Problem 3.

a) Nyquist Theorem.

Lets say we have a sinusoidal wave $\Rightarrow$

$\Rightarrow \exp(2\pi f_m n t_s)$

Sinusoids have period $= 2\pi$

$$\exp(2\pi f_m n t_s) = \exp(2\pi n t_s f_m + 2\pi K)$$

$$= \exp\left(2\pi \left(f_m + \frac{K}{n t_s}\right) n t_s\right)$$

$$= \exp\left(2\pi \left(f_m + p f_s\right) n t_s\right)$$

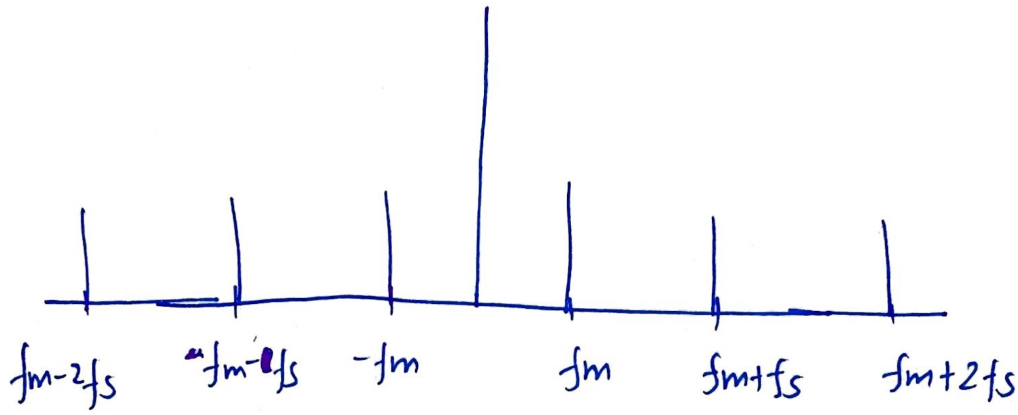
$$(K/n = p)$$

$$f_m + p f_s = f'$$

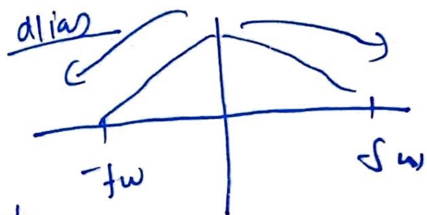
$$= \exp(2\pi f' n t_s)$$

$\hookrightarrow$ multiples of $f_m + p f_s$

Therefore, any sampled sinusoid is equivalent to another sampled sinusoid with a frequency that is an integer multiple of sampling frequency away from the original frequency.



To avoid, aliasing, the alias must lie outside of band width = $2 f_m$



∴ $f_s \geq 2 f_m$ for it to not have aliasing?

That is f_w should alias to left of $-f_w$ and $-f_w$ should alias to right of f_w

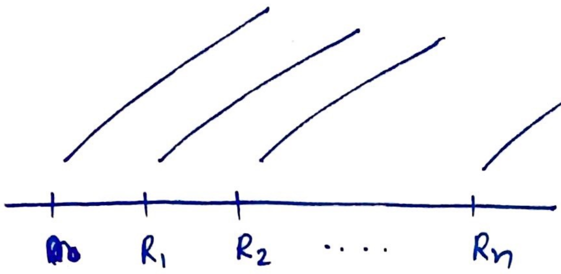
$\Rightarrow f_w - f_s < -f_w \Rightarrow -f_w + f_s > f_w$

$\Rightarrow f_s > 2 f_w$

Hence, proved.

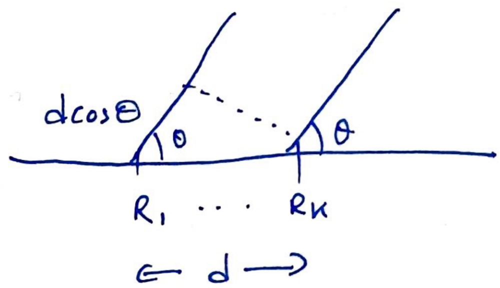
Problem 3.

b)



Let received signal at receiver array be $y(t)$

$$y(t) = \begin{bmatrix} y_{R_1} \\ y_{R_2} \\ y_{R_3} \\ y_{R_4} \\ \vdots \\ y_{R_N} \end{bmatrix}$$

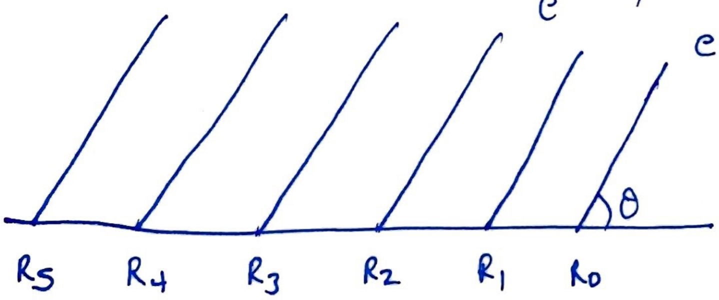


path difference $= d\cos\theta$

phase difference $= \frac{d\cos\theta}{\lambda} \cdot 2\pi = \phi$

$e^{j2\pi 1.\phi}$

$e^{j2\pi 0} \cdot e^{j2\pi 0.\phi}$



$e^{j2\pi.2\phi}$

$e^{j2\pi.3\phi}$

$$
\begin{bmatrix} y_{R_1} \\ y_{R_2} \\ y_{R_3} \\ \vdots \\ y_{R_N} \end{bmatrix}
=
\begin{bmatrix} \cos 2\pi f t \\ \cos 2\pi f t + \phi \\ \cos 2\pi f t + 2\phi \\ \vdots \\ \cos 2\pi f t + (n-1)\phi) \end{bmatrix}
= e^{j2\pi f t}
\begin{bmatrix} 1 \\ e^{j\phi} \\ e^{j2\phi} \\ e^{j3\phi} \\ \vdots \\ e^{j(n-1)\phi} \end{bmatrix}
$$

Steering matrix.

in case of mutiple signal:

$$
M = \begin{bmatrix} e^{j0} & e^{j0} \\ e^{j\phi_1} & e^{j\phi_2} \\ e^{j2\phi_1} & e^{j2\phi_3} \\ e^{j3\phi_1} & e^{j3\phi_2} \\ \vdots & \vdots \\ e^{j(n-1)\phi_1} & e^{j(n-1)\phi_2} \end{bmatrix}
$$

Problem 3.

c) HMM $\quad P(s_k \mid m_{1:n})$

$$P(s_k \mid m_{1:n}) = \frac{P(s_k, m_{1:n})}{P(m_{1:n})} \approx P(s_k, m_{1:n})$$

$P(m_{1:n}) \Rightarrow$ doesn't matter

$$P(s_k, m_{1:n}) = P(s_k, m_{1:k}, m_{k+1:n})$$

Using chain rule,

$\Rightarrow$

$$P(m_{k+1:n} \mid s_k, m_{1:k})\, P(s_k, m_{1:k})$$

$$= P(m_{k+1:n} \mid s_k, m_{1:k})\, P(s_k \mid m_{1:k})\, P(m_{1:k})$$

$P(m_{1:k}) \Downarrow$ doesn't matter

$$= \underbrace{P(m_{k+1:n} \mid s_k)}_{\text{backward prediction}}\ \underbrace{P(s_k \mid m_{1:k})}_{\text{forward prediction}}$$

Problem 3.

d) chain rule on $P(A,B,C,D)$

$$P(A,B,C,D) = P(A|BCD)\,P(BCD)$$

$$\Downarrow$$

$$P(B|CD)\,P(CD)$$

$$\Downarrow$$

$$P(C|D)\,P(D)$$

$$= P(A,B,C,D) = P(A|BCD)\,P(B|CD)\,P(C|D)\,P(D)$$

$$= P(A|BCD)\,P(B|CD)\,P(C|D)\,P(D)$$