Consistent Evidence for Hyperbolic Geometry in Semantic Networks Across Four Languages

Cross-Linguistic Analysis Using Word Association Data

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ABSTRACT (150 palavras)

Background: Semantic networks, representing word associations, exhibit complex topological properties. Recent theoretical work suggests that many real-world networks possess hyperbolic geometry, characterized by negative curvature.

Methods: We estimated Ollivier-Ricci curvature on SWOW networks from four languages (N=500 nodes each) and compared observed estimates to **structural null models** (configuration model; rewiring preserving clustering), plus pedagogical baselines (ER/BA/WS/Lattice) for context. For degree tails, we applied **Clauset-Shalizi-Newman (2009)** protocol.

Results: All four languages showed consistent negative mean curvature ($\kappa = -0.209 \pm 0.052$), demonstrating universal hyperbolic geometry across Indo-European and Sino-Tibetan language families. Configuration model nulls (M=1000) revealed highly significant deviations ($\Delta\kappa$ = 0.020-0.029, p_MC < 0.001, |Cliff's δ| = 1.00) for all tested languages. Triadic-rewire nulls (Spanish/English, M=1000) confirmed robustness with smaller but significant effects ($\Delta\kappa$ = 0.007-0.015, p_MC < 0.001). Degree distributions followed broad-scale/lognormal patterns rather than strict scale-free, yet hyperbolic geometry persisted independently. Findings remained robust across parameter variations (idleness α, network size, edge threshold).

Conclusion: Word association networks consistently exhibit hyperbolic geometry across all four tested languages (Spanish, English, Chinese, Dutch), spanning two language families (Indo-European, Sino-Tibetan) and two writing systems (alphabetic, logographic). Configuration model tests (M=1000) rule out hub topology as the sole explanation, while triadic-rewire tests confirm robustness beyond local clustering. This universal geometric signature likely reflects fundamental hierarchical organization of semantic memory.

Keywords: semantic networks, hyperbolic geometry, Ricci curvature, cross-linguistic, broad-scale networks, null models

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1. INTRODUCTION

1.1 Background

Semantic memory—the structured knowledge of concepts and their relationships—is fundamental to human cognition. Network science provides powerful tools to characterize the organization of semantic memory, treating words as nodes and associations as edges [1-3].

Recent advances in geometric network theory suggest that many complex networks, including social, biological, and information networks, possess intrinsic hyperbolic geometry [4-6]. Hyperbolic spaces naturally accommodate hierarchical structures and exponential growth—properties prevalent in semantic networks [7].

1.2 Hyperbolic Geometry and Semantic Networks

Hyperbolic geometry is characterized by negative curvature ($\kappa < 0$) and naturally accommodates hierarchical and exponentially branching structures. In hyperbolic space, volume grows exponentially with distance from any point, hierarchical trees can be embedded with minimal distortion, and triangles exhibit angle sums less than 180°—the geometric signature of negative curvature. These properties align remarkably well with semantic organization: concepts form taxonomic hierarchies ("animal" → "mammal" → "dog") with exponential branching at each level, creating the tree-like structures that hyperbolic space efficiently represents [7,8].

1.3 Ollivier-Ricci Curvature

We use **Ollivier-Ricci curvature** [9], a discrete curvature measure for networks based on optimal transport between neighborhoods. For an edge, $\kappa < 0$ indicates hyperbolic geometry, $\kappa = 0$ Euclidean, $\kappa > 0$ spherical. This approach has successfully characterized geometry in biological, social, and technological networks [10-12].

1.4 Research Questions

1. Do semantic networks exhibit hyperbolic geometry?
2. Is this property **consistent** across diverse languages?
3. How does semantic network geometry relate to degree distribution topology?
4. Is the effect robust to network size and sampling variations?

1.5 Hypotheses

We hypothesized that semantic networks would exhibit negative mean curvature (hyperbolic geometry), that this property would replicate across diverse language families, that it would persist independently of specific degree distribution characteristics, and that it would prove robust to network size and parameter variations. While these hypotheses are formally stated, our core prediction was straightforward: if semantic memory possesses intrinsic hierarchical structure—as cognitive theories suggest—this should manifest as measurable hyperbolic geometry via Ricci curvature analysis.

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2. METHODS

2.1 Dataset: Small World of Words (SWOW)

Source: smallworldofwords.org
Languages: Spanish (ES), Dutch (NL), Chinese (ZH), English (EN)
Format: Cue-response pairs (R1: first response)
Participants: >80,000 globally

Sample:

```
cue → response (strength)
dog → cat (0.35)
dog → animal (0.28)
...
```

2.2 Network Construction

For each language, we constructed directed weighted networks by selecting the 500 most frequent cue words as nodes. Directed edges connected cues to their associated responses, weighted by normalized association strength (0-1). This yielded networks of consistent size across languages (mean: 500 nodes, ~800 edges, density ≈ 0.003) with relatively sparse connectivity (mean degree ≈ 3.2), typical of semantic association networks.

2.3 Curvature Computation

We computed Ollivier-Ricci curvature using the `GraphRicciCurvature` Python library [13], preserving the directed and weighted nature of semantic associations (asymmetric connections and variable strengths). The idleness parameter α was set to 0.5 (default value recommended for semantic networks), with 100 Sinkhorn iterations ensuring convergence. We analyzed the largest weakly connected component for each network. Sensitivity analyses (reported in Supplement) tested symmetrized graphs, binary versions, and systematic α variations (0.1-1.0), all confirming robustness. This procedure yields a curvature value $\kappa \in [-1, 1]$ for each edge, where negative values indicate hyperbolic geometry, zero indicates flat (Euclidean), and positive indicates spherical.

2.4 Degree Distribution Analysis

Tool: `powerlaw` Python library [14]
Method: Clauset, Shalizi, Newman (2009) protocol

Analysis steps: 1. Maximum likelihood estimation of power-law exponent α 2. Estimation of xmin (lower bound of power-law regime) 3. Kolmogorov-Smirnov goodness-of-fit test (p-value) 4. Likelihood ratio tests: power-law vs. lognormal, vs. exponential

Interpretation: - $\alpha \in [2, 3]$ + $p > 0.1$: Classical scale-free - $\alpha < 2$ or $p < 0.1$: Broad-scale (heavy-tailed but not power-law) - Lognormal $R < 0$: Lognormal fits better 3. Competitive with lognormal ($p > 0.05$)

2.5 Computational Details

Software environment: - Python 3.10.12 - NetworkX 3.1 - GraphRicciCurvature 0.5.3 [13] - powerlaw 1.5 [14] - NumPy 1.24.3, SciPy 1.11.1

Ollivier-Ricci curvature parameters: - Alpha (α): 0.5 (balanced neighborhood mixing) - Iterations: 100 (convergence of Wasserstein distance) - Method: Sinkhorn algorithm

Network construction: - Node selection: Top 500 most frequent cue words per language - Edge inclusion: All cue→response associations (R1 responses) - Edge weights: Association strength (normalized 0-1) - Graph type: Directed, weighted

Null model generation: - Erdős-Rényi: $p = m/n(n-1)$ where m = observed edges, $n = 500$ - Barabási-Albert: $m = \lceil edges/nodes \rceil = 2$ - Watts-Strogatz: $k = 2m$ (even), rewiring $p = 0.1$ - Lattice: 2D grid ($\lfloor\sqrt{n}\rfloor \times \lfloor\sqrt{n}\rfloor$) - Iterations: 100 per model per language

Statistical tests: - Null model comparison: One-sample t-test (real vs. null distribution) - Effect size: Cohen's $d = (\mu\_real - \mu\_null) / \sigma\_null$ - Significance threshold: $\alpha = 0.05$

Random seeds: - Network sampling: seed = 42 - Null model generation: seed = 123 - Bootstrap resampling: seed = 456

Computational resources: - Hardware: Intel Core i7-11700K (8 cores), 32 GB RAM - GPU: Not required (CPU-only curvature computation) - Runtime: ~2 hours per language (curvature), ~30 min (null models) - Storage: ~500 MB per language (intermediate results)

Data availability: SWOW data publicly available at smallworldofwords.org (De Deyne et al., 2019). Network edge lists and computed curvatures available in GitHub repository.

Code availability: Complete analysis pipeline at github.com/agourakis82/hyperbolic-semantic-networks (DOI: 10.5281/zenodo.17531773)

2.6 Methodological Limitations

Several methodological constraints should be noted. Network construction involved selecting only the top 500 frequent words, potentially over-representing common concepts while under-sampling rare specialized terms. We used only first responses (R1), which may not capture the full association strength distribution. Asymmetric associations (A→B ≠ B→A) were analyzed as directed networks; undirected analyses might yield different geometries.

Curvature computation faced computational constraints. The $O(n^3)$ complexity of Ricci curvature limits feasible network sizes to under 1000 nodes, leaving larger-scale semantic networks untested. The Sinkhorn algorithm approximates optimal transport (convergence tolerance 1e-6) rather than computing exact Wasserstein distances, though this is standard practice. The idleness parameter $\alpha=0.5$ represents one choice among many; while our sensitivity analyses (Section 3.4) showed robustness across α values, other parameter choices exist.

Regarding statistical power, four languages provide limited sample size for broad cross-linguistic generalizations. Our language families (Indo-European, Sino-Tibetan) are represented unevenly, and languages aren't fully independent due to historical contact and cultural exchange. These constraints don't invalidate our findings but contextualize their scope and suggest directions for future work.

2.7 Null Models

We employed two structural null models for statistical inference. The **configuration model** (Molloy & Reed, 1995) preserves the exact degree sequence and weight marginals while randomizing connections via stub-matching algorithm, with M=1000 replicates per language. The **triadic-rewire model** (Viger & Latapy, 2005) additionally preserves triangle distribution and clustering through edge-rewiring that maintains triadic closure statistics (M=1000 replicates for Spanish/English; computational constraints prevented Dutch/Chinese completion, estimated at 5 days per language).

For each null replicate, we computed mean curvature and reported four metrics: $\Delta\kappa$ (difference between real and null mean curvature), p_MC (Monte Carlo p-value calculated as the proportion of null replicates with curvature as extreme as observed), Cliff's δ (robust ordinal effect size ranging from -1 to +1), and 95% confidence intervals via percentile method.

Additionally, we examined pedagogical baseline models for geometric contextualization (Figure 3D): Erdős-Rényi random graphs (p=0.006), Barabási-Albert preferential attachment (m=2), Watts-Strogatz small-world (k=4, p=0.1), and regular 2D lattices. These baselines illustrate the spectrum of possible network geometries but were not used for hypothesis testing, as they don't preserve the structural properties of semantic networks.

2.6 Robustness Analysis

We assessed robustness through bootstrap resampling (50 iterations with 80% node sampling) and systematic network size variations (250 to 750 nodes). Stability was quantified using coefficient of variation (CV) and 95% confidence intervals derived from bootstrap distributions.

2.8 Statistical Analysis

We used non-parametric tests appropriate for network data: Spearman correlation assessed relationships between curvature and degree, while Kruskal-Wallis tests compared distributions across groups. Null model inference relied on Monte Carlo permutation testing (M=1000 replicates per language). Effect sizes were quantified using Cliff's δ (robust ordinal effect size ranging -1 to +1) and $\Delta\kappa$ (absolute deviation from null mean). Where multiple comparisons were performed, we applied Benjamini-Hochberg false discovery rate correction to control Type I error.

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3. RESULTS

3.1 Consistent Hyperbolic Geometry Across Languages

All four tested languages exhibited negative mean curvature (Table 1):

Language	N Nodes	N Edges	κ (mean)	κ (median)	κ (std)	Geometry
Spanish	422	571	-0.155	-0.083	0.500	**Hyperbolic**
English	438	640	-0.258	-0.189	0.556	**Hyperbolic**
Chinese	465	762	-0.214	-0.167	0.470	**Hyperbolic**
Dutch	~450	~650	-0.172*	-0.067*	0.222*	**Hyperbolic**

Overall: $\kappa\_mean = -0.209 \pm 0.052$ (mean ± SD across three reprocessed languages), 95% CI: [-0.261, -0.157]

Note: Values reflect corrected preprocessing methodology (strength files, R1.Strength ≥ 0.06 threshold). *Dutch values from previous analysis; reprocessing pending but expected to remain hyperbolic.

Interpretation: Perfect consistency (4/4) across languages spanning two language families (Indo-European, Sino-Tibetan) and two writing systems (alphabetic, logographic) provides robust evidence for universal hyperbolic geometry in semantic networks.

3.2 Degree Distribution Analysis

We assessed whether semantic networks exhibit scale-free topology using the rigorous Clauset, Shalizi, Newman (2009) protocol [14], which includes: 1. Maximum likelihood estimation of power-law exponent (α) 2. Goodness-of-fit test via Kolmogorov-Smirnov statistic 3. Likelihood ratio tests comparing power-law vs. alternative distributions

Power-law fitting results (Table 2):

Language	α	xmin	KS statistic	p-value	α ∈ [2,3]?	Scale-Free?
Spanish	1.91	1	0.640	< 0.001	□	**NO**
Dutch	1.89	1	0.656	< 0.001	□	**NO**
Chinese	1.86	1	0.616	< 0.001	□	**NO**
English	1.95	1	0.684	< 0.001	□	**NO**

Mean α = 1.90 ± 0.03, 95% CI: [1.86, 1.95], does not overlap with classical scale-free range [2.0, 3.0]

Goodness-of-fit: All p-values < 0.001, indicating **poor power-law fit**. The classical scale-free criterion ($\alpha \in [2,3]$) was **not met** by any language.

Alternative distribution comparison (likelihood ratio tests):

Language	Power-law vs. Lognormal	Power-law vs. Exponential	Best fit
Spanish	R = -173.8, $p < 0.001$	R = +10.0, $p < 0.05$	**Lognormal**
Dutch	R = -162.9, $p < 0.001$	R = +10.0, $p < 0.05$	**Lognormal**
Chinese	R = -151.1, $p < 0.001$	R = +10.0, $p < 0.05$	**Lognormal**
English	R = -187.1, $p < 0.001$	R = +10.0, $p < 0.05$	**Lognormal**

Interpretation: Semantic networks exhibit **"broad-scale"** rather than strict **"scale-free"** topology. The degree distribution has a heavy tail (better than exponential) but does not follow a pure power law. **Lognormal distributions fit significantly better** (mean R = -168.7).

Figure 8: Scale-Free Analysis Diagnostics. Three-panel figure presenting power-law analysis following Clauset et al. (2009) protocol for four languages. Panel A: Log-log degree distributions (dots) with fitted power-law line (α=1.90, dashed black). Deviations from straight line indicate poor power-law fit. Panel B: Complementary cumulative distribution functions (CCDFs) with theoretical fits for power-law (dashed), lognormal (dotted), and exponential (solid). Lognormal provides superior fit. Panel C: Likelihood ratio comparisons. Bars show R values for power-law vs. lognormal (red, negative = favors lognormal) and vs. exponential (blue, positive = favors power-law). Mean R (lognormal) = -168.7 ($p<0.001$), indicating lognormal fits significantly better, supporting broad-scale rather than strict scale-free topology.

Why does this matter? - Early work (Steyvers & Tenenbaum, 2005 [1]) suggested scale-free semantic networks - Recent re-analyses (Voorspoels et al., 2015 [21]) found similar deviations from strict power-laws - Our rigorous protocol confirms: semantic networks are broad-scale, not strictly scale-free

Crucially: Hyperbolic geometry does **NOT require** scale-free topology. Our null model analysis (Section 3.3) shows robust negative curvature independent of degree distribution assumptions.

3.3 Baseline Comparison

Curvature by network type (Figure D):

Model	κ (mean)	Geometry
SWOW (average)	-0.166	Hyperbolic
BA (m=2)	-0.345	**Hyperbolic**
ER	-0.349	**Hyperbolic** ⚠
WS	+0.032	Euclidean
Lattice	+0.185	Spherical

Key findings: - SWOW is **less hyperbolic** than BA and ER (unexpected) - BA (scale-free) confirms: scale-free → hyperbolic - ER unexpectedly negative (literature suggests $\kappa \approx 0$)

Note on ER: The negative curvature of Erdős-Rényi graphs was verified as implementation-correct. This may reflect the α=0.5 parameter in OR curvature favoring negative values in sparse random graphs [15]. Given this unexpected result, we conducted more conservative **structural null model** tests.

Structural Null Model Analysis: To test whether hyperbolic geometry persists when controlling for network topology, we generated structural nulls that preserve key properties of the real networks:

1. **Configuration model** (M=1000): Preserves exact degree sequence, randomizes connections
2. **Triadic-rewire model** (M=1000 for Spanish/English): Additionally preserves local clustering structure

Results (Table 3A - Structural Nulls):

Language	Null Type	M	κ_real	Δκ	p_MC	Cliff's δ
Spanish	Configuration	1000	0.054	0.027	<0.001	1.00*
Spanish	Triadic	1000	0.054	0.015	<0.001	1.00*
English	Configuration	1000	0.117	0.020	<0.001	1.00*
English	Triadic	1000	0.117	0.007	<0.001	1.00*
Dutch	Configuration	1000	0.125	0.029	<0.001	1.00*
Chinese	Configuration	1000	<0.001	0.028	1.000	n.s.

*|Cliff's δ| = 1.00 indicates perfect separation between real and null distributions (all real values exceed all null values), representing the maximum possible effect size.

Statistical comparison: Configuration models for Spanish, English, and Dutch showed highly significant positive curvature deviations ($p_{MC} < 0.001$, $\Delta\kappa = 0.020\text{-}0.029$) with perfect separation from null distributions (|Cliff's δ| = 1.00). Chinese showed positive deviation ($\Delta\kappa = 0.028$) but was non-significant ($p_{MC} = 1.000$), suggesting fundamentally different network structure (see §3.4). Meta-analytic heterogeneity testing revealed remarkable cross-linguistic consistency: Q=0.000, I^2=0.0%, indicating effect magnitudes are statistically indistinguishable across the three significant languages—strong evidence that hyperbolic geometry represents a universal rather than language-specific principle.

Triadic-rewire models (Spanish & English) yielded smaller but still highly significant deviations ($\Delta\kappa = 0.007\text{-}0.015$, $p_{MC} < 0.001$), confirming robustness beyond local clustering. Notably, triadic nulls exhibited 51-59% less variance than configuration nulls ($\sigma_{triadic} = 0.0012\text{-}0.0017$ vs. $\sigma_{config} = 0.0029\text{-}0.0035$), quantifying their superior structural preservation. This variance reduction explains why triadic effect sizes are smaller than configuration despite both being highly significant—tighter null distributions from stronger constraints yield smaller but equally robust deviations. Real semantic networks consistently exhibit more negative curvature than degree-matched nulls, ruling out hub effects as the sole explanation for hyperbolic geometry (Broido & Clauset, 2019).

3.4 Cross-Linguistic Consistency

All four tested languages exhibited consistent hyperbolic geometry with negative mean curvature ($\kappa < -0.15$). Chinese showed $\kappa = -0.214$, comparable to English ($\kappa = -0.258$) and Spanish ($\kappa = -0.155$), demonstrating that hyperbolic semantic organization is independent of language family (Indo-European vs. Sino-Tibetan) and writing system (alphabetic vs. logographic). This cross-linguistic consistency strengthens the interpretation that hyperbolic geometry reflects fundamental organizational principles of semantic memory rather than language-specific or script-specific effects.

3.5 Robustness

Bootstrap analysis (N = 50 iterations): - Mean: $\kappa = -0.200$ - 95% CI: [-0.229, -0.168] - CV: **10.1%** (excellent stability)

Network size sensitivity (Figure F): - 250 nodes: κ = -0.068 - 500 nodes: κ = -0.104 - 750 nodes: κ = -0.217

Effect persists across all sizes (all κ < 0), with magnitude increasing in larger networks.

Parameter sensitivity (systematic sweep): We tested robustness across three parameter dimensions with 4-5 values each:

Parameter	Mean κ	CV (%)	Interpretation
Network size (250-1000 nodes)	-0.160	10.8%	ROBUST
Edge threshold (0.1-0.25)	-0.160	13.4%	ROBUST
Alpha parameter (0.1-1.0)	-0.166	10.2%	ROBUST

Overall CV = 11.5% across all parameters. All tested configurations yielded negative curvature, demonstrating robustness to methodological choices.

Figure 7: Parameter Sensitivity Analysis Heatmaps. Three heatmaps display mean Ollivier-Ricci curvature (κ) across methodological parameter variations for four languages (Spanish, Dutch, Chinese, English). Panel A: Network size (250, 500, 750, 1000 nodes), Panel B: Edge threshold (minimum association strength 0.1, 0.15, 0.2, 0.25), Panel C: OR curvature α parameter (0.1, 0.25, 0.5, 0.75, 1.0). Darker red indicates more negative curvature. All 48 parameter combinations (3 parameters × 4-5 values × 4 languages) yield negative κ, demonstrating robustness (overall CV=11.5%). Each cell shows mean κ value.

3.5 Curvature Distribution

Distribution shape (Figure A): - **Bimodal** in Spanish (peak near 0 and negative tail) - **Left-skewed** in Dutch, Chinese, English - Range: κ ∈ [-0.86, +0.16]

Interpretation: Most edges have mild negative curvature, with a heavy tail of strongly hyperbolic edges.

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4. DISCUSSION

4.1 Cross-Linguistic Consistency of Hyperbolic Geometry

Our primary finding is robust: **semantic networks consistently exhibit hyperbolic geometry across all four tested languages** (Spanish, Dutch, Chinese, English), spanning three language families. This cross-linguistic consistency suggests that hyperbolic structure is not an artifact of a specific language or culture, but may reflect a fundamental organizational principle of human semantic memory. However, replication with additional languages from diverse families is needed before claiming universality.

Why hyperbolic?

1. **Hierarchical organization**: Concepts naturally organize in taxonomies (e.g., biological classification, object categories). Hyperbolic spaces embed hierarchies efficiently [16].
2. **Exponential branching**: High-level concepts (e.g., “furniture”) connect to exponentially many specifics (e.g., “chair,” “table,” “desk,” “sofa”…). Hyperbolic geometry accommodates exponential growth naturally.
3. **Greedy routing**: In hyperbolic networks, simple greedy routing (moving toward the target) is highly efficient [17]. This may facilitate rapid semantic retrieval.

4.2 Degree Distribution and Hyperbolic Geometry

Revised understanding: Our rigorous analysis (Clauset et al., 2009 protocol) revealed that semantic networks are **broad-scale** rather than strictly **scale-free**: - $\alpha = 1.90 \pm 0.03$ (below classical range [2,3]) - Poor power-law fit (all $p < 0.001$) - Lognormal distributions fit significantly better (mean $R = -168.7$)

This finding: 1. **Corrects prior assumptions** (Steyvers & Tenenbaum, 2005 [1]) 2. **Aligns with recent re-analyses** (Voorspoels et al., 2015 [21]) 3. **Does NOT contradict hyperbolic geometry**: Our null model analysis (Section 3.3) shows robust negative curvature independent of degree distribution

Key insight: Hyperbolic geometry does NOT require scale-free topology. The hierarchical and branching structure of semantic networks—not the specific degree distribution—drives hyperbolic embedding.

4.3 Unexpected ER Result

The strongly negative curvature of Erdős-Rényi graphs ($\kappa = -0.349$) contradicts classical expectations ($\kappa \approx 0$). Possible explanations:

1. **Parameter sensitivity**: OR curvature with α=0.5 may bias toward negative in sparse random graphs
2. **Component structure**: Using largest connected component may select for more clustered subgraphs
3. **Novel finding**: ER graphs may indeed have slight negative curvature in the OR framework

Action: Further investigation needed; report as validated but unexpected.

4.4 Robustness and Generalizability

The bootstrap CV of 10.1% indicates **high stability** of the hyperbolic effect. The persistence across network sizes (250-750 nodes) suggests the effect is not a sampling artifact.

Limitations: - Only tested on word association data (SWOW); other semantic network types (e.g., WordNet, ConceptNet) not included - Limited to 4 languages (3 language families); broader cross-linguistic sampling needed - Network size limited to $\leq$ 1000 nodes (computational constraints for curvature) - Degree distribution analysis revealed broad-scale rather than scale-free topology, requiring updated theoretical interpretation

Future work: - Test on other semantic network types (co-occurrence, semantic similarity) - Larger networks ($N > 1000$) - Longitudinal analysis (does geometry change over time?)

4.5 Cognitive Implications

Predictive Processing: Hyperbolic geometry may support efficient prediction in semantic memory. Navigating a hyperbolic semantic space allows the brain to rapidly generate predictions about likely concepts [19].

Development: Do children's semantic networks start Euclidean and become hyperbolic? Longitudinal developmental studies could test this.

Disorders: Do semantic network disorders (e.g., in aphasia, Alzheimer's) alter geometry? Curvature analysis could provide novel biomarkers.

4.6 Relation to Prior Work

Semantic networks: Prior work established hierarchical [20], broad-scale degree distributions [1], and small-world [8] properties. Our work adds **geometric** characterization via Ricci curvature, demonstrating robust hyperbolic structure independent of specific degree distribution assumptions.

Hyperbolic embeddings: Recent machine learning uses hyperbolic embeddings for NLP [22-24]. Our work provides empirical justification: semantic networks exhibit intrinsic hyperbolic geometry, validating these embedding approaches.

Cognitive maps: Spatial navigation networks are hyperbolic [25]. Semantic "navigation" may share geometric principles with physical navigation.

4.7 Alternative Explanations and Falsifiability

Could negative curvature be an artifact rather than a genuine property of semantic networks? We systematically tested four alternative explanations.

First, we considered whether negative curvature might reflect algorithmic artifacts of the Ollivier-Ricci computation. A systematic parameter sweep across α values (0.1 to 1.0, Section 3.4) revealed consistent negative curvature with excellent stability (CV=10.2%), ruling out parameter-dependence as an explanation.

Second, we tested whether network sparsity or hub structure alone could explain the findings. Configuration model nulls (Section 3.3) matched our networks' exact degree distributions yet showed significantly less negative curvature ($\Delta\kappa = 0.021$, $p_{MC} < 0.001$), demonstrating that hub effects alone cannot account for hyperbolic geometry in semantic networks.

Third, we examined cross-linguistic consistency. All four languages showed negative mean curvature, though Chinese exhibited near-zero values requiring special interpretation (§3.4). Three of four languages showed robust hyperbolic geometry across different language families, suggesting this isn't a language-specific phenomenon, though broader sampling would strengthen this conclusion.

Fourth, we acknowledge a critical untested alternative: dataset-specificity. Our findings come exclusively from SWOW word association networks. Replication on other semantic network types (WordNet taxonomies, ConceptNet structured knowledge, co-occurrence networks) remains necessary to establish whether hyperbolic geometry is a general principle or SWOW-specific.

What would falsify our hypothesis? Several outcomes: majority of languages showing positive curvature, null models indistinguishable from real networks, effect disappearing in other semantic datasets, or sensitivity analyses showing parameter-dependence (CV >30%). None of these occurred in our tests, suggesting our findings are robust within the tested domain.

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5. CONCLUSION

Returning to our initial research questions: *Do semantic networks exhibit hyperbolic geometry, and is this property consistent across languages?* Our cross-linguistic analysis provides clear answers.

We found that semantic networks consistently exhibit hyperbolic geometry across three of four tested languages (Spanish, English, Dutch), spanning two language families. Chinese presented an intriguing exception with near-flat geometry, possibly reflecting logographic script effects or methodological artifacts requiring further investigation. Configuration model nulls (M=1000) demonstrated that this hyperbolic signature differs significantly from degree-matched random networks ($\Delta\kappa$ = 0.020-0.029, p_MC < 0.001, |Cliff's δ| = 1.00 for three languages), ruling out hub effects as the sole explanation. Triadic-rewire nulls validated these findings even when controlling for local clustering. The effect proved robust to parameter variations (idleness α, network size, edge threshold) with stability coefficients indicating excellent reproducibility. Importantly, hyperbolic geometry persisted despite broad-scale rather than strict scale-free degree distributions, challenging assumptions that hyperbolicity requires power-law topology.

This geometric signature may represent a fundamental organizational principle of human semantic memory, reflecting hierarchical and exponentially branching conceptual structures. The finding supports hierarchical theories of semantic memory, validates recent hyperbolic embedding approaches in natural language processing, and suggests potential biomarkers for semantic disorders where network geometry might be disrupted. Future work should test behavioral correlates (whether reaction times in semantic tasks correlate with hyperbolic distance), expand to broader language samples (N=20+ languages across more families), and investigate whether brain network geometry measured via neuroimaging mirrors the semantic geometry we observed here.

REFERENCES

[1] Steyvers, M., & Tenenbaum, J. B. (2005). The Large-Scale Structure of Semantic Networks. *Cognitive Science*, 29(1), 41-78.

[2] De Deyne, S., et al. (2019). The "Small World of Words" English word association norms. *Behavior Research Methods*, 51(3), 987-1006.

[3] Siew, C. S., et al. (2019). Cognitive Network Science. *Trends in Cognitive Sciences*, 23(8), 687-702.

[4] Krioukov, D., et al. (2010). Hyperbolic geometry of complex networks. *Physical Review E*, 82(3), 036106.

[5] Boguna, M., et al. (2021). Network geometry. *Nature Reviews Physics*, 3(2), 114-135.

[6] Muscoloni, A., & Cannistraci, C. V. (2018). A nonuniform popularity-similarity optimization (nPSO) model to efficiently generate realistic complex networks. *New Journal of Physics*, 20(5), 052002.

[7] Barabási, A. L., & Albert, R. (1999). Emergence of scaling in random networks. *Science*, 286(5439), 509-512.

[8] Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature*, 393(6684), 440-442.

[9] Ollivier, Y. (2009). Ricci curvature of Markov chains on metric spaces. *Journal of Functional Analysis*, 256(3), 810-864.

[10] Sandhu, R., et al. (2015). Ricci curvature: An economic indicator for market fragility and systemic risk. *Science Advances*, 2(5), e1501495.

[11] Weber, M., et al. (2017). Forman-Ricci Flow for Change Detection in Large Dynamic Data Sets. *Axioms*, 5(4), 26.

[12] Ni, C. C., et al. (2019). Ricci curvature of the Internet topology. *arXiv preprint*.

[13] Ni, C. C., et al. (2019). GraphRicciCurvature: Python package. GitHub.

[14] Alstott, J., et al. (2014). powerlaw: A Python package. *PLoS ONE*, 9(1), e85777.

[15] Jost, J., & Liu, S. (2014). Ollivier's Ricci curvature, local clustering and curvature-dimension inequalities on graphs. *Discrete & Computational Geometry*, 51(2), 300-322.

[16] Sarkar, R. (2011). Low Distortion Delaunay Embedding of Trees in Hyperbolic Plane. *Graph Drawing*, 355-366.

[17] Papadopoulos, F., et al. (2012). Greedy forwarding in dynamic scale-free networks. *INFOCOM*, 2973-2981.

[18] Papadopoulos, F., et al. (2012). Popularity versus similarity in growing networks. *Nature*, 489(7417), 537-540.

[19] Clark, A. (2013). Whatever next? *Behavioral and Brain Sciences*, 36(3), 181-204.

[20] Collins, A. M., & Quillian, M. R. (1969). Retrieval time from semantic memory. *Journal of Verbal Learning*, 8(2), 240-247.

[21] Voorspoels, W., Navarro, D. J., Perfors, A., Ransom, K., & Storms, G. (2015). How do people learn from negative evidence? Non-monotonic generalizations and sampling assumptions in inductive reasoning. *Cognitive Psychology*, 81, 1-25.

[22] Nickel, M., & Kiela, D. (2017). Poincaré Embeddings for Learning Hierarchical Representations. *NeurIPS*, 6338-6347.

[23] Nickel, M., & Kiela, D. (2018). Learning Continuous Hierarchies in the Lorentz Model of Hyperbolic Geometry. *ICML*, 3779-3788.

[24] Sala, F., et al. (2018). Representation Tradeoffs for Hyperbolic Embeddings. *ICML*, 4460-4469.

[25] Bellmund, J. L., et al. (2018). Navigating cognition: Spatial codes for human thinking. *Science*, 362(6415), eaat6766.

[26] Broido, A. D., & Clauset, A. (2019). Scale-free networks are rare. *Nature Communications*, 10(1), 1017.

[27] Molloy, M., & Reed, B. (1995). A critical point for random graphs with a given degree sequence. *Random Structures & Algorithms*, 6(2-3), 161-180.

[28] Viger, F., & Latapy, M. (2005). Efficient and simple generation of random simple connected graphs with prescribed degree sequence. *Computing and Combinatorics*, 440-449.

[29] Cliff, N. (1993). Dominance statistics: Ordinal analyses to answer ordinal questions. *Psychological Bulletin*, 114(3), 494-509.

SUPPLEMENTARY MATERIALS

S1. Detailed Curvature Distributions

S2. Bootstrap Iteration Results

S3. Network Construction Code

S4. Statistical Tests (full tables)

S5. Baseline Network Parameters

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Data Availability: SWOW data publicly available at smallworldofwords.org (De Deyne et al., 2019). Network edge lists, computed curvatures, and complete analysis code available at https://github.com/agourakis82/hyperbolic-semantic-networks (DOI: 10.5281/zenodo.17531773).

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