

- Q1.** (a) Find the first four terms, in ascending powers of  $x$ , of the expansion of  $(1 + \frac{x}{2})^{12}$ , simplifying each coefficient.  
 (b) Use your expansion with a suitable value of  $x$  to find an approximation for  $1.05^{12}$ , giving your answer to 3 decimal places.
- Q2.** In the binomial expansion of  $(k + x)^n$ , where  $k$  is a positive constant and  $n$  is a positive integer, the first three terms in ascending powers of  $x$  are 64,  $960x$ , and  $6000x^2$ . Find the values of  $k$  and  $n$ .
- Q3.** Find the term independent of  $x$  in the expansion of  $(x^2 - \frac{3}{x})^9$ .
- Q4.** (a) Expand  $(1 - 2x)^5$  in ascending powers of  $x$ , simplifying each coefficient.  
 (b) Hence, find the coefficient of  $x^2$  in the expansion of  $(2 + 3x)(1 - 2x)^5$ .
- Q5.** The circle  $C$  has the equation  $x^2 + y^2 - 8x + 4y + 10 = 0$ .  
 (a) The point  $P(3, 1)$  lies on the circle. Find the equation of the tangent to  $C$  at point  $P$ .  
 (b) Find the area of the region enclosed by the x-axis, y-axis and the tangent at  $P$ .
- Q6.** (a) A circle has centre  $M(2, 3)$  and passes through the point  $(5, -1)$ . The line  $l$  has the equation  $y = 3x + k$ , where  $k$  is a constant. Given that the line  $l$  is a tangent to the circle, find the two possible values of  $k$ .  
 (b) Find the coordinates of the point of contact between the line and the circle, using the value of  $k$  which maximises the distance between the point of contact and the origin.
- Q7.** A set of  $n$  numbers  $x_1, x_2, \dots, x_n$  has a mean of 15 and a standard deviation of 4. A new observation,  $x_{n+1} = 25$ , is added to the set. Given that the new mean is 16, find the value of  $n$ , and find the new standard deviation.
- Q8.** The events  $A$  and  $B$  are such that  $P(A) = 0.35$  and  $P(A \cup B) = 0.82$ .  
 (a) Find  $P(B)$  given that  $A$  and  $B$  are mutually exclusive.  
 (b) Find  $P(B)$  given that  $A$  and  $B$  are independent.
- Q9.** The discrete random variable  $Y$  has a probability mass function given by:

$$P(Y = y) = \begin{cases} k(y^2 - 1) & y = 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

where  $k$  is a positive constant.

- (a) Show that  $k = \frac{1}{26}$ .

(b) Two independent observations of  $Y$ ,  $Y_1$  and  $Y_2$ , are taken. Find  $P(Y_1 + Y_2 = 5)$ .

- Q10.** A manufacturer of seeds claims that 80% of their sunflower seeds will germinate. A gardener suspects that the germination rate is actually lower than this. They plant 20 seeds and 13 of them germinate. Perform a hypothesis test at the 5% level of significance to determine whether there is sufficient evidence to support the gardener's suspicion.
- Q11.** A standard six-sided die is rolled 30 times to test if it is biased. The number 6 appears 10 times. Perform a hypothesis test at the 10% level of significance and find the critical region for this test. Based on your critical region, determine whether there is significant evidence to suggest the die is biased.