

# Continuous Consistency in the Coordination of Airborne and Ground-Based Agents

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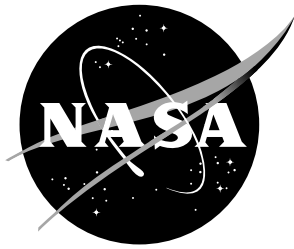
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## Abstract

The System Wide Safety (SWS) program has been investigating how crewed and uncrewed aircraft can safely operate in shared airspace, taking disaster response scenarios as a motivating use case. Enforcing safety requirements for distributed agents requires coordination by passing messages over a communication network. However, the operating environment will not admit reliable high-bandwidth communication between all agents, introducing theoretical and practical obstructions to global consistency that make it more difficult to maintain safety-related invariants. This self-contained memo discusses some of the distributed systems challenges involved in system-wide safety, focusing on the practical shortcomings of both strong and weak consistency models for shared memory. Then we survey two *continuous* consistency models that come from different parts of the literature. Unlike weak consistency models, continuous consistency models provides hard upper bounds on the “amount” of inconsistency observable by clients. Unlike strong consistency, these models are flexible enough to accomodate real-world conditions, such as by providing liveness during brief network partitions or tolerating disagreements between sensors in a sensor network. We conclude that continuous consistency models are appropriate for analyzing safety-critical systems that operate without strong guarantees about network performance.

# Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	Layout of this document . . . . .	4
<b>2</b>	<b>Distributed systems and consistency models</b>	<b>5</b>
2.1	System model . . . . .	7
2.2	Linearizability and the CAP theorem . . . . .	11
2.3	Sequential consistency . . . . .	16
2.4	Causal and FIFO (PRAM) consistency . . . . .	19
<b>3</b>	<b>Desiderata</b>	<b>21</b>
<b>4</b>	<b>Continuous consistency and the conit framework</b>	<b>22</b>
4.1	TACT system model . . . . .	23
4.2	Measuring consistency on conits . . . . .	24
4.3	Enforcing inconsistency bounds . . . . .	25
4.4	Future work . . . . .	25
<b>5</b>	<b>Sheaf theoretic data fusion</b>	<b>25</b>
5.1	Data fusion . . . . .	25
5.2	Presheaves . . . . .	26
5.3	Sheaves and assignments . . . . .	26
5.4	The consistency radius . . . . .	26
<b>6</b>	<b>Conclusion</b>	<b>26</b>
	<b>Bibliography</b>	<b>26</b>

# 1 Introduction

Civil aviation has traditionally focused primarily on the efficient and safe transportation of people and goods via the airspace. Owing to the application of sound engineering practices and conservative operating procedures, flying is today the safest mode of transport. The desire not to compromise this safety makes it challenging to introduce uncrewed vehicles into the airspace. To that end, the NASA Aeronautics’ Airspace Operations and Safety Program (AOSP) System Wide Safety (SWS) project has been investigating how crewed and uncrewed aircraft may safely operate in shared airspace. Wildfire suppression and hurricane relief are being studied as motivating use cases, in part because the rules for operating in the US national airspace are typically relaxed during natural disasters and relief efforts.

Disaster response scenarios present unique challenges for safe operations. Of fundamental importance is the need for system-wide coordination in spite of an unpredictable communications environment. Designing systems that are resilient to these sorts of environments is a challenge for distributed computing, a subdiscipline of computer science. This purpose of this memorandum is to enumerate some of the considerations involved in coordinating air- and ground-based elements from a distributed computing perspective, identifying challenges, potential requirements, and frameworks that suggest possible solutions.

Traditionally, civil aviation has employed simple communication patterns between airborne and ground-based agents and among aircraft. For instance, aircraft equipped with Automatic Dependent Surveillance-Broadcast (ADS-B) monitor their location using GPS and periodically broadcast this information to air traffic controllers and nearby aircraft. The use cases under consideration demand more sophisticated coordination schemes between airborne and ground-based elements to collectively accomplish goals such as navigating safely in close proximity, delivering resources to remote locations, and suppressing fires.

Unfortunately, the operating environment cannot generally be expected to provide reliable, high-bandwidth internet connections that would allow any group of system nodes to exchange lots of information quickly. For instance, obstructions like distance, terrain, smoke, and weather mean we should expect network packets to be dropped or delayed in unpredictable ways. We also expect the network characteristics to vary between deployments and to evolve dynamically in time, with connections varying in strength as agents move around the environment. These factors make network performance difficult to predict and control.

Weak guarantees about network performance make it difficult to coordinate distributed agents and offer strong safety guarantees. So-called strong consistency models, the subject of most of Section 2, can enforce strong safety guarantees, but they are unworkably brittle—they can only be provided under ideal network conditions unless severe performance penalties are incurred. An exemplary result is Brewer’s CAP theorem (Theorem 2.1), which implies that neither *atomic* nor *sequential* consistency (C) can be guaranteed by an eventually-available (A) system in the presense of network partitions (P). Partitions, or transient drops in network connectivity, are virtually guaranteed to occur in the environments under consideration. The CAP theorem therefore implies that we cannot use strong consistency to

enforce safety without sacrificing system performance, meaning the system’s ability to respond to clients’ requests in a reasonable amount of time.

On the other hand, weak consistency models such as *causal* and *eventual* consistency can be provided by real-world systems. However, these models are too weak to ensure strong safety guarantees. In particular, they do not bound the overall divergence between two replicas of a shared data structure, so they provide few assurances about the mutual consistency of the data observed by different clients.

At face value, the CAP theorem would seem to imply that either consistency (hence safety) or availability must be sacrificed by distributed systems deployed in the field. A more nuanced view is that the theorem observes a fundamental *trade-off* between consistency and availability; this tradeoff is amplified by suboptimal network performance. While the CAP theorem rules out highly idealized systems that maintain strong consistency and high availability except under perfect network conditions, it does not inherently rule out systems that maintain adequate levels of both consistency and performance under realistic conditions.

What does it mean to have an “amount” of consistency? The idea is made precise by continuous consistency models, or formal measures of (in)consistency as a continuous value rather than a Boolean condition. This memo describes two continuous consistency models in the literature. Both define consistency as an upper bound on the amount of *inconsistency* between objects, though the models are concerned with different kinds of objects. One model, the theory of *conits*, comes from research into distributed shared memory. The other, *sheaf-theoretic data fusion*, comes from research in data integration and sensor networks. Both define consistency as something which, in principle, varies smoothly. At one extreme, both models describe a form of “perfect” consistency that cannot usually be expected in real applications. At the other extreme, the models enforce no guarantees. In the middle, they place upper bounds on the divergence between related data objects.

Broadly speaking, it stands to reason that quantitative measurements of consistency should in turn offer quantitative measurements of safety. One potential application of having a continuous consistency model is therefore to compute the amount of safety provided by a deployed system and enforce this value to within tolerable limits. As we see in Section 2, the CAP theorem implies that network performance can become so poor that a system cannot provide tolerable safety levels while maintaining availability. When adequate safety margins cannot be enforced, authorities can decide to take fewer risks. What is centrally important is to know *how much* safety one has, and that is (hopefully) what is provided by the models described in this document.

## 1.1 Layout of this document

This memo focuses on two (prima facie unrelated) notions of continuous consistency developed in the literature. This document aims to be reasonably self-contained and readable to a broad technical audience. It is laid out as follows.

Section 2 provides a high-level introduction to distributed systems and memory consistency models. We define two strong models, atomic and sequential consistency, both of which provide highly desirable safety guarantees; we contrast these with the



weaker guarantees implied by the causal consistency model. Then we turn our attention to the CAP theorem (Brewer 2000) (Gilbert and Lynch 2002), which captures a fundamental consistency/availability tradeoff in the presense of network partitions. We observe that the theorem effectively prohibits both forms of strong consistency in our intended use case. This raises the question of how, if at all, one can rigorously enforce safety properties without compromising system performance beyond acceptable levels.

Informed by the previous discussion, Section 3 offers a list of three desiderata of distributed applications in the contexts under consideration. These have been selected as especially desirable and relevant for our use cases, and they provide a basis for assessing the applicability of frameworks and techniques to overcome the apparent limitations imposed by the CAP theorem.

Section 4 explains how the *conit* framework (Yu and Vahdat 2002) quantifies the C/A tradeoff with respect to three metrics: numerical error, order commit error, and real-time error. We summarize how to enforce consistency up to some real-valued  $\epsilon \geq 0$  for each of these metrics separately. The *conit* framework allows applications to specify their own consistency semantics with respect to these measurements, and to mark updates as having a greater or lesser impact on consistency. Each replica, indeed each system request, can set its own bounds on observable inconsistency, while a general-purpose middleware library transparently enforces these requirements. At the extreme, the framework can enforce either atomic or sequential consistency by setting  $\epsilon = 0$  for appropriately-defined *conits*. For  $\epsilon > 0$ , the framework offers neither CAP-consistency nor CAP-availability; in return, applications can provide limited amounts of availability, possibly during network partitions, while strictly bounding levels of inconsistency. One use case for this is to “smooth out” intermittent fluctuations in network performance, a desirable feature for safety-critical systems operating without strict assumptions about the network.

Section 5 is an introduction to applied sheaf theory, which provides a highly general framework for measuring the mutual consistency of “overlapping” observations. Intuitively, these are observations which we expect to be correlated if not equal, such as the data generated by nearby sensors in a sensor network. We discuss an simulated example, due to (Robinson 2017), where sheaves are used to integrate heterogeneous sensor data, thereby improving an estimated location for a crashed aircraft. Unlike many introductions to the subject, we emphasize sheaves as “topologically-flavored” presheaves, viewing presheaves as highly generalized transition systems. Our expectation is that this approach makes the subject more accessible to computer scientists and serves to highlight themes common to Sections 4 and 5. It may even indicate the possibility of re-grounding *conit* theory in the principled mathematical framework of sheaf theory.

Section 6 concludes with suggestions for future work.

## 2 Distributed systems and consistency models

A distributed system, broadly construed, is a collection of independent entities that cooperate to solve a problem that cannot be individually solved (Kshemkalyani and

Singhal 2008). In the context of computing, (Singhal and Shivaratri 1994) offer the following definition.

“A collection of computers that do not share common memory or a common physical clock, that communicate by message passing over a communication network, and where each computer has its own memory and runs its own operating system.”

A fundamental goal for distributed computing systems is to “[appear] to the users of the system as a single coherent computer” (Tanenbaum and Steen 2007). This can be understood as the requirement that all nodes present a *mutually-consistent* view of the world, e.g. the state of a globally-maintained database, to system clients.

When *strong consistency* is enforced, clients cannot tell whether they are all interacting with a single central computer, or a complex system of independent computers acting in tandem. This abstraction shields clients and application developers from complexity and makes it simpler to reason about system behavior. For example, if a client modifies a data structure by submitting a write request to one system node, a strongly consistent system must ensure that other nodes reflect the update as well. If some nodes were to reflect the update while others did not, the abstraction of a shared world would be broken, the clients’ mental model of the system can become invalid, and safety requirements may be violated. A few examples of the perils of inconsistency:

- A bank client would be unhappy if deposits that appear in their account online are not reflected when they check their balance at an ATM, or if they disappear after refreshing the webpage.
- Air traffic controllers cannot safely coordinate the movement of aircraft if they are presented with conflicting or out-of-date information about their positions and velocities.
- Potentially dangerous misunderstandings can arise in a group messaging application if messages appear in different orders to different clients.
- Resource-tracking systems cannot be trusted if they do not reflect the true information about the availability and location of resources.

What exactly constitutes consistency? There are different consistency models, and the most appropriate model for an application depends on the semantics it expects, which must be weighed against other requirements. All other things being equal, one wants to have as much consistency as possible. Below, we shall see that naïve notions of system coherence are brittle in the sense that they generally cannot be guaranteed for theoretical and practical reasons; that is, unless one is willing to pay with significant performance penalties, including applications that fail to respond to users under some conditions. Therefore, real-world applications must tolerate weaker notions of consistency. This makes applications more difficult to reason about, as their behavior may depend on uncontrollable factors in the

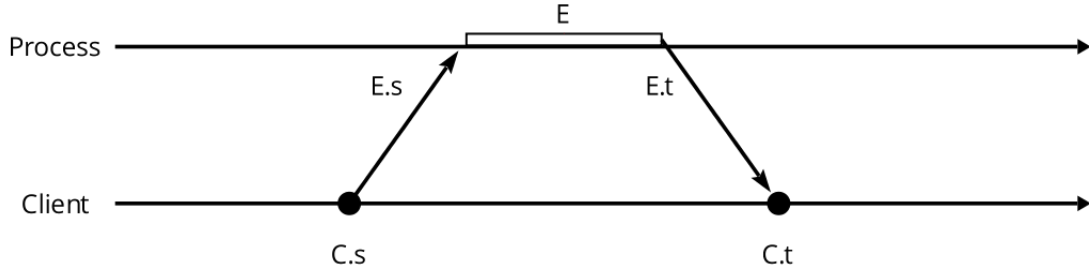


Figure 1: Lifetime of a client request

environment. As fewer behaviors can be ruled out, it becomes more difficult to ensure the system maintains safety-related invariants.

The difficulty of enforcing strong consistency is that it requires system nodes to coordinate by exchanging information. Absence of a common memory implies that inter-process communication takes place over the network (whereas processes on the same machine have the option to share data by writing it to a common memory location). A foundational assumption of distributed systems, and an especially apt one in the context of disaster response scenarios, is that the network is less than perfectly reliable. Message delivery is not instantaneous, and delivery times may be unpredictable. The network may be allowed to silently drop network packets, reorder them, or deliver them several times. In the case of *Byzantine faults* (CITE) the network may even act like a malicious adversary (though we will not consider this scenario in this document). Imperfections in communications represent obstructions to system consistency and make it challenging to enforce safety requirements.

We shall now make this discussion more precise by introducing formal definitions.

## 2.1 System model

A distributed system consists of a set  $\mathcal{P} = \{P_i\}_{i \in I}$  of *processes*, which we think of these as executing on independent, often geographically dispersed computers that communicate by message-passing.

Processes take requests from clients, such as to read or write a value in a database. The lifecycle of a typical request is depicted in Figure 1. At some physical time (i.e. wall-clock time)  $C.s \in \mathbb{R}$  (client start time), a client sends a message to a process. At time  $E.s$ , which we'll call the *start time* of the event, the message is accepted by the process. The request is processed until some strictly greater time  $E.t > E.s$  when a response is sent back to the client. The value  $E.t - E.s$  is the *duration* of the event.

While handling the request, the process may coordinate with other processes in the background by sending and receiving more messages. For example, the process may propagate the client's request to other processes, retrieve up-to-date values from other processes to give to the client, or delay handling the client's request in order to handle other requests.

To discuss consistency models, we shall be less interested in the details of

message-passing and more interested just in the responses observed by clients. We shall consider the full set of events across a distributed system, such as shown in Figure 2. This is called an *execution*. Consistency models constrain the set of allowable return values in response to clients' requests.

As is often the case, we shall assume that requests handled by a single process do not overlap in time. This can be enforced with local serialization methods such as two-phase locking (CITE) that can be used to isolate concurrent transactions from each other, providing the abstraction of a system that handles requests one at a time. On the other hand, any two processes may handle two events at the same physical time, so that there is no obvious total order of events across the system. Instead, one has a partial order called *external order*. Intuitively, it is the partial order of events that would be witnessed by an observer recording the real time at which systems begin and finish responding to requests.

**Definition 2.1.** Let  $E$  be an execution. Request  $E1$  *externally precedes* request  $E2$  if  $E1.t < E2.s$ . That is, if the first request terminates before the second request is accepted. This induces an irreflexive partial order called *external order*.

Recall that an irreflexive partial order is a binary relation  $<$  such that  $A \not< A$ ,  $A < B \implies B \not< A$ , and  $A < B, B < C \implies A < C$ .

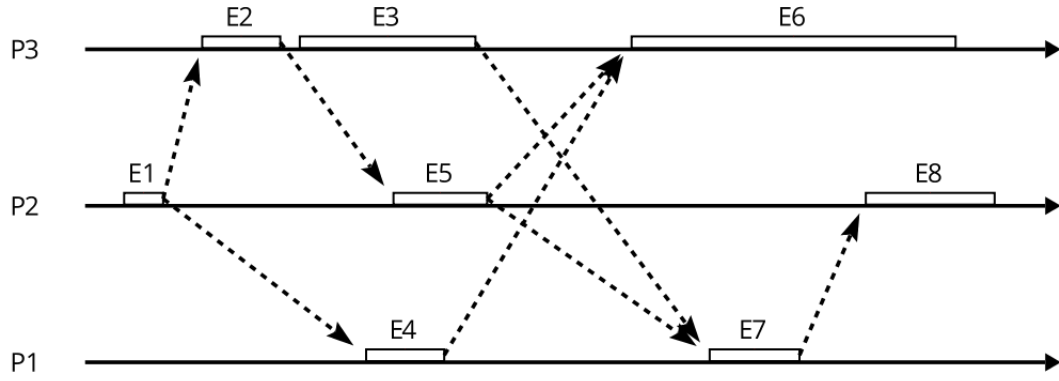
Because we assume processes handle events one-at-a-time, the events handled at any one process are totally ordered by external order—one event cannot start before another has finished. If  $E1$  and  $E2$  are events at *different* processes, they need not be comparable by external order, i.e. neither  $E1.t < E2.s$  nor  $E2.t < E1.s$ , making them *physically concurrent*.

**Definition 2.2.** If two events overlap in physical time (equivalently, if they are not comparable by external order), we call the events *physically concurrent* and write  $E1||E2$ .

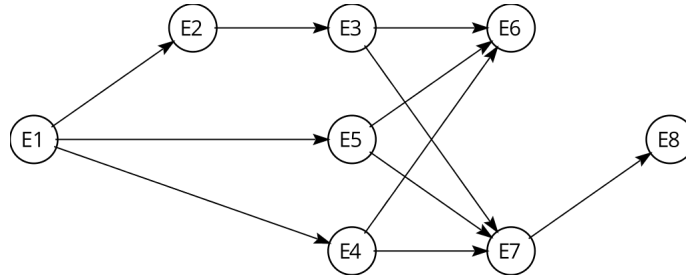
Physical concurrency is a reflexive and symmetric—but usually not transitive—binary relation. Such structures are often called *compatibility relations*. The general intuition is that anything is compatible with itself (reflexivity), and the compatibility of two objects does not depend on their order (symmetry). But if  $A$  and  $B$  are compatible with  $C$ , it need not be the case that  $A$  and  $B$  are compatible with each other.

Figure 2a shows the external order relation for an execution. To save space we elide arrows between two events of the same process and arrows that can be inferred by transitivity. This corresponds to the directed acyclic graph structure shown in 2b.

The reader may wonder if we can consider events to be totally ordered, say by pairing them with a timestamp that records their physical start time to resolve ties like  $x||y$ . This order is not generally useful for a couple reasons. First, we assume processes have only loosely synchronized clocks, so timestamps from two different processes may not be comparable. Additionally, even systems that enforce linearizable consistency (c.f. Section 2.2) do not necessarily handle requests in order of their physical start times.



(a) Depiction of external order between concurrent events across three processes. Intra-process and transitive edges are not depicted.



(b) The directed acyclic graph (DAG) induced by external order.

Figure 2: External order

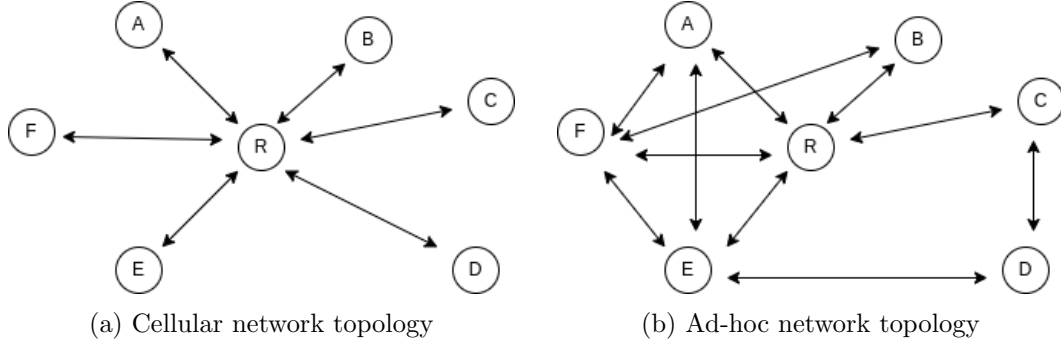


Figure 3: Network topology models for geodistributed agents. Edges represent communication links (bidirectional for simplicity).

### 2.1.1 Physical communications

The details of the physical communication between processes is outside the scope of this memo. We make just a few high-level observations about the possibilities, as the details of the network layer are likely to have an impact on distributed applications, such as the shared memory abstraction we discuss below and in Section 4. For such applications, it may be important to optimize for the sorts of usage patterns encountered in real scenarios, which are affected by (among other things) the low-level details of the network.

The *cellular* model (Figure 3a) assumes nodes are within range of a powerful, centralized transmission station that performs routing functions. Message passing takes place by transmitting to the base station (labeled *R*), which routes the message to its destination. Such a model could be supported by the ad-hoc deployment of portable cellphone towers transported into the field, for instance.

The *ad-hoc* model (Figure 3b) assumes nodes communicate by passing messages directly to each other. This requires nodes to maintain information about things like routing and the approximate location of other nodes in the system, increasing complexity and introducing a possible source of inconsistency. However, it may be more workable given (i) the geographic mobility of agents in our scenarios (ii) difficult-to-access locations that prohibit setting up communication towers (iii) the inherent need for system flexibility during disaster scenarios.

One can also imagine hybrid models, such as an ad-hoc arrangement of localized cells. In general, one expects more centralized topologies to be simpler for application developers to reason about, but to require more physical infrastructure and support. On the other hand, the ad-hoc model is more fault resistant, but more complicated to implement and potentially offering fewer assurances about performance. In either case, higher-level applications such as shared memory abstractions should be tuned for the networking environment. It would be even better if this tuning can take place dynamically, with applications reconfiguring manually or automatically to the particulars of the operating environment. This requires examining the relationship between the application and networking layers, rather than treating them as separate blackboxes.

An interesting possibility is for the *network* to automatically configure itself to the quality-of-service needs of the application. For example, a client that receives a lot of requests may be marked as a preferred client and given higher-priority access to the network. If UAV vehicles can be used to route messages by acting as mobile transmission base stations, one can imagine selecting a flight pattern based on networking needs. For example, if the communication between two firefighting teams is obstructed by a geographical feature, a UAV could be dispatched to provide overhead communication support. Such an arrangement could greatly blur the line between the networking and application layers.

## 2.2 Linearizability and the CAP theorem

A fundamental distributed application is the *shared distributed memory* abstraction. We shall assume that all processes maintain a local replica of a globally shared data object, as replication increases system fault tolerance. For simplicity, we shall discuss the data store as a simple key-value store, but it could be something else like a database, filesystem, persistent object, etc.

We assume clients submit two types of requests to processes. A *read request* is a request to lookup the current value of a variable. A request to read the variable  $x$  that returns value  $a$  is written  $R(x, a)$ . A *write request* is a request to set the current value of a variable. Notation  $W(x, a)$  represents writing value  $a$  to  $x$ . We assume all processes provide access to the same set of shared variables.

A *memory consistency model* formally constrains the allowable system responses during executions. *Strong* consistency models are generally understood as ones provide the illusion that all clients are accessing just one globally shared replica. As we will see, this still leaves room for different possible behaviors (i.e. allows non-determinism in the execution of a distributed application), but the allowable behavior is tightly constrained.

*Linearizability* (Herlihy and Wing 1990) is essentially the strongest common consistency model. It is known variously as atomic consistency, strict consistency, and sometimes external consistency. In the context of database transactions (which come with other guarantees, like isolation, that are more specific to databases), the analogous condition is called strict serializability. A linearizable execution is defined by three features:

- All processes act like they agree on a single, global total order defined across all accesses.
- This sequential order is consistent with the actual external order.
- Responses are semantically correct, meaning a read request  $R(x, a)$  returns the value of the most recent write request  $W(x, a)$  to  $x$ .

We can also phrase this in terms of *linearizations*.

**Definition 2.3.** A *linearization point*  $t \in \mathbb{R} \in [E.s, E.t]$  for an event  $E$  is a time between the event’s start and termination. An execution is *linearizable* if and only if there is a choice of linearization point for each access, which induces a total order called a *linearization*, such that  $E$  is equivalent to the serial execution of events when totally ordered by their linearization points.

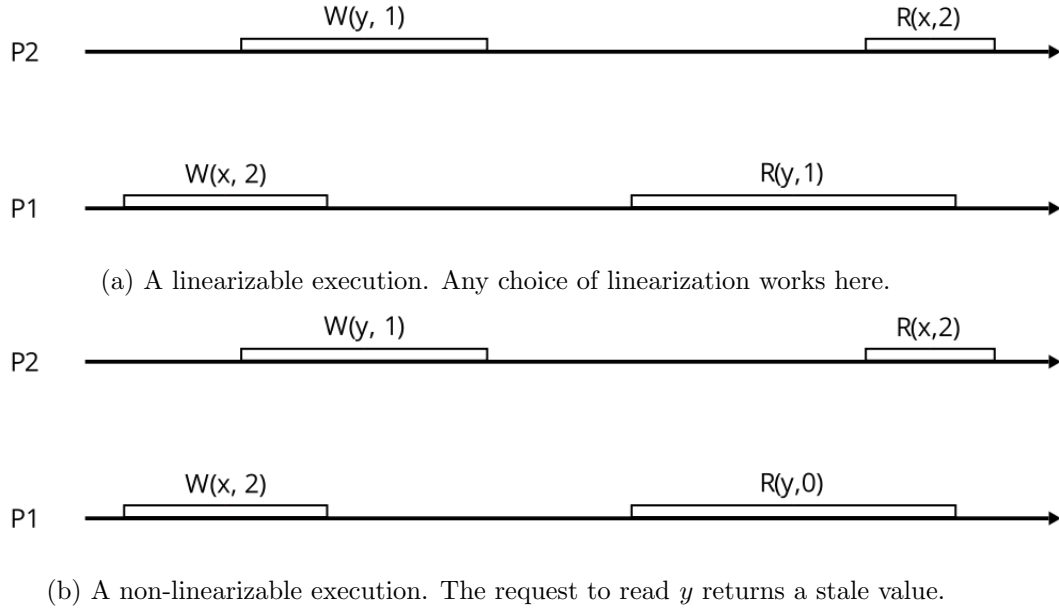


Figure 4: A linearizable and non-linearizable execution.

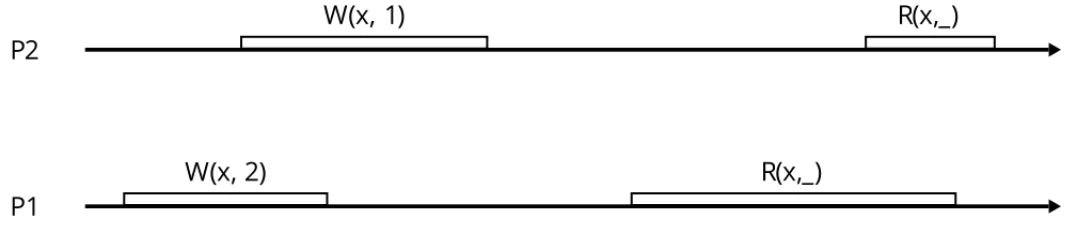
Intuitively, it should appear to an external observer that each access instantaneously took effect at some point between its start and end time. It is assumed no distinct access can have the same linearization point, so that we get a total order. We say an entire system is linearizable when all possible executions of the system are linearizable.

Figure 4a shows a prototypical example of a linearizable execution. We assume that all memory locations are initialized to 0 at the system start time. Figure 4b shows an execution that is not linearizable because the read access on  $y$  on  $P1$  returns stale data instead of reflecting the write access to  $y$  on  $P2$ .

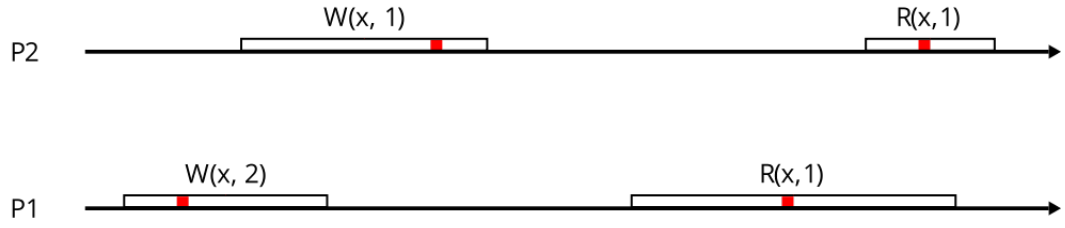
Linearization points are demonstrated in Figure 5. The figure shows different linearizable behaviors in response to the same underlying set of accesses. This demonstrates that linearizability still leaves some room for non-determinism in the execution of distributed applications. In this example, the requests must both return 1 or 2. The constraint is that the values must agree—linearizability forbids the situation in which one client reads 1 and another reads 2 (Figure 6).

**Enforcing linearizability** Linearizability can be enforced with a total order broadcast mechanism (CITE). Total order broadcast is a means for processes to come to a consensus about the order of a set of events. One can imagine that the total order broadcast API implements a routine that accepts a message and notifies all other processes of this message in such a way that all processes see all messages in the same order. To maintain linearizability, it suffices that each replica applies database actions in the order they are announced in the total order broadcast. A subtle point is that a process does not need to handle read requests originally sent to other clients, so these may be ignored. However, the originating replica must

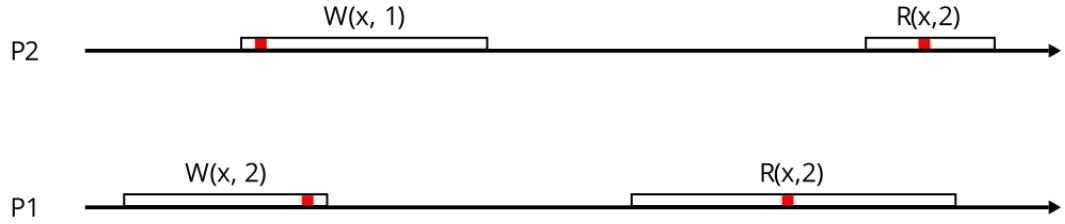




(a) An execution with read responses left unspecified.

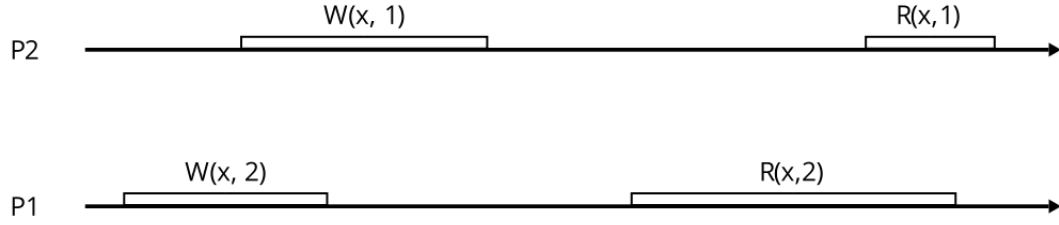


(b) A linearizable execution for which both reads return 1.

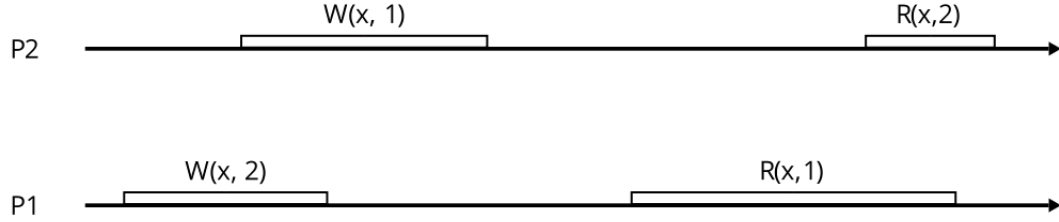


(c) A linearizable execution for which both reads return 2.

Figure 5: Two linearizable executions of the same underlying events that return different responses. Possible linearization points are shown in red.



(a) A nonlinearizable execution with the read access returning disagreeing values. We will see later (Figure 7) that this execution is still sequentially consistent.



(b) Another nonlinearizable execution with read access values swapped. This execution is not sequentially consistent.

Figure 6: Two non-linearizable executions of the same events shown in Figure 5.

not handle its own read requests until *after* they appear in the total order broadcast, rather than at the time they are first submitted to the total order broadcast mechanism.

### 2.2.1 The CAP Theorem

Real-world systems often fall short of behaving as a single perfectly coherent system. The root of this phenomenon is a deep and well-understood tradeoff between system coherence and performance. Enforcing consistency comes at the cost of additional communications, and communications impose overheads, often unpredictable ones.

Fox and Brewer (Fox and Brewer 1999) are credited with observing a particular tension between the three competing goals of consistency, availability, and partition-tolerance. This tradeoff was precisely stated and proved in 2002 by Gilbert and Lynch (Gilbert and Lynch 2002). The theorem is often somewhat misunderstood, as we discuss, so it is worth clarifying the terms used.

**Consistency** Gilbert and Lynch define a consistency system as one whose executions are always linearizable.

**Availability** A CAP-available system is one that will definitely respond to every client request at some point.

**Partition tolerance** A partition-tolerant system continues to function, and ensure whatever guarantees it is meant to provide, in the face of arbitrary partitions

in the network (i.e., an inability for some nodes to communicate with others). It is possible that a partition never recovers, say if a critical communications cable is permanently severed.

A partition-tolerant CAP-available system cannot indefinitely suspend handling a request to wait for network activity like receiving a message. In the event of a partition that never recovers, this would mean the process could wait indefinitely for the partition to heal, violating availability. On the other hand, a CAP-consistent system is not allowed to return anything but the most up-to-date value in response to client requests. Keep in mind that any (other) process may be the originating replica for an update. Some reflection shows that the full set of requirements is unattainable—a partition tolerant system simply cannot enforce both consistency and availability.

**Theorem 2.1** (The CAP Theorem). *In the presense of indefinite network partitions, a distributed system cannot guarantee both linearizability and eventual-availability.*

*Proof.* Technically, the proof is almost trivial. We give only the informal sketch here, leaving the interested reader to consult the more formal analysis by Gilbert and Lynch. The key technical assumption is that a processes' behavior can only be influenced by the messages it actually receives—it cannot be affected by messages that are sent to it but never delivered.

In Figure 4a, suppose the two processes are on opposite sides of a network partition, so that no information can be exchanged between them (even indirectly through a third party). If we just consider the execution of  $P_2$  by itself, without  $P_1$ , linearizability would require it to read the value 2 for  $y$ . If we do consider  $P_1$ , linearizability requires that the read access to  $y$  must return 1. But if  $P_2$  cannot send messages to  $P_1$ , then  $P_2$ 's behavior cannot be influenced by the write access to  $y$ , so it would still have to return 2, violating consistency. Alternatively, it could delay returning any result until it is able to exchange messages with  $P_1$ . But if the partition never recovers,  $P_1$  will wait forever, violating availability.  $\square$

While the proof of the CAP theorem is simple, its interpretation is subtle and has been the subject of much discussion in the years since (Brewer 2012). It is sometimes assumed that the CAP theorem claims that a distributed system can only offer two of the properties C, A, and P. In fact, the theorem constrains, but does not prohibit the existence of, applications that apply some relaxed amount of all three features. The CAP theorem only rules out their combination when all three are interpreted in a highly idealized sense.

In practice, applications can tolerate much weaker levels of consistency than linearizability. Furthermore, network partitions are usually not as dramatic as an indefinite communications blackout. Real conditions in our context are likely to be chaotic, featuring many smaller disruptions and delays and sometimes larger ones. Communications between different clients may be affected differently, with nearby agents generally likely to have better communication channels between them than agents that are far apart. Finally, CAP-availability is a suprisingly weak condition. Generally one cares about the actual time it takes to handle user requests, but the CAP theorem exposes difficulties just ensuring the system handles requests at all.

Altogether, the extremes of C, A, and P in the CAP theorem are not the appropriate conditions to apply to many, perhaps most, real-world applications.

Broadening our perspective, the tension between consistency and availability is a prototypical example of a deeper tension in computing: that between safety and liveness properties (Davidson, Garcia-Molina, and Skeen 1985; Gilbert and Lynch 2012). These terms can be understood as follows.

- **Safety** properties ensure that a system avoids doing something “bad” like violating a consistency invariant. Taken to the extreme, one way to ensure safety is to do nothing. For instance, we could enforce safety by never responding to read requests in order to avoid offering information that is inconsistent with that of other nodes.
- **Liveness** properties ensure that a system will eventually do something “good”, like respond to a client. Taken to the extreme, one very lively behavior would be to immediately respond to user requests, without taking any steps to make sure this response is consistent with that of other nodes.

Note that in our use cases, an unresponsive system could arguably be “unsafe.” The distinction between the terms in this narrow context is that “safety” constrains a system’s allowable responses to clients, if one is even given, while liveness requires giving responses. The fact that both of these have implications for human safety is one more reason to prefer continuous consistency models over CAP-unavailable models like linearizability.

Because of the tension between them, building applications that provide both safety and liveness features is challenging. The fundamental tradeoff is that if we want to increase how quickly a system can respond to requests, eventually we must relax our constraints on what the system is allowed to return.

Next we consider a slightly more relaxed consistency model that admits a greater range of system behaviors while maintaining the total order guarantees of atomic consistency.

### 2.3 Sequential consistency

Enforcing atomic consistency means that an access  $E$  at process  $P_i$  cannot return to the client until every other process has been informed about  $E$ . For many applications this is an unacceptably high penalty. A weaker model that is still strong enough for most purposes is *sequential* consistency. This is an appropriate model if a form of strong consistency is required, but the system is agnostic about the precise physical time at which events start and finish, provided they occur in a globally agreed upon order.

A sequentially consistent system ensures that any execution is equivalent to some global serial execution, even if this serial order is not the one suggested by the real-time ordering of events. When real-time constraints are not important, this provides essentially the same benefits as linearizability. For example, it allows programmers to reason about concurrent executions of programs because the result

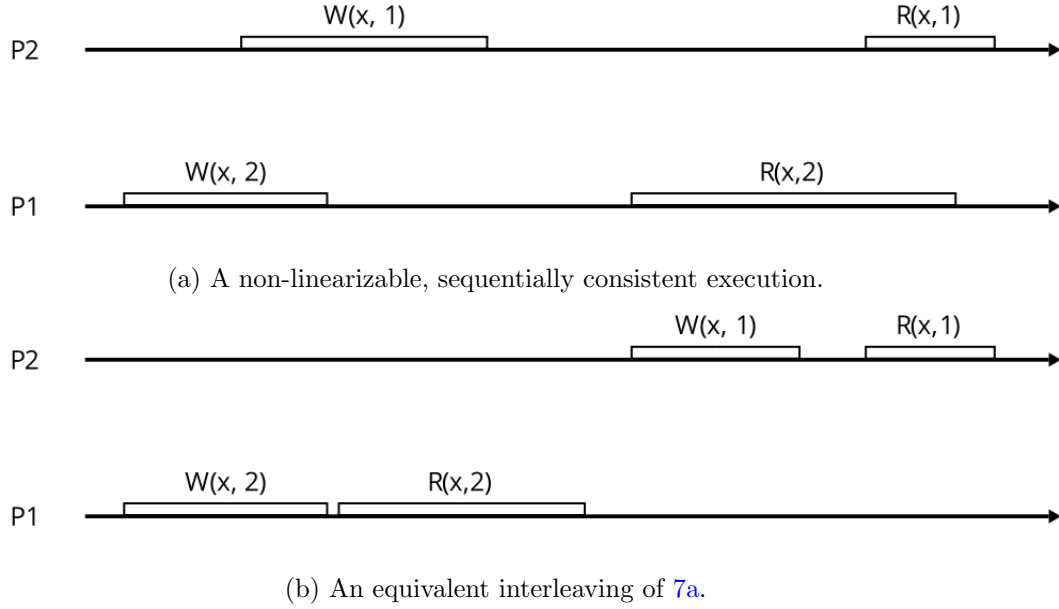


Figure 7: A sequentially consistent execution and a possible interleaving.

is always guaranteed to represent some possible interleaving of instructions, never allowing instructions from one program to execute out of order.

Processes in a sequentially consistent system are required to agree on a total order of events, presenting the illusion of a shared database from an application programmer's point of view. However, this order need not be given by external order. Instead, the only requirement is that sequential history must agree with process order, i.e. the events from each process must occur in the same order as they do in the process.

**Definition 2.4.** A *sequentially consistent* execution is characterized by three features:

- All processes act like they agree on a single, global total order defined across all accesses.
- This sequential order is consistent with the program order of each process.
- Responses are semantically correct, meaning reads return the most recent writes (as determined by the global order)

This is nearly the definition of linearizability, except that external order has been replaced with merely program order. We immediately get the following lemma.

**Lemma 2.2.** A *linearizable execution* is *sequentially consistent*.

*Proof.* This follows because process order is a subset of external order.  $\square$

Visually, sequential consistency allows reordering an execution by sliding events along each process' time axis like beads along a string. Two events from the same

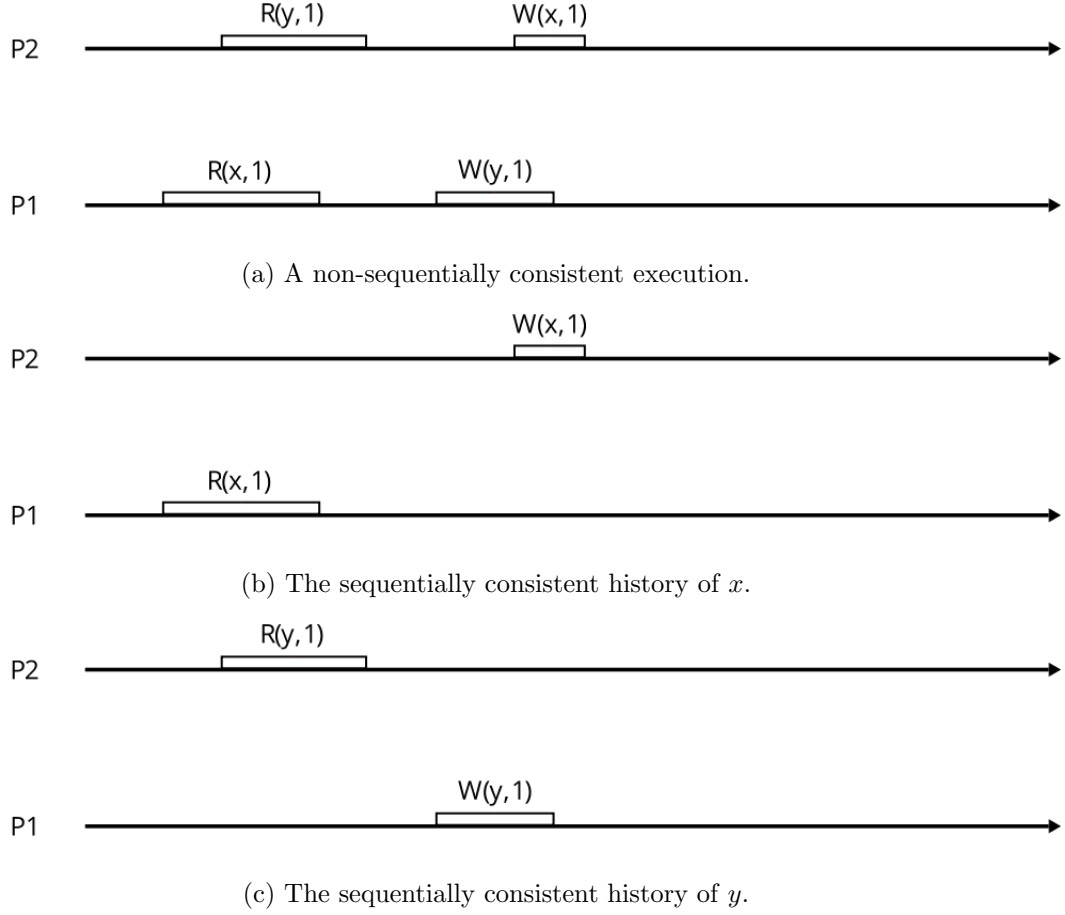


Figure 8: A non-sequentially consistent execution with sequentially-consistent executions at each variable.

process cannot pass over each other as this would violate program order, but events on different processes may be commuted past each other, violating external order. This sliding allows defining an arbitrary interleaving of events, a totally ordered execution with no events overlapping. From this perspective, while linearizability requires the existence of a linearization, sequential consistency requires the existence of an equivalent interleaving.

The converse of Lemma 2.2 does not hold. For example, Figure 7a was previously shown (Figure 6a) as a nonlinearizable execution. However, it is sequentially consistent, as evidenced by the interleaving in Figure 7b that slides the events  $W(x, 1)$  and  $R(x, 2)$  past each other.

**Enforcing sequential consistency** Notably, to enforce sequential consistency for the whole system, it is not enough to enforce it at the level of individual variables. Figure 8a shows a history that is not sequentially consistent. However, the histories of accesses to individual variables (Figures 8b and 8c) are sequentially consistent. This is a key difference from linearizability (CITE).

Like linearizability, sequential consistency can also be enforced with a total order broadcast mechanism. All write requests are first broadcast, and replicas only apply updates in the order they appear in the total order broadcast. The crucial difference from linearizability is that each process can handle requests immediately, returning its local replica value, instead of waiting for the broadcast mechanism to assign a global order to the read request.

**CAP unavailability** Sequential consistency is a relaxation of atomic consistency, but not by much. The model is still too strict to enforce under partition conditions.

**Lemma 2.3.** *An eventually-available system cannot provide sequential consistency in the presense of network partitions.*

*Proof.* The proof is an adaptation of Theorem 2.1. Suppose  $P_1$  and  $P_2$  form of CAP-available distributed system and consider the following execution:  $P_1$  reads  $x$ , then assigns  $y$  the value 1.  $P_2$  reads  $y$ , then assigns  $x$  the value 1. (Note that this is the sequence of requests shown in Figure 8a, but we make no assumptions about the values returned by the read requests). By availability, we know the requests will be handled (with responses sent back to clients) after a finite amount of time. Now suppose  $P_1$  and  $P_2$  are separated by a partition so they cannot read each other's writes during this process. For contradiction, suppose the execution is equivalent to a sequential order.

If  $W(y, 1)$  precedes  $R(y)$  in the sequential order, then  $R(y)$  would be constrained to return to 1. But  $P_2$  cannot pass information to  $P_1$ , so this is ruled out. To avoid this situation, suppose the sequential order places  $R(y)$  before  $W(y, 1)$ , in which case  $R(y)$  could correctly return the initial value of 0. However, by transitivity the  $R(x)$  event would occur after  $W(x, 1)$  event, so it would have to return 1. But there is no way to pass this information from  $P_1$  to  $P_2$ . Thus, any attempt to consistently order the requests would require commuting  $W(y, 1)$  with  $R(x)$  or  $W(x, 1)$  with  $R(y)$ , which would violate program order.  $\square$

As discussed in (Muñoz-Escóí et al. 2019), this stronger theorem was essentially proved by Birman and Friedman (Friedman and Birman 1996), before the CAP theorem.

## 2.4 Causal and FIFO (PRAM) consistency

Causal consistency is that each clients is consistent with a total order that contains the happened-before relation. It does not put a bound on divergence between replicas. Violations of causal consistency can present clients with deeply counterintuitive behavior.

- In a group messaging application, Alice posts a message and Bob replies. On Charlie's device, Bob's reply appears before Alice's original message.
- Alice sees a deposit for \$100 made to her bank account and, because of this, decides to withdraw \$50. When she refreshes the page, the deposit is gone and her account is overdrawn by 50. A little while later, she refreshes the page

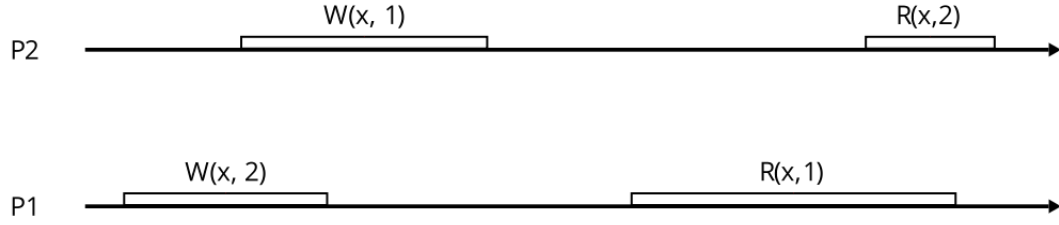


Figure 9: A causally consistent, non-sequentially-consistent execution

and the deposit reappears, but a penalty has been assessed for overdrawing her account.

In these scenarios, one agent takes an action *in response to* an event, but other processes observe these causally-related events taking place in the opposite order. In the first example, Charlie is able to observe a response to a message he does not see, which does not make sense to him. In the second example, Alice’s observation at one instance causes her to take an action, but at a later point the cause for her actions appears to have occurred after her response to it. Both of these scenarios already violate atomic and sequential consistency because those models enforce a system-wide total order of events. Happily, they are also ruled out by causally consistent systems. The advantage of the causal consistency model is that it rules out this behavior without sacrificing system availability, as shown below.

Causal consistency enforces a global total order on events that are *causally related*. Here, causal relationships are estimated very conservatively: two events are potentially causally if there is some way that the outcome of one could have influenced another.

**Lemma 2.4.** *Sequential consistency implies causal consistency.*

*Proof.* This is immediate from the definitions. Sequential consistency requires all processes to observe the same total order of events, where this total order must respect program order. Causal consistency only requires processes to agree on events that are potentially causally related. Program order is a subset of causal order, so any sequential executions also respects causal order.  $\square$

However, causal consistency is not nearly as strong as sequential consistency, as processes do not need to agree on the order of events with no causal relation between them. This weakness is evident in the fact that the CAP theorem does not rule out highly available systems that maintain causal consistency even during network partitions.

**Lemma 2.5.** *A causally consistent system need not be unavailable during partitions.*

*Proof.* Suppose  $P_1$  and  $P_2$  maintain replicas of a key-value store, as before, and suppose they are separated by a partition. The strategy is simple: each process



immediately handles read requests by reading from its local replica, and handles write requests by applying the update to its local replica. It is easy to see this leads to causally consistent histories. Intuitively, the fact that no information flows between the processes also means the events of each process are not related by causality, so causality is not violated.  $\square$

Note that in this scenario, a client’s requests are always routed to the same processor. If a client’s requests can be routed to any node, causal consistency cannot be maintained without losing availability. One sometimes says that causal consistency is “sticky available” because clients must stick to the same processor during partitions.

The fact that causal consistency can be maintained during partitions suggests it is too weak. Indeed, there are no guarantees about the difference in values for  $x$  and  $y$  across the two replicas.

**Definition 2.5.** FIFO consistency. This also called *pipelined random access memory* or PRAM.

It is easy to see that causal consistency already implies FIFO consistency. Figure REF demonstrates that the reverse need not hold. As a weaker model, FIFO consistency cannot bound the divergence between two replicas of a data object.

### 3 Desiderata

Having discussed some of the fundamental distributed systems issues that arise under real-world network conditions, we turn our attention to three desiderata we will use to frame and analyze the models discussed in Sections 4 and 5.

The CAP theorem, and others like it, place fundamental limitations on the consistency of real-world distributed systems. In the absence of a “perfect” system, engineers are forced to make tradeoffs. Ideally, these tradeoffs should be tuned for the specific application in mind—a protocol that works well in a datacenter might not work well in a heterogeneous geodistributed setting. This section lists three desirable features of distributed systems and frameworks for reasoning about or implementing them. We chose this set based on the particular details of civil aviation and disaster response, where safety is a high priority and usage/communication patterns may be unpredictable.

#### 3.0.1 D1: Quantifiable bounds on inconsistency

*A distributed application should quantify the amount of consistency it delivers. That is, it should (1) provide a mathematical way of measuring inconsistency between system nodes, and (2) bound this value while the system is available.*

The CAP theorem implies that an available data replication application cannot bound inconsistency in all circumstances. When bounded inconsistency cannot be guaranteed, a system satisfying D1 may become unavailable. Alternatively, a reasonable behavior would be to continue providing some form of availability, but alert the user that due to network and system use conditions the requisite level of

consistency cannot be guaranteed by the application, leaving the user with the choice to assess the risk and continue using the system with a weaker safety guarantees.

### 3.0.2 D2: Accommodation of heterogeneous nodes

*An application should not assume that there is a typical system node. Instead, the system should accommodate a diverse range of heterogeneous clients presenting different capabilities, tasks, and risk-factors.*

One can expect a variety of hardware in the field. For example, wildfires often involve responses from many different fire departments, and it must be assumed that they are not always using identical systems. Different participants in the system may be solving different tasks, with different levels of access to the network, and they present different risks. With these sorts of factors in mind, one should hope for frameworks that are as general as possible to accommodate a wide variety of clients.

### 3.0.3 D3: Optimization for a geodistributed wide area network

*An application should be optimized for the sorts of communication patterns that occur in geodistributed wide area networks (WANs) under real-world conditions.*

Consider two incidents. Wouldn't want to enforce needless global consistency, particularly if the agents in one area do not have the same consistency requirements for another area.

Network throughput has some (perhaps approximately linear) relationship with throughput. Communications patterns are likely far from uniform too. In fact, these two things likely coincide—it is often that nodes which are nearby have a stronger need to coordinate their actions than nodes which are far away. For example, consider manoeuvring airplanes to avoid crash.

## 4 Continuous consistency and the conit framework

Strong consistency is a discrete proposition: an application provides strong consistency or it does not. For many real-world applications, it evidently makes sense to work with data that is consistent up to some  $\epsilon \in \mathbb{R}^{\geq 0}$ . Thus, we shift from thinking about consistency as an all-or-nothing condition, towards consistency as a bound on inconsistency.

The definition of  $\epsilon$  evidently requires a more or less application-specific notion of divergence between replicas of a shared data object. Take, say, an application for disseminating the most up-to-date visualization of the location of a fire front. It may be acceptable if this information appears 5 minutes out of date to a client, but unacceptable if it is 30 minutes out of date. That is, we could measure consistency with respect to *time*. One should expect the exact tolerance for  $\epsilon$  will be depend very much on the client, among other things. For example, firefighters who are very close to a fire have a lower tolerance for stale information than a central client keeping only a birds-eye view of several fire fronts simultaneously.

Now suppose many disaster-response agencies coordinate with to update and propagate information about the availability of resources. A client may want to

lookup the number of vehicles of a certain type that are available to be dispatched within a certain geographic range. We may stipulate that the value read by a client should always be 4 of the actual number, i.e. we could measure inconsistency with respect to some numerical value.

In the last example, the reader may wonder we should tolerate a client to read a value that is incorrect by 4, when clearly it is better to be incorrect by 0. Intuitively, the practical benefit of tolerating weaker values is to tolerate a greater level of imperfection in network communications. For example, suppose Alice and Bob are individually authorized to dispatch vehicles from a shared pool. In the event that they cannot share a message.

Or, would could ask that the the value is a conservative estimate, possibly lower but not higher than the actual amount. In these examples, we measure inconsistency in terms of a numerical value.

As a third example,

By varying  $\epsilon$ , one can imagine consistency as a continuous spectrum. In light of the CAP theorem, we should likewise expect that applications with weaker consistency requirements (high  $\epsilon$ ) should provide higher availability, all other things being equal.

Yu and Vahdat explored the CAP tradeoff from this perspective in a series of papers (Yu and Vahdat 2000a, 2000c, 2000b, 2001, 2002). They propose a theory of *conits*, a logical unit of data subject to their three metrics for measuring consistency. By controlling the threshold of acceptable inconsistency of each conit as a continuous quantity, applications can exercise precise control the tradeoff between consistency and performance, trading one for the other in a gradual fashion.

They built a prototype toolkit called TACT, which allows applications to specify precisely their desired levels of consistency for each conit. An interesting aspect of this work is that consistency can be tuned *dynamically*. This is desirable because one does not know a priori how much consistency or availability is acceptable.

The biggest question one must answer is the competing goals of generality and practicality. Generality means providing a general notion of measuring  $\epsilon$ , while practicality means enforcing consistency in a way that can exploit weakened consistency requirements to offer better overall performance.

- The tradeoff of CAP is a continuous spectrum between linearizability and high-availability. More importantly, it can be tuned in real time.
- TACT captures neither CAP-consistency (i.e. neither atomic nor sequential consistency) nor CAP-availability (read and write requests may be delayed indefinitely if the system is unable to enforce consistency requirements because of network issues).

#### 4.1 TACT system model

As in Section 2, we assume a distributed set of processes collaborate to maintain local replicas of a shared data object such as a database. Processes accept read and write requests from clients to update items, and they communicate with each other to ensure to ensure that all replicas remain consistent.

However, access to the data store is mediated by a middleware library, which sits between the local copy of the replica and the client. At a high level, TACT will allow an operation to take place if it does not violate user-specific consistency bounds. If allowing an operation to proceed would violate consistency constraints, the operation blocks until TACT synchronizes with one or more other remote replicas. The operation remains blocked until TACT ensures that executing it would not violate consistency requirements.

$$\text{Consistency} = \langle \text{Numerical error}, \text{Order error}, \text{Staleness} \rangle.$$

Processes forward accesses to TACT, which handles committing them to the store. TACT may not immediately process the request—instead it may need to coordinate with other processes to enforce consistency. When write requests are processed (i.e. when a response is sent to the originating client), they are only committed in a *tentative* state. Tentative writes eventually become fully committed at some point in the future, but when they are committed, they may be reordered. After fully committing, writes are in a total order known to all processes.

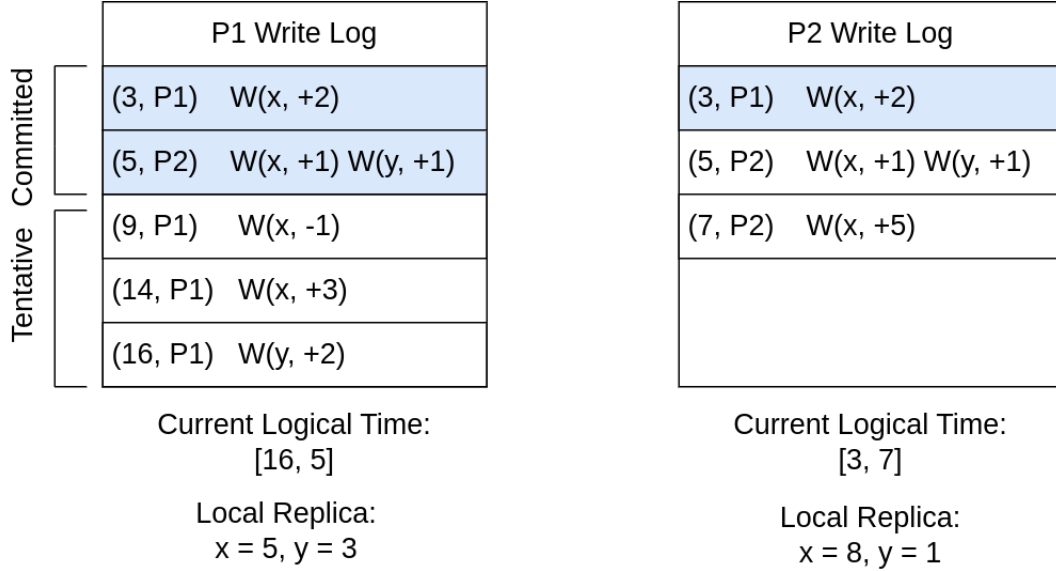


Figure 10: Snapshot of two local replicas using TACT

A write access  $W$  can separately quantify its *numerical weight* and *order weight* on conit  $F$ . Application programmers have multiple forms of control:

Consistency is enforced by the application by setting bounds on the consistency of read accesses. The TACT framework then enforces these consistency levels.

## 4.2 Measuring consistency on conits

### Numerical consistency

**Order consistency** When the number of tentative (uncommitted) writes is high, TACT executes a write commitment algorithm. This is a *pull-based* approach which

pulls information from other processes in order to advance  $P_i$ 's vector clock, raising the watermark and hence allowing  $P_i$  to commit some of its writes.

## **Real time consistency**

### **4.3 Enforcing inconsistency bounds**

**Numerical consistency** We describe split-weight AE. Yu and Vahdat also describe two other schemes for bounding numerical error. One, compound AE, bounds absolute error trading space for communication overhead. In their simulations, they found minimal benefits to this tradeoff in general. It is possible that for specific applications the savings are worth it. They also consider a scheme, Relative NE, which bounds the relative error.

## **Order consistency**

## **Real time consistency**

### **4.4 Future work**

## **5 Sheaf theoretic data fusion**

Strong consistency models provide the abstraction of an idealized global truth. In the case of conits, the numerical, commit-order, and real-time errors are measured with respect to an idealized global state of the database. This state may not exist on any one replica, but it is the state each replica would converge to if it were to see all remaining unseen updates.

We consider distributed applications that receive data from many different sources, such as from a sensor network (broadly defined). It will often be the case that some sources of data should be expected to agree with each other, but they may not. A typical scenario, we want to integrate these data into a larger model of some kind. Essentially take a poll, and attempt to synthesize a global picture that agrees as much as possible with the data reported from the sensor network.

Here, we need a consistency model to measure how successful our attempts are to synthesize a global image. And to tell us how much our sensors agree. Ideally, we could use this system to diagnose disagreements between sensors, identifying sensors that appear to be malfunctioning, or to detect aberrations that necessitate a response.

### **5.1 Data fusion**

To be written.

## 5.2 Presheaves

**Definition 5.1.** A *partially order-indexed family of sets* is a family of sets indexed by a partially-ordered set, such that orders between the indices correspond to functions between the sets.

We can also set  $(P, \leq)$  acts on the set  $\{S_i\}_{i \in I}$ .

**Definition 5.2.** A *semiautomaton* is a monoid paired with a set.

This is also called a *monoid action* on the set.

**Definition 5.3.** A copresheaf is a \*category acting on a family of sets\*.

**Definition 5.4.** A presheaf is a \*category acting covariantly on a family of sets\*.

## 5.3 Sheaves and assignments

To be written.

## 5.4 The consistency radius

To be written.

## 6 Conclusion

To be written

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14. ABSTRACT The System Wide Safety (SWS) program has been investigating how crewed and uncrewed aircraft can safely operate in shared airspace, taking disaster response scenarios as a motivating use case. Enforcing safety requirements for distributed agents requires coordination by passing messages over a communication network. However, the operating environment will not admit reliable high-bandwidth communication between all agents, introducing theoretical and practical obstructions to global consistency that make it more difficult to maintain safety-related invariants. This self-contained memo discusses some of the distributed systems challenges involved in system-wide safety, focusing on the practical shortcomings of both strong and weak consistency models for shared memory. Then we survey two <i>continuous</i> consistency models that come from different parts of the literature. Unlike weak consistency models, continuous consistency models provides hard upper bounds on the "amount" of inconsistency observable by clients. Unlike strong consistency, these models are flexible enough to accommodate real-world conditions, such as by providing liveness during brief network partitions or tolerating disagreements between sensors in a sensor network. We conclude that continuous consistency models are appropriate for analyzing safety-critical systems that operate without strong guarantees about network performance.						
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