Q1. Let the function f be defined by:

$$f(x) = \frac{\sqrt{x^2 - 4x + 3}}{x - 5}$$

.

(a) Determine the largest possible domain $D \subseteq \mathbb{R}$ for $f(x)$.

(b) Find the range of f over this domain, identifying any horizontal asymptotes and the behavior of the function near $x = 5$.

(c) If we restrict the domain to $x \in [3, \infty) \setminus \{5\}$, determine if the function is injective.

Q2. The function $k(x)$ is defined as:

$$k(x) = \frac{ax + b}{cx + d}$$

where a, b, c, d are non-zero constants.

(a) Show that if $ad - bc \neq 0$, the range of k is $\mathbb{R} \setminus \{\frac{a}{c}\}$.

(b) Suppose $ad - bc = 0$, describe function k.

(c) For $k(x) = \frac{x+3}{x-2}$, find the range of the composite function $k \circ k(x)$ and determine its domain.

Q3. The function $f(x) = \frac{x+1}{x-1}$ is defined for $D = \{x \in \mathbb{R} : x \neq 1\}$.

(a) Show that $f(f(x)) = x$ for all $x \in D$.

(b) What does this result imply about the symmetry of the graph $y = f(x)$?

(c) Let $g(x) = f(f(f(x)))$. State the domain and range of g.

Q4. Consider the functions $h(x) = \sqrt{x - 2}$ for $x \geq 2$ and $k(x) = x^2 + c$, where c is a constant.

(a) Find the condition on c such that the composite function $h \circ k(x)$ can be formed for all $x \in \mathbb{R}$.

(b) Given that $c = 3$, find the range of $h(k(x))$.

(c) Determine the expression for the composite function $k(h(x))$ and state its domain.

(d) Explain why the domains of $h(k(x))$ and $k(h(x))$ differ.

Q5. Consider the following functions:

(a) $y = \sin^4(3x^2 + 5)$.

  i. Rewrite the expression in the form $y = u(v(x))^4$

  ii. Apply the chain rule twice to evaluate $\frac{dy}{dx}$

(b) $f(x) = (x^3 + 2x)e^{5x}$.

  i. Let $f(x) = u(x)v(x)$, where $u(x)$ is a polynomial. State $u(x)$ and $v(x)$.

  ii. Find $v'(x)$ by identifying it as a composite function of the form $e^{g(x)}$.

  iii. Use the product rule to evaluate $f'(x)$ and simplify your answer.

(c) $y = \left(x^4\sqrt{5x - 2}\right)^3$.

  i. Find $\frac{dy}{dx}$

(d) $h(x) = (2x^2 + 7)\cos^3(4x)$.

  i. Find $h'(x)$