Introduction to Gravitational Wave Data Analysis

Chia-Jui Chou

National Yang Ming Chiao Tung University, Taiwan

General Relativity: Einstein Equation

Einstein Equation

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

- $g_{\mu\nu}$: metric tensor,
- $R_{\mu\nu}$: Ricci tensor,
- $R = g^{\mu\nu} R_{\mu\nu}$: Ricci scalar,
- $T_{\mu\nu}$: stress-energy tensor.

Measure of interval in spacetime

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu.$$

General Relativity: Linearized Theory

Perturbation of metric tensor

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1.$$

Linearized Einstein Equation

$$\Box \bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^\rho \partial^\sigma \bar{h}_{\rho\sigma} - \partial^\rho \partial_\nu \bar{h}_{\mu\rho} - \partial^\sigma \partial_\mu \bar{h}_{\nu\rho} = -\frac{16\pi G}{c^4} T_{\mu\nu}.$$

- $h = \eta^{\mu\nu} h_{\mu\nu}$,
- $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$.

Harmonic Gauge and Transverse-Traceless Gauge

Harmonic Gauge

$$\partial^{\nu}\bar{h}_{\mu\nu} = 0.$$

Linearized Einstein Equation

$$\Box\bar{h}_{\mu\nu} = 0,$$

$T_{\mu\nu} = 0$ if we observe the GW far away from the source.

Transverse-Traceless Gauge

$$h^{0\mu}, \quad h^{i}_{i} = 0, \partial^{j} h_{ij} = 0,$$

which can only be chosen away from the source.

Solution of the Gravitational Wave

We can choose the propagation direction along $\hat{z}$, with the wave vector:

$$k^\mu = (\omega/c, \mathbf{k}),$$

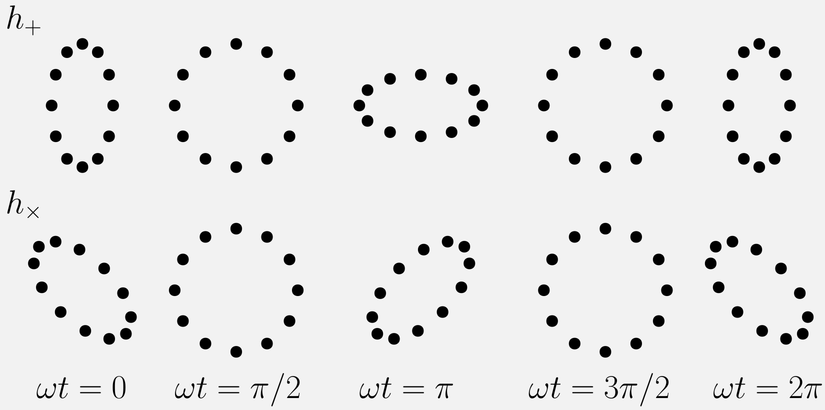
and the solution of GW:

$$h_{\mu\nu}^{\mathrm{TT}}(t,z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cos\left[\omega(t - z/c)\right].$$

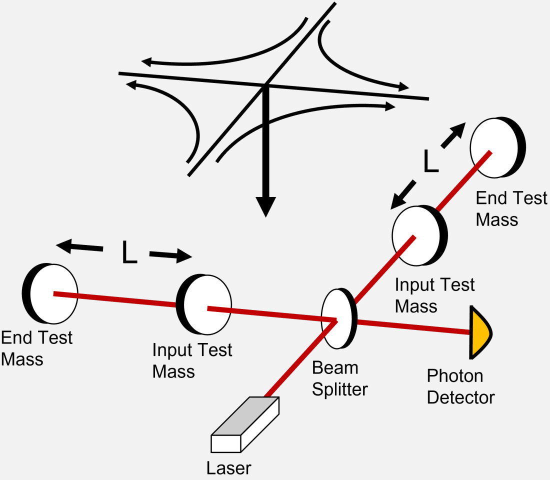
The we have the perturbed metric tensor:

$$\begin{aligned} ds^2 = &- c^2 dt^2 + dz^2 + \{1 + h_+ \cos[\omega(t - z/c)]\}\, dx^2 \\ &- \{1 + h_+ \cos[\omega(t - z/c)]\}\, dy^2 + 2h_\times \cos[\omega(t - z/c)] dxdy. \end{aligned}$$

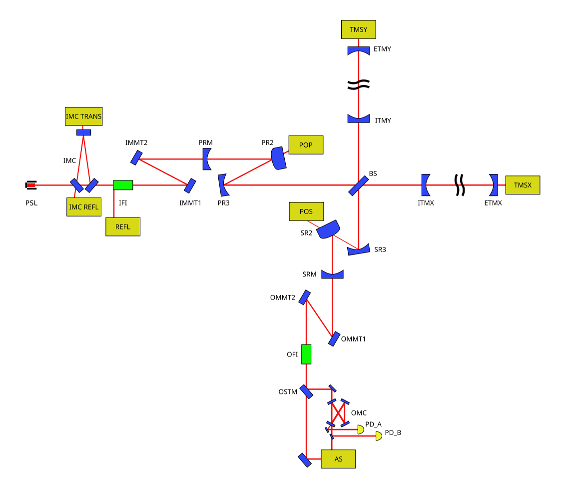
Gravitational Wave Interacting with Test Masses



Gravitational Wave Detector



Gravitational Wave Detector



Gravitational Wave Detector

$$0 = -c^2 dt^2 + (1 + h_+)\, dx^2, \quad \rightarrow cdt \simeq \left(1 + \frac{h_+}{2}\right) |dx|\,.$$

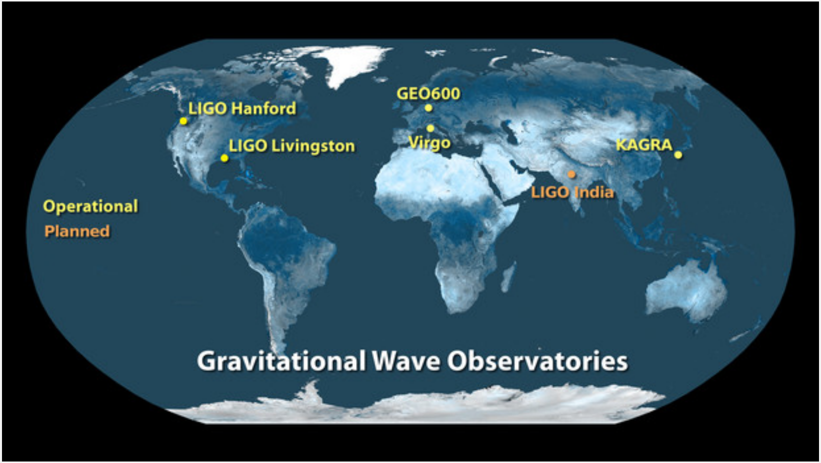
$$\Delta T_x = \frac{h_+ L_x}{2c}, \quad \Delta L_x = \frac{h_+ L_x}{2}.$$

$$0 = -c^2 dt^2 + (1 - h_+)\, dy^2, \quad \rightarrow cdt \simeq \left(1 - \frac{h_+}{2}\right) |dy|\,.$$

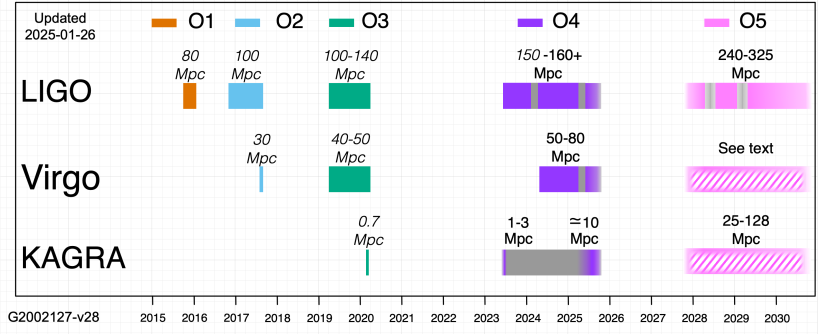
$$\Delta T_y = -\frac{h_+ L_y}{2c}, \quad \Delta L_y = -\frac{h_+ L_y}{2}.$$

$$\Delta L = \Delta L_x - \Delta L_y = h_+ \cdot \frac{L_x + L_y}{2} \equiv h_+ L.$$

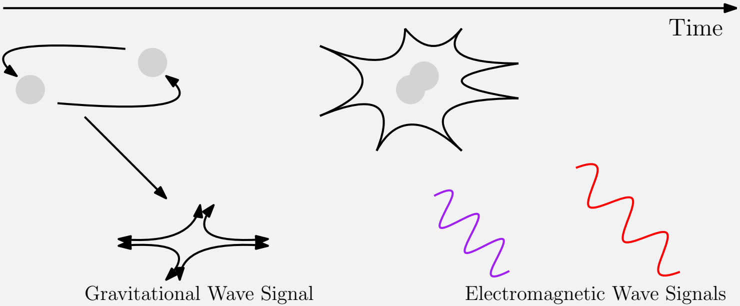
Gravitational Wave Detectors



Observation Runs



Multi-Messenger Astronomy (MMA)



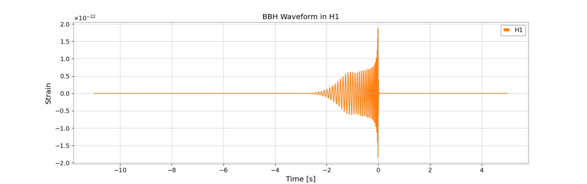
Sky localization of the "Known GW Sources" from their GW signals and send out alerts in low latency for the EM telescopes to capture the follow-up EM wave signals.

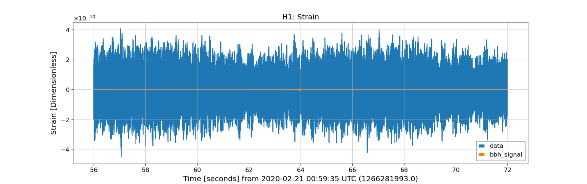
Sources of Gravitational Waves

- Compact Binary Coalescence:
  - Black Hole-Black Hole Mergers,
  - Neutron Star-Neutron Star Mergers,
  - Black Hole-Neutron Star Mergers.
- Bursts:
  - Core-collapse Supernovae, Fast Radio Burst, Gamma Ray Burst, etc.
- Continuous Waves:
  - Spinning Neutron Stars,etc.
- Stochastic Background:
  - Cosmological background, Astrophysical background, etc.

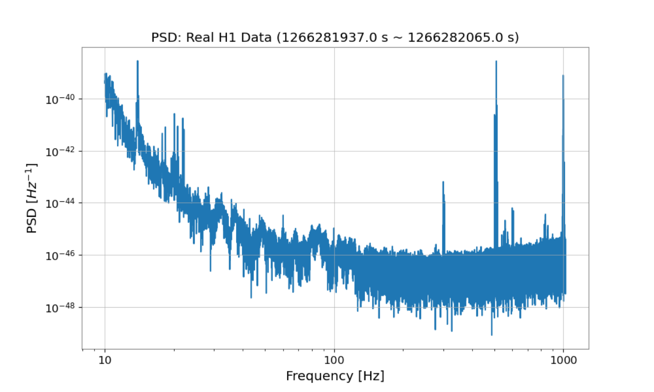
Gravitational Waveforms and Background Noise

GW strain: $h(t) = \frac{\Delta L(t)}{L} \sim 10^{-22}$.

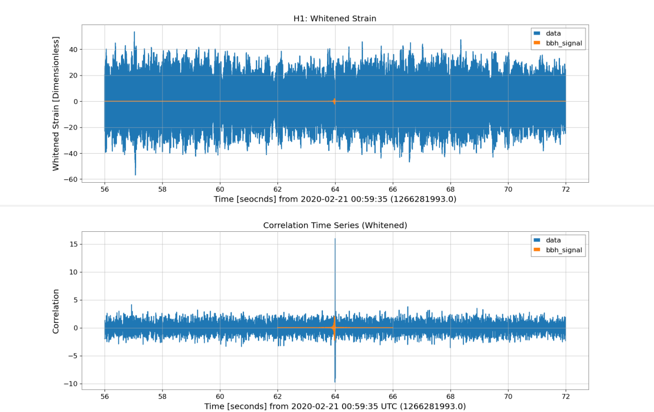




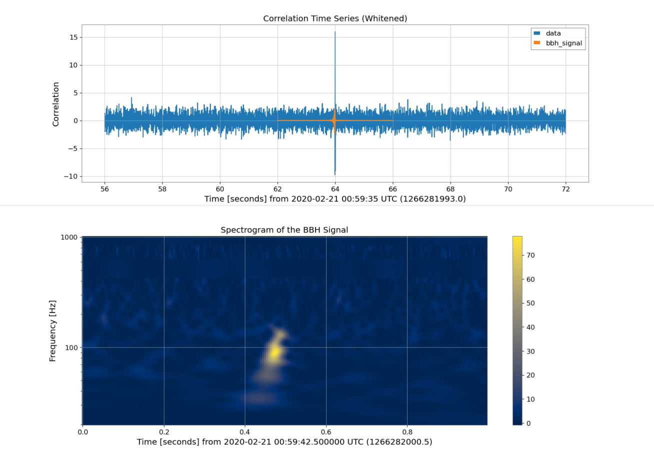
Power Spectral Density (PSD)



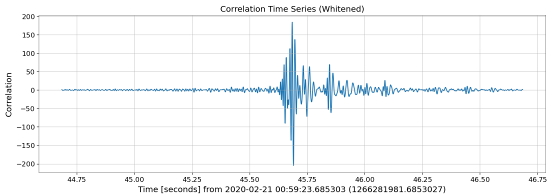
Finding BBH Signals

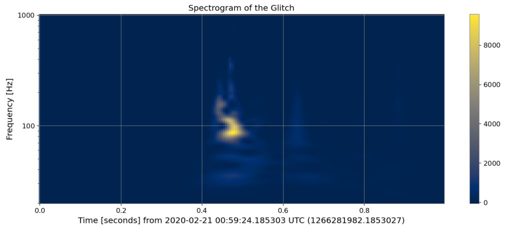


Finding BBH Signals (Matched Filtering)



Glitches (Matched Filtering)





Thank you