Lab 3

Problem 1:

i) Since (1,1) is a root, we obtain (1,1) on iterating with initial value as (1,1).
Starting with (0,0) we get NaN as output in Matlab , because jacobian matrix is a zero matrix in this case and hence singular, so it doesn't have an inverse.

ii) Starting with (1,1) and performing two iterations we obtain x(1) = 1.6314 and x(2) = -0.0489.

Starting with (0,0) and performing two iterations we obtain x(1) = -0.5132 and x(2) = -0.0184.

Problem 2: Using x = $(0,0,0)^t$, the following values were obtained on iteration:

n	x(1)	x(2)	x(3)	f1(x)	f2(x)	f3(x)
1	0.5	-0.18614233	-0.523598776	0.009491694	0.124719	0.29262
2	0.49815781	-0.199606825	-0.528826395	7.89E-05	-1.27E-06	-1.48E-05
3	0.498144685	-0.199605896	-0.528825978	4.10E-13	6.81E-11	5.62E-11
4	0.498144685	-0.199605896	-0.528825978	-2.22E-16	1.11E-16	-3.55E-15

Here x(1) , x(2) , x(3) represent the components of vector x and f1(x), f2(x) and f3(x) are the functions given in the question.

Problem3: Starting with initial value x_0 = 1, the following results were obtained:

n	x	y	f1(x,y)	f2(x,y)
1	1	0.635848	-7.77307	0
2	2.545187	0.233995	-5.13556	0
3	2.83091	0.177473	0.576922	-8.88E-16
4	2.804987	0.181745	0.005794	-4.44E-16
5	2.804721	0.18179	5.99E-07	8.88E-16
5	2.804721	0.18179	7.11E-15	-4.44E-16

Problem 4:

a) The multiplicity obtained is 4.

The table of values by Newton's method is:

TABLE

n	x_n	f(x_n)
0	-1.000000	0.001220
1	-1.103850	0.000385
2	-1.181566	0.000122
3	-1.239780	0.000039
4	-1.283411	0.000012
5	-1.316121	0.000004
6	-1.340648	0.000001
7	-1.359041	0.000000
8	-1.372835	0.000000
9	-1.383180	0.000000
10	-1.390938	0.000000
11	-1.396757	0.000000
12	-1.401121	0.000000
13	-1.404394	0.000000
14	-1.406849	0.000000
15	-1.408690	0.000000
16	-1.410071	0.000000
17	-1.411107	0.000000
18	-1.411883	0.000000

19	-1.412466	0.000000
20	-1.412903	0.000000
21	-1.413230	0.000000
22	-1.413475	0.000000
23	-1.413658	0.000000
24	-1.413795	0.000000
25	-1.413894	0.000000

The table of values by modified Newton's method is:

TABLE

n	x_n	f(x_n)
0	-1.000000	0.001220
1	-1.415401	0.000000
2	-1.414215	-0.000000

b) The multiplicity obtained is three.
The table of values by Newton's method is:

TABLE

n	x_n	f(x_n)
0	-1.000000	-0.173565
1	-0.313049	-0.003980
2	-0.263651	-0.001091
3	-0.234578	-0.000309
4	-0.216562	-0.000089
5	-0.205082	-0.000026
6	-0.197646	-0.000008
7	-0.192780	-0.000002
8	-0.189575	-0.000001
9	-0.187455	-0.000000
10	-0.186050	-0.000000
11	-0.185116	-0.000000
12	-0.184495	-0.000000
13	-0.184082	-0.000000

14	-0.183806	-0.000000
15	-0.183623	-0.000000
16	-0.183501	-0.000000
17	-0.183419	-0.000000
18	-0.183365	-0.000000
19	-0.183329	-0.000000
20	-0.183305	-0.000000
21	-0.183289	-0.000000
22	-0.183278	-0.000000
23	-0.183271	-0.000000
24	-0.183266	-0.000000
25	-0.183263	-0.000000

The table of values by modified Newton's method is:

TABLE

n	x_n	f(x_n)
0	-1.000000	-0.173565
1	1.060852	448.082374
2	0.602381	18.458682
3	0.206271	0.546351
4	-0.064309	0.006452
5	-0.170166	0.000006
6	-0.183087	0.000000
7	-0.183256	0.000000

Therefore, it can be seen that Modified Newton's method is a lot faster than simple Newton's method in cases when the multiplicity of root is >1.

Problem 5:

a) Using modified Newton's method the tabulated values are as follows:

TABLE

n	x_n	f(x_n)
0	1.300000	0.026730
1	1.006667	0.000000
2	1.000004	0.000000

b) Using modified Newton's method the tabulated values are as follows:

i) For double root p=1:

TABLE

n	x_n	f(x_n)
0	0.000000	2.000000
1	0.571429	0.086750
2	0.866995	0.000897
3	0.980086	0.000000
4	0.999433	0.000000
5	1.000000	0.000000
6	1.000000	0.000000
7	1.000000	0.000000
8	1.000000	0.000000
9	1.000001	0.000000
10	1.000000	0.000000
11	0.999976	0.000000
12	1.000000	0.000000
13	0.999996	0.000000
14	1.000000	0.000000

ii) For triple root p =2:

TABLE

n	x_n	f(x_n)
0	3.000000	80.000000
1	2.250000	10.375977

2	2.029412	4.524787
3	2.000550	4.009354
4	2.000000	4.000003
5	2.750000	44.549823
6	2.166667	7.719266
7	2.014493	4.252325
8	2.000137	4.002325
9	1.999998	3.999973
10	2.005669	4.097285
11	2.000021	4.000361
12	1.999990	3.999825
13	1.999856	3.997552
14	1.999999	3.999991