

Orbit Design Around Asteroids

Literature Study

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"If you wish to make an apple pie from scratch, you must first invent the universe."

Carl Sagan

PREFACE

After 45 years since the day man landed on the Moon, mankind created history, yet again. For the first time ever, a spacecraft was put into an orbit around a comet and a lander was deployed to its surface. This was the Rosetta mission; launched in March 2004, the spacecraft took an astonishing 10 years to travel to the comet 67P/Churyumov-Gerasimenko, finally arriving at the comet in August 2014. This is an immense achievement for the scientists and engineers involved in the Rosetta mission because space missions to small irregular bodies in our solar system, both comets and asteroids, pose significant dynamical challenges. For scientists, missions to comets and asteroids are of great interest since in-situ exploration of these small bodies can provide insight into the birth of our Solar System and answer some very important and fundamental questions such as those about the origins of life on Earth. Now even the private space industry is interested in these small bodies, such as in mining the vast reserves of untapped natural resources within the small bodies. For a student, designing and assessing orbits around a small irregular body, and in our case an asteroid, turns out to be one of the toughest problems in astrodynamics, making it a perfect research topic for an MSc Thesis.

This report serves to be a *Literature Study* in the framework of the Master's program at the Faculty of Aerospace Engineering, Delft University of Technology. It paves way for the upcoming thesis project, where the actual research work shall be carried out. I am grateful I could do this literature study under the supervision of my supervisor Ir. Ron Noomen and with support from Dr. Jinglang Feng. Their experience in the subject matter has been of tremendous help to me. In writing this report, I have tried my very best to ensure that the material in the report is presented in a manner which is pleasant to read and understand. I hope you can gain some valuable knowledge from reading this report.

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LIST OF SYMBOLS

LATIN LETTERS

Symbol	Units	Description
a	m	Semi-major axis of ε_0
$a_{ip}, b_{ip}, c_{ip}, d_{ip}$	–	Polynomial coefficients in the Ellipsoidal harmonic expansion, where $i = 1, 2, \dots$
b	m	Semi-major axis of ε_0
B	kg/m ²	Mass-to-area ratio of the particle orbiting the binary asteroid system
c	m	Semi-major axis of ε_0
C_{lm}	–	Gravitational field harmonic coefficient (also called Stokes Coefficient)
\hat{d}	–	Unit vector joining the centre of Sun to binary asteroid barycentre and pointing away from the sun
E_n^p	–	Lamé's Function of the first kind
E_e	–	Edge dyad computed as the summation of two outer products where each outer product involves the face normal vector and the edge normal vector
E_x, E_y, E_e	m	Focal lengths of the triaxial ellipsoid as used in elliptic integral model for gravitational potential
F_f	–	Face dyad computed as the outer product of the face normal vector with itself
\mathbf{F}_θ^A	N	Force acting on body A due to a second body B
\mathbf{F}_θ^B	N	Force acting on body B due to a second body A
F_n^p	–	Lamé's function of the second kind
G	m ³ kg ⁻¹ s ⁻²	Universal gravitational constant
g_{SRP}	m/s ²	Magnitude of the solar radiation pressure perturbing acceleration
h	m	The focal length of ε_0
I_0, I_1, I_2, I_3	–	Basic integrals used in the evaluation of the constant density ellipsoid potential
k	m	The focal length of ε_0
K_n^p	–	The Lamé's function of the first kind
L_n^p	–	The Lamé's function of the first kind
l	–	Degree of spherical harmonics expansion
L_e	–	Dimensionless per-edge factor used in polyhedron modeling
m	–	Order of spherical harmonics expansion
M	kg	Mass of a body
M_n^p	–	The Lamé's function of the first kind
n	–	Degree in Ellipsoidal Harmonic expansion
\hat{n}_f	–	Normal vector to a face 'f' of the polyhedron
\hat{n}_e^f	–	Normal vector to the edge 'e' of a face 'f' of the polyhedron

N_n^p	–	The Lamé's function of the first kind
p	–	Order, respectively, in Ellipsoidal Harmonic expansion
P	–	Transformation matrix mapping from body-fixed frame of body A to inertial frame
\mathbf{P}	kg-m/s	Relative linear momentum in the full two-body problem
P_{lm}	–	Associated Legendre function of degree l and order m
\mathbf{q}	–	6-element vector containing barycentre coordinates of polyhedron facet vertices
r	m	position vector magnitude
\mathbf{r}	m	position vector
\mathbf{R}	m	Relative position vector between the binary asteroid centroids, defined in inertial frame
$\vec{\mathbf{r}}$	m	second representation for position vector
R_{celt}	m^2/s^2	Perturbing potential from other celestial bodies
R_{SRP}	m^2/s^2	Solar Radiation Pressure perturbing potential
S	–	Transformation matrix mapping from body-fixed frame of body B to inertial frame
S_{lm}	–	Gravitational field harmonic coefficient (also called Stokes Coefficient)
s_1, s_2, s_3	m^2	Ellipsoidal coordinates as used in elliptic integral model for gravitational potential
T	–	Transformation matrix mapping from body-fixed frame of body B to body-fixed frame of body A
U	m^2/s^2	Gravitational potential
u	–	Barycentre coordinate, used in mutual potential formulation for two polyhedrons
u	m	Alternate variant of ellipsoidal coordinates used in elliptic integral model for gravitational potential
U_{SE}	m^2/s^2	Gravitational potential for the sphere-ellipsoid binary asteroid model
U_s	m^2/s^2	Gravitational potential for the sphere in sphere-ellipsoid binary asteroid model
$U_{e(1,2)}$	m^2/s^2	Gravitational potential for the ellipsoids in both the sphere-ellipsoid and ellipsoid-ellipsoid binary asteroid model
v	–	Barycentre coordinate, used in mutual potential formulation for two polyhedrons
w	–	Barycentre coordinate, used in mutual potential formulation for two polyhedrons
w_f	–	Dimensionless per-face factor used in polyhedron modelling
x	m	position in X direction
$\hat{\mathbf{x}}$	–	unit vector along the X axes in the Cartesian coordinate system
y	m	position in Y direction
$\hat{\mathbf{y}}$	–	unit vector along the Y axes in the Cartesian coordinate system
z	m	position in Z direction

$\hat{\mathbf{z}}$	–	unit vector along the Z axes in the Cartesian coordinate system
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GREEK

Symbol	Units	Description
α	rad	True anomaly of the Sun with respect to the binary asteroid system
α_{np}	–	Ellipsoidal Harmonic Coefficient
β	rad	Alternate variant of ellipsoidal coordinates used in elliptic integral model for gravitational potential
δ	rad	spherical coordinate called latitude
γ_n^p	–	Normalization constant for ellipsoidal harmonic expansion
Γ_A	kg-m ² /s	Angular momentum of the primary asteroid expressed in its own body-fixed frame
Γ_B	kg-m ² /s	Angular momentum of the secondary asteroid expressed in the primary asteroid's body-fixed frame
λ	rad	spherical coordinate called longitude
$\lambda_1, \lambda_2, \lambda_3$	m	Ellipsoidal Coordinates
λ_1^{ref}	m	Largest semi-major axis of ε_0
λ	rad	Alternate variant of ellipsoidal coordinates used in elliptic integral model for gravitational potential
μ_A	Nm	Torque acting on body <i>A</i> as expressed in the frame fixed to body <i>A</i>
μ_B	Nm	Torque acting on body <i>B</i> as expressed in the frame fixed to body <i>A</i>
Ω_A	rad/s	Angular velocity of the primary asteroid expressed in its own body frame
Ω_B	rad/s	Angular velocity of the secondary asteroid expressed in its own body frame
ρ	–	Reflectivity of a particle orbiting the binary asteroid system
ε_0	–	Reference ellipsoid used in the Ellipsoid Harmonics gravitational potential model

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INTRODUCTION

Asteroids are small rocky bodies in our solar system that are orbiting the Sun. These small bodies are basically the remnants from the process that formed the inner planets in our Solar System [2]. Asteroids are mainly found in an orbit between Jupiter and Mars and as such are classified as MBO (Main-Belt Objects). These MBO range in size from a few meters to hundreds of kilometers, the largest one being 1 Ceres with a diameter of 948 km. A subset of the MBO, called the NEA (Near-Earth Asteroids), are asteroids whose orbits come extremely close to, and sometimes even cross, the orbit of the Earth [1]. Other small bodies in our small system, classified as asteroids when broadly speaking, are the Trojans (small bodies captured at Jupiter's Lagrange points 4 and 5), the TNO (Trans-Neptunian Objects) (small bodies whose orbits around the Sun go beyond Neptune), the Centaurs (small bodies whose orbits lie in between Jupiter and Neptune) [1]. The asteroids in the main-belt tend to be more rocky in nature, however the small bodies beyond Jupiter tend to have a more icy-composition due to their relatively larger distance from the Sun [1]. A histogram plot depicting the distribution of MBO is shown in Figure 1.1. The gaps in the plot depict resonance in mean-motion between Jupiter and an asteroid [1].

Asteroids don't only exist as single bodies in the Solar System, but they are also found in local multi-body systems consisting of two to even three asteroids. With advanced asteroid detection methods, astrophysicists have found over 190 multiple asteroid systems in the Solar System [3]. Contrary to intuition, these multiple asteroid systems exhibit a wide diversity in terms of the size ratios of the components, their mutual orbits and separation, implicating that the individual components evolved differently over time [3]. If a multi-asteroid system consists of two or three components, which are bound gravitationally, then it is termed as *binary asteroids* or *triple asteroids* respectively. Triple asteroids are also sometimes termed as *trinary* or *ternary* [4]. Asteroid components that are not gravitationally bound but are genetically related, are termed as *asteroid pairs*. Asteroid pairs where the larger asteroid is a binary or a triple asteroid, are termed as *paired binaries* or *paired triples*, respectively. The larger component in a binary or triple asteroid system or an asteroid pair, is referred to as the *primary* and similarly the smaller component is referred to as the *secondary* [4]. Asteroids are further classified based on their dimensions and thermal properties, for which the reader should read the publication in [3].

We now know what asteroids are and the different ways in which they are found in our Solar System, but is it important to study them? There are three major, and most commonly expressed, reasons to study asteroids in our solar system, and not just from a distance such as through radar telescopes placed on Earth, but also through in-situ exploration involving spacecrafts and surface probes. These reasons are mentioned as follows.

- Asteroids are basically the material left-over from formation of planets in our Solar System. Thus, they are the perfect source to study and understand the origins of the Solar System, as

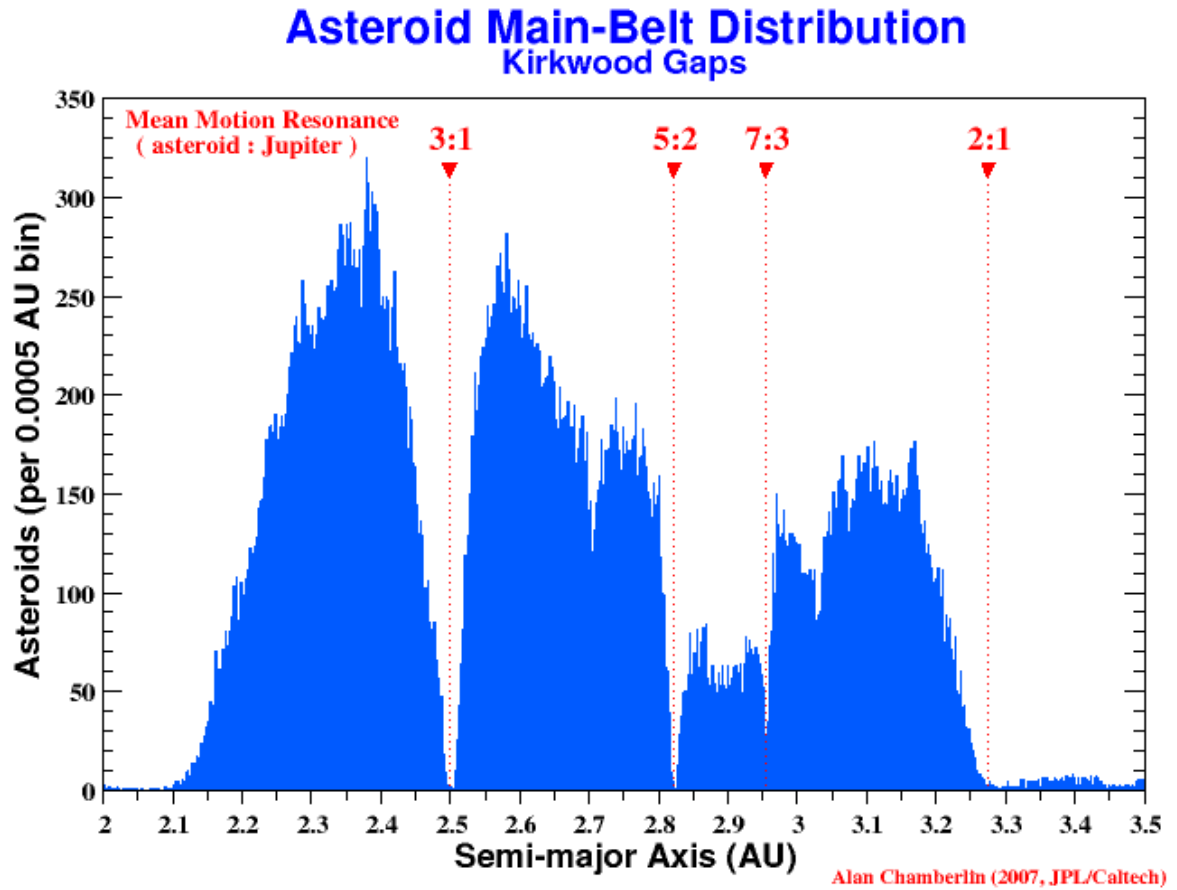


Figure 1.1: Histogram plot depicting distribution of semi-major axis of 156,929 main-belt asteroids, created in June 2007 [1].

they have remained in the same pristine form since the birth of the Solar System, unlike the planets which have undergone massive topographical and atmospheric changes after their formation. The asteroids can provide valuable information on the chemical composition and initial conditions which led to the formation of planets, including Earth some 4.6 billion years ago. Several scientists have also hypothesized that water and life could have been brought about on Earth through an asteroid or comet and hence exploration of these small bodies could provide a definite answer to an age old question of how life began on Earth [2].

- Asteroids have been hypothesized to have brought complex molecules to the surface of Earth that eventually resulted in life, but lately they have also been linked to the extinction of dinosaurs due to its impact with Earth. Earth is continuously bombarded with very small interplanetary material, most of which doesn't reach the surface of the Earth but gets evaporated in its atmosphere. However, every few 100 years, an asteroid spanning some tens of meter could impact Earth resulting in widespread damage, in the present case to life and property. But the impact from those will not cause the human race to extinct. But every 100,000 years or so, larger asteroids, spanning over tens of kilometer would impact the Earth, which will lead to extinction of life as we know it now. Although the probability of getting hit by an asteroid on such a large scale is low, it is still a statistical possibility and to be able to devise strategies for active deflection of such asteroids, it is imperative that we understand more of the dynamics, properties and composition of the asteroids [2].

- The third most important reason for us to study asteroids, is the fact that these small bodies are rich in raw materials or minerals. NEA can be exploited for the resources that they possess and use it to build space structures or generate fuel for spacecrafts to enable human space exploration in farther reaches of the Solar System. By studying the asteroids, we can develop methods to tap the vast reservoirs of raw materials residing in them [\[2\]](#).

1.1 RESEARCH PROBLEM

1.2 STRUCTURE OF THE REPORT

2

RESULTS

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