filter by the impulse sequence $\delta(n)$, then from (4.11) and using $\mathcal{Z}[\delta(n)] = 1$, the output of the filter will be $x(n)$. (This is a numerical approach of computing the inverse z-transform; we will discuss the analytical approach in the next section.) We can compare this output with the given $x(n)$ to verify that $X(z)$ is indeed the transform of $x(n)$. This is illustrated in Example 4.6. An equivalent approach is to use the `impz` function discussed in Chapter 2.

4.2.1 SOME COMMON z-TRANSFORM PAIRS

Using the definition of z-transform and its properties, one can determine z-transforms of common sequences. A list of some of these sequences is given in Table 4.1.

TABLE 4.1 *Some common z-transform pairs*

Sequence	*Transform*	*ROC*
$\delta(n)$	1	$\forall z$
$u(n)$	$\dfrac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u(-n-1)$	$\dfrac{1}{1-z^{-1}}$	$\|z\| < 1$
$a^n u(n)$	$\dfrac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-b^n u(-n-1)$	$\dfrac{1}{1-bz^{-1}}$	$\|z\| < \|b\|$
$[a^n \sin \omega_0 n]\, u(n)$	$\dfrac{(a \sin \omega_0) z^{-1}}{1-(2a \cos \omega_0) z^{-1} + a^2 z^{-2}}$	$\|z\| > \|a\|$
$[a^n \cos \omega_0 n]\, u(n)$	$\dfrac{1-(a \cos \omega_0) z^{-1}}{1-(2a \cos \omega_0) z^{-1} + a^2 z^{-2}}$	$\|z\| > \|a\|$
$na^n u(n)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-nb^n u(-n-1)$	$\dfrac{bz^{-1}}{(1-bz^{-1})^2}$	$\|z\| < \|b\|$

☐ **EXAMPLE 4.6** Using z-transform properties and the z-transform table, determine the z-transform of

$$x(n) = (n-2)(0.5)^{(n-2)} \cos\left[\frac{\pi}{3}(n-2)\right] u(n-2)$$

Solution Applying the sample-shift property,

$$X(z) = \mathcal{Z}[x(n)] = z^{-2}\mathcal{Z}\left[n(0.5)^n \cos\left(\frac{\pi n}{3}\right) u(n)\right]$$