Solution of problem 1

$$P(\eta=i) = \sum_{k=i}^{\infty} P(\eta=i \mid \xi=k) P(\xi=k) =$$

$$= \sum_{k=i}^{\infty} \binom{k}{i} p^i (1-p)^{k-i} e^{-\lambda} \frac{\lambda^k}{k!} =$$

$$= e^{-\lambda} \frac{p^i}{(1-p)^i} \sum_{k=i}^{\infty} \binom{k}{i} (1-p)^k \frac{\lambda^k}{k!} =$$

$$= e^{-\lambda} \frac{p^i}{(1-p)^i} \sum_{k=i}^{\infty} \frac{k!}{i!(k-i)!} (1-p)^k \frac{\lambda^k}{k!} =$$

$$= e^{-\lambda} \frac{p^i}{i!(1-p)^i} \sum_{k=i}^{\infty} \frac{1}{(k-i)!} (1-p)^k \lambda^k =$$

$$= e^{-\lambda} \frac{p^i}{i!(1-p)^i} \sum_{q=0}^{\infty} \frac{1}{q!} (1-p)^{q+i} \lambda^{q+i} =$$

$$= e^{-\lambda} \frac{(\lambda p)^i}{i!} \sum_{q=0}^{\infty} \frac{(1-p)^q \lambda^q}{q!} =$$

(using exponential series)

$$= e^{-\lambda} \frac{(\lambda p)^i}{i!} e^{\lambda(1-p)} =$$

$$= e^{-\lambda p} \frac{(\lambda p)^i}{i!}$$

which is a PMF for Poisson distribution with λp parameter.