<u>Solution of problem 1</u>

$$P(\eta=i) \;=\; \sum_{k=i}^{\infty} P(\eta=i \;|\; \xi=k)\,P(\xi=k) \;=$$
$$=\; \sum_{k=i}^{\infty} \binom{k}{i} p^i (1-p)^{k-i} e^{-\lambda} \frac{\lambda^k}{k!} \;=$$
$$=\; e^{-\lambda} \frac{p^i}{(1-p)^i} \sum_{k=i}^{\infty} \binom{k}{i} (1-p)^k \frac{\lambda^k}{k!} \;=$$
$$=\; e^{-\lambda} \frac{p^i}{(1-p)^i} \sum_{k=i}^{\infty} \frac{k!}{i!(k-i)!} (1-p)^k \frac{\lambda^k}{k!} \;=$$
$$=\; e^{-\lambda} \frac{p^i}{i!(1-p)^i} \sum_{k=i}^{\infty} \frac{1}{(k-i)!} (1-p)^k \lambda^k \;=$$
$$=\; e^{-\lambda} \frac{p^i}{i!(1-p)^i} \sum_{q=0}^{\infty} \frac{1}{q!} (1-p)^{q+i} \lambda^{q+i} \;=$$
$$=\; e^{-\lambda} \frac{(\lambda p)^i}{i!} \sum_{q=0}^{\infty} \frac{(1-p)^q \lambda^q}{q!} \;=$$

(using exponential series)

$$=\; e^{-\lambda} \frac{(\lambda p)^i}{i!} e^{\lambda(1-p)} \;=$$
$$=\; e^{-\lambda p} \frac{(\lambda p)^i}{i!}$$

which is a PMF for Poisson distribution with λp parameter.