Q1. A transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ consists of a reflection in the line $y = x$ followed by a stretch by a factor of 3 in the y-direction. By considering the final positions of the standard basis vectors or otherwise, find the single matrix $\mathbf{A}$ that represents this composite transformation.

Q2. The complex number z satisfies the equation $|z - 4| = |z - 2 - 2i|$.

(a) Describe the locus of z.

(b) A second locus is defined by $\arg(z) = \alpha$. Find the value of α such that the two loci intersect at a right angle.

Q3. The transformation T is represented by the matrix $\mathbf{B} = \begin{pmatrix} 5 & -4 \\ 3 & -2 \end{pmatrix}$. Find the invariant lines of $\mathbf{B}$, and determine if there are any lines of invariant points.

Q4. Let $\mathbf{M} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ be a rotation matrix and $\mathbf{N} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ be a shear matrix.

Consider the statement: "there exist values of k and θ such that the composite transformation $\mathbf{MN}$ is equivalent to a reflection across the line $y = x$."

(a) Is this statement true? If so, find the values for k and θ. If not, justify why $\mathbf{MN}$ can never represent the reflection across the line $y = x$.

(b) $\star$ Can you suggest all the possible reflections such that there exist values of k and θ where the statement is true? If there are none, prove why.

Q5. Consider the matrix $\mathbf{C}$. It is known that the line $2x - 3y = 0$ is a line of invariant points, and the point $(1, 1)$ is mapped to the point $(3, \frac{7}{3})$.

(a) Find the matrix $\mathbf{C}$.

(b) Calculate the determinant of $\mathbf{C}$. What type of transformation does matrix $\mathbf{C}$ represent?

(c) The general form of a shear matrix is $\mathbf{S} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ where $k \neq 0$.

Prove that shear matrices must always have a line of invariant points.

Q6. The matrix $\mathbf{D} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ represents a transformation that maps the unit square in the first quadrant S to a quadrilateral with vertices at $(0, 0)$, $(3, 1)$ and $(4, 4)$.

(a) What are the values of a, b, c and d?

(b) What are the coordinates of the final vertex of the quadrilateral? Thus, state the shape of the image of S.

(c) The unit circle centered at the origin C is transformed by matrix $\mathbf{D}$. Let the image of C be C'

i. What shape is C'?

ii. What is the exact area of C'?

(d) $\star$ How many possible matrices $\mathbf{D}$ satisfy the conditions?

Q7. The matrix $\mathbf{E} = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$.

(a) Find the invariant lines of $\mathbf{E}$.

(b) $\star$ Consider a line L_0 where $L_0 : y = x + c$ with $c \neq 0$. Let $L_n = \mathbf{E}L_{n-1}$ where $L_n : y = x + k$. Find k in terms of c and n.