Chapter 3

3.1

Derivative of a Constant Function $\frac{d}{dx}(c) = 0$

Power Rule $\frac{d}{dx}(x)^n = nx^{n-1}$

Constant Multiple Rule $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

Sum Rule $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

Difference Rule $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

Definition of the Number e $\lim\limits_{h\to 0}\dfrac{e^h - 1}{h} = 1$

Derivative of the Natural Exponential Function $\frac{d}{dx}(e^x) = e^x$

Derivative of the Exponential Function $\frac{d}{dx}(a^x) = a^x \ln a$

3.2

The Product Rule $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

The Quotient Rule $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \dfrac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

Table of Differentiation Formulas

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$
$(cf)' = cf'$	$(f+g)' = f' + g'$	$(f-g)' = f' - g'$
$(fg)' = f'g + fg'$	$(\frac{f}{g})' = \dfrac{f'g - fg'}{g^2}$	$(a^x)' = a^x \ln a$

3.3

Trig Identities

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$

Derivatives of Trigonometric Functions

$$\tfrac{d}{dx}\sin(x) = \cos(x) \qquad \tfrac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\tfrac{d}{dx}\cos(x) = -\sin(x) \qquad \tfrac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\tfrac{d}{dx}\tan(x) = \sec^2(x) \qquad \tfrac{d}{dx}\cot(x) = -\csc^2(x)$$

3.4

The Chain Rule

$$F = f \circ g = f(g(x))$$

$$F'(x) = f'(g(x)) * g'(x)$$

$$y = f(u), u = g(x)$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

3.5

Derivatives of Inverse Trigonometric Functions

$$\tfrac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \tfrac{d}{dx}\csc^{-1}(x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\tfrac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}} \qquad \tfrac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\tfrac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \tfrac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$

3.6

Derivatives of Logorithmic Functions

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a} \qquad \frac{d}{dx}(\ln x) = \frac{x'}{x} \qquad \frac{d}{dx}\ln|x| = \frac{1}{x} \qquad \frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x)$$

Applying the Chain Rule to Derivatives of Logorithmic Functions

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx} \qquad \frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$$

The number e as a Limit

$$e = \lim_{x \to 0}(1+x)^{1/2} \qquad e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n$$

Logarithmic Differentiation

1. Take natural logarithms of both sides of an equation and use Log Laws to Simplify.

2. Differentiate implicitly with respect to x.

3. Solve the resulting equation for y'

Log Rules

$$\log_b(mn) = \log_b(m) + \log_b(n)$$
$$\log_b \tfrac{m}{n} = \log_b(m) - \log_b(n)$$
$$\log_b(m^n) = n\log_b(m)$$

3.10

Linear Approximation

$$f(x) \approx f(a) + f'(a)(x-a)$$
$$L(x) = f(a) + f'(a)(x-a)$$

Differentials

$$dy = f'(x)dx$$
$$\Delta y = f(x + \Delta x) - f(x)$$

3.11

Definition of Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$
$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$
$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

Hyperbolic Identities

$$\sinh(-x) = -\sinh x \qquad \cosh(-x) = \cosh x$$
$$\cosh^2 x - \sinh^2 x = 1 \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$
$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$

3.11

Derivatives of Hyperbolic Functions

$\frac{d}{dx}(\sinh\ x) = \cosh\ x$	$\frac{d}{dx}(\operatorname{csch}\ x) = -\operatorname{csch}\ x\ \ \coth\ x$
$\frac{d}{dx}(\cosh\ x) = \sinh\ x$	$\frac{d}{dx}(\operatorname{sech}\ x) = -\operatorname{sech}\ x\ \ \tanh\ x$
$\frac{d}{dx}(\tanh\ x) = \operatorname{sech}^2 x$	$\frac{d}{dx}(\coth\ x) = -\operatorname{csch}^2 x$

Inverse Hyperbolic Functions

$$y = \sinh^{-1} x \Leftrightarrow \sinh\ y = x$$

$$y = \cosh^{-1} x \Leftrightarrow \cosh\ y = x \quad \text{and} \quad y \geq 0$$

$$y = \tanh^{-1} x \Leftrightarrow \tanh\ y = x$$

Inverse Hyperbolic Functions in Logarithmic Terms

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}) \qquad x \in \mathbb{R}$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \qquad x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) t \qquad -1 < x < 1$$

Derivatives of Inverse Hyperbolic Functions

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$	$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{\lvert x \rvert\sqrt{x^2+1}}$
$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$	$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$
$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$	$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$