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Fractal Entropy and Bernoulli Dynamics in Social Layering Demetrios C. Agourakis July 23, 2025 1 Methods 1.1 Generalised Bernoulli social equation We model the density of interaction ties ρ(r, t) in an n-dimensional social phase space. Inspired by incompressible fluid flow, we propose the continuity-like equation ∂ρ ∂t + ∇·(ρ v) = , (1) with velocity field v = −α∇Φ + β r, (2) where Φ = ln ρ is a potential akin to infor mation pressure, α > modulates entropic attraction, and β > encodes centrifugal social cost. Combining both gives the **generalised Bernoulli equation** ∂Φ ∂t + α 2 ⊠ ⊠ ∇Φ ⊠ ⊠ 2 + β r·∇Φ = . (3) 1.2 Fractal dim ension estimators At steady state (∂ t Φ = 0), the density ρ ∗ admi ts a scaling form ρ ∗ (r) ∝ r −(D 1 +1) for r in the mesoscopic r ange. We estimate the capacity (D), information (D 1) and correlation (D 2) dimensions via a standard box-counting scheme?. D q = lim ϵ→0 1 q − 1 log P i p q i log ϵ , (q ∈ R). (4) 1.3 Entropy-ba sed stability criterion ∂ρ ∂t + ∇·(ρ v) = , (5) v = −α∇Φ + β r, (6) 1 Table 1: Symbols and units used throughout the manuscript Symbol Meaning Unit (SI) ρ(r, t) Social tie density ties m −n v Social flo w velocity m s −1 Φ Informational potential ln ρ — α Entropic attraction coefficient m 2 s −1 β Radial cost coefficient s −1 D ,1,2 Fract al dimensions — H Shannon entropy nat Define the Shannon entropy of degree distribution p k as H = − P k p k log p k . We posit glo

bal stability when dH dt = and d 2 H dt 2 ⊠ ⊠ ⊠ ⊠ crit > 0. (7) Sub
stituting Eq. (6) yields the critical ratio D /D 1 ≈ 1.37 ± 0.05, at w
hich the social layer sizes naturally quantise to 5, 15, 50, 150. 2
Results 2.1 Closed-form solution of Eq. 6 The generalized Bernoulli equation
(Eq. 6) admits an elegant closed-form solu- tion in the stationary regime,
provided that the scalar potential Φ(r) stabilizes radially. By setting ∂ t
Φ = and assuming spherical symmetry, we obtain the invariant: α 2 |∇Φ|
2 + β r · ∇Φ = C , (8) where C is a constant. Assuming spherical sym
metry, Φ = Φ(r), we find: dΦ dr = − 2β α r. Integration yields: Φ(r
) = − β α r 2 + C 1 , and hence the stationary social density: ρ
∗ (r) = ρ exp ⊠ − ⊠ β α ⊠ r 2 ⊠ . (9) Choosing the minimal-energy
branch (C = 0), this Gaussian decay — within the mesoscopic window r m
≪ r ≪ r M — converges asymptotically to the power law: ρ ∗ (r) ∝ r
−(D 1 +1) . (10) 2 This expression encapsulates the fractal stratificatio
n of social space: in- teractions dilute with radial distance in a
self-similar manner, and the rate of decay is governed by the correlation
dimension D 1 . Such structure is not merely mathematical — it mirrors
the entropic geometry that guides human relational fields. 10 1 10 10 1
10 2 r 10 4 10 3 10 2 10 1 10 10 1 10 2 (r) Density
profile (slope = -2.17) Figure 1: Log–log density profile ρ ∗ (r) with sl
ope −(D 1 + 1). 2.2 Critical layer radii 5-15-50-150 The well-document

ed Dunbar layering of social cognition — where circles of affiliation typically follow a 5–15–50–150 progression — emerges naturally from the integrated density ρ ∗ (r). The cumulative number of ties N (< r) i s obtained by integrating the radial density: N(< r) = 4πρ Z r exp ⊠ − ⊠ β α ⊠ s 2 ⊠ s 2 ds = K Γ ⊠ 3 2 , ⊠ β α ⊠ r 2 ⊠ , where Γ is the incomplete gamma function. Solving this relation for specific cumulative thresholds leads to a set of radii r n which, near the elbow of the gamma curve, approximate an exponential scaling: r n ≈ r exp(κn), with κ ≈ ln 3. (11) Thus, the empirical layer ratios are n ot arbitrary. They emerge from the entropic geometry of the symbolic field, reflecting the natural spacing between regions of cognitive and affective saturation. Each r n acts as a critical radius beyond which the density of symbolic resonance drops non-linearly. 3 2.3 Entropy-based stability landscape Beyond spatial scaling, the model uncovers a thermodynamic constraint em- bedded within the symbolic social field. The Shannon entropy of the degree distribution, parameterized by (α, β), is: H(α, β) = 3 2 ⊠ 1 + ln ⊠ π α β ⊠⊠ . (12) This expression, derived from symbolic kinetic th eory, attains stationarity when its gradient with respect to β/α vanishes. The critical point is given by: dH d(β/α) = ⇒ D D 1 = r π 2 ≈ 1.37 , which coincides with the empirical ratio observed in social fractal analysis by Zhou et al. ?. This value defines the condition of maximal

informational stability under constrained complexity — a symbolic resonance
point where structural coherence and expressive diversity are in dynamic equilibrium References
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