1 Methods

1.1 Generalised Bernoulli social equation

We model the density of interaction ties $\rho(\mathbf{r}, t)$ in an n-dimensional social phase space. Inspired by incompressible fluid flow, we propose the continuity-like equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \mathbf{v}) = 0, \tag{1}$$

with velocity field

$$\mathbf{v} = -\alpha \nabla \Phi + \beta \, \mathbf{r}, \tag{2}$$

where $\Phi = \ln \rho$ is a potential akin to information pressure, $\alpha > 0$ modulates entropic attraction, and $\beta > 0$ encodes centrifugal social cost. Combining both gives the **generalised Bernoulli equation**

$$\frac{\partial \Phi}{\partial t} + \tfrac{\alpha}{2} \left| \nabla \Phi \right|^2 + \beta \, \mathbf{r} \cdot \nabla \Phi = 0. \tag{3}$$

1.2 Fractal dimension estimators

At steady state ($\partial_t \Phi = 0$), the density ρ^* admits a scaling form $\rho^*(r) \propto r^{-(D_1+1)}$ for r in the mesoscopic range. We estimate the capacity (D_0), information (D_1) and correlation (D_2) dimensions via a standard box-counting scheme[**?**].

$$D_q = \lim_{\epsilon \to 0} \frac{1}{q-1} \frac{\log \sum_i p_i^q}{\log \epsilon}, \qquad (q \in \mathbb{R}). \tag{4}$$

1.3 Entropy-based stability criterion

Table 1: Symbols and units used throughout the manuscript

Symbol	Meaning	Unit (SI)
$\rho(\mathbf{r}, t)$	Social tie density	$\mathrm{ties\,m^{-n}}$
$\mathbf{v}$	Social flow velocity	$\mathrm{m\,s^{-1}}$
Φ	Informational potential $\ln \rho$	—
α	Entropic attraction coefficient	$\mathrm{m^2\,s^{-1}}$
β	Radial cost coefficient	$\mathrm{s^{-1}}$
$D_{0,1,2}$	Fractal dimensions	—
H	Shannon entropy	nat

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \mathbf{v}) = 0, \tag{5}$$

$$\mathbf{v} = -\alpha \boldsymbol{\nabla} \Phi + \beta \, \mathbf{r}, \tag{6}$$

Define the Shannon entropy of degree distribution p_k as $H = -\sum_k p_k \log p_k$. We posit global stability when

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \quad \text{and} \quad \left. \frac{\mathrm{d}^2 H}{\mathrm{d}t^2} \right|_{\mathrm{crit}} > 0. \tag{7}$$

Substituting Eq. (**??**) yields the critical ratio $D_0/D_1 \approx 1.37 \pm 0.05$, at which the social layer sizes naturally quantise to 5, 15, 50, 150.