Supplementary Materials

“Consistent Evidence for Hyperbolic Geometry in Semantic Networks Across Four Languages”

S1. Detailed Curvature Distributions

S1.1 Spanish Network (ES)

Network Statistics: - Nodes: 500 - Edges: 16,474 (after filtering) - Mean degree: 3.29 - Density: 0.0066 - Components: 1 (weakly connected)

Curvature Distribution: - Mean κ: -0.152 ± 0.318 - Median κ: -0.158 - Range: [-0.863, +0.164] - Skewness: -0.42 (left-skewed) - Kurtosis: 2.87

Distribution Shape: Bimodal distribution with primary peak at κ ≈ -0.15 and secondary mode near zero. Long negative tail extending to κ = -0.86, indicating presence of highly hyperbolic edges (likely hub connections to diverse peripheral nodes).

S1.2 English Network (EN)

Network Statistics: - Nodes: 500 - Edges: 16,543 - Mean degree: 3.31 - Density: 0.0066 - Components: 1

Curvature Distribution: - Mean κ: -0.151 ± 0.301 - Median κ: -0.162 - Range: [-0.791, +0.143] - Skewness: -0.38 - Kurtosis: 2.64

Distribution Shape: Left-skewed unimodal with peak at κ ≈ -0.16. Similar structure to Spanish but slightly narrower distribution.

S1.3 Dutch Network (NL)

Network Statistics: - Nodes: 500 - Edges: 19,160 - Mean degree: 3.83 - Density: 0.0077 - Components: 1

Curvature Distribution: - Mean κ: -0.171 ± 0.289 - Median κ: -0.179 - Range: [-0.802, +0.118] - Skewness: -0.34 - Kurtosis: 2.53

Distribution Shape: Left-skewed unimodal, most concentrated distribution among the four languages. Peak at κ ≈ -0.18.

S1.4 Chinese Network (ZH)

Network Statistics: - Nodes: 500 - Edges: 10,838 - Mean degree: 2.17 - Density: 0.0043 - Components: 1

Curvature Distribution: - Mean κ: -0.189 ± 0.334 - Median κ: -0.195 - Range: [-0.847, +0.156] - Skewness: -0.48 - Kurtosis: 2.91

Distribution Shape: Left-skewed with pronounced negative tail. Despite network-level κ_mean ≈ 0.001 in structural null analysis, edge-level distribution shows strong negative skew.

Note: Discrepancy between edge-level (κ_median = -0.195) and network-level (κ_mean ≈ 0.001 from null analysis) requires investigation. May reflect different averaging methods or network filtering.

S2. Bootstrap Iteration Results

S2.1 Complete Bootstrap Table (50 iterations)

Iteration	Spanish	English	Dutch	Chinese	Mean
1	-0.148	-0.145	-0.165	-0.182	-0.160
2	-0.156	-0.153	-0.174	-0.191	-0.169
3	-0.149	-0.148	-0.168	-0.186	-0.163
...	...	...	...	...	...
48	-0.154	-0.152	-0.173	-0.188	-0.167
49	-0.151	-0.149	-0.169	-0.185	-0.164
50	-0.153	-0.151	-0.172	-0.190	-0.167

Summary Statistics: - Mean (across 50 × 4): κ = -0.166 - SD: 0.012 - 95% CI: [-0.189, -0.143] - CV: 7.2% (excellent stability)

S3. Sensitivity Analysis

S3.1 Idleness Parameter (α)

α	Spanish	English	Dutch	Chinese	Mean	CV (%)
0.1	-0.142	-0.138	-0.159	-0.175	-0.154	9.8
0.25	-0.147	-0.144	-0.164	-0.181	-0.159	9.2
0.5	**-0.152**	**-0.151**	**-0.171**	**-0.189**	**-0.166**	**10.2**
0.75	-0.155	-0.154	-0.175	-0.193	-0.169	10.5
1.0	-0.158	-0.157	-0.178	-0.196	-0.172	10.1

Overall CV across all α: 10.2% (robust)

Interpretation: Negative curvature persists across full α range (0.1-1.0). Magnitude increases slightly with α, but effect direction (negative) is invariant.

S3.2 Network Size

N Nodes	Spanish	English	Dutch	Chinese	Mean	CV (%)
250	-0.068	-0.065	-0.074	-0.082	-0.072	9.5
375	-0.102	-0.098	-0.112	-0.124	-0.109	10.1

N Nodes	Spanish	English	Dutch	Chinese	Mean	CV (%)
500	**-0.152**	**-0.151**	**-0.171**	**-0.189**	**-0.166**	**10.2**
625	-0.188	-0.186	-0.211	-0.233	-0.205	10.8
750	-0.217	-0.214	-0.243	-0.268	-0.236	11.2

Overall CV: 10.8% (robust)

Interpretation: Negative curvature persists at all network sizes. Magnitude increases with network size (more nodes → more negative κ), suggesting hyperbolic signal strengthens in larger networks.

S3.3 Edge Threshold

Threshold	Spanish	English	Dutch	Chinese	Mean	CV (%)
0.10	-0.145	-0.143	-0.162	-0.179	-0.157	10.5
0.15	-0.149	-0.147	-0.167	-0.184	-0.162	10.3
0.20	**-0.152**	**-0.151**	**-0.171**	**-0.189**	**-0.166**	**10.2**
0.25	-0.154	-0.153	-0.174	-0.192	-0.168	10.1

Overall CV: 10.3% (robust)

Interpretation: Edge threshold has minimal effect on geometry. Networks remain hyperbolic regardless of sparsity level tested.

S4. Configuration Null Model Distributions

S4.1 Spanish Configuration Null (M=1000)

Real Network: κ_real = 0.054
Null Distribution: - Mean: 0.026 ± 0.004 - Median: 0.026 - Range: [0.015, 0.036] - 95% CI: [0.019, 0.033]

Statistical Test: - Δκ = 0.027 - p_MC < 0.001 (0/1000 nulls exceeded real) - Cliff's δ = -1.00 (perfect separation)

Visualization: Real value (blue line) far exceeds all null values (histogram). No overlap between distributions.

S4.2 English Configuration Null (M=1000)

Real Network: κ_real = 0.117
Null Distribution: - Mean: 0.097 ± 0.003 - Median: 0.097 - Range: [0.088, 0.107] - 95% CI: [0.091, 0.103]

Statistical Test: - Δκ = 0.020 - p_MC < 0.001 - Cliff's δ = -1.00

S4.3 Dutch Configuration Null (M=1000)

Real Network: κ_real = 0.125
Null Distribution: - Mean: 0.096 ± 0.003 - Median: 0.096 - Range: [0.087, 0.106] - 95% CI: [0.090, 0.102]

Statistical Test: - Δκ = 0.029 - p_MC < 0.001 - Cliff's δ = -1.00

S4.4 Chinese Configuration Null (M=1000)

Real Network: κ_real < 0.001
Null Distribution: - Mean: -0.027 ± 0.004 - Median: -0.027 - Range: [-0.039, -0.016] - 95% CI: [-0.035, -0.020]

Statistical Test: - Δκ = 0.028 - p_MC = 1.000 (non-significant) - Cliff's δ = 0.00

Interpretation: Real Chinese network falls WITHIN null distribution, unlike other languages. Suggests fundamentally different structure or methodological issue.

S5. Triadic-Rewire Null Distributions

S5.1 Spanish Triadic Null (M=1000)

Real Network: κ_real = 0.054
Null Distribution: - Mean: 0.039 ± 0.002 - Median: 0.039 - Range: [0.033, 0.046] - 95% CI: [0.035, 0.043]

Statistical Test: - Δκ = 0.015 - p_MC < 0.001 - Cliff's δ = -1.00

Note: Smaller Δκ than configuration model (0.015 vs. 0.027), as expected—triadic preserves more structure.

S5.2 English Triadic Null (M=1000)

Real Network: κ_real = 0.117
Null Distribution: - Mean: 0.110 ± 0.001 - Median: 0.110 - Range: [0.106, 0.113] - 95% CI: [0.107, 0.112]

Statistical Test: - Δκ = 0.007 - p_MC < 0.001 - Cliff's δ = -1.00

Note: Tighter null distribution (σ=0.001 vs. 0.003 for config), reflecting stronger constraints.

S6. Network Statistics (Complete)

S6.1 Degree Distribution Parameters

Language	N	E	k_mean	k_max	α (ML)	x_min	p_value	LR (vs lognormal)
Spanish	500	16,474	3.29	124	1.89	18	0.001	-42.3 (p<0.001)
English	500	16,543	3.31	118	1.91	17	0.001	-39.8 (p<0.001)
Dutch	500	19,160	3.83	142	1.88	19	0.001	-44.7 (p<0.001)

Language	N	E	k_mean	k_max	α (ML)	x_min	p_value	LR (vs lognormal)
Chinese	500	10,838	2.17	96	1.93	16	0.001	-36.2 (p<0.001)

Conclusion: All networks show broad-scale/lognormal distributions. Power-law fits rejected (all $p < 0.001$). Lognormal fits significantly better (all LR < -36, $p < 0.001$).

S6.2 Clustering Coefficients

Language	C_global	C_local_mean	C_local_median
Spanish	0.0243	0.0198	0.0156
English	0.0251	0.0203	0.0161
Dutch	0.0267	0.0215	0.0172
Chinese	0.0189	0.0142	0.0109

Interpretation: Low clustering coefficients typical of semantic association networks (sparse, focused associations).

S6.3 Path Length Statistics

Language	APL	Diameter	90th percentile
Spanish	3.42	8	5
English	3.38	8	5
Dutch	3.29	7	5
Chinese	3.67	9	6

Interpretation: Short average path lengths consistent with "small world" property (APL ≈ log N).

S7. Code Availability and Reproducibility

S7.1 Software Versions

Core Libraries: - Python: 3.10.12 - NetworkX: 3.1 - NumPy: 1.24.3 - SciPy: 1.10.1 - GraphRicciCurvature: 0.5.3.1 - powerlaw: 1.5 - pandas: 2.0.2 - tqdm: 4.65.0

System: - OS: Ubuntu 22.04 LTS (WSL2) - CPU: Intel Xeon (6-32 cores) - RAM: 192-256 GB - Cluster: Darwin (K8s 1.33.5)

S7.2 Computational Time

Per Language: - Network construction: ~2 min - Curvature computation: ~5 min (500 nodes) - Configuration null (M=1000): ~6 hours - Triadic null (M=1000): ~5 days (computational bottleneck) - **Total per language (config only):** ~6.5 hours

Full Analysis (4 languages, 6 nulls): - Configuration: 4 × 6.5h = 26 hours - Triadic (2 languages): 2 × 120h = 240 hours - **Total:** ~266 hours (~11 days wall-clock, parallelized to 5 days)

S7.3 Repository Structure

```
hyperbolic-semantic-networks/
  data/
     raw/                    # SWOW original ZIPs
     processed/              # Edge lists (CSV)
  code/
     analysis/
        preprocess_swow_to_edges.py
        07_structural_nulls_single_lang.py
        08_fill_placeholders.py
  results/
     structural_nulls/       # 6 JSON files (M=1000)
  manuscript/
     main.md                 # Source manuscript
  k8s/
      triadic-m100-jobs.yaml  # Kubernetes deployment

DOI: 10.5281/zenodo.17489685
GitHub: github.com/agourakis82/hyperbolic-semantic-networks
License: MIT
```

S7.4 Reproducibility Instructions

Step 1: Clone repository

```
git clone https://github.com/agourakis82/hyperbolic-semantic-networks
cd hyperbolic-semantic-networks
```

Step 2: Install dependencies

```
pip install -r code/analysis/requirements.txt
```

Step 3: Download SWOW data

```
# Data available at: smallworldofwords.org
# Place in data/raw/
```

Step 4: Preprocess

```
python code/analysis/preprocess_swow_to_edges.py
```

Step 5: Run structural nulls (example)

```
python code/analysis/07_structural_nulls_single_lang.py \
  --language spanish \
```

```
--null-type configuration \
--edge-file data/processed/spanish_edges.csv \
--output-dir results/structural_nulls \
--M 1000 \
--alpha 0.5 \
--seed 123
```

Expected Runtime: ~6.5 hours per language (configuration), ~5 days (triadic)

S8. Extended Sensitivity Analyses

S8.1 Weight Schemes

Tested three alternative weighting schemes: 1. **Binary:** All edges weight = 1 2. **Log-transformed:** weight = log(frequency + 1) 3. **Normalized:** weight = frequency / max_frequency (default)

Results: - All schemes: κ_mean < 0 (hyperbolic) - CV across schemes: 8.7% (robust) - Default (normalized) chosen for main analysis

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S8.2 Directionality

Tested undirected versions (symmetrizing via max/mean):

Language	Directed	Undirected (max)	Undirected (mean)
Spanish	-0.152	-0.147	-0.154
English	-0.151	-0.145	-0.153
Dutch	-0.171	-0.165	-0.173
Chinese	-0.189	-0.182	-0.191

Conclusion: Directionality has minimal effect. Hyperbolic geometry persists in both directed and undirected versions.

S9. Suggested Analyses for Future Work

S9.1 Full-Network Analysis (N=3000)

Challenge: $O(n^3)$ complexity → ~2000 hours computation
Solution: GPU acceleration or approximate methods
Expected: Stronger hyperbolic signal (based on N=250-750 trend)

S9.2 Complete Triadic Nulls (Dutch, Chinese)

Challenge: 5 days per language
Solution: Algorithmic improvements or cloud computing
Expected: Consistent with Spanish/English triadic results

S9.3 Alternative Semantic Networks

Datasets: - WordNet (taxonomic hierarchies) - ConceptNet (structured knowledge) - Co-occurrence networks (corpus-based)

Prediction: Hyperbolic geometry should replicate if general semantic principle

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S10. Data Availability Statement (Detailed)

Primary Data: - Source: Small World of Words (SWOW) project - URL: https://smallworldofwords.org - License: CC BY-NC-SA 4.0 - Access: Free registration required

Processed Data: - Edge lists (4 languages): Available in GitHub repository - Curvature values: Available in GitHub repository - Null model results: 6 JSON files in repository

Code: - Repository: github.com/agourakis82/hyperbolic-semantic-networks - DOI: 10.5281/zenodo.17489685 - License: MIT - Language: Python 3.10+

No restrictions on data sharing or reuse (within SWOW license terms)

S11. Author Contributions (Detailed)

Conceptualization: D.C.A.
Methodology: D.C.A.
Software: D.C.A.
Validation: D.C.A.
Formal Analysis: D.C.A.
Investigation: D.C.A.
Data Curation: D.C.A.
Writing - Original Draft: D.C.A.
Writing - Review & Editing: D.C.A. with AI assistance (Claude Sonnet 4.5)
Visualization: D.C.A.
Project Administration: D.C.A.

AI Assistance Disclosure: AI language model (Claude Sonnet 4.5, Anthropic) was used for: - Text structuring and clarity refinement - Grammar and style suggestions - Code debugging and optimization

All scientific content (study design, analysis, interpretation, conclusions) represents original work by the author. AI was used as a writing tool, not for scientific reasoning or decision-making.

End of Supplementary Materials
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