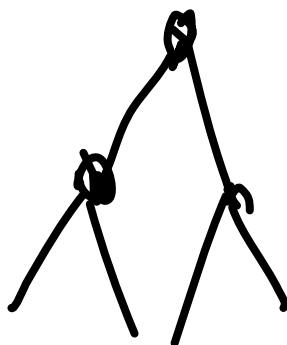


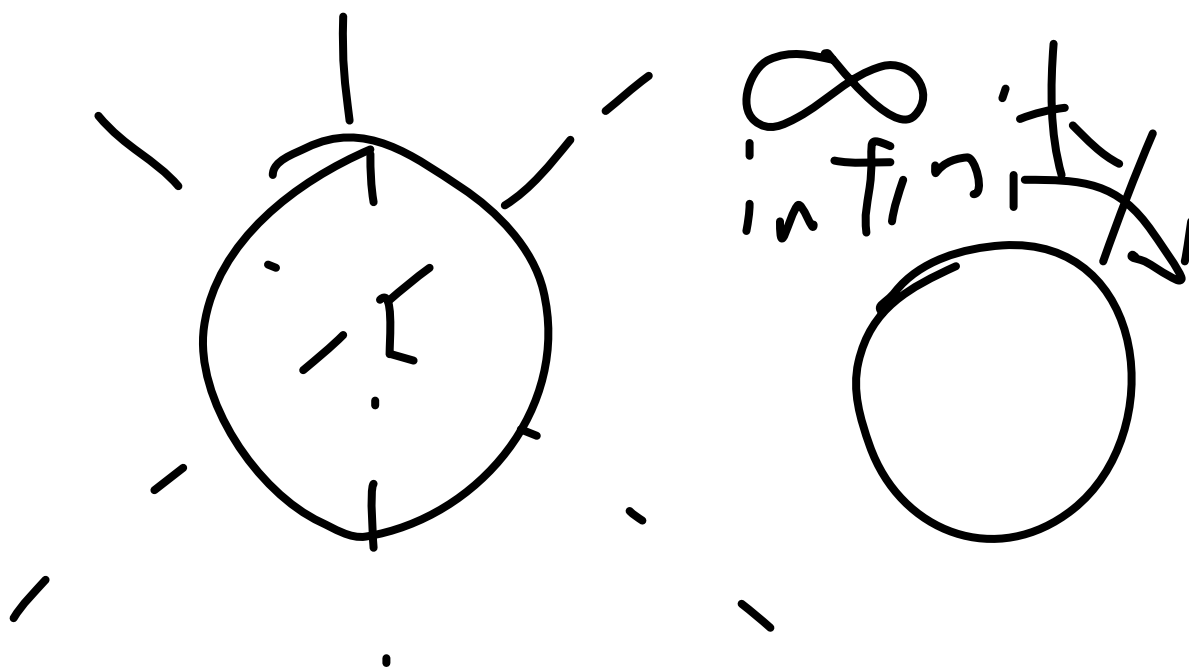
M106 Summer 2020 Recitation Lectures: Symmetries

6/6 Practice Lecture

Alex Goddard
M106

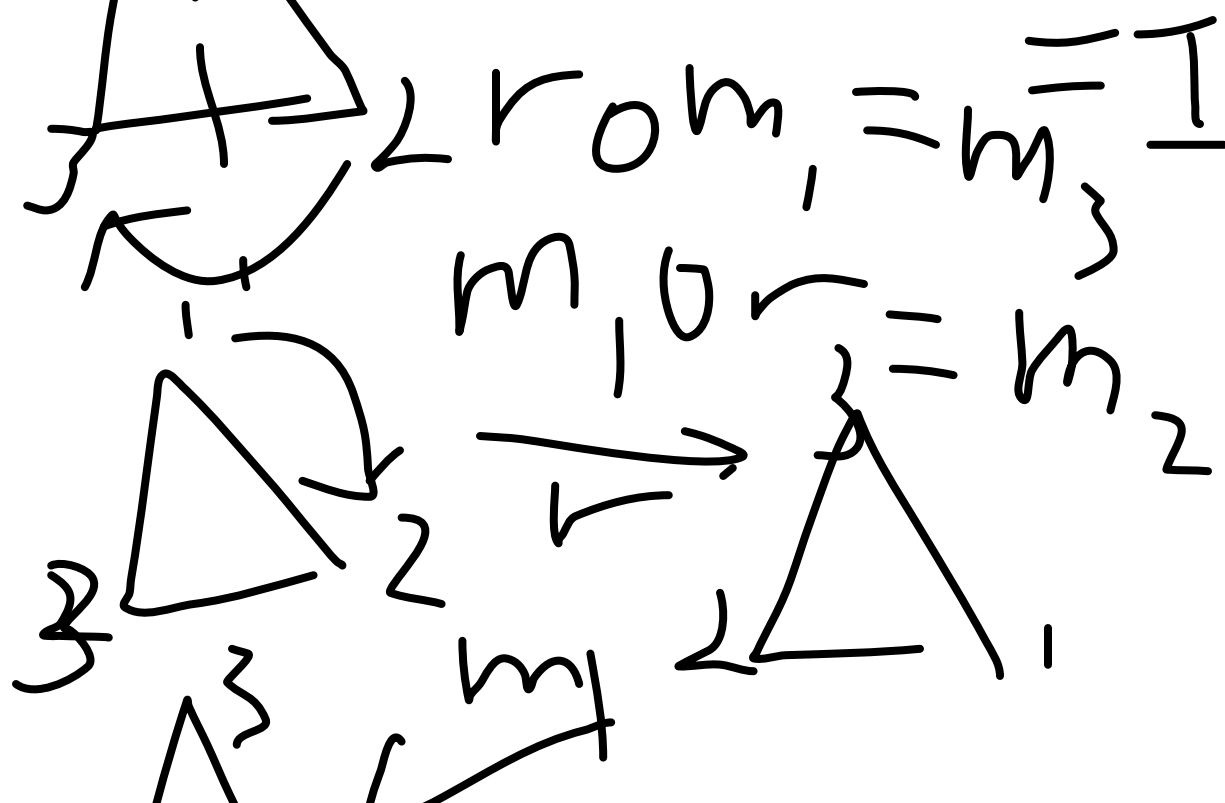
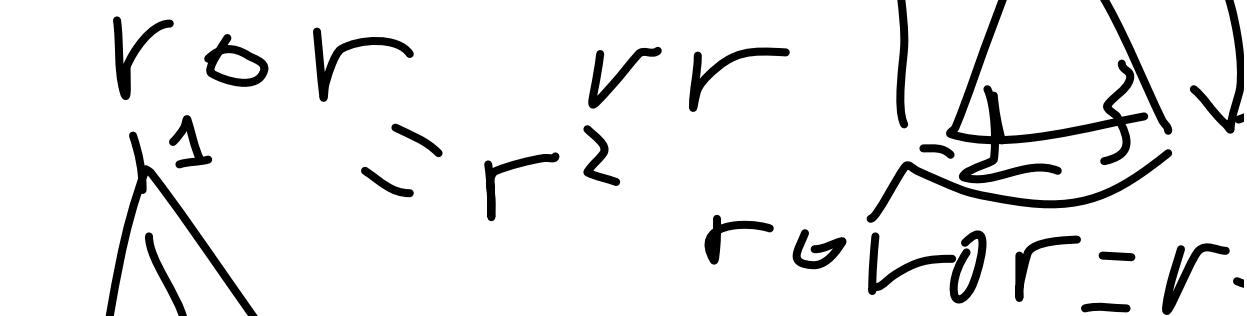


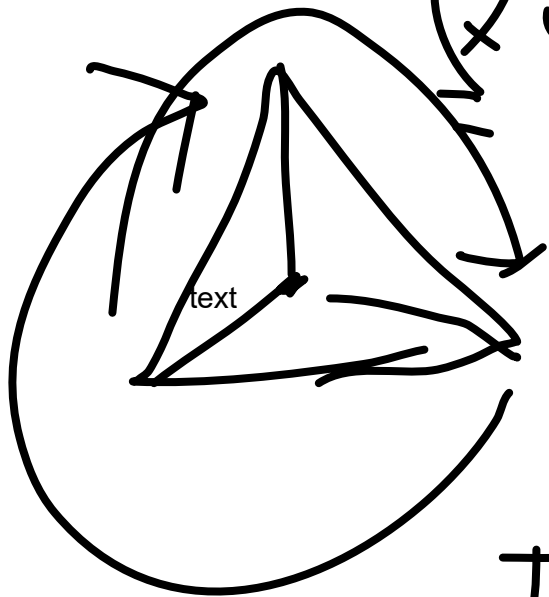
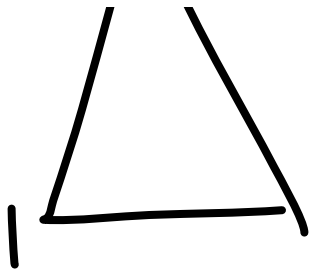
6/9 Lecture



Theorem 2.6 (Symmetries of Finite 2D Figures). Every symmetry of a finite 2D-figure is one of the following: ☐ Trivial symmetry ☐ Rotation ☐ Reflection

$$I \quad \Sigma T \quad r^1 \quad r^2$$





$(x \ 0 \ 0 \ 0 \ 2 \ I \ 0 \ X = X$
 $\sum x \ 0 \ I_1 = X$
 $\sum I \ r \ r$

the columns apply first

$0 \ I \ r \ r^2$

$r^1 \ 0 \ r^2$ rows go second

$I \ I \ r^1 \ r^2$
 $H \ r \ r^2 \ T$
 $r^2 \ I \ r^1$

$r^2 \ 0 \ r^2 = r^4$

$x \ 0 \ y \ 0 \ 2$

$(x \ 0 \ y) \ 0 \ 2 = x \ 0 \ (y \ 0$

$\checkmark \ r \dots + \checkmark \checkmark \checkmark$

$$\wedge \cup \gamma \neq \gamma \cup \wedge$$

$$X^h \cup \gamma^h \neq (X \cup \gamma)^h$$

$$X \cup \gamma = \gamma \cup X$$



$$(X \cup \gamma)^h = X^h \cup \gamma$$

6/10 Lecture:

SHUT DOWN STEM STRIKE

6/11 Lecture:

Recap (on 6/9):

- Compositions (Class 3)
- Composition Tables (Class 4)

Homework/Exam:

- Homework 1 due TONIGHT (11:59 pm)
- Homework 2 due TOMORROW NIGHT
- EXAM on Monday (6/15)

-Homework 3 due MONDAY (6/15)

Questions on ANY homework?

-Homework 2 question 1

$r^3 \circ l \circ m^4 \circ r^{-1} \circ m^{-2} \circ l$

$r^3 * l * m^3$

$$(r^3 \circ l \circ m^4 \circ r^{-1} \circ m^{-2} \circ l)$$

1. Associative

$$= r^3 \circ m^4 \circ r^{-1} \circ m^{-2} \circ l$$

$$= r^3 \circ m^4 \circ r^{-1} \circ m^{-2}$$

$$m \circ m = I$$

$$m^{-1} = I$$

m_3

$$r^4 = I$$

$$m^{-2} = I$$

\downarrow
 $\begin{pmatrix} r & 3 & 0 & I & 0 & r & -1 \\ r & 3 & 0 & r^{-1} & - & r & \end{pmatrix}$

$r \circ r^{-1} = I = I$
 $x \circ x^{-1} = I$

$\begin{matrix} & & m_c \\ & & \downarrow \\ \begin{pmatrix} 1 & 2 \\ 4 & 4 \end{pmatrix} & r & \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix} \end{matrix}$

$\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} m$

< a

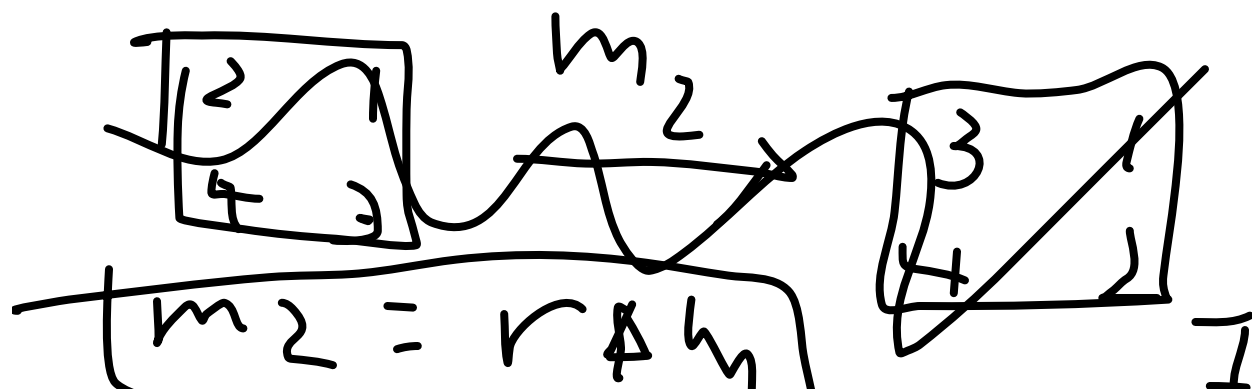
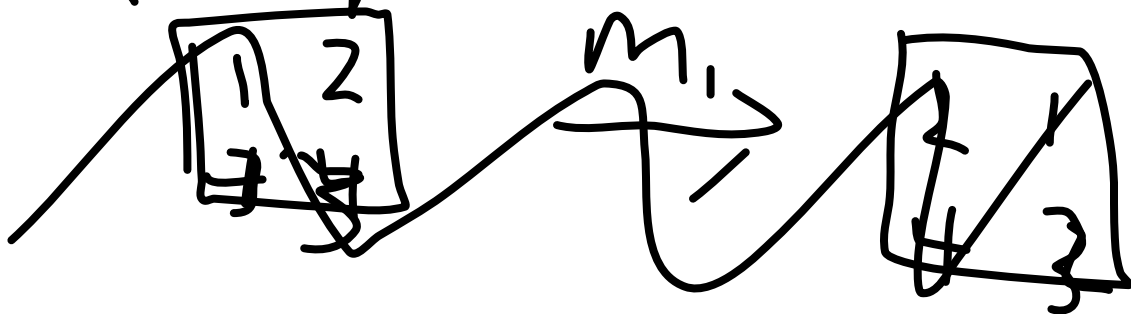
$$\{1, r, r^2, r^3\}$$

$$r^3 * r = r^4 = I$$

$$r^m * r^n = r^{m+n}$$

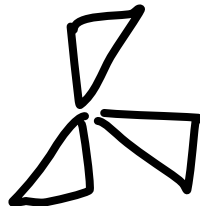
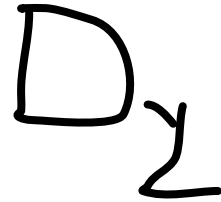
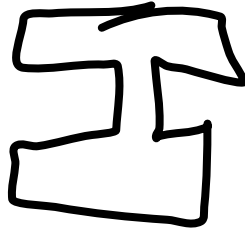
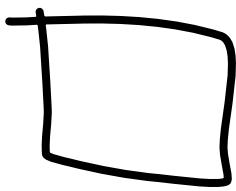
$$x^m * x^n = x^{m+n}$$

$$m_2 * m_1$$



$$\begin{aligned}
 & \cancel{r h_2 \times m_1 = r \times m_1} \\
 & = r \\
 & m_2 = m \\
 & m_3 \times m_4 \\
 & \cancel{h_4} \\
 & m_4 = r \times m \\
 & m_3 = m_4 \times r
 \end{aligned}$$

A polygon has the same SYMMETRY TYPE if it has the same composition table



D_n is an n -gon

C_n the symmetry type of an n -pinwheel

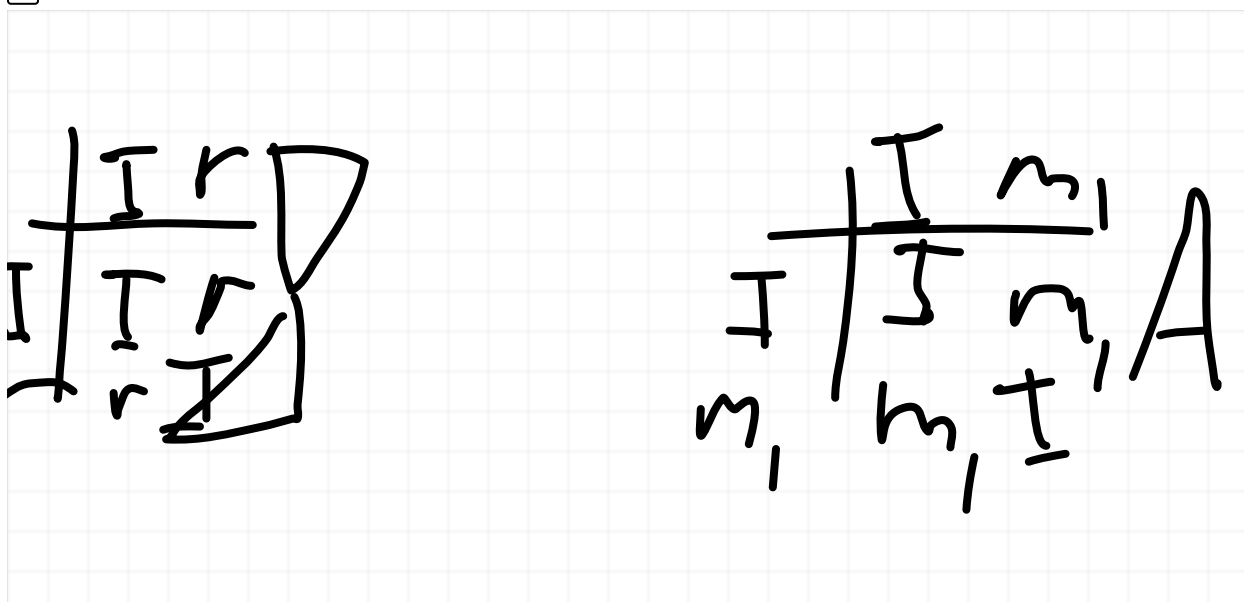
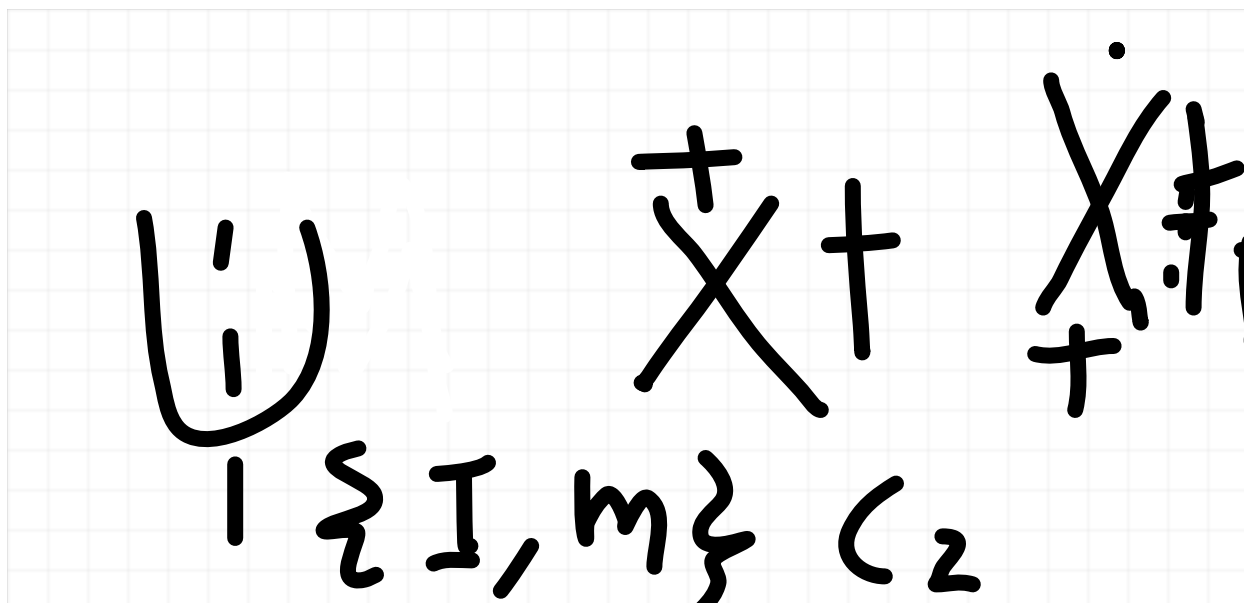
Theorem: Every finite figure has symmetry type D_n or C_n

C_1

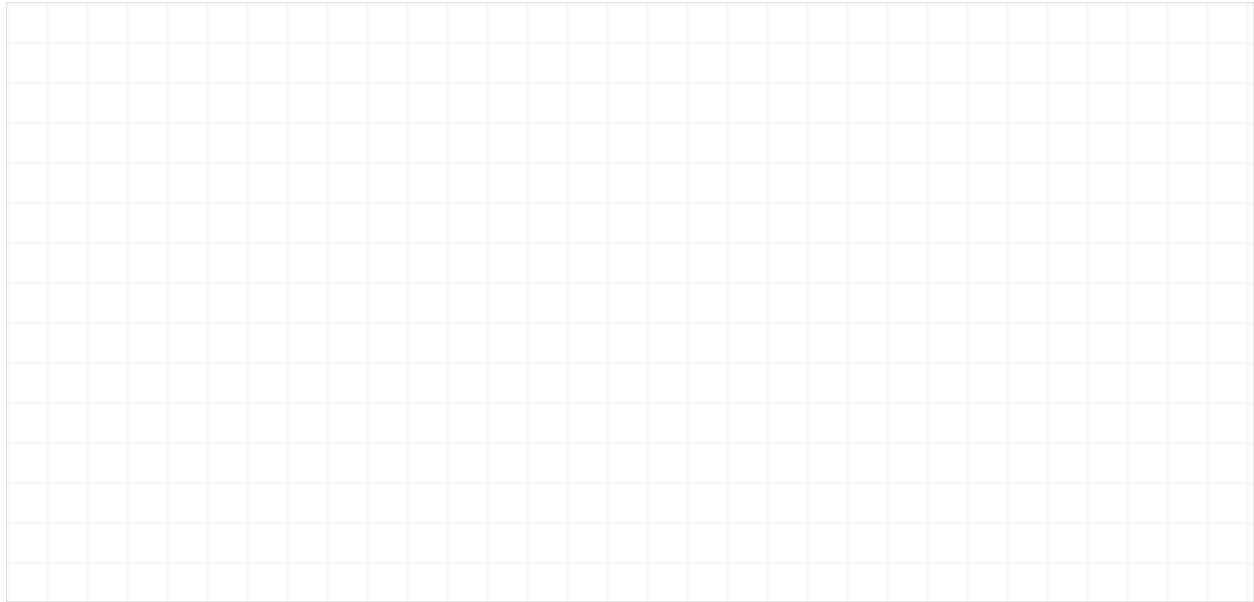
In Homework 2 Question 2:

Any letter has symmetry type D_n or C_n

Symmetry type of letter A: $\{I, m_1\} C_2$



C_2 is a special case of the "pinwheel" symmetry type, where any shape containing only one reflection OR only one rotation has that same symmetry type.



Plan for Tomorrow:

Infinite symmetries (Class 6-7)

Frieze patterns (Class 8)

Homework 3 is based on those classes

6/12 Lecture

Announcements:

Office hour doodle poll AFTER CLASS

Review Session THIS WEEKEND (doodle poll for that AFTER CLASS as well)

Game Theory is up

Upcoming Assignments:

-Homework 2 (due Tonight)

-EXAM ON MONDAY AT CLASS TIME

*Given on Canvas (you will download PDF)

*2 hours AFTER USUAL CLASS TIME (factoring in for any tech difficulties)

*WILL NOT BE PROCTORED

*DON'T CHEAT

-Homework 3 (due Monday night)

Previously (on the exam):

- *Symmetries of finite figures (class 2)
- *Composing symmetries (class 3)
- *Composition Tables (class 4)
- *Symmetry Types (class 5)

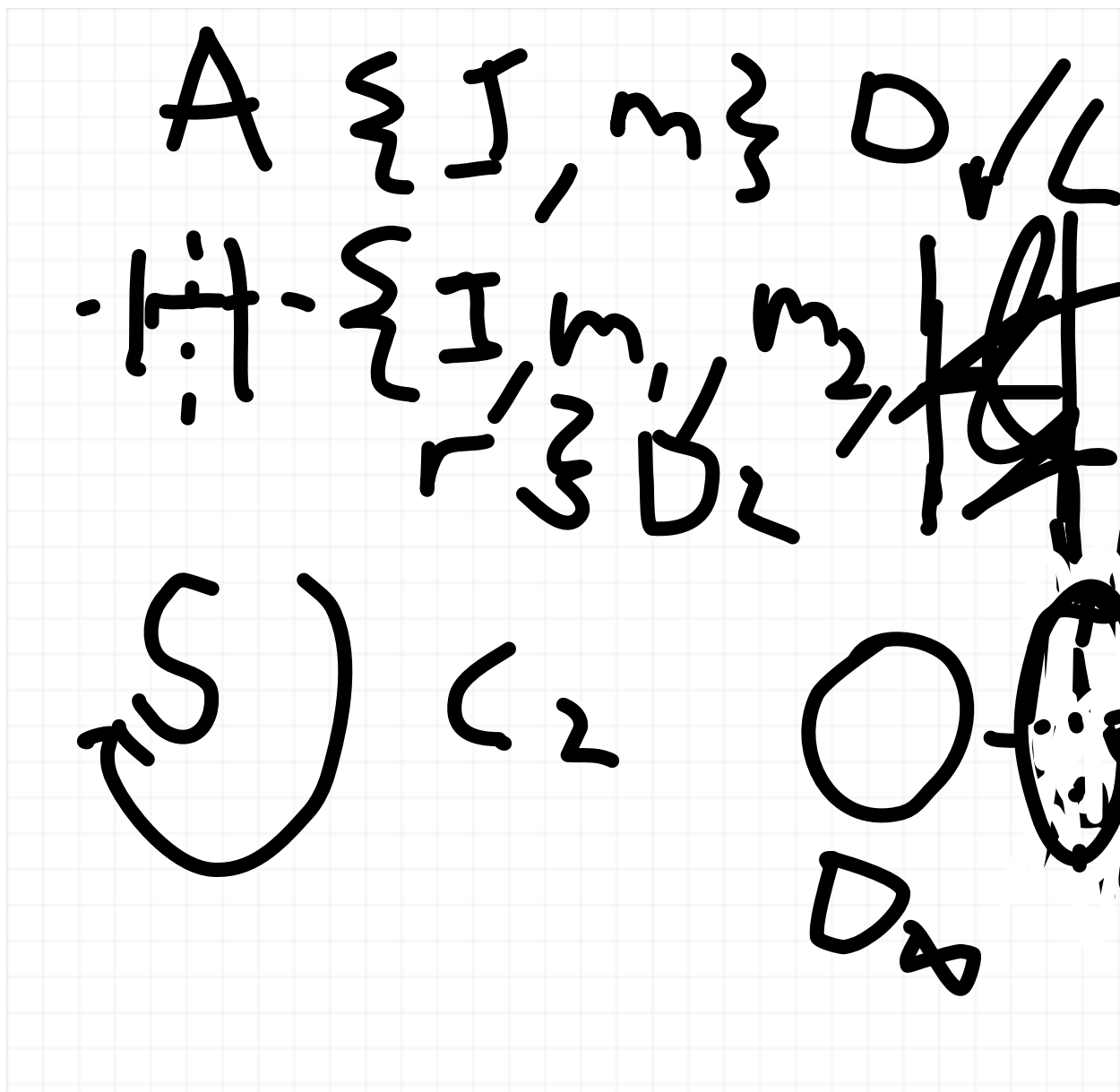
Upcoming:

- *Infinite Symmetries (class 6-7)
- *Frieze Patterns (class 8)

Questions Homework 2&3?

Expect grades for Homework 1&2 Saturday

Homework 2 Question 2



NOTE: Symmetry types WILL be used more than once

NOTE FOR HOMEWORK 1 QUESTION 1 B: The trivial symmetry will only be used only once. More generally speaking,

$$\begin{aligned}
 X^n &= X \circ X \circ \dots \circ X \\
 m_2^4 &= (m_2)^4 \quad \text{4 times} \\
 &= m_2 \circ m_2 \circ m_2 \circ m_2 = \\
 m_2 \circ m_2 &= r^2 \circ m_2 = \\
 &= h_2
 \end{aligned}$$

Infinite Symmetries:

Classes 1-5 talked symmetries of finite shapes

Classes 6-8 deal with infinite symmetries

As before infinite symmetries have rotations (including trivial symmetry and nontrivial rotations) and reflections

With infinite symmetries, THERE IS TWO MORE KINDS OF SYMMETRIES

-Translation symmetries

-Gliding symmetries

Theorem: Symmetries of an xy-plane consist of the following

*trivial

*translations

- *reflections (with respect some line)
- *rotations (within a single center)
- *glide reflections

Theorem: Every frieze pattern always has the trivial and translational symmetries. Only possible additional symmetries include the following:

- (R) Rotation by $1/2$ -turn
 - (V) Reflection in vertical lines
 - (H) Reflection in horizontal line
 - (G) Glide reflection in the horizontal line
- Symmetry represented sub-string of TRVHG
- T
 - TR
 - TV
 - TRVH
 - TRVHG
 - And so on...