Solution to the Inviscid Burgers' Equation by the Lax-Friedrichs Scheme*

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1 Introduction

2 Body

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta x}\left(\mathcal{F}(U_j^n, U_{j+1}^n) - \mathcal{F}(U_{j-1}^n, U_{j+1}^n)\right)$$

$$U_j^{n+1} = \frac{1}{2}\left(U_{j-1}^n + U_{j+1}^n\right) - \frac{\Delta t}{2\Delta x}\left(f(U_{j+1}^n) - f(U_{j-1}^n)\right)$$

$$\mathcal{F}(U_j^n, U_{j+1}^n) := \frac{\Delta t}{2\Delta x}(U_j^n - U_{j+1}^n) + \frac{1}{2}\left(f(U_j^n) + f(U_{j+1}^n)\right)$$

$$U_j^n \sim \bar{u}_j^n \equiv \frac{1}{\Delta x}\int_{x_{j-1/2}}^{x_{j+1/2}} u(x, t_n)dx$$

$$\mathcal{F}(U_j^n, U_{j+1}^n) \sim \frac{1}{\Delta x}\int_{t_n}^{t_{n+1}} f(u(x_{j+1/2}, t))dt$$

$$\mathcal{F}(\bar{u}, \bar{u}) = f(\bar{u})$$

$$u_t + f(u)_x = 0$$

$$F_{j-1/2}^n = \mathcal{F}(U_{j-1}^n, U_j^n) \qquad F_{j+1/2}^n = \mathcal{F}(U_j^n, U_{j+1}^n)$$

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} + \frac{F_{j+1/2}^n - F_{j-1/2}^n}{\Delta x} = 0$$

*Placeholder title!

3 Conclusion

References

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