Q1. Sketch the locus of points z in the Argand diagram satisfying the equation $|z-3-4i| = 5$. Determine the Cartesian equation of this locus and identify the coordinates of the points where the locus intersects the real axis.

Q2. Given that $(x-2)$ and $(x+3)$ are factors of $P(x) = x^4 + ax^3 - 11x^2 + bx + 30$, find the values of a and b. Hence, evaluate all four roots of the polynomial.

Q3. The locus of points z is defined by the equation:

$$|z-6| = 2|z-3i|$$

Show that this locus represents a circle, and find its center and radius.

Q4. A cubic equation $x^3 + px^2 + qx + r = 0$ has roots α, β, and γ. It is known that:

$$\alpha + \beta + \gamma = 2$$
$$\alpha\beta\gamma = 4$$
$$\alpha^2 + \beta^2 + \gamma^2 = 0$$

(a) How many real roots does the equation have?

(b) Determine the values of p, q, and r.

Q5. A complex number z satisfies the inequality:

$$\frac{\pi}{6} \leq \arg(z + 2 - 2i) \leq \frac{\pi}{2}$$

Illustrate this region on an Argand diagram. Calculate the minimum value of $|z|$.

Q6. The roots of the quartic equation $x^4 - 3x^3 + 5x^2 - 2 = 0$ are α, β, γ, and δ. Find the value of:

(a) $\sum \alpha^2$

(b) $\sum \frac{1}{\alpha}$

Q7. Sketch the locus of z such that:

$$\arg\left(\frac{z-4}{z-4i}\right) = \frac{\pi}{2}$$

(a) Describe the shape of this locus precisely, including any points that must be excluded from the set.

(b) $\star$ By rewriting the left-hand side of the equation, can you suggest a geometrical explanation for your answer?

Q8. The roots of the equation $x^3 - 4x^2 + 2x - 7 = 0$ are α, β, γ.

(a) Find the cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$, and $\frac{1}{\gamma}$.

(b) State the relationship between the coefficients of the new equation and the original equation. Are there any conditions for this relationship to be true?