

- Q1.** (a) Find the first four terms, in ascending powers of x , of the expansion of $(1 + \frac{x}{2})^{12}$, simplifying each coefficient.
(b) Use your expansion with a suitable value of x to find an approximation for 1.05^{12} , giving your answer to 3 decimal places.
- Q2.** In the binomial expansion of $(k + x)^n$, where k is a positive constant and n is a positive integer, the first three terms in ascending powers of x are 64 , $960x$, and $6000x^2$. Find the values of k and n .
- Q3.** Find the term independent of x in the expansion of $(x^2 - \frac{3}{x})^9$.
- Q4.** (a) Expand $(1 - 2x)^5$ in ascending powers of x , simplifying each coefficient.
(b) Hence, find the coefficient of x^2 in the expansion of $(2 + 3x)(1 - 2x)^5$.
- Q5.** The circle C has the equation $x^2 + y^2 - 8x + 4y + 10 = 0$.
(a) The point $P(3, 1)$ lies on the circle. Find the equation of the tangent to C at point P .
(b) Find the area of the region enclosed by the x-axis, y-axis and the tangent at P .
- Q6.** (a) A circle has centre $M(2, 3)$ and passes through the point $(5, -1)$. The line l has the equation $y = 3x + k$, where k is a constant. Given that the line l is a tangent to the circle, find the two possible values of k .
(b) Find the coordinates of the point of contact between the line and the circle, using the value of k which maximises the distance between the point of contact and the origin.
- Q7.** A set of n numbers x_1, x_2, \dots, x_n has a mean of 15 and a standard deviation of 4. A new observation, $x_{n+1} = 25$, is added to the set. Given that the new mean is 16, find the value of n , and find the new standard deviation.
- Q8.** The events A and B are such that $P(A) = 0.35$ and $P(A \cup B) = 0.82$.
(a) Find $P(B)$ given that A and B are mutually exclusive.
(b) Find $P(B)$ given that A and B are independent.
- Q9.** The discrete random variable Y has a probability mass function given by:

$$P(Y = y) = \begin{cases} k(y^2 - 1) & y = 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

- (a) Show that $k = \frac{1}{26}$.

- (b) Two independent observations of Y , Y_1 and Y_2 , are taken. Find $P(Y_1 + Y_2 = 5)$.

Q10. A manufacturer of seeds claims that 80% of their sunflower seeds will germinate. A gardener suspects that the germination rate is actually lower than this. They plant 20 seeds and 13 of them germinate. Perform a hypothesis test at the 5% level of significance to determine whether there is sufficient evidence to support the gardener's suspicion.

Q11. A standard six-sided die is rolled 30 times to test if it is biased. The number 6 appears 10 times. Perform a hypothesis test at the 10% level of significance and find the critical region for this test. Based on your critical region, determine whether there is significant evidence to suggest the die is biased.