# OCR Further Mathematics A (2018)

## 1.04 Binomial Theorem

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# 1 Preface

## 1.1 Introduction

The *binomial theorem* is a method to quickly work out the coefficients of factorised polynomials of the form $(a + bx)^n$.

**Definition 1** (Binomial)**.** *A binomial is an expression with two terms combined with a + or −. For example,* $1 + x$, $x^2 - x^4$, $x + y$, $2a + b$ *are all binomials.*

The binomial theorem states:

$$(a + bx)^n = \sum_{r=0}^{n} \binom{n}{r} a^r (bx)^{n-r}$$

## 1.2 Notation

### 1.2.1 Summation

The $\Sigma$ is called *sigma* and means sum. The numbers above and below $\Sigma$ are the upper and lower bounds of the sum respectively. For example:

$$\sum_{r=2}^{5} r = 2 + 3 + 4 + 5 = 14$$

### 1.2.2 Choose function

The $\binom{n}{r}$ is called the *choose function* and means "how many ways can you select r objects from a total of n objects?".

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Where $x! = x \times (x-1) \times ... \times 2 \times 1$ i.e. "*x factorial*". For example:

$$\binom{9}{6} = \frac{9!}{6!3!} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$$

So there are 84 unique ways of choosing 6 objects from a selection of 9.

## 2 Example

Find the coefficient of $x^3$ in the expansion of $(2-3x)^{10}$.

*Proof.* By the binomial theorem,

$$(2-3x)^{10} = \sum_{r=0}^{10} \binom{10}{r} 2^r (-3x)^{10-r}$$

Let $C$ be the coefficient of $x^3$ in the expansion. We consider the term in the sum when $r = 7$, as this means our power of $x$ is 3. Then:

$$C = \binom{10}{7} 2^7 (-3)^3 = 120 \times 128 \times (-27) = -414720$$

□

## 3 Problems

1. Find the coefficient of $x^6$ in the expansion of $(1 + \frac{1}{2}x)^8$.
2. Write the expansion of $(2-x)^{12}$, up to and including the $x^3$ term.
3. What is the smallest positive integer $n$ such that the coefficient of $x$ in the expansion of $(2+x)^n$ is greater than 1000?