M211 Recitation Notes Ch. 2

Section 2.1-2.2

1/27 office hours

2.1 Office Hour Homework Questions

<u>WebWork4 Sec2.1and2.2 Problem 1.</u>

If a ball is thrown straight up into the air with an initial velocity of 40 ft/s, it height in feet after t second is given by y=40t−16t^2. Find the average velocity for the time period begining when t=2 and lasting

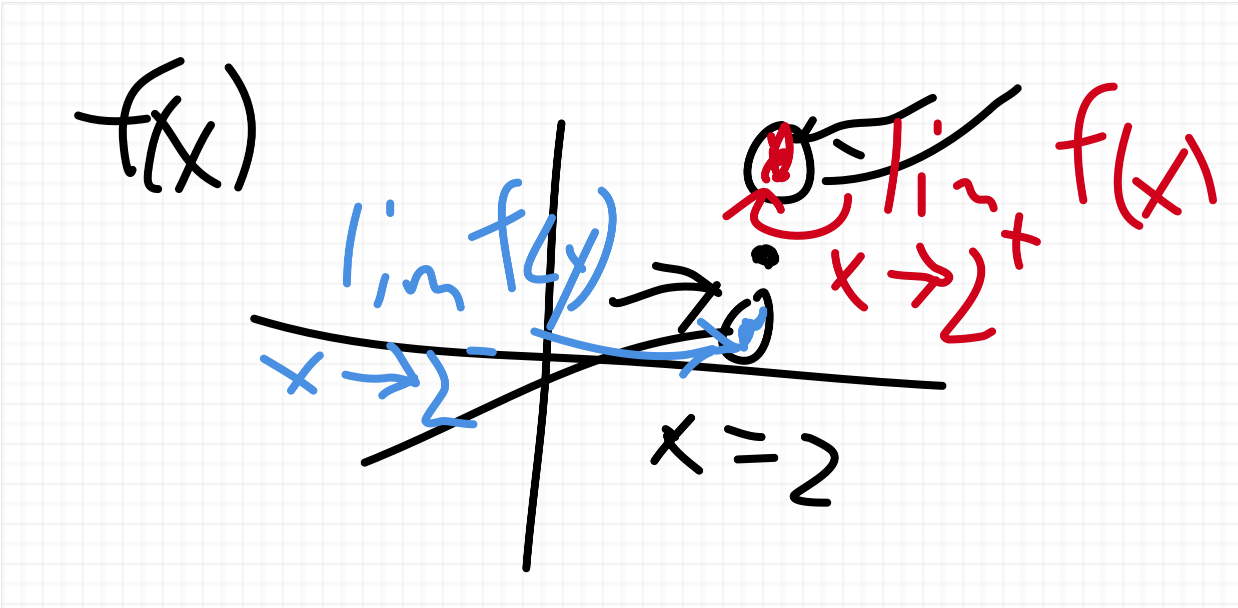
$t_i = 2$
$y_i = 40(2) - 16(2)^2 = 80 - 64 = 16$
average velocity $= \dfrac{\Delta y}{\Delta t} = \dfrac{y_f - y_i}{t_f - t_i}$

(i) $\Delta t = .1$ seconds

$t_f = t_i + \Delta t = 2 + .1 = 2.1$
$y_f = 40(2.1) - 16(2.1)^2$

average velocity $= \dfrac{40(2.1) - 16(2.1)^2 - 16}{.1}$

2.2 Office Hour Expostion



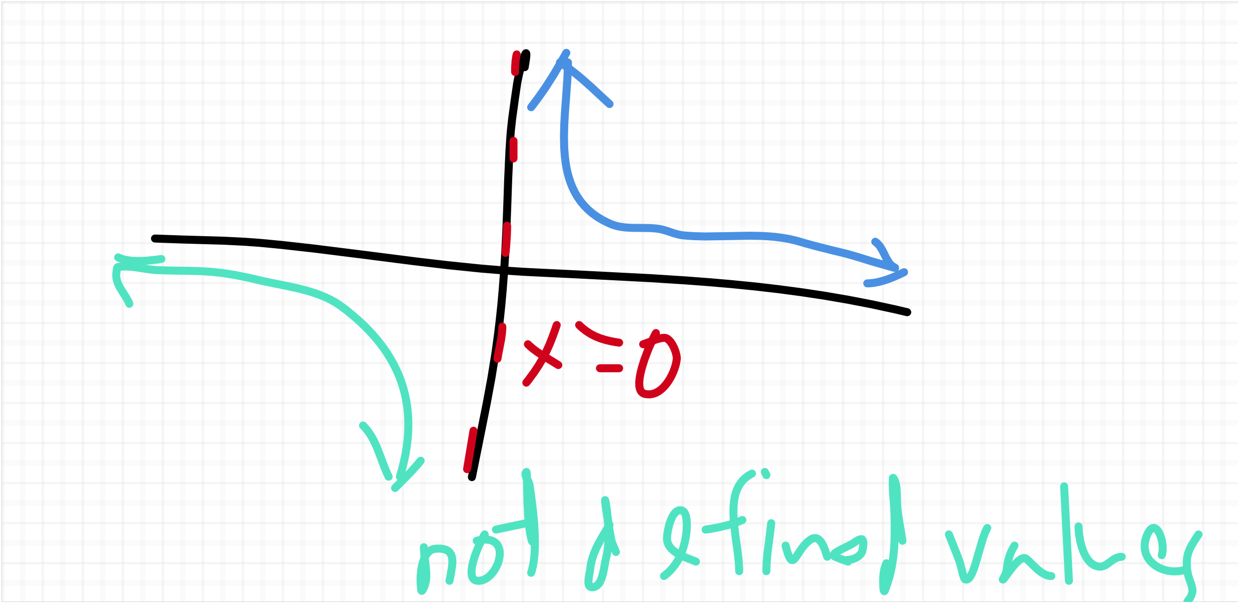
Definition. $\lim_{x \to a} f(x)$ exists precisely when the right hand limit $\lim_{x \to a^+} f(x)$ and left hand limit $\lim_{x \to a^-} f(x)$ exists. In the above drawing, $\lim_{x \to 2} f(x)$ DNE (acronym for "does not exist"), because

$$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$$

$1/x$ defined on $(0, \infty)$

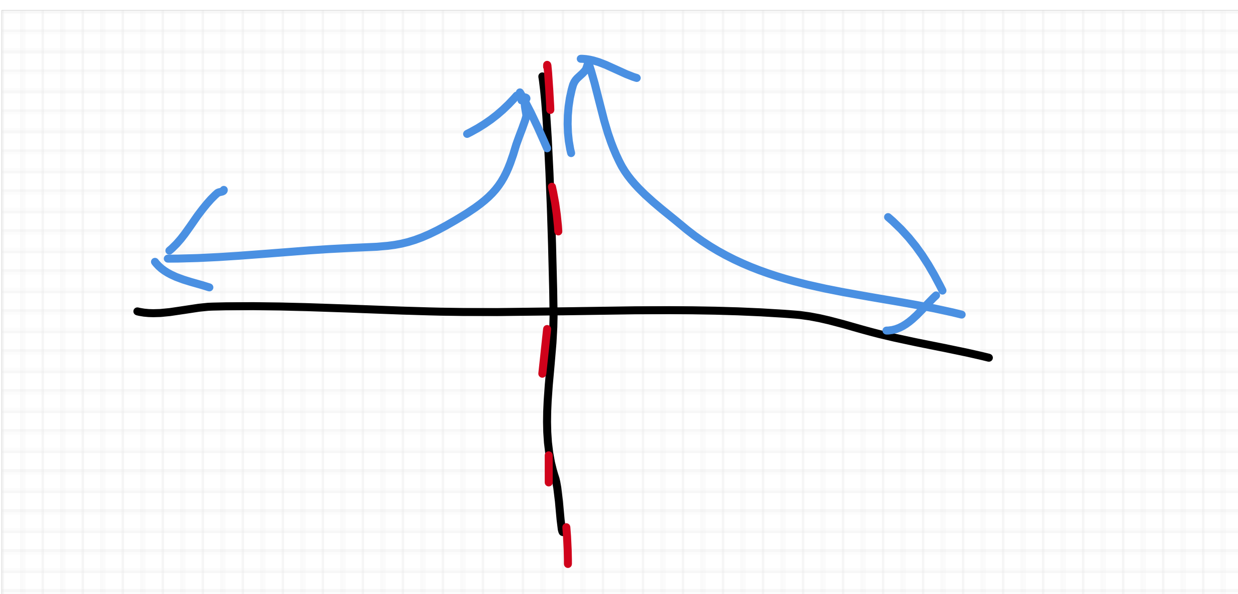
$$\lim_{x \to 0} 1/x$$

Because it blows up on one side



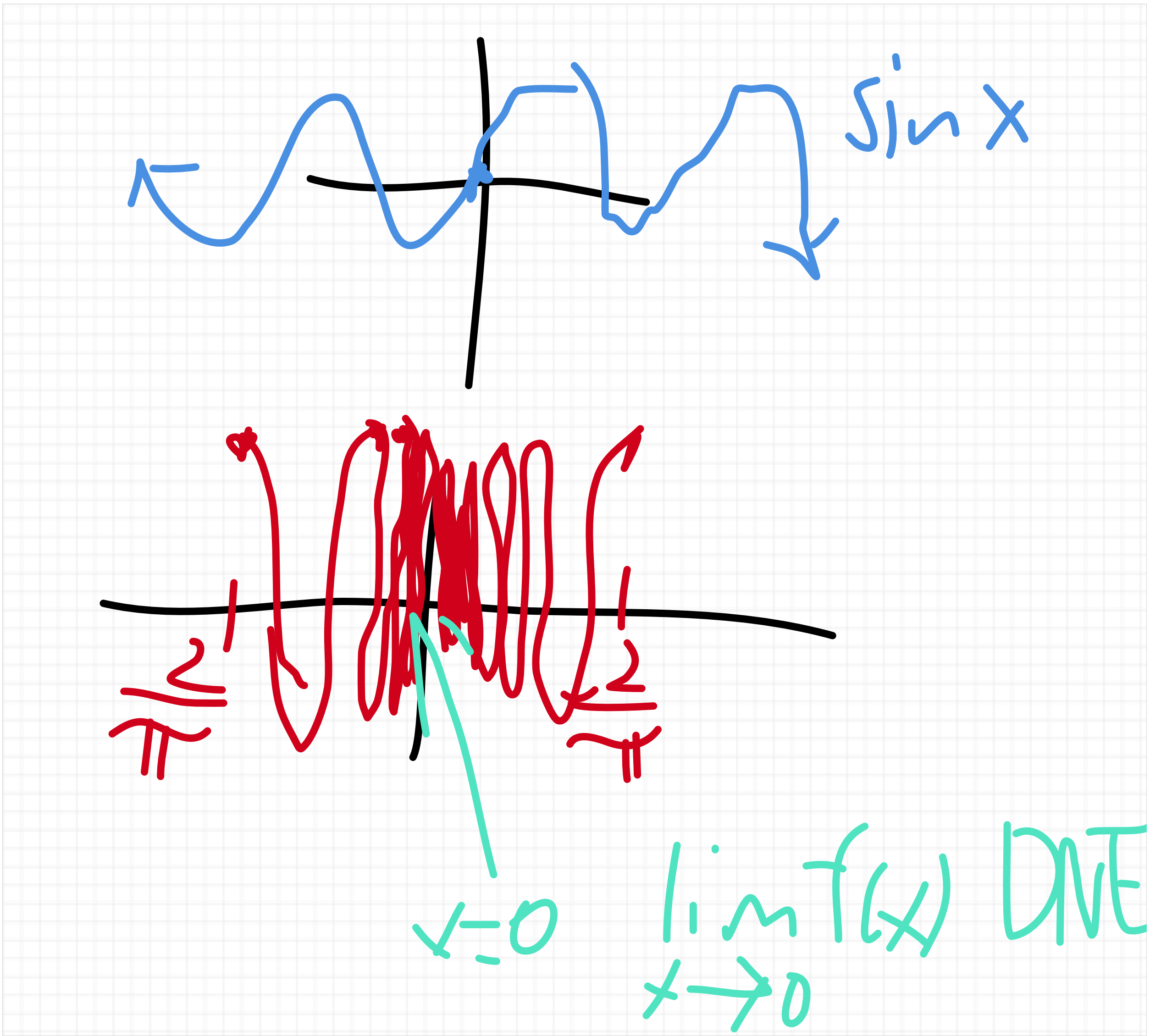
let's say we have $f(x) = \left|\frac{1}{x}\right|$ defined on $(0, \infty) \cup (-\infty, 0)$ (every point except for 0.

$\lim_{x \to 0} f(x)$ DNE because it blows up on both sides.



$f(x)$ may also "oscilate" very wildly

Let's say $f(x) = \sin\left(\frac{1}{x}\right)$ is defined at $(0, \infty) \cup (-\infty, 0)$



The limit as $x \to 0$ for $f(x) = \sin(1/x)$ DNE because it oscilates on both sides (and note moreover that $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$ do not exist either.

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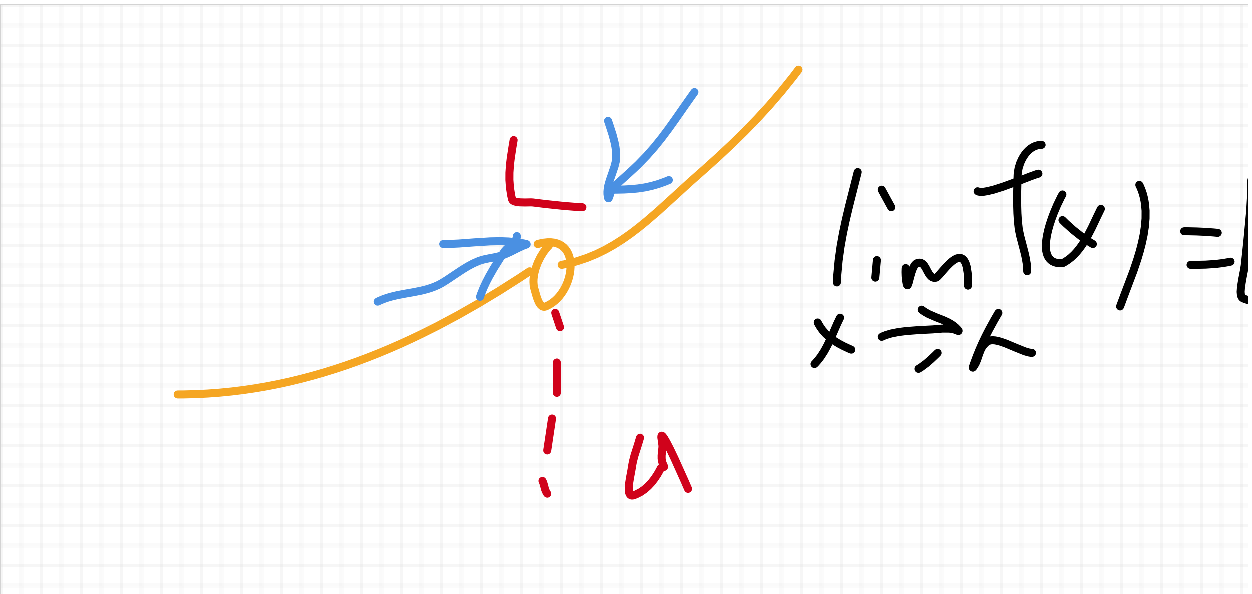
2.2 Expostion

What is a limit?

Definition. A limit of a value $a \in \mathbb{R}$ is the value that $f(x)$ approaches as x gets arbitrarily close to a

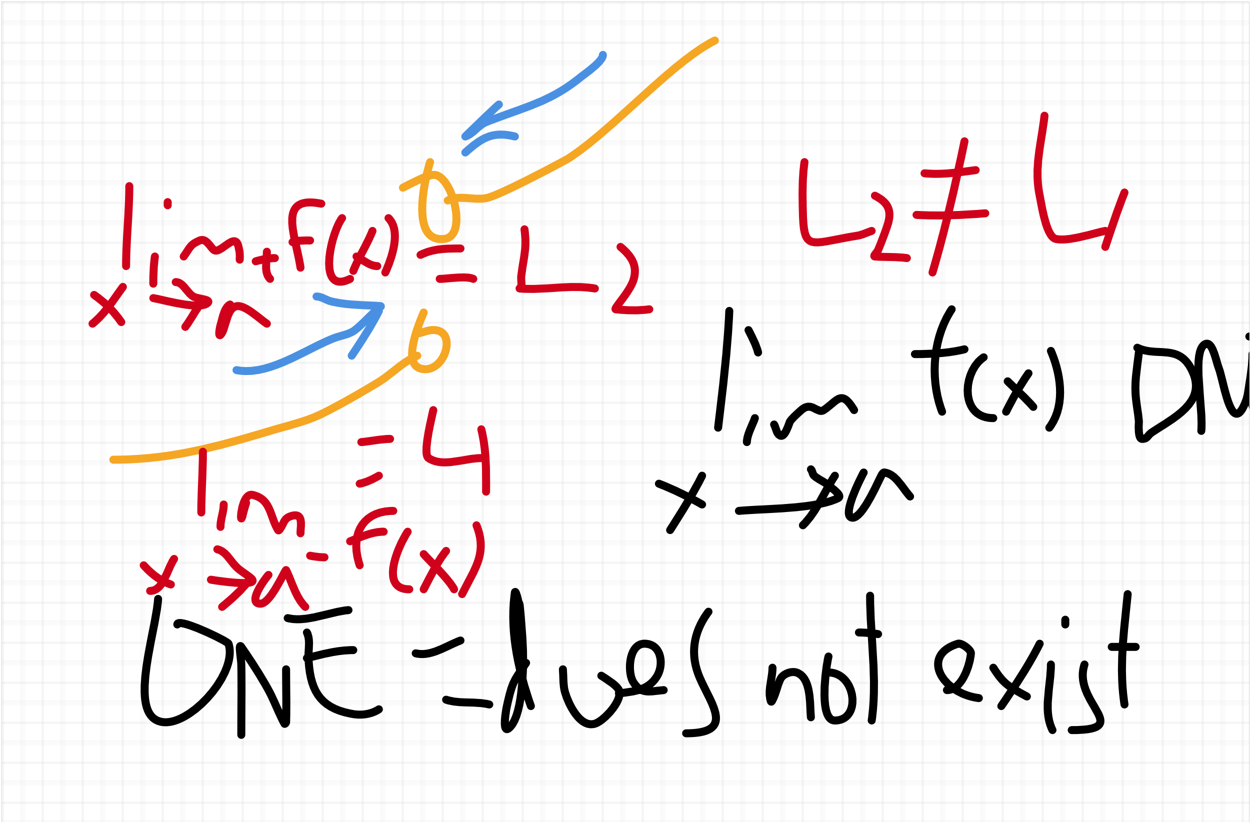
NOTE: The limit may or may not exist

Exists when a consistent value is approached



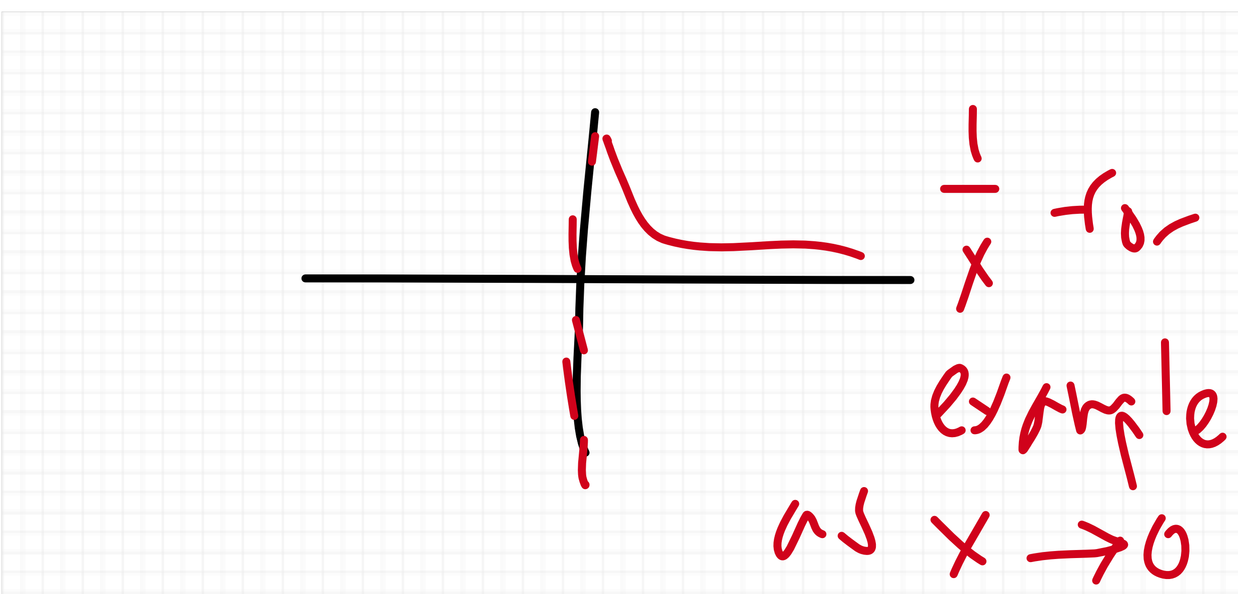
Limit may not exist when:

1. When different one-sided limits exist (when a different value is approached from the left and right)



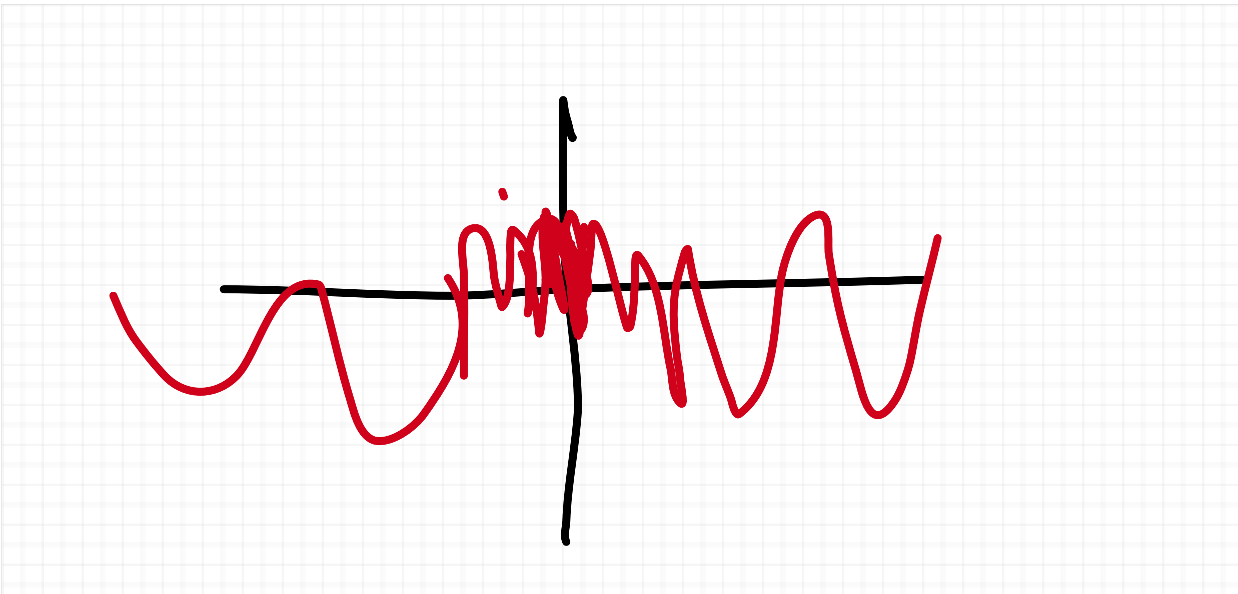
Takeaway: $\lim_{x \to a} f(x)$ exists precisely when $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$ exist and are equal.

2. The limit on one side may blow up (this happens when a vertical asymptote is there)



In this situation, the limit "doesn't exist" but an "infinite limit" may exist (see page 89)

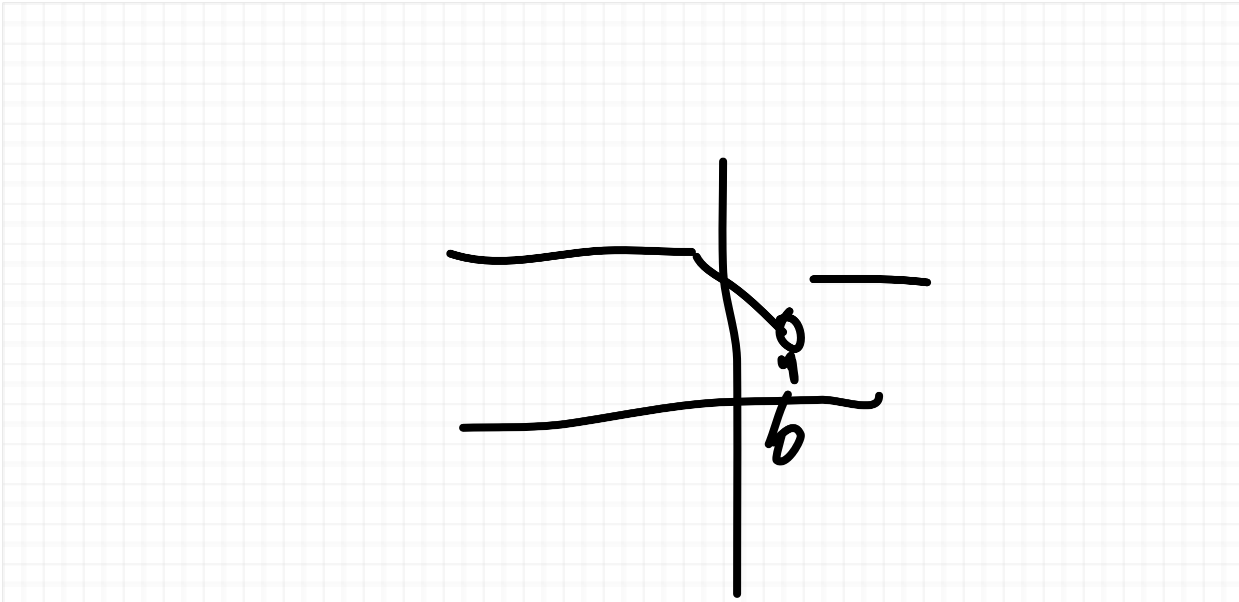
3. It may oscilate (refer to example 4 on page 86) $\sin \pi / x$



2.2 Homework Questions

<u>WebWork4 Sec2.1and2.2 Problem 4.</u>

1. $\lim_{x \to 4^-} f(x) = 15$
4. $\lim_{x \to 6^-} f(x) = 5$



2/1 office hours

<u>WW6Sec2.4 Problem 1.</u>

$0 < |x-9| < \delta \;\; y = |\sqrt{x}-3| < .2$

Triangle Inequality:
$|x+y| \leq |x| + |y|$

$$\begin{aligned} |x-9| &= |\left(\sqrt{x}-3\right)\left(\sqrt{x}+3\right)| = |\sqrt{x}-3| \cdot |\sqrt{x}+3| = |\sqrt{x}-3| \cdot |\sqrt{x}-3+6| \\ &\leq |\sqrt{x}-3| \cdot \left(|\sqrt{x}-3|+3\right) = y \cdot (y+6) = y^2 + 6y < (.2)^2 + 6(.2) = .04 + 1.2 = 1.24 \end{aligned}$$

NOTE: I redo this problem in the notes below

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2.3-2.5 Expostion

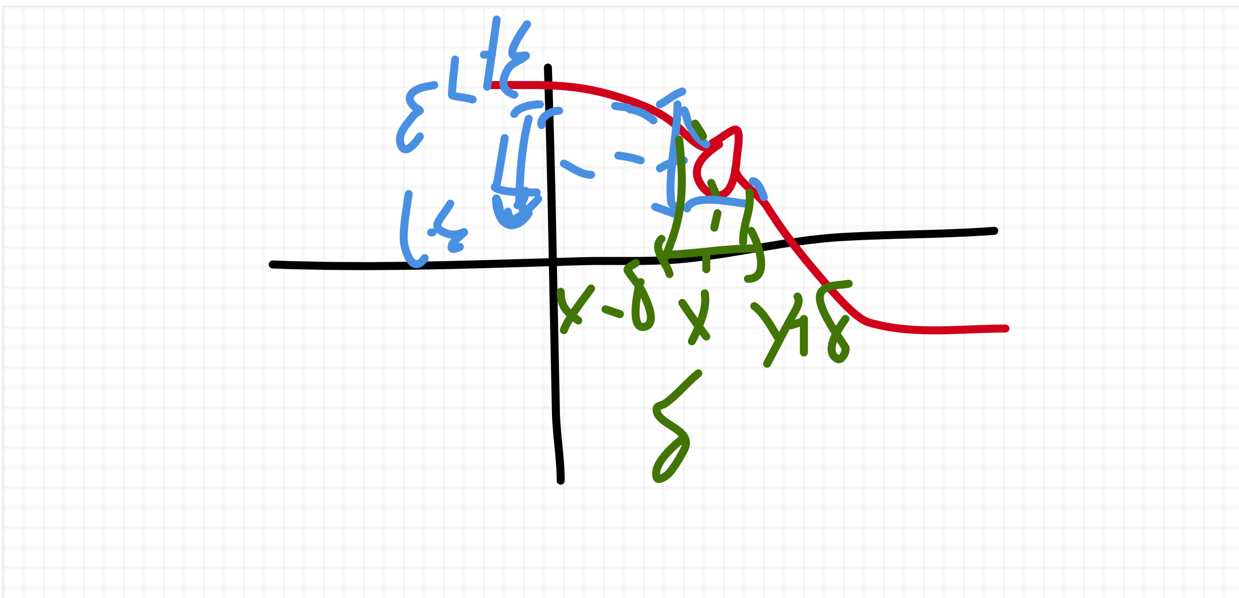
Precise definition of a limit:

Main idea: "the δ and ϵ kinda thing. Using the absolute value of the distance to figure out the existence of the limit" (paraphrasing)

f is a function is defined on an open interval I

Definition. We say that the limit of $f(x)$ as x approaches a is L (and that limit exists if such an L exists) if for every $\epsilon > 0$ there exists $\delta > 0$ such that

$$0 < |x - a| < \delta \Longrightarrow |f(x) - L| < \epsilon$$



WW6Sec2.4 Problem 1.

$$0 < |x - 9| < \delta \quad y = |\sqrt{x} - 3| < .2$$

Find the maximum δ that works

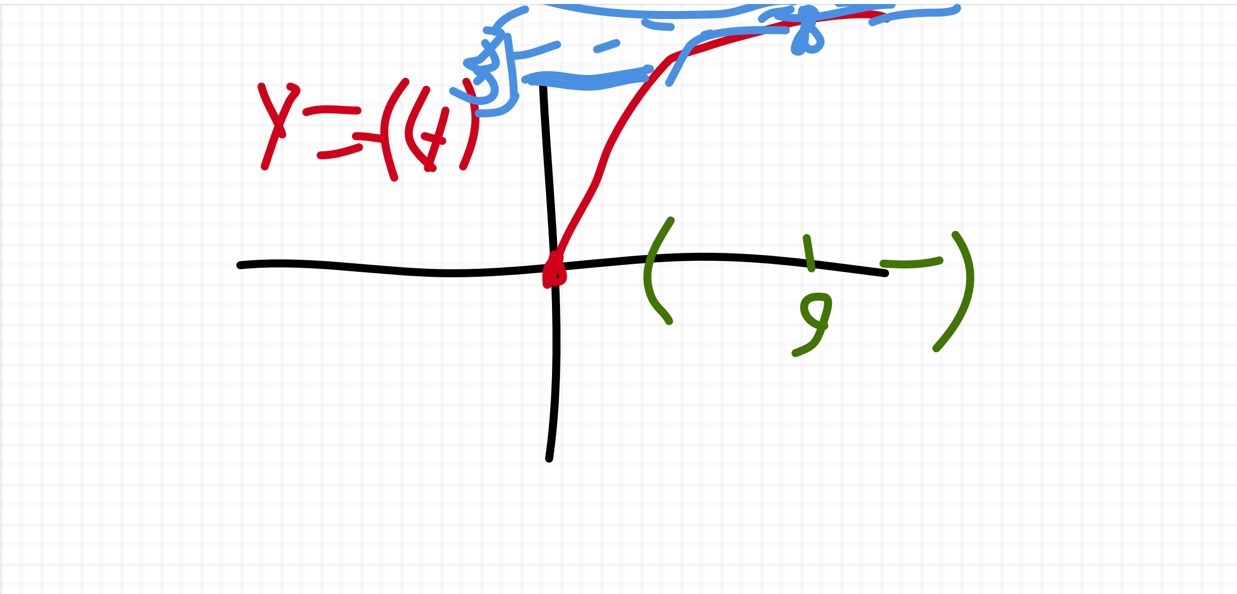
$$f(x) = \sqrt{x}, \quad f(x)^2 = x$$

$$f\left(x^-\right) = 2.8$$
$$f\left(x^+\right) = 3.2$$

$$x^- = f\left(x^+\right)^2 = 3.2^2 = 10.24$$
$$x^- = f\left(x^-\right)^2 = 2.8^2 = 7.84$$
$$9 - 7.84 = 1.16$$
$$10.24 - 9 = 1.24$$

For the δ that works, we want to take the smaller value, so $\delta = 1.16$

NOTE: the precise definition of a limit it two-sided and we need to check both sides there actually is a precise definition of one sided limits, and the two-sided precise definition is equivalent to both one-sided definitions being satisfied (as it is with the intuitive definition)



Definition. A function is continuous if

$$\lim_{x \to a} f(x) = f(a).$$

Explicit examples of continuous function:

$f(x) = x$

Polynomial functions continuous on $\mathbb{R} = (-\infty, \infty)$

Rational functions $\dfrac{f(x)}{g(x)}$ ($f(x), g(x)$ polynomials) are continuous wherever they're defined (i.e. where $g(x) \neq 0$)

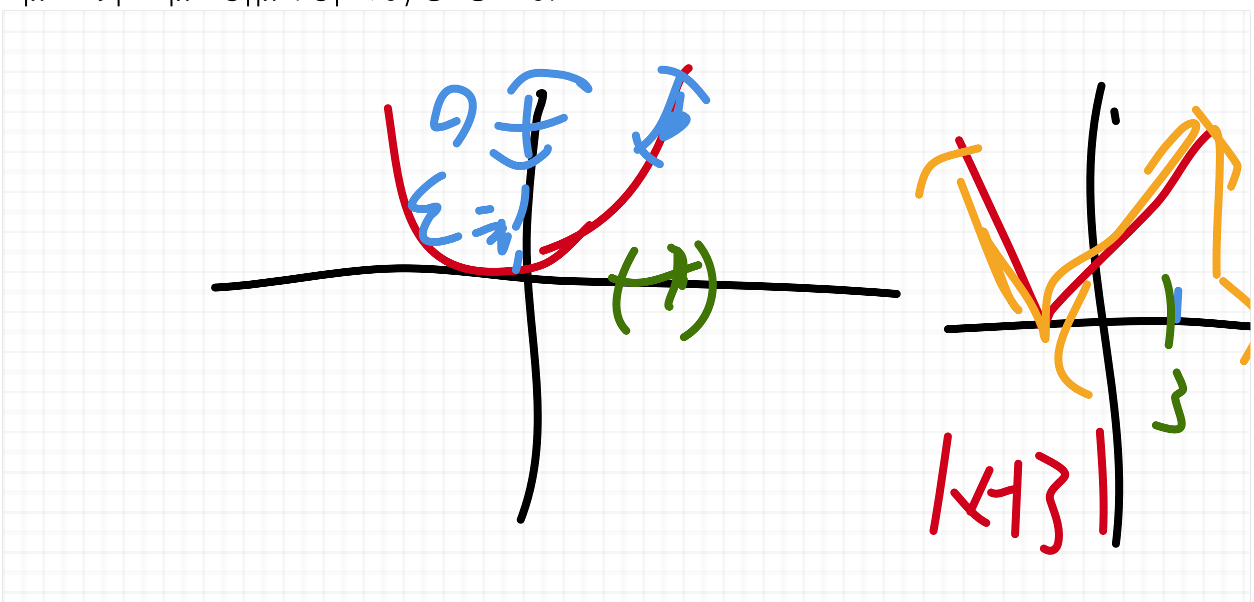
root functions, exponenential functions, logorithmic functions.

Example 4 Section 2.4.

Given $\epsilon > 0$, we want δ such that $|x-3| < \delta \implies |x^2 - 9| < \epsilon$

As we zoom in to an arbitrary interval containing 3, we can bound that interval by some constant C such that $|x+3| < C$. Now we can set $\delta = \epsilon / C$. Now $|x-3| < \delta = \epsilon / C$

$$|x^2 - 9| = |x-3||x+3| < \epsilon / C \cdot C = \epsilon.$$



2.3-2.5 Homework Questions

<u>WW7Sec2.5 Problem 2.</u>

$ct + 7 = ct^2 - 7$ plug in the point $t = 3$ (because we want the value to agree)

$c(3) + 7 = c(9) - 7$
$3c + 7 = 9c - 7$
$14 = 6c$
$14/6 = c$

2.6-2.8 Expostion

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2.6-2.8 Homework Questions

2/8 office hours

WW9Sec2.7 Problem 2.

$$\lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h}$$

$$\begin{aligned}
\lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h} &= \lim_{h \to 0} \frac{(2+h)(2+h)^2 - 2^3}{h} = \lim_{h \to 0} \frac{(2+h)\left(h^2 + 4h + 4\right) - 2^3}{h} \\
&= \lim_{h \to 0} \frac{2\left(h^2 + 4h + 4\right) + h\left(h^2 + 4h + 4\right) - 2^3}{h} \\
&= \lim_{h \to 0} \frac{(0+1)h^3 + (2+4)h^2 + (8+4)h + (8+0) - 2^3}{h} \\
&= \lim_{h \to 0} \frac{h^3 + 6h^2 + 12h + 8 - 8}{h} \\
&= \lim_{h \to 0} \frac{h^3}{h} + \frac{6h^2}{h} + \frac{12h}{h} + \frac{0}{h} \\
&= \lim_{h \to 0} \left(h^2 + 6h + 12\right) \\
&=
\end{aligned}$$

Definition of a derivative in terms of a function $f(x)$ is defined to be the following limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative of $f(x) = x^3$ at 2 is the following limit (using the definition above):

$$\lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h}$$

WW9Sec2.7 Problem .

$f(x) = 2x^2 + 7x + 3$ at $(4, 63)$

$m(x) = f(x) + g(x)$
$m'(x) = f'(x) + g'(x)$

$$m'(x) = \lim_{h \to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h} = \lim_{h \to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h}$$
$$= \lim_{n \to 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} = \lim_{n \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{n \to 0} \frac{g(x+h) - g(x)}{h}$$
$$= f'(x) + g'(x)$$

$f'(x) = \left(2x^2\right)' + (7x)' + (3)'$ at $(4, 63)$

$$\left(2x^2\right)' = 2\left(x^{2\prime}\right) = \lim_{h \to 0} \frac{(4+h)^2 - 4^2}{h} = \lim_{h \to 0} \frac{h^2 + 2 \cdot 8h^1 + 16 - 16}{h}$$
$$= \lim_{h \to 0} \frac{h^2}{h} + \frac{16h}{h}$$
$$= \lim_{n \to 0} h + 16$$
$$= 16$$

$(7x)' = 7$

$(3)' = 0$

$f'(x) = 16 + 7 + 0 = 7$

To find the equation of the tangent line $y = mx + b$, we know the slope is $m = f'(x) = 23$, we want to find b, which we can do by plugging in $y = 63$ and solving for b as follows:

$63 = 23(4) + b$
$63 = 92 + b$
$-92 \quad -92$
$-29 = b$

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