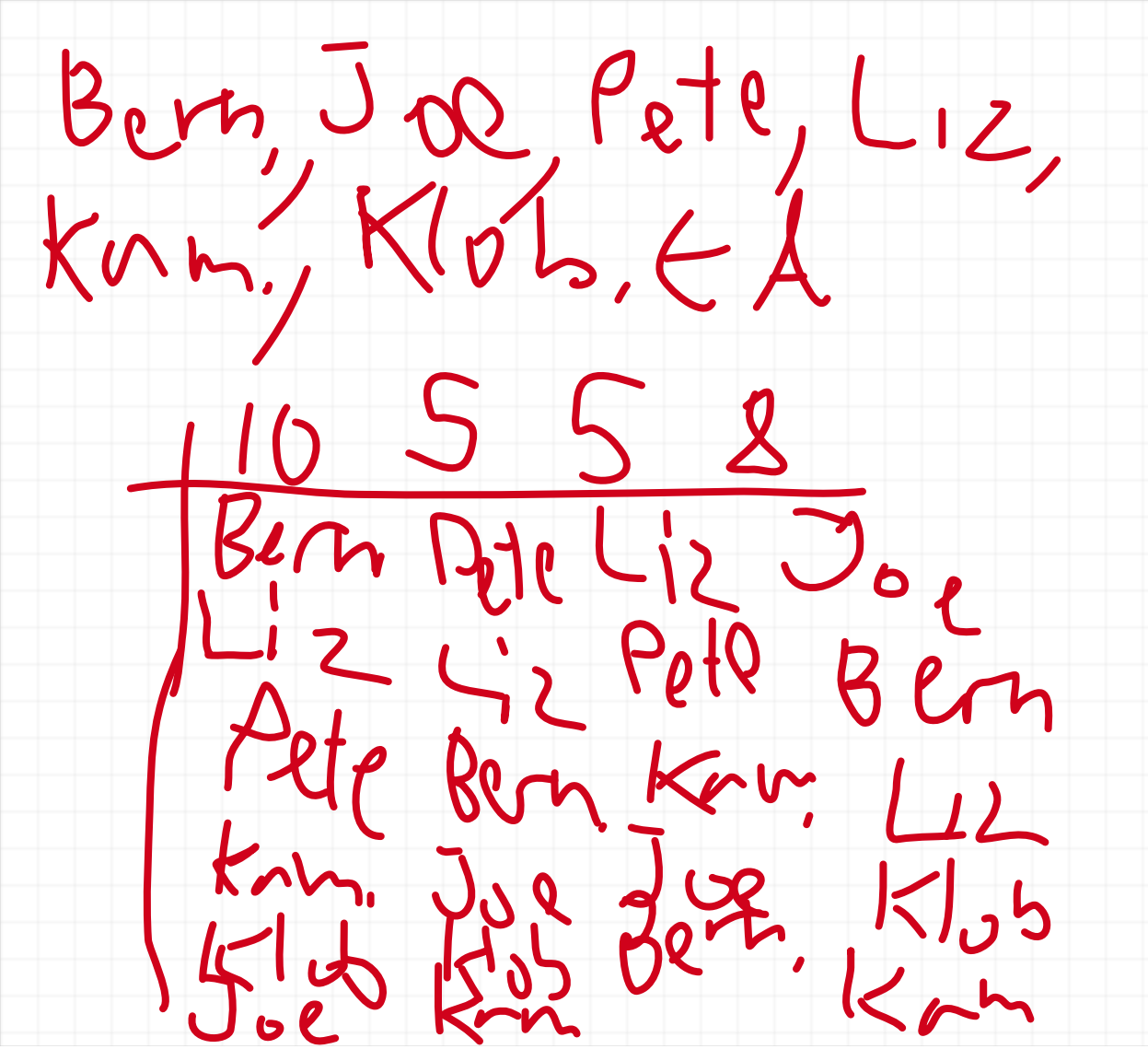
Here are some voting systems in Class 3 (Def. 3.1-3.3 of your notes):

In the **dictatorship** voting system, the outcome of the election is determined by one particular voter



Let's say that voter 29 who has $Vern \succ_{29} Jo$ is the dictator. Then the social ranking $\succ$ is $Vern \succ Jo$.

In the **plurality** voting system, the alternative with the most top preferences comes first, the alternative with the second most comes second, etc. (and a tie yields indifference)



In a plurality, we would have:
Bernie with 10 votes
Pete with 5 votes
Liz with 5 votes
Joe with 8 votes

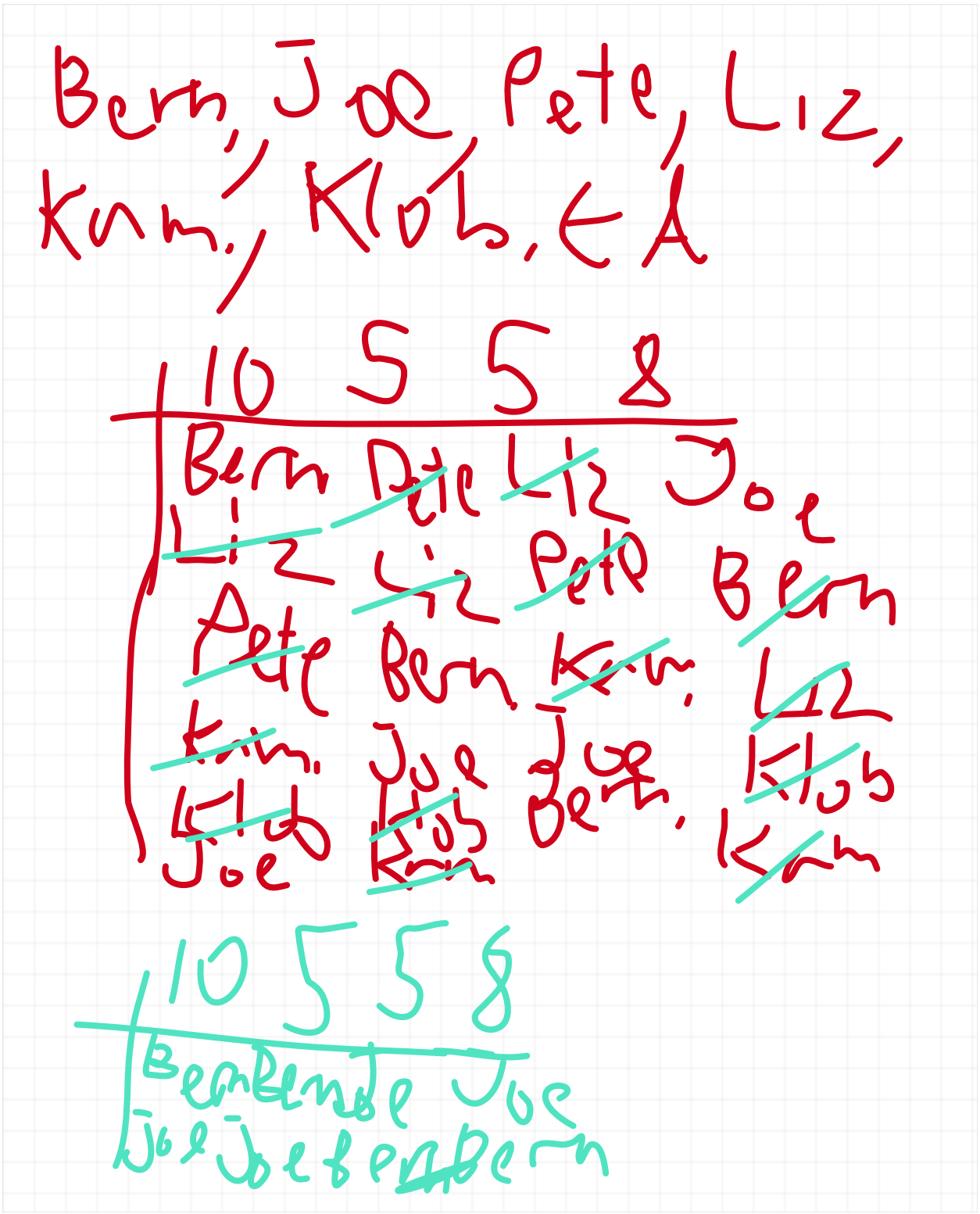
As a result, Bernie wins in a plurality voting system, and the social ranking is as follows: $Bern \succ Joe \succ Pete \sim Liz \succ Kam.$

The **two round run off** voting system, which selects the winner in two rounds:

Round 1: The two alternatives with the most top preference votes move on to the second round

Round 2: The votes of the knocked-out alternatives are redistributed to thge remaining two alternatives. The alternative with the most top preference votes wins.

To do a two-round voting system, we do round 1 as if it was a plurality, and NOTE that if a single alternative A has over 50% of the votes in the first round, then A wins already. Then we do round 2 as a election between the top two candidates that remain (and base the votes off of who prefers each person more)
In the illustration below, the first round of voting gives us Joe and Bernie, so then we do an election between them



Bernie gets 15 votes
Joe gets 13 votes
So we have $Bern \succ Joe$

NOTE: The plurality is NEED NOT be the two-round runoff winner

Example 3.4 (Highway Referendum).
Consider the highway referendum (Example 1.3). The voter profile is given by:

Votes	28	9	2	30	0	31
Preferences	*I*	*I*	*S*	*S*	*N*	*N*
	S	*N*	*I*	*N*	*I*	*S*
	N	*S*	*N*	*I*	*S*	*I*

In plurality:
I gets 28+9=37 votes
S gets 2+30=32 votes
N gets 31+0=31 votes
so we have $I \succ S \succ N$

BUT in a two round runoff:
1. We are left with I and S facing off
2.
I gets 28+9+0=37
S gets 2+30+31=63
3. As a result S wins in a landslide, and $S \succ I$ in a two round runoff

Different voting systems yield different winners.

We'll end recitation by defining some voting systems in class 4:

Def. 4.1 (Instant Runoff):
The i**nstant runoff** voting system is a multi-round system:
*In reach round, eliminate the alternative with the fewest top preference votes
*Redistribute the eliminated alternatives votes to the remaining alternatives
*Repeat until there is only one alternative left
Also called the **single transferable vote** or the **alternative vote**

Like runoff, except one-by-one elimination and multiple rounds (same as the two round runoff with three players).

Def. 4.7 (Agenda Voting):
An **agenda voting system** is a voting system where the alternatives compete in a series of head-to-head elections to determine the winner.

The particular order in which they compete is called the **agenda**

NOTE: The agenda more or less the US voting system where political parties nominate their candidates through individual partywide elections, and then those winners face off in the second round in the general election.

7/13 Lecture

Announcements:

Exam 3 grade ETA tonight
Homework 1-5 all uploaded; due dates are also ALL announced
Homework 1 due Tues.
Homework 2 due Wed.
Homework 3 due Thurs.
Homework 4 due Fri.
Homework 5 due Sun.
Exam 4 Mon.
Best we can hope ("the gauntlet module")

Questions on Homework 1-2:

NOTE: "Boda" is a typo and they mean "Borda"

Previously:

We talked about different voting systems:

Class 3:
dictatorship (one vote decides everything)
plurality (who every has the most top preferences wins)
two round runoff (elimate everyone except who has the first and second top preference, and then the head-to-head vote winner of that wins)

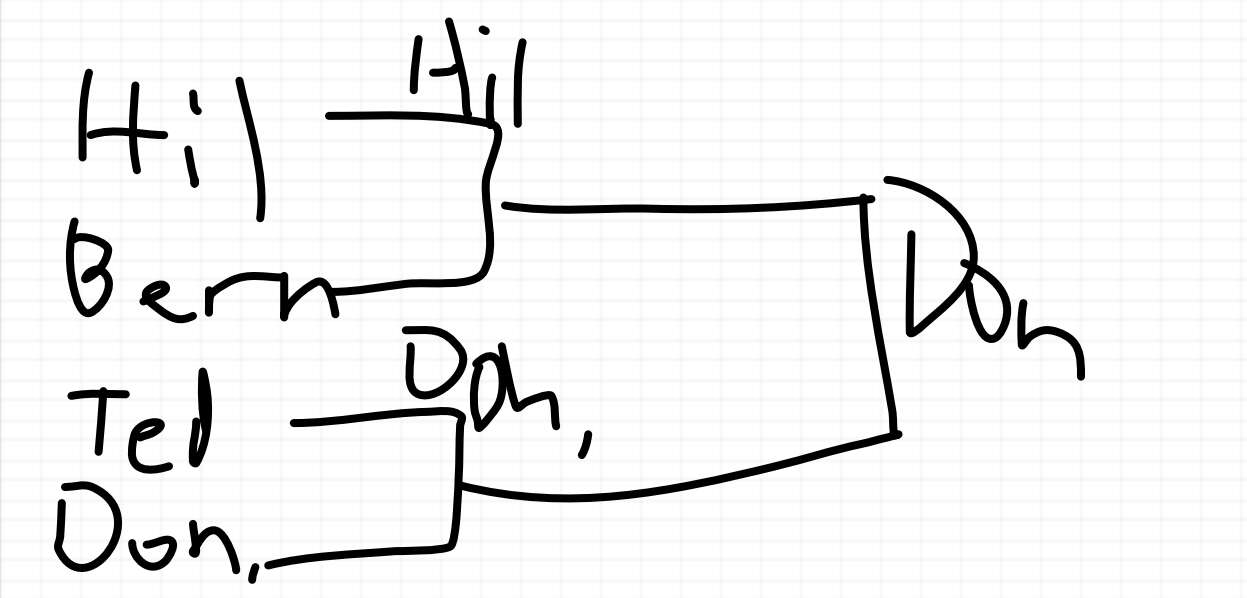
Class 4:
instant runoff (we eliminate whoever has the least top preferences, and then we look at a new election without that eliminated, and repeat)
agenda voting (we have bracket voting tournament and we do a series of head-to-head

votes)

Class 3-5: Voting Systems

Two more systems (that is covered in class): Borda voting systems, Baldwin voting system "Ranked-Choice system":

I INCORRECTLY stated last class that the agenda voting system is the American voting system. That's not exactly true, since there are more candidates in primary voting than two candidates.



Don is the winner, and we have $Don \succ Hil$ and $Hil \succ Bern$ but $Bern \sim Ted$

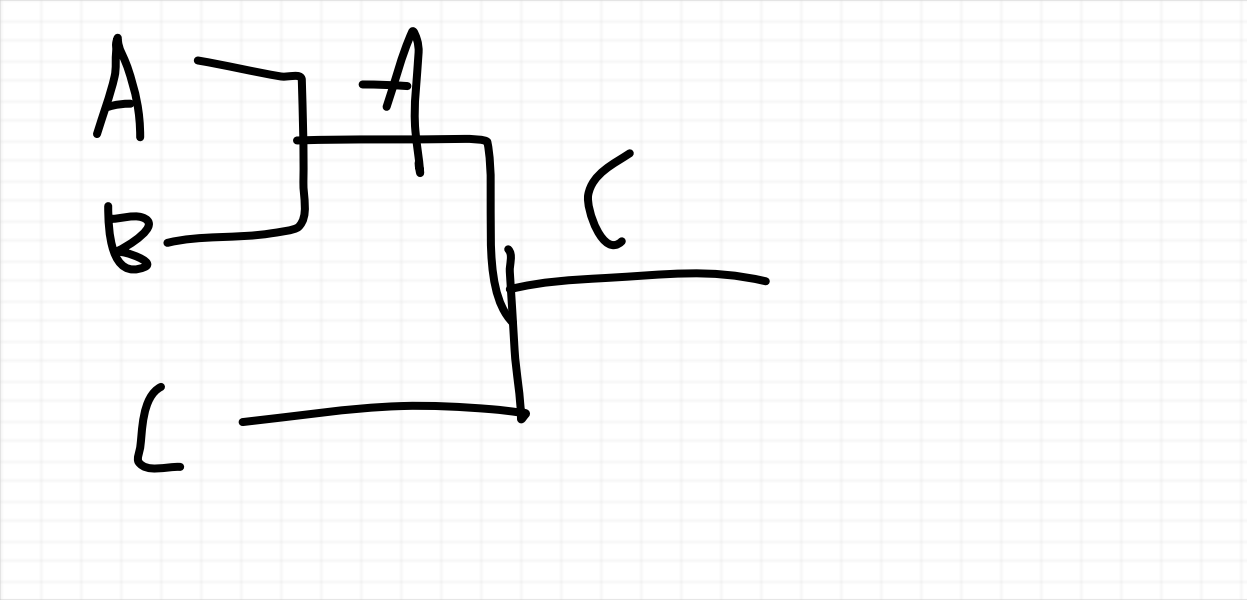
Example 4.9 (Agenda - No Condorcet Winner).
Consider the following voter profile:

Votes	5	2	3	6	5	1
Preferences	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

A vs. B:
A has 5+2+5=12
B has 3+6+1=10

A advances to the next round
A vs. C:
A has 5+2+3=10
C has 6+5+1=12

C wins that particular agenda



$C \succ A \succ B$

NOTE: There are are other agendas (refer to notes) where potentially a different candidate wins.

Example 4.3 (Instant Runoff).

A possible profile is:

Votes	12	13	11	16	12	8
Preferences	A	B	B	C	D	D
	C	A	A	A	C	B
	D	D	C	D	A	C
	B	C	D	B	B	A

Instant runoff

Round 1:

A has 12 votes

B has 24 votes
C has 16 votes
D has 20 votes

We eliminate A

Round 2:

We do the election with C eliminated and C votes reassigned
We have B, C, D

B has 24 votes
C has 28 votes
D has 20 votes

D is eliminated

Round 3:

B and C going head-to-head

B has 13+11+8=32
C has 12+16+11=37

C wins

We'll define two more voting systems (Def. 5.1, Def. 5.5):

The **Borda voting system** assigns points ranked in order of least preferred to most preferred, based on how the voter ranked them, and the alternative with the most points wins.

The **Balwin voting system** is a multi-round voting system that is done as followed:
In each round, the alternatives are ranked in order of least preferred to most preferred (like in the Borda voting system), the one with the *least points* is eliminated, and the process continues (like in the instant runoff)

NOTE: There is this popular voting system called "ranked choice voting", and the Baldwin voting system seems like rank choice, but they are different voting systems, and note furthermore that rank choice voting isn't covered

The **Rank Choice voting system** is multi-round voting system that is done by tallying candidate preferences from most preferred to least preferred and a voter with over 50% of the most preferred votes wins automatically, but in the case where that doesn't happen, the candidate with the *highest tally* gets eliminated.

Example 5.6 (Baldwin Voting System - No Condorcet Winner).
Consider the following voter profile:

Votes	5	2	3	6	5	1	
Preferences	A	A	B	B	C	C	2 points
	B	C	A	C	A	B	1 point
	C	B	C	A	B	A	0 points

A has 10+4+3+5=22
B has 5+6+12+1=24
C has 2+6+10+2=20

In the Borda voting system, B wins
$B \succ A \sim B$
In the Baldwin system
A has 5+2+5=12
B has 3+6+1=10
$A \succ B \succ C$

Next Time: We'll talk "fairness principles" (from class 4 and 5, and then insincere voting in class 7)

7/14 Lecture

Announcements:

Graph Theory Exam Grades ETA DEFINITELY this afternoon
Homework 1 due tonight
Homework 2 due tomorrow (tax day!)
Homework 3 Thurs. 7/16
Homework 4 Fri. 7/17
Homework 5 Sun. 7/19 (my birthday!)

Questions on Homework 1-2:

Previously:

We touched on the remaining systems of voting from classes 3-5. Two days ago, we talked the "simple ones"

dictatorship
plurality
two round runoff
instant runoff

yesterday, we talked about two new voting methods

agenda-Series of head-to-head votes; essentially a tournament bracket.
borda-cont-Rank the votes from least preferred (starting with 0) to most preferred, and tally all the votes; the candidate with the most vote wins.
baldwin-multi-rounded voting system, where we count all the votes similar to the borda-cont, except like the instant runoff, we eliminate the candidate with the least votes.

I also mention "rank choice voting", which is not covered in class, but it's a voting system very much worth discussing.

Class 5-6: Condorcet Criterion and "Donkey Voting"

Def. 6.1 and Def. 6.6

The **Condorcet winner** is an alternative that beats every other alternative in head-to-head elections.

We say that a voting system satisfies the **condorcet criterion** if it always chooses the Condorcet winner whenever one exists.

NOTE:

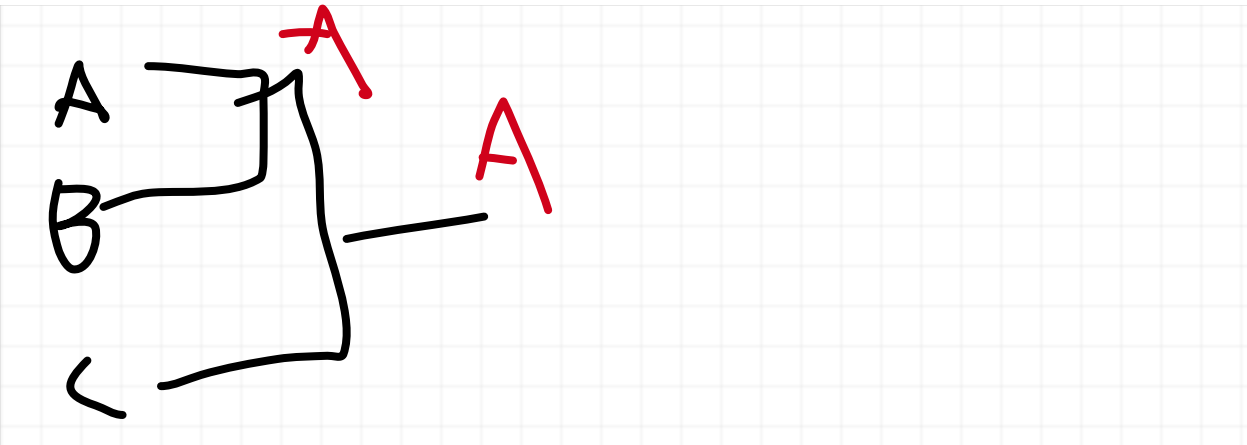
*To show that the condorcet criterion is not satisfied, we show a voter profile that has a winner such there is another candidate that beats the other candidate head-to-head.

Example 6.7 (Condorcet Winner).
Consider the following voter profile:

Votes	5	1	14	1	16	2
Preferences	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

A vs. B, 5+1+16=22 votes for A and 14+1+2=17 votes for B , A wins
A vs. C, 5+1+14=20 votes for A and 1+16+2=19 votes for C, A wins

A is the condorcet winner



The winner of the above agenda voting system is also the condorcet winner of that voting profile. Therefore, the above voting system is condorcet efficient.

In fact, every agenda voting system is condorcet efficient.

A voting is **neutral** (Def. 4.11) if none of the alternatives is given any sort of special advantage over any of the others

Thm. 4.12: The agenda voting system is not neutral.

Think of the NBA and the issue with tournaments

Back to condorcet efficiency/winners

Example 6.3 (No Condorcet Winner).
Consider the following voter profile:

Votes	1	5	7	3	5	2
Preferences	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

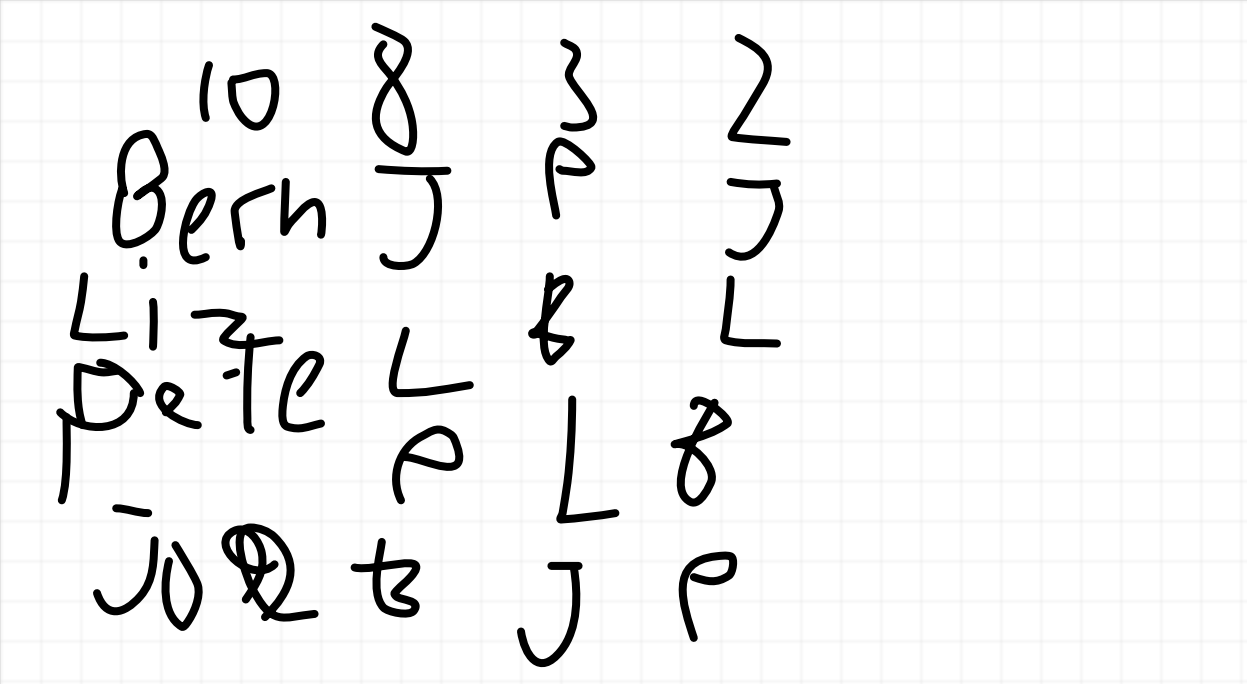
B vs. A: A wins
A vs. C: C wins
B vs. C: B wins

Since any candidate can be beaten by another candidate (think like rock paper scissors), there is no condorcet winner.

Theorem 6.11 (Condorcet Methods).

Condorcet Methods	*Non-Condorcet Methods*
Agenda	*Plurality*
Dodgson	*Two Round Runoff*
Baldwin	*Instant Runoff*
	Borda

NOTE: Ranked-Choice voting is also non-Condorcet it takes a lot of effort to construct such an example and for voters to know how to game that system, as we get into next class, but an example like the one below does exist.



In this ranked choice voting system, Bernie Sanders is the Concordet winner, but in this sytem Liz Warren wins the election.

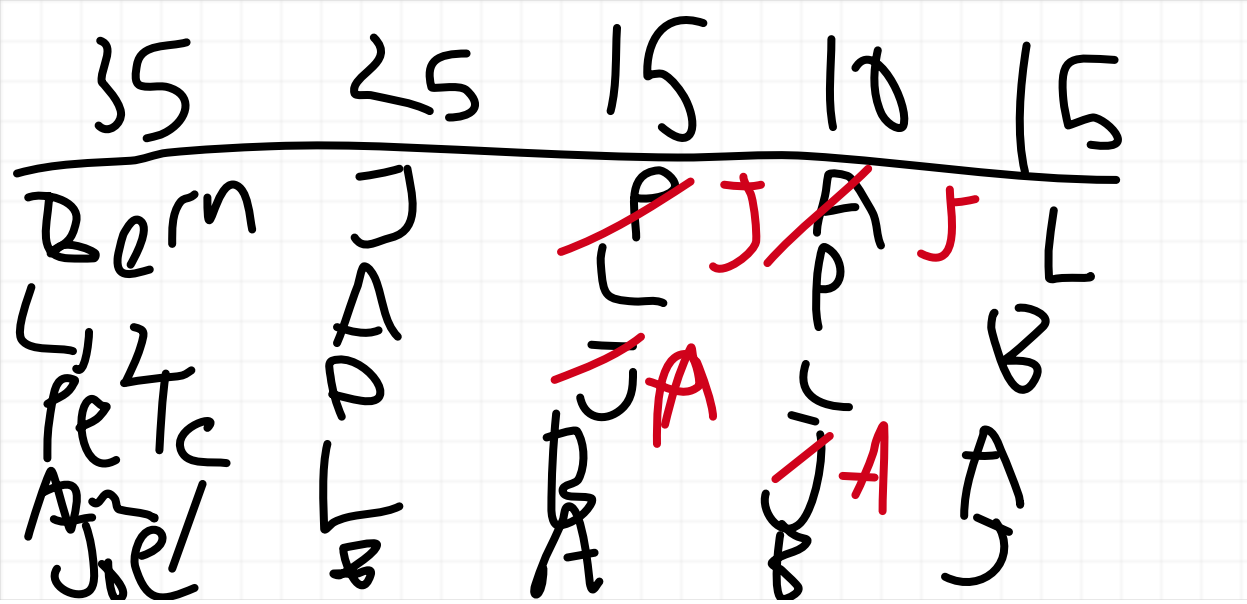
Rank voting has yet another flaw (alas *sigh*), which is a phenomenon known as "donkey voting" (Def 5.8): It phenomon not knowing all the candidates, hence not being able to sufficiently, so in a situation where you're rank candidates, you just blindly rank them (typically by the order they appear on the ballot).

It's a problem for Baldwin, Borda, and Rank choice voting

This has known to be an issue in Australia.

Class 7-8: Insincere Voting

This is a profile that models the Democrat 2020 primary:



Bernie wins ordinarity plurality
However, if all 15 voters for Pete and all 10 voters for Amy decided to vote for Joe, then Joe clearly wins.
More or less what happened in the 2020 primary was this strategic voting.

This is just one classic example of strategic/insincere voting.

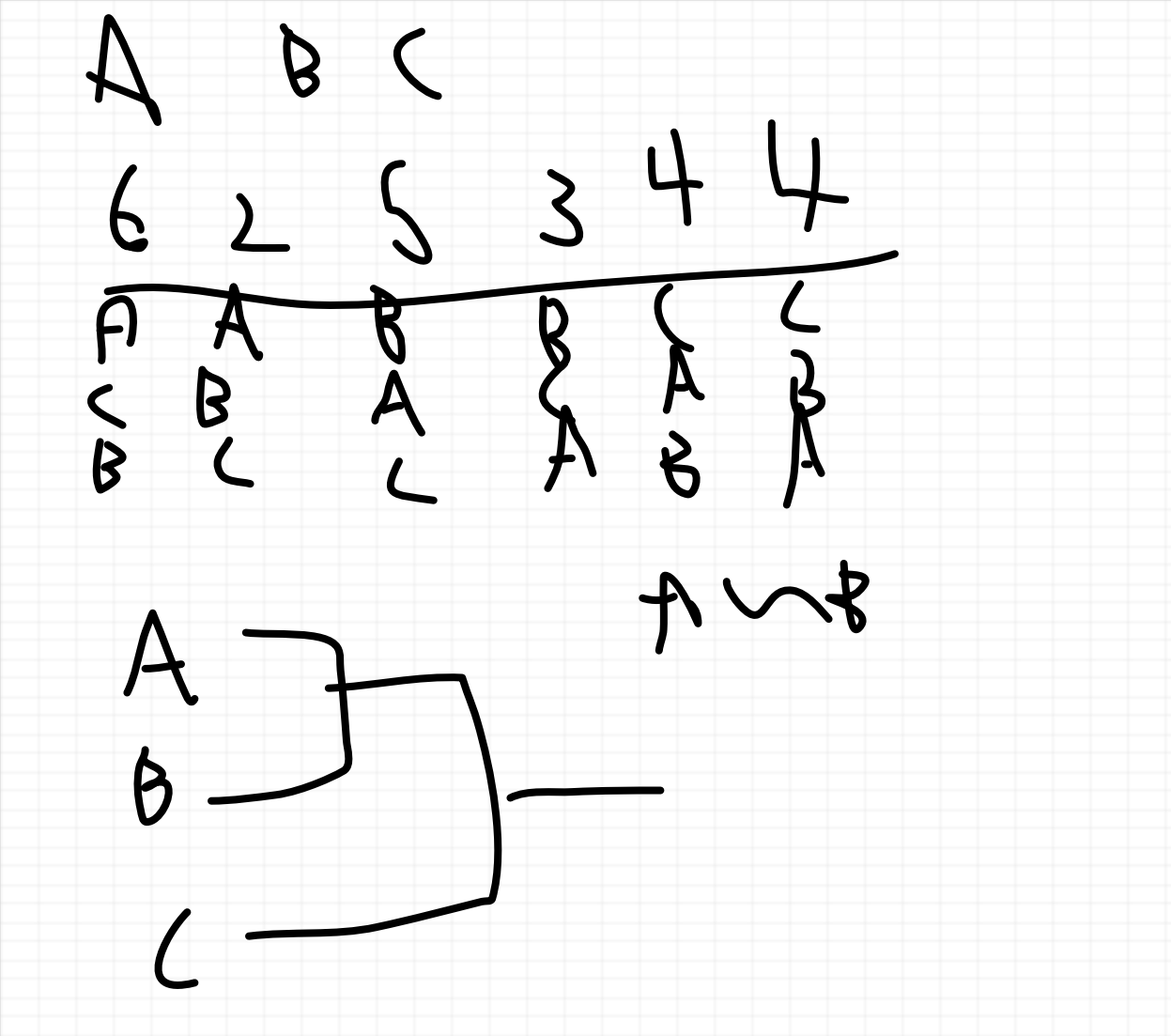
Def. 7.1-7.2

Insincere voting is when a voter casts a ballot that does not reflect how they actually feel about the alternatives. When a voter casts insincere vote with the intent of changing the election result to a preferred outcome is called **strategic voting** or **tactical voting**

Three important types of tactical voting (Def. 8.1)
*__Compromise__-a group of voters vote for a candidate they prefer less so that the winner is a candidate they prefer more than what otherwise would have happened.
*__Pushover__-where voters change their preference to knock out a candidate in a multi-rounded system they would otherwise beat.
*__Burrying__-Where in a ranked voting system, voters rank an alternative lower (for example make it count less in the Borda system) in the hopes of defeating it.

NOTE: sometimes tactical voting employs more than one such strategy at the same time.

Next time: We'll give specific examples (from the notes) of these voting tactics in action, and we'll get into fairness principles in classes 9-10.



If the tie happens before the final round, the agenda can't be completed and there is no social ranking (stack overflow error!).

If the tie is in the final round, then there is a social ranking and the alternatives both are the winner in the sense that they are preferred over every other alternative but between each other are indifferent.

7/15 Lecture

Announcements:

Happy tax day!
Exam 3 is graded, solutions are uploaded
Exam went "ok", average about 74%
Homework 1 grade ETA tonight

Questions on Homework 2-3:

"Some of the questions in hw 2 have a tie in the agenda.
I never specifically addressed this issue in my videos, or my notes.
I recommend the following way of dealing with ties in agendas:
If there is a tie in an agenda (not in the final round), then we say that the agenda can't be completed, and there is no social ranking. If the tie is in the final round, then the alternatives that tie come joint first, everyone else below them."

"Does not compute" "stack overlow"

Previously:

-Talked about condorcet winner and the condorcet criterion (on HW 3)
-Classified voting systems that satisfy and don't satisfy the condorcet criterion

Theorem 6.11 (Condorcet Methods).

Condorcet Methods	*Non-Condorcet Methods*
Agenda	*Plurality*
Dodgson	*Two Round Runoff*
Baldwin	*Instant Runoff*
	Borda

NOTE: The dogson method is a method that picks the "closest" candidate to the condorcet winner (see Def. 6.10). (we don't really cover the Dogson on the homework or probably not the exam)

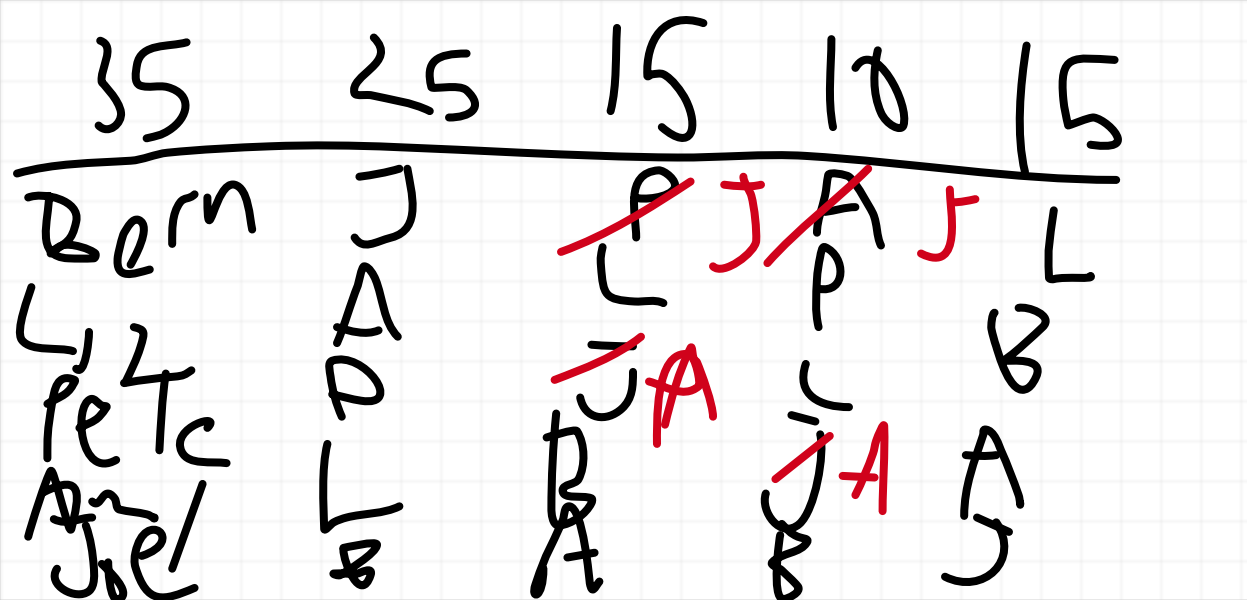
We defined **insincere voting** and **tactical voting**:

Three types of tactical voting:
*__Compromise__-a group of voters vote for a candidate they prefer less so that the winner is a candidate they prefer more than what otherwise would have happened.
*__Pushover__-where voters change their preference to knock out a candidate in a multi-rounded

system they would otherwise beat.
*__Burrying__-Where in a ranked voting system, voters rank an alternative lower (for example make it count less in the Borda system) in the hopes of defeating it.

Class 7-8 (Cont.): Examples of Insincere Voting

Let's discuss (one more time) the 2020 Democrat primary:



The situation where Amy and Pete coalesced around Biden is an example of compromise.

Example 7.3 (Tactical Voting - Compromise - Plurality).
Consider the following voter profile:

Votes	40	45	15
Preferences	A	B	C
	C	C	A
	B	A	B

Under the plurality system, the social ranking would be: $B \succ A \succ C$.

Imagine if the 6 voters who voted $C \succ A \succ B$ changed their votes to $A \succ C \succ B$.

The new voter profile would look like:

Votes	46	45	9
Preferences	A	B	C
	C	C	A
	B	A	B

NOTE:
-Any non-two candidate voting system is potentially susceptible to compromise
-Any voting system that doesn't satisfy the condorcet criterion is esp. vulnerable to compromise
-Plurality system is the most classically vulnerable to compromise, instant runoff and borda-cont are also vulnerable to compromise, but to a lesser degree.

Example 7.6 (Tactical Voting - Pushover - Instant Runoff).
Consider the following voter profile:

Votes	0	9	0	5	6	0
Preferences	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

Under the instant runoff voting system the social ranking would be: $C \succ A \succ B$.

Alternative A lost because it was faced with a tough opponent in the final round. But if that tough opponent was knocked out before the final round, then A might win.

Votes	0	~~9~~ 7	~~0~~ 2	5	6	0
Preferences	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

Under the instant runoff voting system the new social ranking would be: $A \succ C \succ B$.

NOTE:
-Always in a multi-rounded voting system
-Get rid of a condorcet winner
-most prevelant in instant runoff and agenda but prevalent in other multirounded systems as well

Example 8.2 (Tactical Voting - Burying - Borda).
Consider the following voter profile:

Votes	4	1	3
Preferences	A	B	C
	C	A	A
	B	C	B

The Borda count is: $A : 12, \quad B : 2, \quad C : 10.$
So the social ranking would be: $A \succ C \succ B.$

Votes	4	1	3
Preferences	A	B	C
	C	A	~~B~~
	B	C	~~A~~

The new Borda count is : $A : 9, \quad B : 5, \quad C : 10.$
So the new social ranking would be: $C \succ A \succ B.$

NOTE:
This is most notably an issue with Borda-Cont, but it's also an issue with other forms of ranked voting such as ranked choice voting.

Example 8.4 (Tactical Voting - Compromise and Burying - Borda).
Consider the following voter profile:

Votes	4	5	4	8	3	2	2
Preferences	A	A	B	B	C	C	D
	B	C	A	C	A	B	A
	C	B	C	A	B	A	B
	D	D	D	D	D	D	C

The Borda count is: $A : 55, \quad B : 58, \quad C : 49, \quad D : 6.$
So the social ranking would be: $B \succ A \succ C \succ D.$

Suppose D voters compromise to support their second choice, and also bury the strongest opponent.

Class 9-10: Fairness Principles

neutral
condorcet efficient

"sufficient information" (too little information that we donky voting! We don't want too much information)

Some other fairness principles (mentioned classes 9-10) incude:

egalitalitarian-a system that treats all the voters equally
no dictator
sovereignty-if every possible social ordering can be produced with the right voter profile
unanimity-whenever every voter has the same preference, that is the social ranking
weak Pareto-a property of when every voter prefers $A \succ B$ then the voting system ranks $A \succ B$.
strategy-proof-immune to strategic voting.
monotone-voters gaining support for a candidate doesn't hurt that candidate.

7/16 Lecture

Announcements:

Homework 1 graded
Homework 2 grade ETA tonight
Exam 4 study session Sunday at 12pm

The plan:
Today 7/16: Ties, fairness principles, the impossibility theorem (if time)
Tomorrow 7/17: impossibility thoerems (cont.), gerrymandering
Sun. 7/19 12pm-1-ish-pm: Exam 4 study sesh
Mon. 7/20: Exam 4
Tues. 7/21

Questions on Homework 3-4:

Previously:

We gave examples of insincere voting and tactical voting. Can anyone the types of tactical voting?
Compromise-where voters whose primary candidate won't win decide to vote based what they prefer next most.
Burrying-in rank voting system where you vote an order different from your preference to get

a better alternative.
Pushover-candidates who want their primary guy to win vote different from their preference to eliminate a stronger candidate.

We defined some fairness principles.

Ties:

In the voting lectures and notes, there's not a lot of discussion on ties, so let's remedy that.

By assumption of rational voters, there is no tie in a dictatorship

Ties in singe sound voting systems:
This includes plurality and borda-cont
Social are determined *purely* by the numeric score of the *voting tally*

Ties in multiround voting systems;
baldwin, instant runoff, two round runoff, agenda

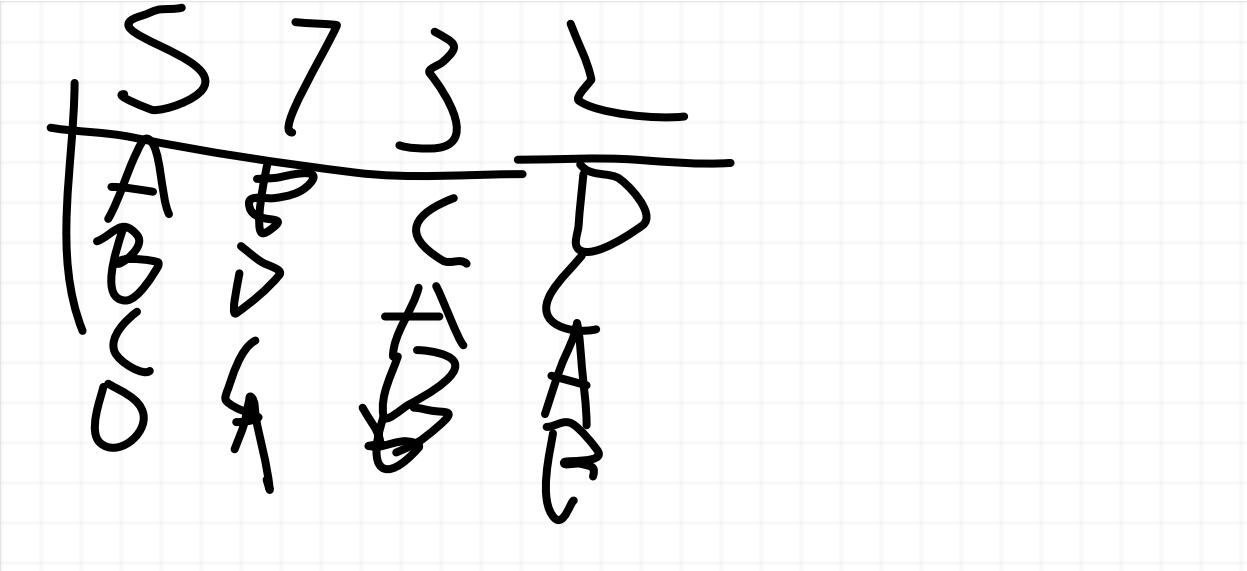
A **glitch** in the voting system is where an error happens in the voting system that makes it so it cannot continue.

Voting systems are algorithms, glitches could happen.

<u>General rules of Multiround voting systems:</u>

1. Any tie in the final means that both candidates tie for the winner (and hence are preferred over every other alternative and are indifferent between each other)
2. Any tie for elimination means that the voting system cannot continue.

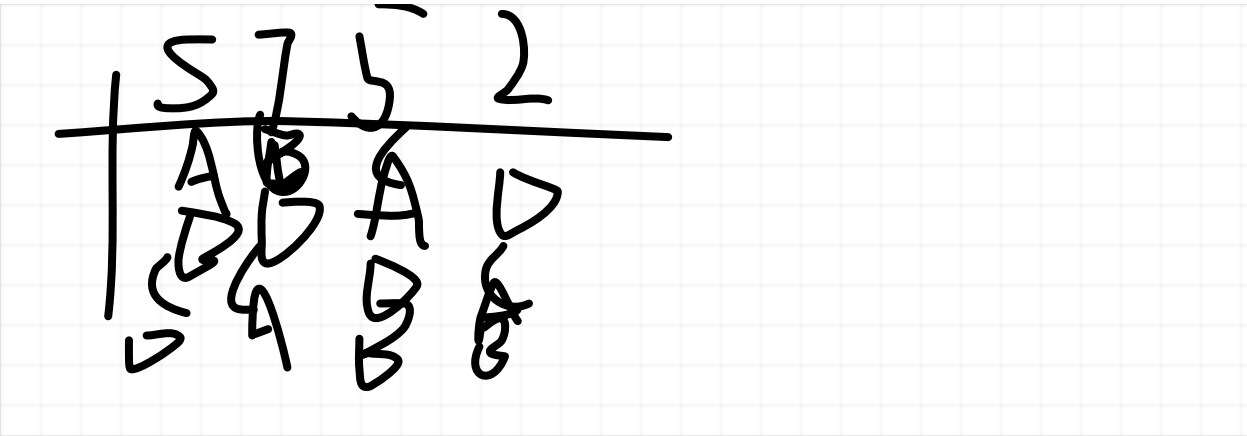
Example-Instant runoff



1st round:
A has 5 votes, B has 7 votes, C has 3 votes, D has 2 votes
D is the clear last place candidate, so D gets elminated

2nd round:
A has 5 votes, B has 7 votes, C has 5 votes
The elimination condition is not satisfied, and no social ranking exists.

Example-Two Round Runoff



1st round:
There is no top two candidates since A and C tie for second, so then we can't eliminate all other candidates, and the voting system stops, no social ranking exists.

NOTE: Condorcet winners are determined strictly; in other words, in order for the condorcet winner condition to be met, a candidate has to strictly win (cannot tie) in all its head-to-head

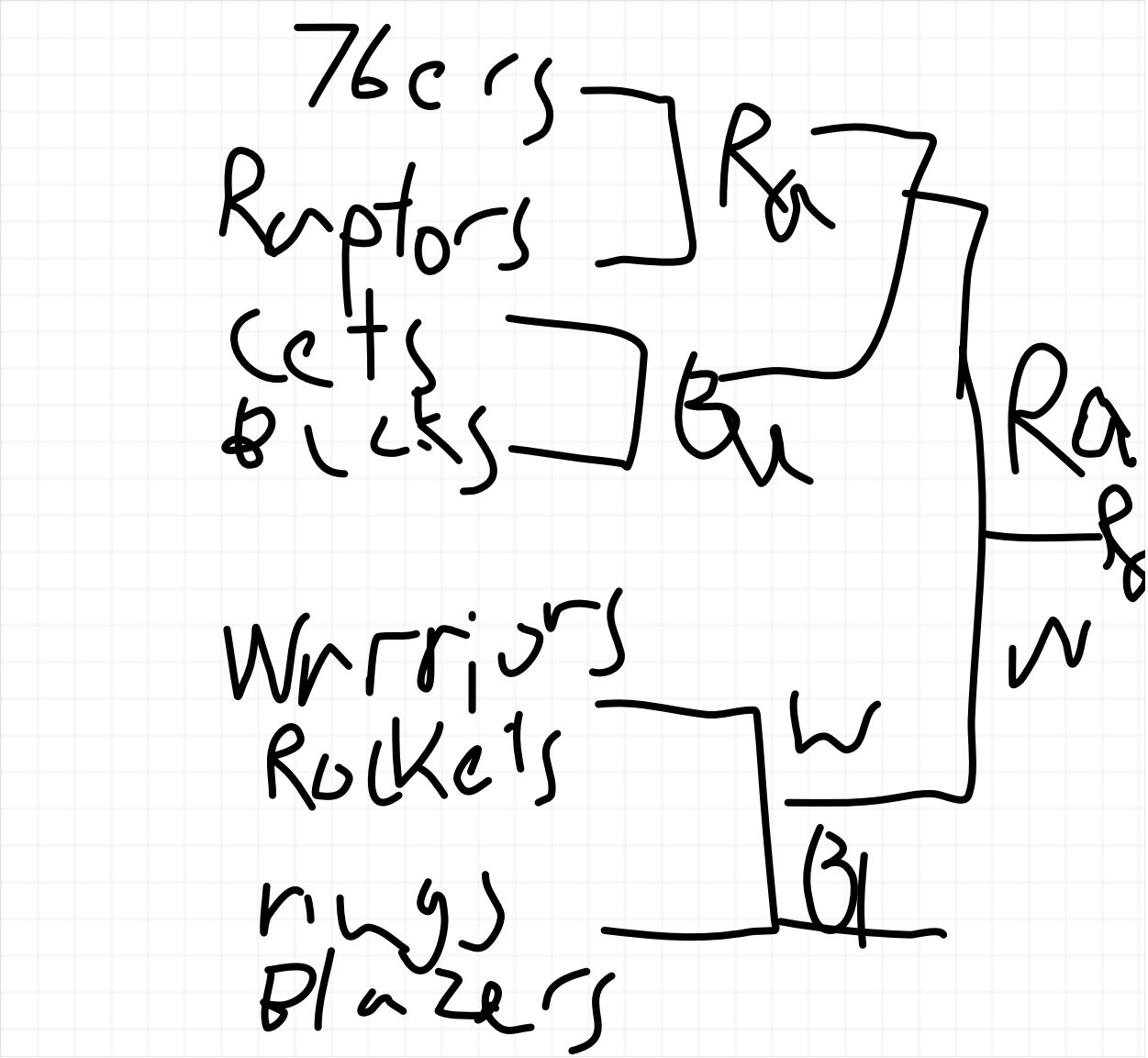
matchups.

FINAL NOTE:

Social rankings in multiround voting systems are determined by how long it takes for candidates to be eliminated, i.e.any candidate eliminated in the second round has higher social ranking than candidates eliminated in the first round.

Candidates that are elminated in the SAME ROUND are socially indifferent.

Example-2019 NBA playoffs (from second round to final round)



$Ra \succ W \succ Bu \sim Bl \succ 76 \sim C \sim Ro \sim N$

Class 9-10 (Cont.):

neutral
condorcet efficient
"sufficient information" (too little information that we donky voting! We don't want too much information)

Some other fairness principles (mentioned classes 9-10) incude:

egalitalitarian-a system that treats all the voters equally
no dictator
sovereignty-if every possible social ordering can be produced with the right voter profile
unanimity-whenever every voter has the same preference, that is the social ranking

NOTE: Definition is slightly wrong due to exceptional circumstances involving ties.

weak Pareto-a property of when every voter prefers $A \succ B$ then the voting system ranks $A \succ B$.
strategy-proof-immune to strategic voting.
monotone-voters gaining support for a candidate doesn't hurt that candidate

All these fairness principles are in class 9

Class 10 definitions

condorcet loser criterion-a voting system winner is never a condorcet loser
majority critereon-one alternative is ranked first by majority voters, then that alternative must win.
indepence of irrelevent alternatives (IIA)-next time
clone proof criterion (IoC)-next time

7/17 Lecture

Announcements:

Homework 2 and 3 grade ETA this afternoon to early evening, solutions to HW 1-3 to follow
Homework 4 grade ETA tomorrow night

Homework 5 early grade ETA Sunday afternoon
otherwise ETA will be Monday morning
Exam 4 Study sesh 12pm-1pm-ish
iu.zoom.us/my/agoodlad

Questions on Homework 4-5:

Previously:

-Talked about what to do with tie situations and how we account for them in voting theory. In particular, we talked about **glitches** in the voting algorithm and how they may not output an actual social ranking.

Class 9-11: Impossibility Theorems

Classes preceding class 9 mentioned the following principles:

neutral
condorcet criterion
"sufficient information" (too little information that we donky voting! We don't want too much information)

Some other fairness principles (mentioned classes 9-10) incude:

egalitalitarian-a system that treats all the voters equally

All voting systems covered in this are egalitarian.

Example 9.2 (Non-egalitarian voting systems)

-The US senate 2 votes per state (each state has unequal population)
-The electoral college
-The U.S. house (though it's more based on population than the senate), technically not egalitarian

no dictator
sovereignty-if every possible social ordering can be produced with the right voter profile

All the voting systems we cover in calss are soverign (except for agenda).

unanimity-whenever every voter has the same preference, that is the social ranking
SCRAP THIS OLD DEFINITION

NOTE: Definition is slightly wrong due to exceptional circumstances involving ties.

For example, in plurality, if everyone votes A>B>C>D, then A wins, but B,C,D all tie for second because they all get zero (top preference) votes. So the social ranking is A>B~C~D. Thus Plurality wouldn't satisfy Unanimity as defined previously.

unanimity actually is when everyone votes the same alternative in first place, then that alternative comes in first place.

weak Pareto-a property of when every voter prefers $A \succ B$ then the voting system ranks $A \succ B$.

strategy-proof-immune to strategic voting.

All the voting systems mentioned (except dictatorship) are NOT strategy proof when the number of alternatives are three or more.

monotone-voters gaining support for a candidate doesn't hurt that candidate

All these fairness principles are in class 9

Class 10 definitions

condorcet loser criterion-a voting system winner is never a condorcet loser

majority critereon-one alternative is ranked first by majority voters, then that alternative must win.

indepence of irrelevent alternatives (IIA)-(Def. 10.7) A voting has IIA if the social preferences between alternatives A and B depend ONLY on individual preferences between A and B (in other words, any other C has no bearing on the social ranking of A and B between each other)

clone proof criterion (IoC)-(Def. 10.11) very similar alternatives neither help nor harm each other.

Check out the definition and Example 10.12-10.13

from page 39 of the notes:

Impossibility Theorems

With two alternative votings sytems, there is basically one unique "fair" voting system, i.e. plurality.

When there three or more alternatives, not only is it hard to comes up with a good voting system, it's literally IMPOSSIBLE to come up with a voting system that has EVERY desired property:

Arrow's Impossibility Theorem (Thm. 11.1)

The only ordinal voting system with at least three alternatives that satisifes *weak pareto* and *IIA* is a dictatorship.

Gibbard's Theorem (Thm. 11.2)

The only system at least three alternatives which satisfies the *unanimity* and *strategy proof* fairness principles is a dictatorship.

FINAL THOUGHT: While there may not be a perfect voting system, there are voting systems that are less susceptible to harmful strategic voting that games the system. In other words, there's that are more *strategy resistent*, just like how there are tougher codes to crack than

others.

Finding a voting system that is harder for the powers that be to game is still a valid question.

Moreover, there are other issues with our democracy worth addressing, namely gerrymandering.

Class 12-13: Gerrymandering

Gerrymandering (Def. 12.1) is used for the purpose manipulating political representation

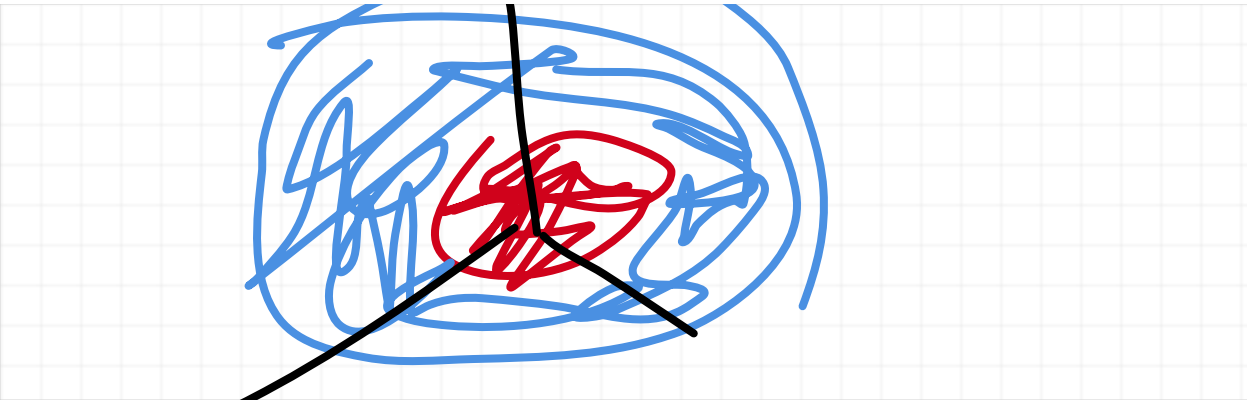
There's various motivations behind doing this, such as:

racial gerrymandering (cultural gerrymandering)
  *racist gerrymandering
  *affirmative gerrymandering
partisan gerrymandering

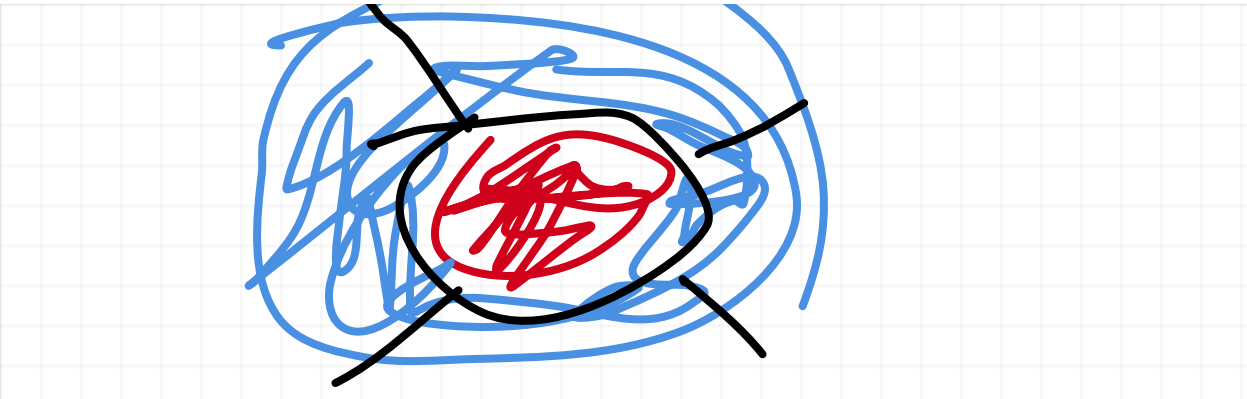
NOTE: those two labels aren't mutually exclusive.

There's three types of gerrymandering (in this module, we only study two types in Def. 12.5)

Cracking-disperse a group of voters into several districts, so that a majority isn't formed



Packing-condensing like-minded voters into one district to prevent them from affecting elections in other districts.



Stacking-put low vote turnout "majority" against a high voter turnout "minority"

Def. 13.2
A **wasted vote** is one of the following:
A losing vote-votes for the loser
A surplus vote-excess votes for the winner.

Def. 13.4
The **efficiency gap** defined to be
$[wasted\ votes\ for\ A - wasted\ votes\ for\ B] / total\ number\ of\ votes$

7/19 Study Sesh

Questions on Homework 5:

Review: