

Chapter 3

3.1

Derivative of a Constant Function $\frac{d}{dx}(c) = 0$

Power Rule $\frac{d}{dx}(x)^n = nx^{n-1}$

Constant Multiple Rule $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

Sum Rule $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

Difference Rule $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

Definition of the Number e $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$

Derivative of the Natural Exponential Function $\frac{d}{dx}(e^x) = e^x$

Derivative of the Exponential Function $\frac{d}{dx}(a^x) = a^x \ln a$

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The Product Rule $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

The Quotient Rule $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

Table of Differentiation Formulas

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$
$(cf)' = cf'$	$(f + g)' = f' + g'$	$(f - g)' = f' - g'$
$(fg)' = f'g + fg'$	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$	$(a^x)' = a^x \ln a$

3.3

Trig Identities

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \qquad \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx} \sin(x) = \cos(x) \qquad \frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x) \qquad \frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x) \qquad \frac{d}{dx} \cot(x) = -\csc^2(x)$$

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The Chain Rule

$$F = f \circ g = f(g(x))$$

$$F'(x) = f'(g(x)) * g'(x)$$

$$y = f(u), u = g(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

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Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \csc^{-1}(x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} \cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx} \cot^{-1}(x) = -\frac{1}{1+x^2}$$

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Derivatives of Logarithmic Functions

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a} \quad \frac{d}{dx}(\ln x) = \frac{x'}{x} \quad \frac{d}{dx} \ln|x| = \frac{1}{x} \quad \frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x)$$

Applying the Chain Rule to Derivatives of Logarithmic Functions

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx} \quad \frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$$

The number e as a Limit

$$e = \lim_{x \rightarrow 0} (1 + x)^{1/2} \quad e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

Logarithmic Differentiation

1. Take natural logarithms of both sides of an equation and use Log Laws to Simplify.
2. Differentiate implicitly with respect to x .
3. Solve the resulting equation for y'

Log Rules

$$\log_b(mn) = \log_b(m) + \log_b(n)$$

$$\log_b \frac{m}{n} = \log_b(m) - \log_b(n)$$

$$\log_b(m^n) = n \log_b(m)$$

3.10

Linear Approximation

$$f(x) \approx f(a) + f'(a)(x - a)$$

$$L(x) = f(a) + f'(a)(x - a)$$

Differentials

$$dy = f'(x)dx$$

$$\Delta y = f(x + \Delta x) - f(x)$$

3.11

Definition of Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{coth} x = \frac{\cosh x}{\sinh x}$$

Hyperbolic Identities

$$\sinh(-x) = -\sinh x \qquad \cosh(-x) = \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

3.11

Derivatives of Hyperbolic Functions

$$\begin{aligned}\frac{d}{dx}(\sinh x) &= \cosh x & \frac{d}{dx}(\operatorname{csch} x) &= -\operatorname{csch} x \coth x \\ \frac{d}{dx}(\cosh x) &= \sinh x & \frac{d}{dx}(\operatorname{sech} x) &= -\operatorname{sech} x \tanh x \\ \frac{d}{dx}(\tanh x) &= \operatorname{sech}^2 x & \frac{d}{dx}(\coth x) &= -\operatorname{csch}^2 x\end{aligned}$$

Inverse Hyperbolic Functions

$$\begin{aligned}y &= \sinh^{-1} x \Leftrightarrow \sinh y = x \\ y &= \cosh^{-1} x \Leftrightarrow \cosh y = x \quad \text{and} \quad y \geq 0 \\ y &= \tanh^{-1} x \Leftrightarrow \tanh y = x\end{aligned}$$

Inverse Hyperbolic Functions in Logarithmic Terms

$$\begin{aligned}\sinh^{-1} x &= \ln(x + \sqrt{x^2 + 1}) \quad x \in \mathbb{R} \\ \cosh^{-1} x &= \ln(x + \sqrt{x^2 - 1}) \quad x \geq 1 \\ \tanh^{-1} x &= \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad -1 < x < 1\end{aligned}$$

Derivatives of Inverse Hyperbolic Functions

$$\begin{aligned}\frac{d}{dx}(\sinh^{-1} x) &= \frac{1}{\sqrt{1+x^2}} & \frac{d}{dx}(\operatorname{csch}^{-1} x) &= -\frac{1}{|x|\sqrt{x^2+1}} \\ \frac{d}{dx}(\cosh^{-1} x) &= \frac{1}{\sqrt{x^2-1}} & \frac{d}{dx}(\operatorname{sech}^{-1} x) &= -\frac{1}{x\sqrt{1-x^2}} \\ \frac{d}{dx}(\tanh^{-1} x) &= \frac{1}{1-x^2} & \frac{d}{dx}(\coth^{-1} x) &= \frac{1}{1-x^2}\end{aligned}$$