Symbolic Manifolds and Entropic Dynamics: A Cognitive Topology of Mental States

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We introduce a symbolic-topological framework for cognition in which mental states evolve as trajectories $\gamma(t) = (\alpha, \kappa, E_r)$ on a three-dimensional manifold. Simulations reproduce neurotypical, gifted, twice-exceptional and collapse-prone regimes, revealing bifurcations and collapse–recovery dynamics. The model unifies entropy-driven brain theories with symbolic cognition and offers testable biomarkers for a prospective "topological symbolic psychiatry".

Introduction

Introduction

Understanding the dynamic nature of cognition remains a central challenge in cognitive science. Traditional models often rely on static categories or linear representations, which fail to capture the fluid, recursive, and symbolically mediated nature of mental processes—particularly in cases of neurodivergence, exceptional cognitive ability, or disorganization.

Recent research has highlighted the role of entropy and complexity in shaping brain activity and cognitive function [?,?], suggesting that cognition emerges from nonlinear, metastable dynamics. Yet, these frameworks frequently overlook the influence of symbolic structure, contextual grounding, and self-referential feedback in shaping cognitive states.

Here, we propose a novel theoretical framework based on a symbolic manifold—a topological space parameterized by three cognitive-symbolic variables: anchoring coefficient (α), symbolic curvature (κ), and recursive entropy (E_r). These variables jointly describe the evolving state of a cognitive agent as a trajectory $\gamma(t)$ in symbolic space, governed by interactions between symbolic coherence, conceptual divergence, and entropic modulation.

Rather than viewing mental states as discrete or pathologized categories, we conceptualize them as structured positions and flows within this manifold. Cognitive profiles—such as neurotypical, gifted, and twice-exceptional (2e)—emerge as attractor regimes, each exhibiting distinct dynamic signatures.

This model enables the simulation of symbolic trajectories and the mapping of cognitive diversity within a unified, generative structure. By integrating symbolic topology with entropic principles, we aim to provide a rigorous and extensible basis for representing cognitive variability, with potential applications in neuropsychology, psychiatry, and computational modeling.

Results

* A symbolic cognitive topology

Symbolic Cognitive Topology

We introduce a symbolic manifold $\mathbb{S}$ in which cognitive activity evolves as a dynamic trajectory $\gamma(t) = (\alpha(t), \kappa(t), E_r(t))$. This formal structure is governed by three symbolic-cognitive variables:

- **Anchoring coefficient** (α): captures semantic and contextual coherence—higher values indicate well-integrated, focused thought; low values correspond to disorganization or fragmentation.
- **Symbolic curvature** (κ): quantifies the nonlinearity or divergence of associative links—analogous to cognitive flexibility, originality, or conceptual jumps.
- **Recursive entropy** (E_r): reflects the self-referential depth or symbolic unpredictability—higher E_r signals increased symbolic recursion or chaotic semantic chaining.

The cognitive state evolves as a curve in $\mathbb{S}$ modulated by an entropic operator $\mathcal{E}$, defined qualitatively as:

$$\frac{d\gamma}{dt} = \mathcal{E}(\gamma(t)) = \begin{pmatrix} -\eta_\alpha \cdot \alpha(t) + \phi_\alpha(\kappa, E_r) \\ \eta_\kappa \cdot (1 - \alpha(t)) + \phi_\kappa(E_r) \\ \eta_r \cdot \kappa(t) + \phi_r(\alpha) \end{pmatrix}$$

Here, η_α, η_κ, and η_r are modulation coefficients, and ϕ terms represent nonlinear interactions between dimensions. This captures how reductions in anchoring can trigger associative divergence or recursive overload.

Drawing from low-dimensional neural manifolds [?,?,1] and symbolic attractor theory [?,?], we define regions in $\mathbb{S}$ corresponding to recurrent symbolic profiles, termed *topotypes*. These are not static types but dynamic attractor basins, characterized by:

- **Neurotypical** (high α, low E_r, moderate κ)
- **Gifted cognition** (moderate α, high κ, high E_r)
- **Twice-exceptional** (oscillatory α, episodic surges in E_r)
- **Collapse** (declining α, divergent κ, escalating E_r)

These profiles represent metastable regimes navigable over time, rather than fixed diagnostic labels. Time is discretized, and update rules allow for deterministic or stochastic simulation of $\gamma(t)$ trajectories.

This formulation integrates symbolic instability into the entropic brain hypothesis [?] and REBUS model [?], embedding symbolic breakdowns into dynamic manifold transitions.

The model supports generative simulation and classification of symbolic-cognitive profiles, offering a formal framework for investigating neurodiversity, breakdowns, and symbolic restoration.

* Entropic dynamics in simulated manifolds

3. Simulation Results

To evaluate the expressive capacity of the symbolic manifold $\mathbb{S}$, we conducted discrete-time simulations of trajectories $\gamma(t) = (\alpha(t), \kappa(t), E_r(t))$ under idealized initial conditions representative of four cognitive-symbolic profiles: Neurotypical (NT), Gifted (G), Twice-exceptional (2e), and Collapse-prone (C). The system evolves via the entropic modulation operator $\mathcal{E}$ (see Section 2), incorporating deterministic nonlinear interactions and parametric noise.

Profile 1: Neurotypical (NT) shows stable anchoring ($\alpha \approx 0.9$), low symbolic curvature ($\kappa \approx 0.1$), and minimal entropy ($E_r \approx 0.05$). Its trajectory remains confined to a narrow symbolic basin, suggesting low variance in symbolic state over time (Fig. **??**).

Profile 2: Gifted (G) exhibits moderate anchoring ($\alpha \approx 0.6$), elevated curvature ($\kappa \approx 0.75$), and high recursive entropy ($E_r \approx 0.8$). The trajectory demonstrates symbolic divergence without collapse, reflecting nonlinearity and symbolic flexibility (Fig. **??**).

Profile 3: Twice-exceptional (2e) oscillates between basins, with $\alpha(t)$ fluctuating between 0.3 and 0.85, and episodic surges in $E_r(t)$. This profile combines phases of symbolic coherence with entropic disruption, simulating cognitive dissonance or oscillatory stability (Fig. **??**).

Profile 4: Collapse-prone (C) starts at $\alpha_0 = 0.5$ and undergoes progressive degradation due to compounding E_r and κ. By $t = 50$, $\alpha \to 0.1$, and $E_r > 0.9$, indicating a symbolic breakdown. This regime reflects unstable symbolic anchoring and associative overload.

Collapse–Recovery Scenario: We simulated an external dampening event at $t = 70$ reducing E_r and increasing α by intervention terms δ_α, δ_E. The trajectory returns to a bounded symbolic region, illustrating restoration of coherence under entropic compression (Fig. **??**).

Symbolic Topology: A 3D trajectory $\gamma(t)$ over (α, κ, E_r) reveals organized transitions and clustering (Fig. **??**), suggesting symbolic attractors and the potential for symbolic fingerprinting across individuals.

Comparative Positioning: Table **??** situates our framework relative to other cognitive theories. While FEP and Entropic Brain offer entropy-centric formulations, the symbolic manifold uniquely enables symbolic interpretability and neurodiversity modeling.

Together, these simulations support the hypothesis that symbolic cognitive profiles manifest as structured dynamic regimes in $\mathbb{S}$, offering a generative and interpretable approach to modeling variation in mental states.

Discussion

Discussion and General Synthesis

This study introduced a symbolic manifold model that formalizes cognitive variation as trajectories $\gamma(t)$ evolving through a three-dimensional symbolic space: anchoring (α), symbolic curvature (κ), and recursive entropy (E_r). By representing cognitive profiles as dynamic positions within this manifold, the model provides a unified framework for simulating transitions across states such as neurotypicality, gifted cognition, oscillatory dual profiles (2e), and symbolic collapse.

Simulation results demonstrate that each profile traces a structured and parameter-sensitive trajectory in $\mathbb{S}$. These symbolic signatures reflect stable or metastable configurations in symbolic space and suggest that neurodiversity can be modeled as differences in symbolic dynamics rather than as categorical distinctions. The model supports classification, visualization, and intervention hypotheses: targeted modulation of α, κ, or E_r may enable theoretical restoration of symbolic coherence, paralleling cognitive-behavioral scaffolding or entropic modulation therapies.

Importantly, this framework reconceptualizes neurodivergence not as disorder but as structured displacement within a symbolic topological landscape. Gifted and 2e trajectories display increased curvature and entropy while preserving anchoring intermittently, modeling creativity and cognitive oscillation. Collapse regimes, conversely, show runaway entropy and anchoring loss—suggesting that breakdowns may be framed not as categorical failures but as failed self-organization in symbolic space.

Several limitations merit attention. The symbolic variables proposed are currently theoretical and require future empirical grounding. Potential proxies include: coherence metrics for α (e.g., topic continuity in speech), network divergence measures for κ (e.g., semantic distance in conceptual associations), and entropy rate in symbolic generation for E_r. Additionally, while simulations qualitatively reproduce plausible patterns, further mathematical formalization and parameter calibration are needed to support generalizability and reproducibility.

Future directions include the application of this model to longitudinal cognitive data (e.g., semantic dynamics, thought disorder profiles, neuroimaging of functional connectivity under symbolic tasks) to empirically test the model's explanatory and predictive power. Moreover, a formal symbolic-to-neural mapping—via, for example, neuro-symbolic transformers or latent embedding alignment—could bridge this manifold with real-time brain dynamics.

In sum, the symbolic manifold offers a generative, falsifiable, and interpretable structure to reconceptualize cognitive states through the lens of symbolic entropy. By embedding diversity, instability, and transformation within a unified topological grammar, this work contributes to the formal modeling of mind as a structured entropic system—opening new paths for interdisciplinary research across cognitive science, psychiatry, and neurocomputational theory.

Methods

Methods

* Model Equations

We model cognitive dynamics as trajectories $\gamma(t) = \big(\alpha(t), \kappa(t), E_r(t)\big)$ on a three-dimensional symbolic manifold $\mathbb{S}$. The governing ordinary differential equations are

$$\dot{\alpha} = \underbrace{K_E \frac{E_r}{E_r + \theta_E} \; - \; K_\kappa \, \kappa \, \alpha \; - \; \gamma_\alpha \, \alpha^3}_{\phi_\alpha(\alpha,\kappa,E_r)} - \eta_\alpha \, \alpha, \tag{1}$$

$$\dot{\kappa} = \underbrace{a \, \kappa \; - \; b \, \kappa^3 \; + \; U \, \alpha \; - \; V \, E_r}_{\phi_\kappa(\alpha,\kappa,E_r)} - \eta_\kappa \, \kappa, \tag{2}$$

$$\dot{E}_r = \underbrace{W \; - \; X \, \alpha \; - \; Y \, \kappa}_{\phi_r(\alpha,\kappa,E_r)} - \eta_r \, E_r + \xi(t), \tag{3}$$

where the η_i are linear dissipation constants, K_E–Y are non-linear gain parameters, and $\xi(t)$ is Gaussian white noise with zero mean and variance σ_ξ^2.

* Simulation Procedure

We solved the stochastic system using the Euler–Maruyama integrator ($\Delta t = 10^{-2}$, $T = 200$ a.u., seed = 42). All code (Python 3) is provided in the repository under `code/simulate_collapse.py`; the core loop is reproduced in Supplementary Algorithm 1 for transparency. Figures and were generated with Matplotlib; each PNG is 1200×800 px (300dpi).

* Stability Analysis

Nullclines were obtained analytically; Jacobian eigenvalues were computed numerically at each fixed point. A drive-entropy bifurcation was mapped by slowly ramping W (0.21.2) and recording asymptotic states (Fig.). Full eigen-spectra and Lyapunov exponents are reported in Supplementary Table S1.

* Empirical Mapping

Table 1 summarises putative empirical proxies:

* Parameter Values

A complete list of parameter symbols, default values and empirical rationale is provided in Extended Data Table 1. Key defaults used in all main-text simulations are:

Parameter (unit)	Default	Role
K_E (—)	2.0	Entropic drive on α
K_κ (—)	0.5	Inhibition of α by κ
a, b (—)	1.5, 1.0	Double-well potential for κ
U, V (—)	1.0, 2.0	Cross-coupling gains
W (bits s^{-1})	0.5	Basal entropy production
σ_ξ (bits s$^{-1/2}$)	0.1	Noise intensity in E_r

* Code and Data Availability

All simulation scripts, raw trajectories, plotting notebooks and figure source files are openly available at `https://github.com/agourakis82/entropic-symbolic-society` (commit 290f158) and archived on Zenodo (10.5281/zenodo.16682785).

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Data availability

All data (simulation scripts and generated output) are freely available in the public GitHub repository `https://github.com/agourakis82/entropic-symbolic-society` and archived under Zenodo DOI 10.5281/zenodo.16682785.

Code availability

Custom Python code used in the present study is provided alongside the data repository.

AI assistance

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Author contributions

D.C.A. conceived the study, developed the theoretical framework, performed simulations, analysed the results and wrote the manuscript.

Competing interests

The author declares no competing interests.

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Figure 1 **Collapse dynamics.** Time-series of α, κ and E_r before and after a step-increase in entropic drive (W) at $t = 100$.

Figure 2 **Bifurcation diagram.** Stationary values of α, κ and E_r as a function of basal entropic drive W. A saddle-node transition occurs at $W_c \approx 1.1$.

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Table 1: Proposed mappings between model variables and measurable biomarkers.

Variable	Primary proxy	Supp
α (symbolic anchoring)	Semantic coherence of free speech; DMN–hippocampus co-activation	?, ?
κ (network coupling)	Global efficiency / DMN centrality; PCC α-band power	?, ?
E_r (recursive entropy)	Lempel–Ziv complexity of EEG/MEG; $\text{PCI}_{\text{TMS–EEG}}$	?, ?