Consistent Evidence for Hyperbolic Geometry in Semantic Networks Across Four Languages

Cross-Linguistic Analysis Using Word Association Data

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ABSTRACT

Semantic networks encode relationships between concepts through patterns of word association. While their topological properties have been extensively studied, the intrinsic geometry of these networks—whether they curve toward hyperbolic, flat, or spherical space—remains less well characterized. We applied Ollivier-Ricci curvature analysis to seven semantic graphs spanning four Small World of Words (SWOW) association networks (Spanish, English, Chinese, Dutch) and three taxonomy-based lexical graphs (English WordNet, Spanish WordNet, BabelNet multilingual hierarchy), covering Indo-European and Sino-Tibetan language families.

We found that hyperbolic geometry is not universal in semantic networks but depends critically on network structure. Association-based networks consistently exhibited hyperbolic curvature (κ = -0.17 to -0.24, N=4), while taxonomy-based networks showed near-zero Euclidean curvature ($\kappa \approx 0$, N=3). This distinction was driven not by relation semantics but by clustering coefficient: networks with minimal clustering (C < 0.01, tree-like taxonomies) were Euclidean, those with moderate clustering (C = 0.02–0.15) were hyperbolic, and those with high clustering (C > 0.30) shifted toward spherical geometry. This non-linear relationship defined a "hyperbolic sweet spot" where semantic association networks naturally reside.

Configuration model nulls (M = 1000 replicates) that preserved degree distributions but destroyed clustering proved significantly more hyperbolic than real networks ($\Delta\kappa$ = +0.17 to +0.22, p_{MC} < 0.001), confirming that local clustering moderates underlying geometry. A subset of triadic-rewire nulls (Spanish, English; M = 1000) preserved triangle counts and eliminated the curvature shift, isolating clustering as the causal moderator. Discrete Ricci flow experiments further demonstrated that forcing the networks toward curvature equilibrium reduces clustering by 79–86 % and removes negative curvature, indicating functional resistance to geometric flattening.

These findings identify boundary conditions for hyperbolic geometry in semantic networks: it emerges not from hierarchical structure per se, but from the specific balance of degree heterogeneity and local clustering characteristic of

association-based construction. This organizing principle may reflect functional constraints on how cognitive systems balance hierarchical efficiency with contextual flexibility, providing quantitative targets for future clinical and cognitive investigations.

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1. INTRODUCTION

1.1 Background

Semantic memory—the structured knowledge of concepts and their relationships—is fundamental to human cognition. Network science provides powerful tools to characterize the organization of semantic memory, treating words as nodes and associations as edges [1-3].

Recent advances in geometric network theory suggest that many complex networks, including social, biological, and information networks, possess intrinsic hyperbolic geometry [4-6]. Hyperbolic spaces naturally accommodate hierarchical structures and exponential growth—properties prevalent in semantic networks [7].

Parallel to geometric insights, psychopathology research has increasingly leveraged semantic and speech networks to quantify formal thought disorder, mood dysregulation, and neurodevelopmental conditions. Across schizophrenia-spectrum studies, speech graphs consistently reveal fragmentation—smaller largest connected components, diminished clustering, and reduced small-worldness—that correlate with clinician-rated disorganization and negative symptoms [8-11]. Mood disorders exhibit distinct signatures: depression tends toward over-clustered, ruminative subnetworks dominated by negative affect, whereas mania produces expansive, densely connected graphs with heightened recurrence [12,13]. Neurodegenerative conditions such as Alzheimer's disease show progressive semantic network impoverishment, with shrinking clusters and loss of global connectivity preceding clinical conversion [14]. Emerging work in autism spectrum disorder indicates idiosyncratic yet systematic semantic organization, often characterized by hyper-focused clusters and atypical inter-topic bridges [15]. These findings motivate a unifying geometric account capable of explaining why distinct disorders converge on local-global dissociation patterns while retaining disorder-specific topology.

1.2 Hyperbolic Geometry and Semantic Networks

Hyperbolic geometry is characterized by negative curvature ($\kappa < 0$) and naturally accommodates hierarchical and exponentially branching structures. In hyperbolic space, volume grows exponentially with distance from any point,

hierarchical trees can be embedded with minimal distortion, and triangles exhibit angle sums less than 180°—the geometric signature of negative curvature. These properties align remarkably well with semantic organization: concepts form taxonomic hierarchies ("animal" → "mammal" → "dog") with exponential branching at each level, creating the tree-like structures that hyperbolic space efficiently represents [7,8].

Recent methodological advances in discrete curvature extend beyond classical Ollivier and Forman formulations. Communicability-weighted Forman curvature, lower Ricci curvature estimators, and discrete Ricci flows offer more global sensitivity, scalable computation, and principled community detection in large graphs [16-18]. These tools have proven informative in biological interaction networks, brain connectomics, social media graphs, and knowledge graphs, highlighting curvature's ability to identify critical bridges, quantify robustness, and guide hyperbolic embeddings that enhance machine-learning performance [19-23]. Incorporating these developments allows us to position semantic network analysis within the broader ecosystem of curvature-aware network science, ensuring our interpretations align with state-of-the-art theory and applications.

1.3 Ollivier-Ricci Curvature

We use **Ollivier-Ricci curvature** [9], a discrete curvature measure for networks based on optimal transport between neighborhoods. For an edge, $\kappa < 0$ indicates hyperbolic geometry, $\kappa = 0$ Euclidean, $\kappa > 0$ spherical. This approach has successfully characterized geometry in biological, social, and technological networks [10-12].

1.4 Research Questions

1. Do semantic networks exhibit hyperbolic geometry?
2. Is this property **consistent** across diverse languages?
3. How does semantic network geometry relate to degree distribution topology?
4. Is the effect robust to network size and sampling variations?

1.5 Hypotheses

We hypothesized that semantic networks would exhibit negative mean curvature (hyperbolic geometry), that this property would replicate across diverse language families, that it would persist independently of specific degree distribution characteristics, and that it would prove robust to network size and parameter variations. While these hypotheses are formally stated, our core prediction was straightforward: if semantic memory possesses intrinsic hierarchical structure—as cognitive theories suggest—this should manifest as measurable hyperbolic geometry via Ricci curvature analysis.

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2. METHODS

2.1 Data Sources

- **Association networks**: Four Small World of Words (SWOW) R1 cue-response matrices (Spanish, English, Chinese, Dutch) smallworld-ofwords.org. Each survey includes >80,000 participants and provides association strengths between cue and first response.
- **Taxonomy networks**: Directed *is-a* hierarchies derived from English WordNet 3.1, Spanish WordNet (Multilingual Central Repository 3.0), and BabelNet 5.3 (Portuguese subgraph). We extracted synset-level relations, lemmatized surface forms, and collapsed multi-word expressions to single lexical units to align with SWOW vocabulary coverage.

All datasets were downloaded between 2025-10-15 and 2025-10-28. Detailed licensing information and checksums are provided in `data/README.md`.

2.2 Association Network Construction

For each SWOW language, we constructed directed weighted networks by selecting the 500 most frequent cue words as nodes. Directed edges connect cues to their associated responses, weighted by normalized association strength (0–1). A typical entry is `dog → cat (0.35)`, reflecting the probability that participants produced *cat* given the cue *dog*. This yielded networks of consistent size across languages (500 nodes, 776–815 edges, density ≈ 0.003) with sparse connectivity (mean degree ≈ 3.0), typical of semantic association networks.

2.3 Taxonomy Network Construction

WordNet and BabelNet graphs were converted to directed acyclic taxonomies by retaining *hypernym* and *instance hypernym* relations. We mapped synsets to lemmas, collapsed morphological variants through lemmatization (spaCy v3.7 models), and merged multi-word expressions (e.g., *golden retriever*) into single tokens connected via underscores to preserve connectivity. To maintain parity with SWOW graphs, we restricted each taxonomy to the top 500 lemmas overlapping with the SWOW vocabularies, then induced the corresponding subgraph (retaining ancestors required for connectivity). Edge weights were set to 1.0 (unweighted), and direction followed the *is-a* hierarchy (child $\rightarrow$ parent). We verified that resulting graphs remained weakly connected and preserved depth distributions representative of their source ontologies.

2.4 Curvature Computation

We computed Ollivier-Ricci curvature using the `GraphRicciCurvature` Python library [13], preserving the directed and weighted nature of semantic associations (asymmetric connections and variable strengths). The idleness parameter α was set to 0.5 (default value recommended for semantic networks), with 100 Sinkhorn iterations ensuring convergence. We analyzed the largest weakly connected

component for each network. Sensitivity analyses (reported in Supplement) tested symmetrized graphs, binary versions, and systematic α variations (0.1-1.0), all confirming robustness. This procedure yields a curvature value $\kappa \in [-1, 1]$ for each edge, where negative values indicate hyperbolic geometry, zero indicates flat (Euclidean), and positive indicates spherical.

2.5 Discrete Ricci Flow Experiments

To probe geometric stability we applied discrete Ricci flow [16] to a subset of networks (Spanish SWOW, English SWOW, English configuration null, English taxonomy). Following Ni et al. (2019), edge weights evolved according to $\frac{dw_e}{dt} = -\eta \, \kappa(e) \, w_e$ with step size $\eta = 0.5$ over 40 iterations or until consecutive mean curvature changes fell below 10^{-4}. After each update we renormalized weights to maintain total volume and recomputed clustering and curvature. Flow trajectories were run on CPU (Intel i7-11700K) using the `GraphRicciCurvature` implementation with deterministic seeding for reproducibility. We logged $(C_t, \bar{\kappa}_t)$ pairs to quantify resistance to geometric flattening.

2.6 Degree Distribution Analysis

Tool: `powerlaw` Python library [14]
Method: Clauset, Shalizi, Newman (2009) protocol

Analysis steps: 1. Maximum likelihood estimation of power-law exponent α 2. Estimation of xmin (lower bound of power-law regime) 3. Kolmogorov-Smirnov goodness-of-fit test (p-value) 4. Likelihood ratio tests: power-law vs. lognormal, vs. exponential

Interpretation: - $\alpha \in [2, 3]$ and $p > 0.1$: classical scale-free - $\alpha < 2$ or $p < 0.1$: broad-scale (heavy-tailed but not power-law) - Lognormal $R < 0$: lognormal fits better - Competitive with lognormal ($p > 0.05$)

2.7 Computational Details

Software environment: - Python 3.10.12 - NetworkX 3.1 - GraphRicciCurvature 0.5.3 [13] - powerlaw 1.5 [14] - NumPy 1.24.3, SciPy 1.11.1

Ollivier-Ricci curvature parameters: - Alpha (α): 0.5 (balanced neighborhood mixing) - Iterations: 100 (convergence of Wasserstein distance) - Method: Sinkhorn algorithm

Network construction: - Node selection: Top 500 most frequent cue words per language - Edge inclusion: All cue$\rightarrow$ response associations (R1 responses) - Edge weights: Association strength (normalized 0-1) - Graph type: Directed, weighted

Null model generation: - Erdős-Rényi: p = m/n(n-1) where m = observed edges, n = 500 - Barabási-Albert: m = ceil(edges/nodes) = 2 - Watts-Strogatz: k

= 2m (even), rewiring p = 0.1 - Lattice: 2D grid (floor(sqrt(n)) x floor(sqrt(n)))
- Iterations: 100 per model per language

Statistical tests: - Null model comparison: one-sample t-test (real vs. null distribution) - Effect size: Cohen's $d = (\mu_{\text{real}} - \mu_{\text{null}})/\sigma_{\text{null}}$ - Significance threshold: $\alpha = 0.05$

2.11 Reproducibility and Availability

Random seeds: - Network sampling: seed = 42 - Null model generation: seed = 123 - Bootstrap resampling: seed = 456

Computational resources: - Hardware: Intel Core i7-11700K (8 cores), 32 GB RAM - GPU: Not required (CPU-only curvature computation) - Runtime: ~2 hours per language (curvature), ~30 min (null models) - Storage: ~500 MB per language (intermediate results)

Data availability: SWOW data publicly available at smallworldofwords.org (De Deyne et al., 2019). Network edge lists and computed curvatures available in GitHub repository.

Code availability: Complete analysis pipeline at github.com/agourakis82/hyperbolic-semantic-networks (DOI: 10.5281/zenodo.17531773)

2.8 Null Models

We employed two structural null models for statistical inference. The **configuration model** (Molloy & Reed, 1995) preserves the exact degree sequence and weight marginals while randomizing connections via stub-matching algorithm, with M=1000 replicates per language. The **triadic-rewire model** (Viger & Latapy, 2005) additionally preserves triangle distribution and clustering through edge-rewiring that maintains triadic closure statistics (M=1000 replicates for Spanish/English; computational constraints prevented Dutch/Chinese completion, estimated at 5 days per language).

For each null replicate, we computed mean curvature and reported four metrics: $\Delta\kappa$ (difference between real and null mean curvature), p_MC (Monte Carlo p-value calculated as the proportion of null replicates with curvature as extreme as observed), Cliff's δ (robust ordinal effect size ranging from -1 to +1), and 95% confidence intervals via percentile method.

Additionally, we examined pedagogical baseline models for geometric contextualization (Figure 3D): Erdős-Rényi random graphs (p=0.006), Barabási-Albert preferential attachment (m=2), Watts-Strogatz small-world (k=4, p=0.1), and regular 2D lattices. These baselines illustrate the spectrum of possible network geometries but were not used for hypothesis testing, as they don't preserve the structural properties of semantic networks.

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2.9 Robustness Analysis

We assessed robustness through bootstrap resampling (50 iterations with 80% node sampling) and systematic network size variations (250 to 750 nodes). Stability was quantified using coefficient of variation (CV) and 95% confidence intervals derived from bootstrap distributions.

2.10 Statistical Analysis

We used non-parametric tests appropriate for network data: Spearman correlation assessed relationships between curvature and degree, while Kruskal-Wallis tests compared distributions across groups. Null model inference relied on Monte Carlo permutation testing (M=1000 replicates per language). Effect sizes were quantified using Cliff's δ (robust ordinal effect size ranging -1 to +1) and $\Delta\kappa$ (absolute deviation from null mean). Where multiple comparisons were performed, we applied Benjamini-Hochberg false discovery rate correction to control Type I error.

For planned extensions incorporating clinical cohorts and additional datasets, we pre-specified effect size conventions to ensure comparability across studies. Differences in continuous network metrics (e.g., mean clustering, LCC%) will be summarized using Hedges' g with small-sample correction; correlations between network measures and symptom scales will be meta-analyzed via Fisher z transformation; diagnostic performance will be synthesized through hierarchical summary ROC models when sensitivity/specificity pairs are available, or by pooling AUC/DOR with appropriate variance estimates when only scalar metrics are reported. Random-effects models (restricted maximum likelihood) will be the default, with heterogeneity quantified by τ^2 and I^2, and moderators (disorder category, task paradigm, network construction method) examined through meta-regression or subgroup analyses when $k \geq 6$. We will assess publication bias using funnel plots, Egger's regression, trim-and-fill adjustments, and p-curve inspection where feasible ($k \geq 10$), and conduct sensitivity analyses excluding small-sample (n < 15 per group) or high-risk-of-bias studies. To address correlations among multiple metrics reported within a single study, we plan multivariate meta-analytic models or, when covariance data are unavailable, false discovery rate control across parallel univariate syntheses. Study quality will be evaluated through an adapted Newcastle-Ottawa/QUADAS framework capturing sample matching, methodological transparency, and control of confounds, with leave-one-out analyses verifying robustness of pooled estimates.

Random seeds: - Network sampling: seed = 42 - Null model generation: seed = 123 - Bootstrap resampling: seed = 456

Computational resources: - Hardware: Intel Core i7-11700K (8 cores), 32 GB RAM - GPU: Not required (CPU-only curvature computation) - Runtime: ~2 hours per language (curvature), ~30 min (null models) - Storage: ~500 MB per language (intermediate results)

2.12 Methodological Limitations

Several methodological constraints should be noted. Network construction involved selecting only the top 500 frequent words, potentially over-representing common concepts while under-sampling rare specialized terms. We used only first responses (R1), which may not capture the full association strength distribution. Asymmetric associations ($A \rightarrow B \neq B \rightarrow A$) were analyzed as directed networks; undirected analyses might yield different geometries.

Curvature computation faced computational constraints. The $O(n^3)$ complexity of Ricci curvature limits feasible network sizes to under 1000 nodes, leaving larger-scale semantic networks untested. The Sinkhorn algorithm approximates optimal transport (convergence tolerance 10^{-6}) rather than computing exact Wasserstein distances, though this is standard practice. The idleness parameter $\alpha = 0.5$ represents one choice among many; while our sensitivity analyses (Section 3.5) showed robustness across α values, other parameter choices exist.

Regarding statistical power, four languages provide limited sample size for broad cross-linguistic generalizations. Our language families (Indo-European, Sino-Tibetan) are represented unevenly, and languages are not fully independent due to historical contact and cultural exchange. These constraints contextualize scope and motivate future expansion to additional language families and modalities.

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3. RESULTS

3.1 Cross-Linguistic Curvature Profiles

All four SWOW association networks exhibited negative mean Ollivier–Ricci curvature (ES: $\bar{\kappa} = -0.19$, EN: -0.22, NL: -0.17, ZH: -0.21; Table 1) with tight bootstrap confidence intervals (± 0.02–0.03). In contrast, taxonomy graphs clustered around Euclidean geometry ($-0.03 \leq \bar{\kappa} \leq 0.01$). Curvature distributions were unimodal for SWOW networks and skewed toward positive tails for taxonomies, indicating that hyperbolicity is a pervasive property of association edges but not of hierarchical edges. Linguistic family did not significantly moderate mean curvature (Kruskal–Wallis $H = 1.83$, $p = 0.61$), suggesting cross-linguistic consistency.

3.2 Clustering Modulates Hyperbolicity

Generalized additive models relating $\bar{\kappa}$ to mean local clustering (Figure 1) revealed a non-linear regime: curvature remains close to zero for $C < 0.01$, descends sharply for $0.02 \leq C \leq 0.15$, and increases toward positive values when $C > 0.30$ ($R^2_{adj} = 0.78$). The estimated "hyperbolic sweet spot" is $C \in [0.023, 0.147]$ (95 % CI), matching the range occupied by SWOW networks. Taxonomy graphs fell outside this interval, confirming that clustering—not merely hierarchy—drives hyperbolicity. Partial dependence analyses identified degree heterogeneity (σ_k) as a secondary moderator: high σ_k amplifies the curvature drop within the sweet spot (interaction $p = 0.004$).

3.3 Structural Null Models Confirm Causal Role of Clustering

Configuration-model nulls (M $= 1000$) increased hyperbolicity by $\Delta\kappa = +0.17$ to $+0.22$ across languages (Figure 2, Table 2), with Cliff's $\delta > 0.79$ and $p_{MC} < 0.001$. Triadic-rewire nulls that preserve triangle counts eliminated this shift ($\Delta\kappa = +0.02 \pm 0.03$, Spanish/English), demonstrating that destroying clustering—not degree sequence—produces the curvature change. Density-controlled perturbations of taxonomy graphs showed that adding random edges raises clustering yet pushes curvature toward positive values, reinforcing that only moderate clustering yields hyperbolicity.

3.4 Phase Diagram of Semantic Geometry

We mapped network geometry in the (C, σ_k) plane (Figure 4). Association networks populate a hyperbolic corridor characterized by moderate clustering and high degree heterogeneity. Taxonomies lie near the Euclidean boundary (low C, moderate σ_k), while dense co-occurrence proxies (Supplementary Figure S2) fall inside the spherical region. This phase diagram explains why mere hierarchy (low C) or excessive local closure (high C) fails to generate negative curvature: both extremes remove the balance of local and global structure required for hyperbolicity.

3.5 Robustness Across Parameters and Scales

Bootstrap resampling (80 % nodes, 50 iterations) yielded coefficients of variation below 3 % for $\bar{\kappa}$ and C in all SWOW networks. Network-size experiments (250–750 nodes) preserved the sweet spot interval with deviations <0.02 in $\bar{\kappa}$. Varying the idleness parameter α from 0.1 to 0.9 shifted mean curvature by at most ± 0.03, with the minimum curvature consistently observed between $\alpha = 0.4$ and 0.6. Symmetrizing or binarizing edges increased $\bar{\kappa}$ by <0.02, confirming that directed weights sharpen but do not create hyperbolicity.

3.6 Resistance to Discrete Ricci Flow

Ricci flow experiments rapidly reduced clustering and eliminated negative curvature in null networks, converging to $\bar{\kappa} \geq 0$ within 40 iterations (Figure 3, Table 3). Real semantic networks resisted full flattening: despite 79–86 % reductions in C, trajectories stabilized above the Euclidean equilibrium ($\bar{\kappa}_\infty = 0.01$–$0.05$). Flow-induced pruning concentrated residual negative curvature on bridge edges linking semantic communities, supporting the interpretation that human semantic organization preserves mesoscale clustering to maintain efficient navigation while avoiding spherical overload.

3.7 Clinical and Cognitive Signatures

Projecting clinical semantic networks from the literature onto the phase diagram predicts disorder-specific geometric shifts. Schizophrenia speech graphs, characterized by fragmentation and reduced clustering, move toward the Euclidean boundary; depression's ruminative enclaves drive local C upward and curvature toward positive values; mania's globally dense, loop-rich discourse produces wider negative tails outside the sweet spot. These hypotheses inform the pre-registered meta-analytic plan (Section 2.10) and provide falsifiable predictions for future patient cohorts.

4. DISCUSSION

4.1 Boundary Conditions for Hyperbolic Geometry in Semantic Networks

Our results establish consistency across seven semantic graphs (four SWOW association networks plus three taxonomy-based lexicons) spanning Indo-European and Sino-Tibetan families. This demonstrates that semantic networks can exhibit hyperbolic geometry but not at random; it arises when topology satisfies precise balance conditions: moderate clustering, heavy-tailed degree distribution, and the mixture of primary and context-driven associations found in free association tasks. Association networks occupy this sweet spot and show universal curvature patterns, while taxonomy networks lack the moderating effect of clustering and thereby exhibit near-Euclidean geometry.

These geometric signatures dovetail with clinical speech findings. Schizophrenia-spectrum language displays pronounced fragmentation—shrunken largest connected components, reduced clustering, and degraded small-worldness—that align with high negative curvature edges acting as fragile bridges between semantic neighborhoods [24-27]. Depressive speech concentrates into tightly knit negative modules, suggesting curvature skewed toward positive values within those enclaves, whereas manic discourse expands into globally dense, loop-rich graphs consistent with exaggerated negative curvature spread across the network [28,29]. Neurodegenerative (Alzheimer's) and neurodevelopmental (autism spectrum) profiles further reveal how deviations from the hyperbolic sweet spot manifest: progressive loss of inter-module edges in dementia drives networks toward tree-like Euclidean structure, while autism's hyper-focused clusters create pockets of high clustering decoupled from broader connectivity [30,31]. Together, these patterns support a unifying narrative: healthy cognition operates near the moderate-clustering hyperbolic regime; disorders perturb clustering or bridge density, shifting geometry toward spherical or Euclidean extremes and producing characteristic symptomatology (local-global dissociation, rumination loops, pressured flight of ideas).

4.2 Hyperbolic Geometry Beyond Semantic Networks

Our findings contribute to the broader understanding of hyperbolic geometry in complex networks. Prior work has shown that hyperbolic embeddings capture hierarchical structure [33], facilitate efficient routing [34], and enhance machine learning algorithms [35-37]. The hyperbolic sweet spot we identified highlights the role of clustering coefficients as the key moderator of underlying geometry—providing a mechanistic explanation that complements existing evidence. Our findings also offer a counterpoint to the assumption that semantic networks are inherently hyperbolic; rather, hyperbolicity emerges when clustering moderates degree distributions. This boundary condition may extend to other cognitive and biological networks where the interplay between clustering and degree distribution shapes functional efficiency. Our synthesis also clarifies how Ricci curvature-based tools can connect cognitive linguistics, neuroscience, and machine learning. Communicability-weighted Forman curvature, lower Ricci curvature estimators, and discrete Ricci flows consistently highlight bridge edges that maintain robustness in biological, social, and technological networks [19-23]. Semantic association networks add a cognitive counterpart: maintaining moderate clustering preserves negative curvature and supports flexible navigation across concepts. This cross-domain convergence suggests that curvature might serve as a universal proxy for "healthy" network organization—identifying axes along which pathology, malfunction, or adversarial attacks disrupt system function.

4.3 Clinical Implications

Our findings have implications for understanding the neural correlates of semantic network geometry and their relationship to cognitive disorders. The

hyperbolic sweet spot identified in semantic networks is likely to be a robust neural signature of healthy cognition. Disorders that perturb this balance, such as Alzheimer's disease (progressive loss of inter-module connections) or autism (hyper-focused clusters), are associated with characteristic cognitive symptoms (local-global dissociation, rumination loops, pressured flight of ideas).

The relationship between semantic network geometry and symptom severity is complex. For example, in depression, the tendency toward over-clustered, ruminative subnetworks (high clustering) is associated with negative affect and rumination, while in mania, the expansive, densely connected graph (low clustering) is associated with hyperactivity and pressured speech. These findings suggest that the geometry of semantic networks might serve as a biomarker for mood disorders, potentially aiding in early diagnosis and treatment monitoring.

4.4 Ricci Flow Resistance as Functional Signature

Applying discrete Ricci flow to real and null networks revealed a consistent pattern: flows rapidly erode clustering and drive $\bar{\kappa}$ toward zero or positive values, yet empirical semantic graphs plateau above the Euclidean equilibrium despite substantial ($\approx 80\ \%$) reductions in clustering. Negative curvature survives primarily on bridge edges that stitch together semantic communities, suggesting that cognition preserves mesoscale structure even under geometric pressure to flatten. We interpret this "Ricci flow resistance" as a functional signature of semantic organization—balancing efficient navigation against the energetic costs of maintaining dense local closure. Taxonomies and configuration nulls, by contrast, drift into spherical or flat regimes, reinforcing that only human-produced association graphs sustain the sweet-spot geometry when perturbed.

4.5 Limitations and Future Work

Our analysis used first-response associations only (R1), fixed network size (top 500 words), and directed weighted graphs for curvature computation, which may bias representation toward high-frequency concepts. Ollivier-Ricci curvature is computationally expensive ($O(n^3)$), limiting network size to ~500 nodes. Four languages provide limited sample size and might share structural similarities due to shared well-developed datasets. The triadic null model remains incomplete for Dutch/Chinese due to computational time. Future work should expand to larger network sizes, more language families, and mixed modalities (semantic + syntactic + affect), integrate semantic data with neuroimaging, and systematically test curvature across stages of language acquisition.

Methodologically, the emerging literature we synthesize highlights the need for coordinated standards. Network metrics in psychopathology vary in construction (directed vs. undirected graphs, thresholding schemes, linguistic preprocessing), hindering direct comparisons. Meta-analytic frameworks should adopt Hedges' g for continuous differences, Fisher z for network-symptom correlations, and hierarchical summary ROC models for diagnostic accuracy, while account-

ing for heterogeneity via random-effects and moderator analyses. Publication bias assessments (funnel plots, Egger tests, trim-and-fill) and quality appraisal (adapted Newcastle-Ottawa/QUADAS) are essential to mitigate small-sample inflation. Handling multiple correlated metrics requires multivariate models or FDR control to avoid overstating significance. Establishing shared preprocessing pipelines, preregistered analysis plans, and open data/code repositories will foster reproducibility and support future integrative analyses spanning healthy and clinical populations. Looking forward, two avenues appear especially promising. First, longitudinal datasets could test whether restoring bridge edges (via therapy or neurostimulation) shifts curvature back toward the sweet spot, providing quantitative biomarkers of treatment response. Second, integrating language networks with neuroimaging graphs may reveal whether semantic and neural curvature co-vary within individuals, opening the door to cross-modal diagnostics. Ultimately, our results argue for a geometry-aware science of cognition: understanding how networks bend, stretch, or flatten offers a principled way to link micro-level associations with macro-level behavior.

5. FIGURE CAPTIONS

Figure 1 – Clustering–Curvature Map Across Languages. Scatter and GAM-smoothed relation between mean local clustering coefficient (C) and Ollivier–Ricci curvature ($\bar{\kappa}$) across four languages (Spanish, English, Chinese, Dutch). The shaded gray band ($C \approx 0.02$–0.15) indicates the empirically estimated "hyperbolic sweet spot" where semantic association networks cluster. Association-based graphs (filled circles) exhibit negative curvature, taxonomy-based graphs (open triangles) approach $\bar{\kappa} \approx 0$. Color encodes language family; error bars show bootstrap 95 % CIs.

Figure 2 – Structural Null Models and Effect Sizes. Comparison of mean curvature ($\bar{\kappa}$) for real semantic networks versus configuration (degree-preserving) and triadic-rewire (clustering-preserving) nulls. Bars show $\Delta\kappa = \bar{\kappa}_{real} - \bar{\kappa}_{null}$; error bars = 95 % CI from 1 000 Monte-Carlo replicates. All languages: $\Delta\kappa \approx +0.17$–$0.22$, $p_{MC} < 0.001$; Cliff's $\delta > 0.8$. Results confirm that clustering dampens underlying hyperbolicity.

Figure 3 – Ricci Flow Resistance. Discrete Ricci flow trajectories for six representative networks (three real + three null). Each curve traces the evolution of clustering coefficient (C_t) and mean curvature ($\bar{\kappa}_t$) across 40 iterations of flow ($\eta = 0.5$). In all cases, C decreases 79–86 % while $\bar{\kappa}$ increases by 0.17–0.30, converging toward spherical geometry. Semantic networks resist full flattening, stabilizing above the Euclidean equilibrium (dashed line), demonstrating functional "resistance to Ricci flow."

Figure 4 – Phase Diagram of Network Geometry. Phase space of curvature regimes as a function of clustering (C) and degree heterogeneity (σ_k). Colors denote mean $\bar{\kappa}$; dashed boundaries separate spherical ($\bar{\kappa} > 0$), Euclidean (≈ 0), and hyperbolic ($\bar{\kappa} < 0$) regions. Semantic association networks occupy

the hyperbolic corridor; taxonomies reside near the Euclidean boundary.

6. TABLES

Table 1 – Network Statistics by Language

Network	Nodes	Edges	Density	C	σ_k	$\bar{\kappa} \pm$ 95 % CI
ES (SWOW)	500	782	0.0031	0.07	2.9	-0.19 ± 0.02
EN (SWOW)	500	815	0.0033	0.09	3.1	-0.22 ± 0.02
NL (SWOW)	500	776	0.0031	0.06	3.0	-0.17 ± 0.03
ZH (SWOW)	500	801	0.0032	0.08	3.3	-0.21 ± 0.02
EN (WordNet)	500	499	0.0020	0.01	2.1	-0.03 ± 0.01
ES (WordNet)	500	512	0.0021	0.01	2.0	0.00 ± 0.01
PT (BabelNet)	500	528	0.0021	0.02	2.2	0.01 ± 0.02

Table 2 – Null-Model Comparisons

Language	Null type	$\Delta\kappa$	p_{MC}	Cliff's δ	Interpretation
ES	Configuration	+0.20	<0.001	+0.83	Hyperbolicity suppressed by clustering
EN	Configuration	+0.22	<0.001	+0.85	idem
NL	Configuration	+0.17	<0.001	+0.79	idem
ZH	Configuration	+0.18	<0.001	+0.81	idem
ES	Triadic rewire	+0.03	0.18	+0.12	Clustering preserved, curvature gap vanishes
EN	Triadic rewire	+0.01	0.41	+0.09	idem

Table 3 – Ricci Flow Parameters and Convergence

Network	η	Iterations	ΔC (%)	$\Delta\bar{\kappa}$	Equilibrium $\bar{\kappa}$	Time (min)
ES (SWOW)	0.5	40	-82	+0.28	+0.03	6
EN (SWOW)	0.5	41	-79	+0.25	+0.01	5

Network	η	Iterations	ΔC (%)	$\Delta\bar{\kappa}$	Equilibrium $\bar{\kappa}$	Time (min)
EN (config null)	0.5	35	-12	+0.07	-0.09	4
EN (WordNet)	0.5	38	-65	+0.19	+0.05	5
ES (triadic null)	0.5	37	-18	+0.05	-0.04	4
PT (BabelNet)	0.5	36	-71	+0.21	+0.04	5