

# Perturbed orbital motion of regolith around Asteroids

MSc Thesis Report

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*"If you wish to make an apple pie from scratch, you must first invent the universe."*

Carl Sagan



# PREFACE

After 45 years since the day man landed on the Moon, mankind created history, yet again. For the first time ever, a spacecraft was put into an orbit around a comet and a lander was deployed to its surface. This was the Rosetta mission; launched in March 2004, the spacecraft took an astonishing 10 years to travel to the comet 67P/Churyumov-Gerasimenko, finally arriving at the comet in August 2014. This is an immense achievement for the scientists and engineers involved in the Rosetta mission because space missions to small irregular bodies in our solar system, both comets and asteroids, pose significant dynamical challenges. For scientists, missions to comets and asteroids are of great interest since in-situ exploration of these small bodies can provide insight into the birth of our Solar System and answer some very important and fundamental questions such as those about the origins of life on Earth. Now even the private space industry is interested in these small bodies, such as in mining the vast reserves of untapped natural resources within the small bodies. For a student, designing and assessing orbits around a small irregular body, and in our case an asteroid, turns out to be one of the toughest problems in astrodynamics, making it a perfect research topic for an MSc Thesis.

This report serves to be a *Literature Study* in the framework of the Master's program at the Faculty of Aerospace Engineering, Delft University of Technology. It paves way for the upcoming thesis project, where the actual research work shall be carried out. I am grateful I could do this literature study under the supervision of my supervisor Ir. Ron Noomen and with support from Dr. Jinglang Feng. Their experience in the subject matter has been of tremendous help to me. In writing this report, I have tried my very best to ensure that the material in the report is presented in a manner which is pleasant to read and understand. I hope you can gain some valuable knowledge from reading this report.

*Abhishek Agrawal*  
*Delft, August 2016*



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# LIST OF SYMBOLS

## LATIN LETTERS

Symbol	Units	Description
$a$	m	Semi-major axis of $\varepsilon_0$
$a_{ip}, b_{ip}, c_{ip}, d_{ip}$	–	Polynomial coefficients in the Ellipsoidal harmonic expansion, where $i = 1, 2, \dots$
$b$	m	Semi-major axis of $\varepsilon_0$
$B$	kg/m <sup>2</sup>	Mass-to-area ratio of the particle orbiting the binary asteroid system
$c$	m	Semi-major axis of $\varepsilon_0$
$C_{lm}$	–	Gravitational field harmonic coefficient (also called Stokes Coefficient)
$\hat{d}$	–	Unit vector joining the centre of Sun to binary asteroid barycentre and pointing away from the sun
$E_n^p$	–	Lamé's Function of the first kind
$E_e$	–	Edge dyad computed as the summation of two outer products where each outer product involves the face normal vector and the edge normal vector
$E_x, E_y, E_e$	m	Focal lengths of the triaxial ellipsoid as used in elliptic integral model for gravitational potential
$F_f$	–	Face dyad computed as the outer product of the face normal vector with itself
$\mathbf{F}_\theta^A$	N	Force acting on body $A$ due to a second body $B$
$\mathbf{F}_\theta^B$	N	Force acting on body $B$ due to a second body $A$
$F_n^p$	–	Lamé's function of the second kind
$G$	m <sup>3</sup> kg <sup>-1</sup> s <sup>-2</sup>	Universal gravitational constant
$g_{SRP}$	m/s <sup>2</sup>	Magnitude of the solar radiation pressure perturbing acceleration
$h$	m	The focal length of $\varepsilon_0$
$I_0, I_1, I_2, I_3$	–	Basic integrals used in the evaluation of the constant density ellipsoid potential
$k$	m	The focal length of $\varepsilon_0$
$K_n^p$	–	The Lamé's function of the first kind
$L_n^p$	–	The Lamé's function of the first kind
$l$	–	Degree of spherical harmonics expansion
$L_e$	–	Dimensionless per-edge factor used in polyhedron modeling
$m$	–	Order of spherical harmonics expansion
$M$	kg	Mass of a body
$M_n^p$	–	The Lamé's function of the first kind
$n$	–	Degree in Ellipsoidal Harmonic expansion
$\hat{n}_f$	–	Normal vector to a face 'f' of the polyhedron
$\hat{n}_e^f$	–	Normal vector to the edge 'e' of a face 'f' of the polyhedron

$N_n^p$	–	The Lamé's function of the first kind
$p$	–	Order, respectively, in Ellipsoidal Harmonic expansion
$P$	–	Transformation matrix mapping from body-fixed frame of body $A$ to inertial frame
$\mathbf{P}$	kg-m/s	Relative linear momentum in the full two-body problem
$P_{lm}$	–	Associated Legendre function of degree $l$ and order $m$
$\mathbf{q}$	–	6-element vector containing barycentre coordinates of polyhedron facet vertices
$r$	m	position vector magnitude
$\mathbf{r}$	$m$	position vector
$\mathbf{R}$	m	Relative position vector between the binary asteroid centroids, defined in inertial frame
$\vec{\mathbf{r}}$	m	second representation for position vector
$R_{celt}$	$m^2/s^2$	Perturbing potential from other celestial bodies
$R_{SRP}$	$m^2/s^2$	Solar Radiation Pressure perturbing potential
$S$	–	Transformation matrix mapping from body-fixed frame of body $B$ to inertial frame
$S_{lm}$	–	Gravitational field harmonic coefficient (also called Stokes Coefficient)
$s_1, s_2, s_3$	$m^2$	Ellipsoidal coordinates as used in elliptic integral model for gravitational potential
$T$	–	Transformation matrix mapping from body-fixed frame of body $B$ to body-fixed frame of body $A$
$U$	$m^2/s^2$	Gravitational potential
$u$	–	Barycentre coordinate, used in mutual potential formulation for two polyhedrons
$u$	m	Alternate variant of ellipsoidal coordinates used in elliptic integral model for gravitational potential
$U_{SE}$	$m^2/s^2$	Gravitational potential for the sphere-ellipsoid binary asteroid model
$U_s$	$m^2/s^2$	Gravitational potential for the sphere in sphere-ellipsoid binary asteroid model
$U_{e(1,2)}$	$m^2/s^2$	Gravitational potential for the ellipsoids in both the sphere-ellipsoid and ellipsoid-ellipsoid binary asteroid model
$v$	–	Barycentre coordinate, used in mutual potential formulation for two polyhedrons
$w$	–	Barycentre coordinate, used in mutual potential formulation for two polyhedrons
$w_f$	–	Dimensionless per-face factor used in polyhedron modelling
$x$	m	position in X direction
$\hat{\mathbf{x}}$	–	unit vector along the X axes in the Cartesian coordinate system
$y$	m	position in Y direction
$\hat{\mathbf{y}}$	–	unit vector along the Y axes in the Cartesian coordinate system
$z$	m	position in Z direction

$\hat{\mathbf{z}}$	–	unit vector along the Z axes in the Cartesian coordinate system
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## GREEK

Symbol	Units	Description
$\alpha$	rad	True anomaly of the Sun with respect to the binary asteroid system
$\alpha_{np}$	–	Ellipsoidal Harmonic Coefficient
$\beta$	rad	Alternate variant of ellipsoidal coordinates used in elliptic integral model for gravitational potential
$\delta$	rad	spherical coordinate called latitude
$\gamma_n^p$	–	Normalization constant for ellipsoidal harmonic expansion
$\Gamma_A$	kg-m <sup>2</sup> /s	Angular momentum of the primary asteroid expressed in its own body-fixed frame
$\Gamma_B$	kg-m <sup>2</sup> /s	Angular momentum of the secondary asteroid expressed in the primary asteroid's body-fixed frame
$\lambda$	rad	spherical coordinate called longitude
$\lambda_1, \lambda_2, \lambda_3$	m	Ellipsoidal Coordinates
$\lambda_1^{ref}$	m	Largest semi-major axis of $\varepsilon_0$
$\lambda$	rad	Alternate variant of ellipsoidal coordinates used in elliptic integral model for gravitational potential
$\mu_A$	Nm	Torque acting on body <i>A</i> as expressed in the frame fixed to body <i>A</i>
$\mu_B$	Nm	Torque acting on body <i>B</i> as expressed in the frame fixed to body <i>A</i>
$\Omega_A$	rad/s	Angular velocity of the primary asteroid expressed in its own body frame
$\Omega_B$	rad/s	Angular velocity of the secondary asteroid expressed in its own body frame
$\rho$	–	Reflectivity of a particle orbiting the binary asteroid system
$\varepsilon_0$	–	Reference ellipsoid used in the Ellipsoid Harmonics gravitational potential model



# LIST OF ACRONYMS

<b>AU</b>	Astronomical Unit
<b>CCW</b>	Counter-Clockwise
<b>MBO</b>	Main-Belt Objects
<b>NEA</b>	Near-Earth Asteroids
<b>SRP</b>	Solar Radiation Pressure
<b>STBE</b>	Solar Third Body Effect
<b>TNO</b>	Trans-Neptunian Objects





# 1

## INTRODUCTION

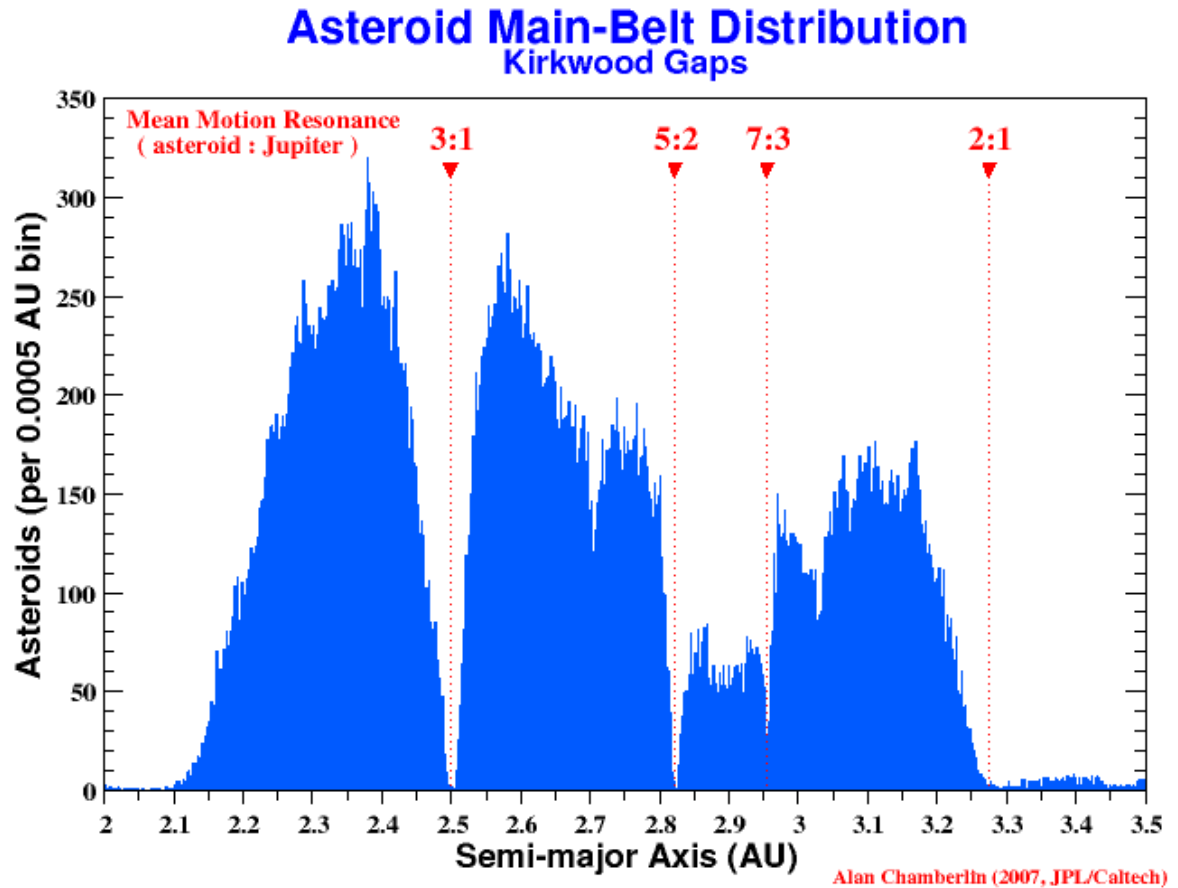
Asteroids are small rocky bodies in our solar system that are orbiting the Sun. These small bodies are basically the remnants from the process that formed the inner planets in our Solar System [2]. Asteroids are mainly found in an orbit between Jupiter and Mars and as such are classified as **MBO (Main-Belt Objects)**. These **MBO** range in size from a few meters to hundreds of kilometers, the largest one being 1 Ceres with a diameter of 948 km. A subset of the **MBO**, called the **NEA (Near-Earth Asteroids)**, are asteroids whose orbits come extremely close to, and sometimes even cross, the orbit of the Earth [1]. Other small bodies in our small system, classified as asteroids when broadly speaking, are the Trojans (small bodies captured at Jupiter's Lagrange points 4 and 5), the **TNO (Trans-Neptunian Objects)** (small bodies whose orbits around the Sun go beyond Neptune), the Centaurs (small bodies whose orbits lie in between Jupiter and Neptune) [1]. The asteroids in the main-belt tend to be more rocky in nature, however the small bodies beyond Jupiter tend to have a more icy-composition due to their relatively larger distance from the Sun [1]. A histogram plot depicting the distribution of **MBO** is shown in Figure 1.1. The gaps in the plot depict resonance in mean-motion between Jupiter and an asteroid [1].

Asteroids don't only exist as single bodies in the Solar System, but they are also found in local multi-body systems consisting of two to even three asteroids. With advanced asteroid detection methods, astrophysicists have found over 190 multiple asteroid systems in the Solar System [3]. Contrary to intuition, these multiple asteroid systems exhibit a wide diversity in terms of the size ratios of the components, their mutual orbits and separation, implicating that the individual components evolved differently over time [3]. If a multi-asteroid system consists of two or three components, which are bound gravitationally, then it is termed as *binary asteroids* or *triple asteroids* respectively. Triple asteroids are also sometimes termed as *trinary* or *ternary* [4]. Asteroid components that are not gravitationally bound but are genetically related, are termed as *asteroid pairs*. Asteroid pairs where the larger asteroid is a binary or a triple asteroid, are termed as *paired binaries* or *paired triples*, respectively. The larger component in a binary or triple asteroid system or an asteroid pair, is referred to as the *primary* and similarly the smaller component is referred to as the *secondary* [4]. Asteroids are further classified based on their dimensions and thermal properties, for which the reader should read the publication in [3].

We now know what asteroids are and the different ways in which they are found in our Solar System, but is it important to study them? There are three major, and most commonly expressed, reasons to study asteroids in our solar system, and not just from a distance such as through radar telescopes placed on Earth, but also through in-situ exploration involving spacecrafts and surface probes. These reasons are mentioned as follows.

- Asteroids are basically the material left-over from formation of planets in our Solar System. Thus, they are the perfect source to study and understand the origins of the Solar System, as





**Figure 1.1:** Histogram plot depicting distribution of semi-major axis of 156,929 main-belt asteroids, created in June 2007 [1].

they have remained in the same pristine form since the birth of the Solar System, unlike the planets which have undergone massive topographical and atmospheric changes after their formation. The asteroids can provide valuable information on the chemical composition and initial conditions which led to the formation of planets, including Earth some 4.6 billion years ago. Several scientists have also hypothesized that water and life could have been brought about on Earth through an asteroid or comet and hence exploration of these small bodies could provide a definite answer to an age old question of how life began on Earth [2].

- Asteroids have been hypothesized to have brought complex molecules to the surface of Earth that eventually resulted in life, but lately they have also been linked to the extinction of dinosaurs due to its impact with Earth. Earth is continuously bombarded with very small interplanetary material, most of which doesn't reach the surface of the Earth but gets evaporated in its atmosphere. However, every few 100 years, an asteroid spanning some tens of meter could impact Earth resulting in widespread damage, in the present case to life and property. But the impact from those will not cause the human race to extinct. But every 100,000 years or so, larger asteroids, spanning over tens of kilometer would impact the Earth, which will lead to extinction of life as we know it now. Although the probability of getting hit by an asteroid on such a large scale is low, it is still a statistical possibility and to be able to devise strategies for active deflection of such asteroids, it is imperative that we understand more of the dynamics, properties and composition of the asteroids [2].

- The third most important reason for us to study asteroids, is the fact that these small bodies are rich in raw materials or minerals. NEA can be exploited for the resources that they possess and use it to build space structures or generate fuel for spacecrafts to enable human space exploration in farther reaches of the Solar System. By studying the asteroids, we can develop methods to tap the vast reservoirs of raw materials residing in them [2].

## **1.1 RESEARCH PROBLEM**

## **1.2 STRUCTURE OF THE REPORT**



# 2

## RESULTS

### 2.1 REGOLITH LAUNCHED FROM THE LONGEST EDGE OF THE ASTEROID

The results that we'll discuss in this section pertain to the case of regolith launched from the longest edge of the asteroid, modeled as an ellipsoid.

#### 2.1.1 DYNAMICS WITHOUT SOLAR PERTURBATIONS

...to be added later...

#### 2.1.2 DYNAMICS WITH SOLAR PERTURBATIONS

In this case, the simulation accounted for perturbations from the irregular gravity field of the asteroid, the **SRP (Solar Radiation Pressure)**, and the **STBE (Solar Third Body Effect)**. Within this category, there are 4 distinct sets of simulations, each for a particle with different Area-to-Mass ratio. These are mentioned in Table 2.1.

For each of the four types of particles mentioned in Table 2.1, the initial conditions for lofting the regolith are varied in the same manner. These initial conditions are mentioned as follows. The asteroid revolves around the Sun in an equatorial circular orbit at a distance of 1.0 **AU (Astronomical Unit)**. Four different initial Solar phase angles were considered for the simulation – 45.0, 135.0, 225.0, 315.0 [deg], to account for the four different quadrants where the Sun could be with respect to the asteroid. For each case in Table 2.1, a total of 72 particles were launched from the surface of the asteroid, each in a different direction (defined using the launch declination and azimuth angles). The launch declination angle, measured from the zenith, was kept constant at 45.0 [deg] for all the particles. The launch azimuth, measured **CCW (Counter-Clockwise)** from the direction pointing to north, was varied at a resolution of 5.0 [deg] starting from 0.0 [deg] all the way up to 355.0 [deg]. Each particle was launched, in their specified direction, with different velocities ranging from 1.0 [m/s] to 16.0 [m/s] (measured with respect to the asteroid-centric rotating frame) at a resolution of 1.0 [m/s]. So basically, every combination of an initial Solar phase angle, initial launch azimuth, and initial launch velocity corresponds to a unique trajectory for a single particle of a given Area-to-

**Table 2.1:** Particle Area-to-Mass ratios

Code	Particle radius [cm]	Density [g/cm <sup>3</sup> ]	Area-to-Mass ratio [m <sup>2</sup> /kg]
LoGSP-1	1.0	3.2	0.0234
LoGSP-2	1.0	2.6	0.0288
LoGSP-3	5.0	3.2	0.0047
LoGSP-4	5.0	2.6	0.0058

Mass ratio; Thus amounting to a total of 4608 unique trajectories. The simulations were subjected to run for a maximum of 270.0 [days] and were terminated earlier if a particular trajectory resulted in escape or surface reimpact.

### CASE LOGSP-1

The density of the regolith was considered to be  $3.2 \text{ [g/cm}^3\text{]}$  with a spherical shape of radius 1.0 [cm]. Figure 2.1 gives a distribution of particles for each of the three different final fates for the regolith i.e. capture, reimpact, and escape, for different initial launch velocities and initial Solar phase angles. Irrespective of the initial Solar phase, initial launch velocities from 1.0 to 3.0 [m/s] results in particles launched in all directions to eventually reimpact the asteroid's surface. Similarly, for initial launch velocities ranging from 14.0 to 16.0 [m/s], we see that the particles always manage to escape the gravitational attraction of the asteroid. However, there is one exception to the former statement, a single particle launched with a velocity of 14.0 [m/s] at a launch azimuth of 90.0 [deg] and at an initial Solar phase angle of 315.0 [deg], reimpacts the asteroid's surface. It is interesting to note that the launch azimuth of the particle is such that it is launched in a direction that is directly opposite to the direction of rotation of the asteroid. Launch velocities from 4.0 to 13.0 [m/s] show a mixed behavior and the final fate distribution trend does not vary drastically for different initial Solar phase angles.

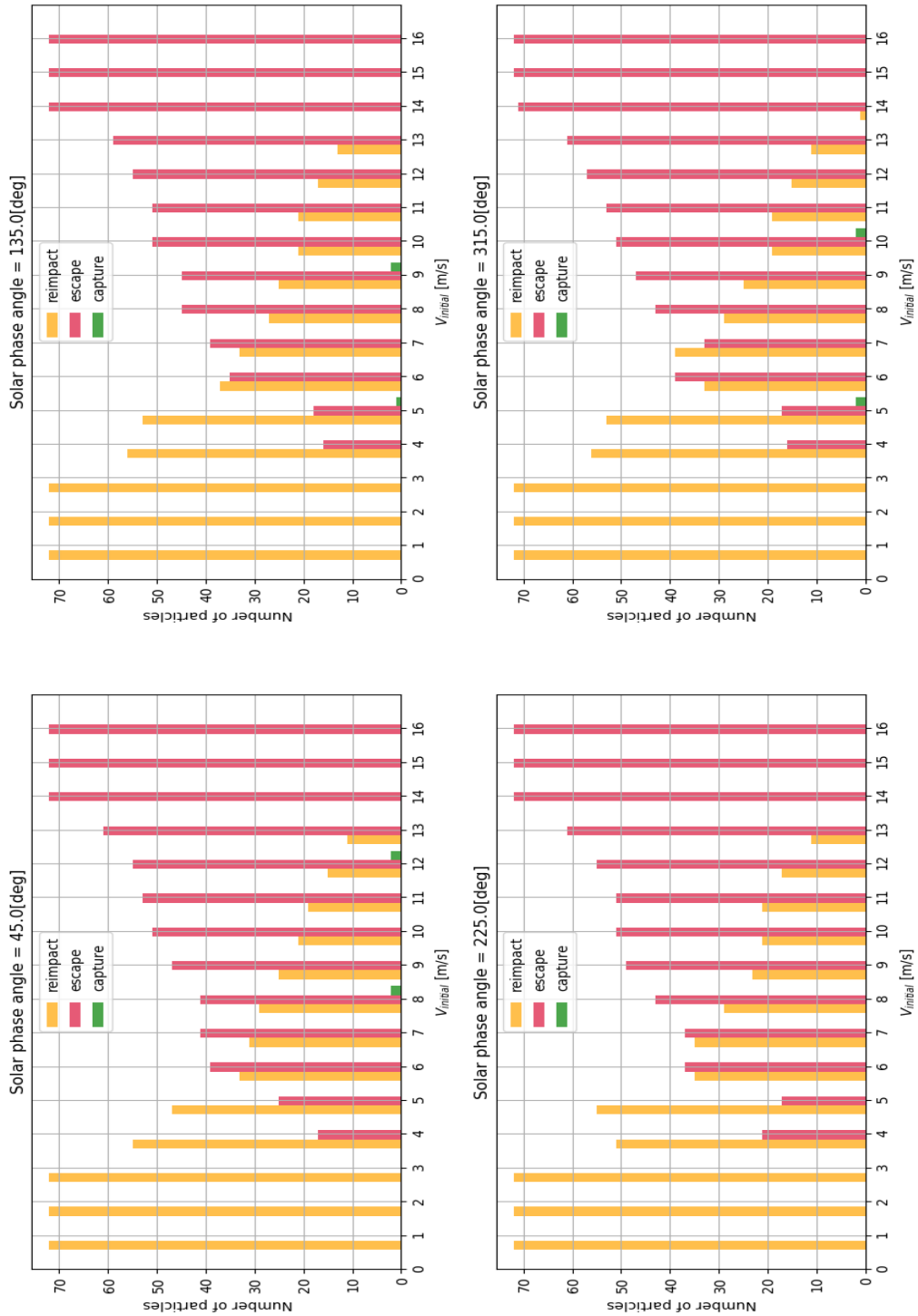
The number of capture cases is far less than those for escape and reimpact. For initial Solar phase of 225.0 [deg], there are no cases of regolith being captured in orbit around the asteroid. All capture cases, arranged in order of increasing launch azimuth angle, are listed in Table 2.2. It is interesting to note that all capture cases result from when the particle is launched in a direction which is against the direction of rotation of the asteroid, bar one exception which is case index-11 in Table 2.2. The capture cases which represent symmetry in terms of the launch azimuth angle are

**Table 2.2:** Initial conditions that resulted in temporary orbital capture of regolith around the asteroid

Index	Launch azimuth [deg]	Launch velocity [m/s]	Initial Solar phase angle [deg]
1	5.0	5.0	315.0
2	10.0	9.0	135.0
3	15.0	8.0	45.0
4	45.0	12.0	45.0
5	45.0	10.0	315.0
6	135.0	12.0	45.0
7	135.0	10.0	315.0
8	165.0	8.0	45.0
9	170.0	9.0	135.0
10	175.0	5.0	315.0
11	185.0	5.0	135.0

highlighted with the same color in Table 2.2. This symmetric behavior results from the combination of two factors. First, the Sun's motion relative to the asteroid is not in an inclined plane, and secondly, the particles are launched from the equatorial tip of the ellipsoid shaped asteroid, which is a point of symmetry on the ellipsoid.

Regolith final fate histogram, Ellipsoid longest edge

**Figure 2.1:** Histogram showing the number of particles that reimpact, escape, or get captured around the asteroid, for different initial launch velocities.



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