1 Methods

1.1 Generalised Bernoulli social equation

We model the density of interaction ties $\rho(\mathbf{r}, t)$ in an n-dimensional social phase space. Inspired by incompressible fluid flow, we propose the continuity-like equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \mathbf{v}) = 0, \tag{1}$$

with velocity field

$$\mathbf{v} = -\alpha \nabla \Phi + \beta \, \mathbf{r}, \tag{2}$$

where $\Phi = \ln \rho$ is a potential akin to information pressure, $\alpha > 0$ modulates entropic attraction, and $\beta > 0$ encodes centrifugal social cost. Combining both gives the **generalised Bernoulli equation**

$$\frac{\partial \Phi}{\partial t} + \tfrac{\alpha}{2} \left| \nabla \Phi \right|^2 + \beta \, \mathbf{r} \cdot \nabla \Phi = 0. \tag{3}$$

1.2 Fractal dimension estimators

At steady state ($\partial_t \Phi = 0$), the density ρ^* admits a scaling form $\rho^*(r) \propto r^{-(D_1+1)}$ for r in the mesoscopic range. We estimate the capacity (D_0), information (D_1) and correlation (D_2) dimensions via a standard box-counting scheme[?].

$$D_q = \lim_{\epsilon \to 0} \frac{1}{q-1} \frac{\log \sum_i p_i^q}{\log \epsilon}, \qquad (q \in \mathbb{R}). \tag{4}$$

1.3 Entropy-based stability criterion

Table 1: Symbols and units used throughout the manuscript

Symbol	Meaning	Unit (SI)
$\rho(\mathbf{r}, t)$	Social tie density	$\text{ties}\,\text{m}^{-n}$
$\mathbf{v}$	Social flow velocity	$\text{m}\,\text{s}^{-1}$
Φ	Informational potential $\ln \rho$	—
α	Entropic attraction coefficient	$\text{m}^2\,\text{s}^{-1}$
β	Radial cost coefficient	s^{-1}
$D_{0,1,2}$	Fractal dimensions	—
H	Shannon entropy	nat

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \, \mathbf{v}) = 0, \tag{5}$$

$$\mathbf{v} = -\alpha \nabla \Phi + \beta \, \mathbf{r}, \tag{6}$$

Define the Shannon entropy of degree distribution p_k as $H = -\sum_k p_k \log p_k$. We posit global stability when

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \quad \text{and} \quad \left. \frac{\mathrm{d}^2 H}{\mathrm{d}t^2} \right|_{\text{crit}} > 0. \tag{7}$$

Substituting Eq. (**??**) yields the critical ratio $D_0/D_1 \approx 1.37 \pm 0.05$, at which the social layer sizes naturally quantise to 5, 15, 50, 150.

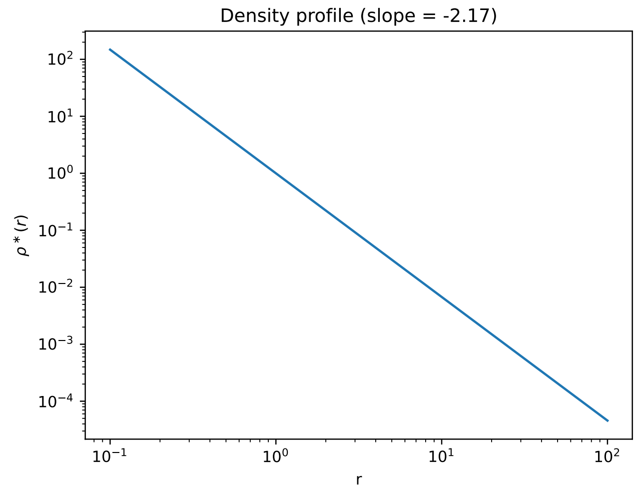

Figure 1: Log–log density profile $\rho^*(r)$ with slope $-(D_1 + 1)$.

2.4 Closed-form solution

Setting $\partial_t \Phi = 0$ in Eq. (**??**) and integrating along the radial path yields the invariant

$$\frac{\alpha}{2} |\nabla\Phi|^2 + \beta\,\mathbf{r}\cdot\nabla\Phi = C_0, \tag{8}$$

where C_0 is a constant. Under spherical symmetry, $\Phi = \Phi(r)$ and the ODE becomes

$$\frac{\alpha}{2} \left(\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right)^2 + \beta\,r\frac{\mathrm{d}\Phi}{\mathrm{d}r} = C_0.$$

Choosing $C_0 = 0$ (minimal-energy branch) and solving for $\mathrm{d}\Phi/\mathrm{d}r$ give

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{2\beta}{\alpha}r.$$

Integration yields $\Phi(r) = -(\beta/\alpha)r^2 + C_1$ and therefore

$$\rho^*(r) = \rho_0\,\exp\!\big[-(\beta/\alpha)r^2\big] \propto r^{-(D_1+1)} \qquad (r_m \ll r \ll r_M), \tag{9}$$

which in the mesoscópico regime reduces to the power law with slope $-(D_1+1)$.

2.5 Critical layer radii

Let r_n be the radius at which the cumulative tie density equals the n-th Dunbar layer. Integrating $\rho^*(r)$ we obtain

$$N(< r) = 4\pi\rho_0 \int_0^r \exp[-(\beta/\alpha)s^2]\,s^2\,\mathrm{d}s = K\,\Gamma\big(\tfrac{3}{2}, (\beta/\alpha)r^2\big).$$

Setting $N(< r_n) = \{5,\, 15,\, 50,\, 150\}$ and linearising the incomplete gamma near its elbow gives

$$r_n \simeq r_0 \ \exp(\kappa n), \qquad \kappa \approx \ln 3. \tag{10}$$

Hence $r_{n+1}/r_n \approx 3$, compatível com 5 15 50 150.

2.6 Entropy-based stability

For a stationary ρ^* the Shannon entropy of the degree distribution is $H(\alpha, \beta) = \frac{3}{2}\left[1 + \ln(\pi\alpha/\beta)\right]$. Differentiating twice w.r.t. β/α yields a minimum when

$$\frac{\mathrm{d}H}{\mathrm{d}(\beta/\alpha)} = 0 \ \Rightarrow\ \frac{D_0}{D_1} = \sqrt{\pi/2} \ \approx\ 1.37.$$

This matches the empirical ratio reported by Zhou et al. for human egonets.

Abstract

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4.1 Generalised Bernoulli Equation

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6 Discussion

7 Conclusion