Lab 8

Problem 1: The tables are as follows:

For h = 0.05000:

t	Approx	Exact	Error
1.000000	0.997428	1.000000	0.002572
2.000000	2.142621	2.143547	0.000926
3.000000	3.348041	3.348370	0.000328
4.000000	4.594682	4.594793	0.000111
5.000000	5.873061	5.873095	0.000033

For h = 0.02500:

t	Approx	Exact	Error
1.000000	0.998739	1.000000	0.001261
2.000000	2.143088	2.143547	0.000459
3.000000	3.348204	3.348370	0.000166
4.000000	4.594735	4.594793	0.000059
5.000000	5.873075	5.873095	0.000020

For h = 0.01250:

t	Approx	Exact	Error
1.000000	0.999398	1.000000	0.000602
2.000000	2.143327	2.143547	0.000220
3.000000	3.348289	3.348370	0.000080
4.000000	4.594764	4.594793	0.000029
5.000000	5.873085	5.873095	0.000010

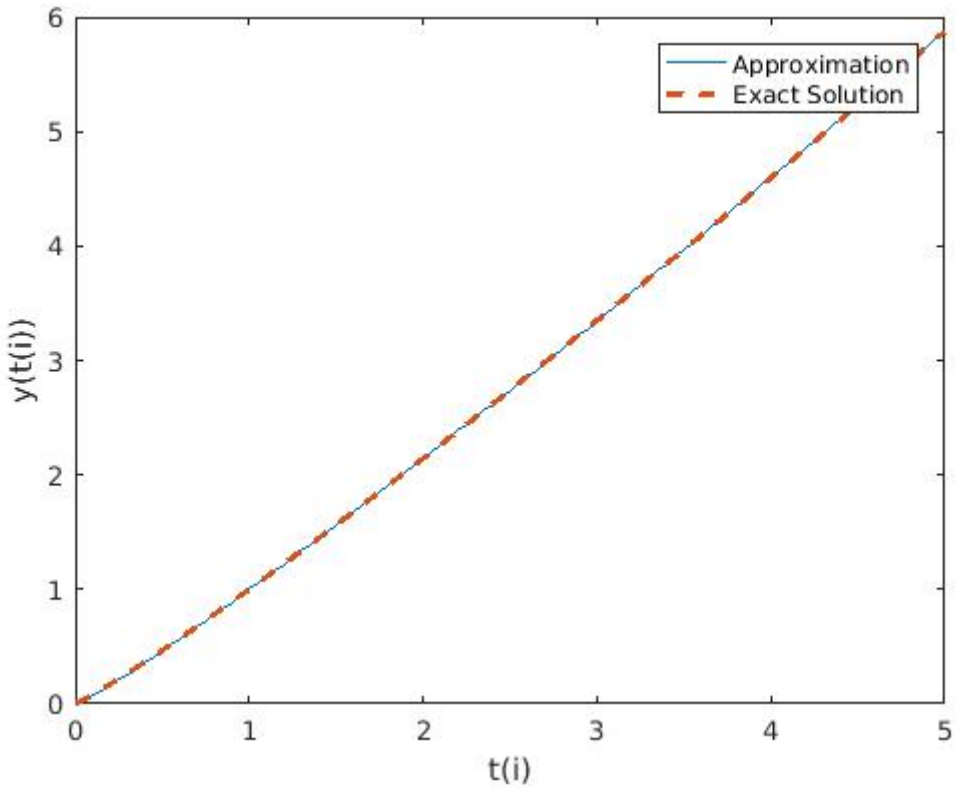
For h = 0.00625:

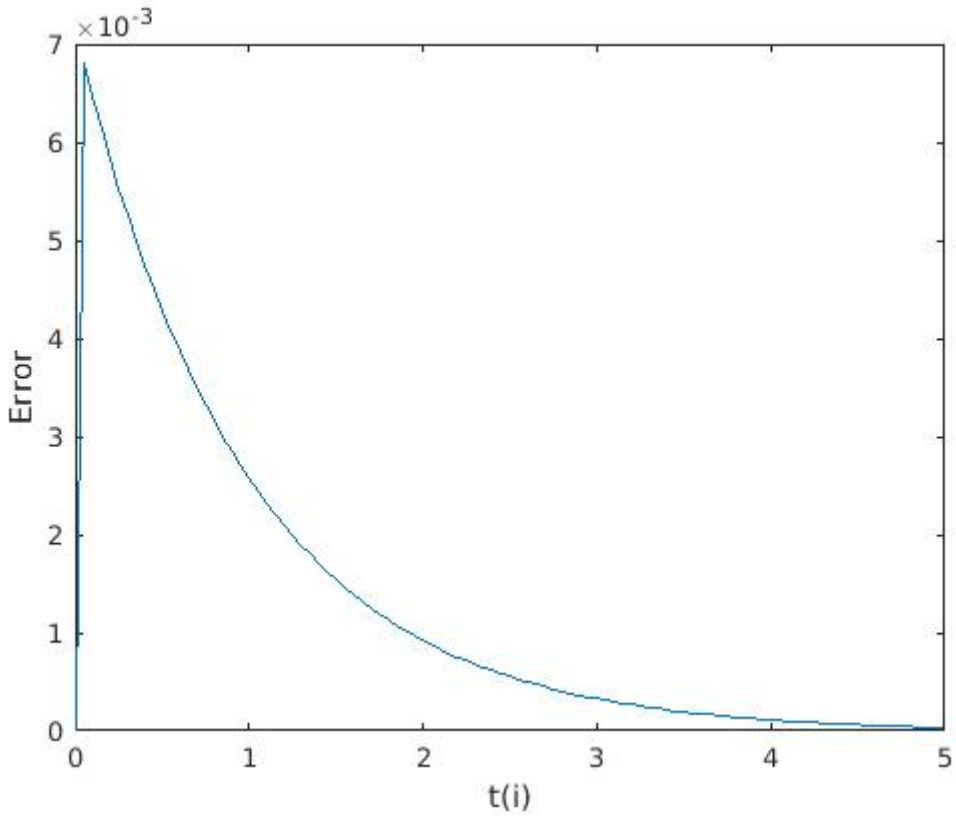
t	Approx	Exact	Error
1.000000	0.999716	1.000000	0.000284
2.000000	2.143443	2.143547	0.000104
3.000000	3.348331	3.348370	0.000038
4.000000	4.594780	4.594793	0.000014
5.000000	5.873090	5.873095	0.000005

h	Max Error	log2(E(n)/E(2n))
0.050000	0.006810	1.008720
0.025000	0.003385	1.056438
0.012500	0.001627	1.078703
0.006250	0.000770	

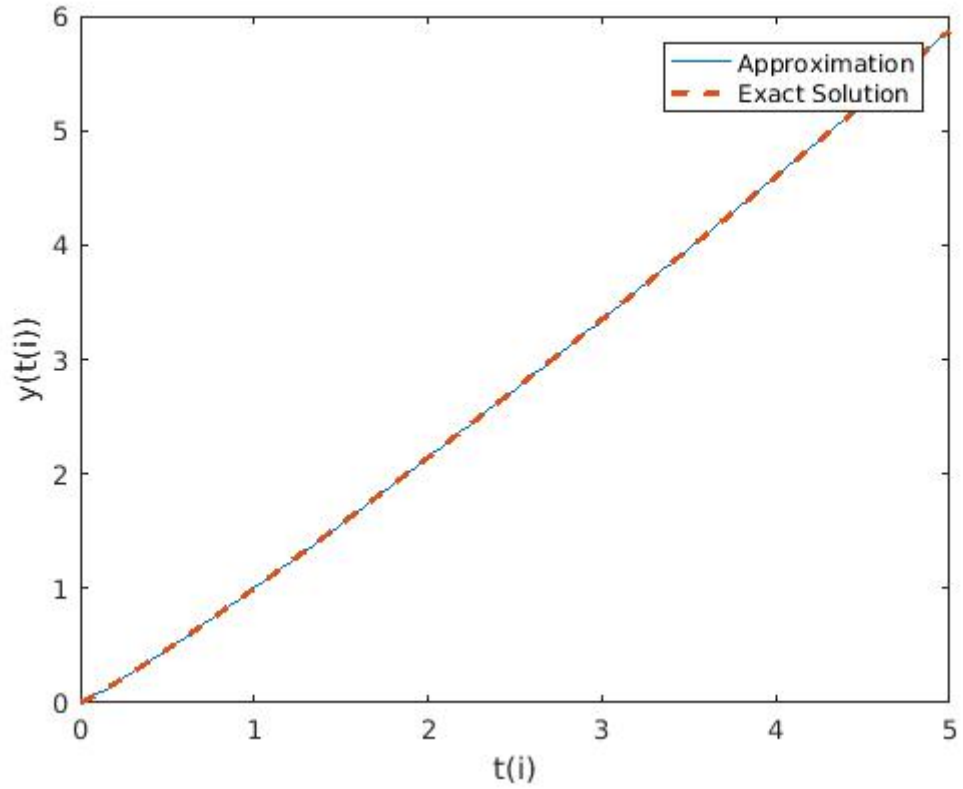
Figures for Approximation vs. Exact solution and Error vs. t(i) for different values of h are as follows:

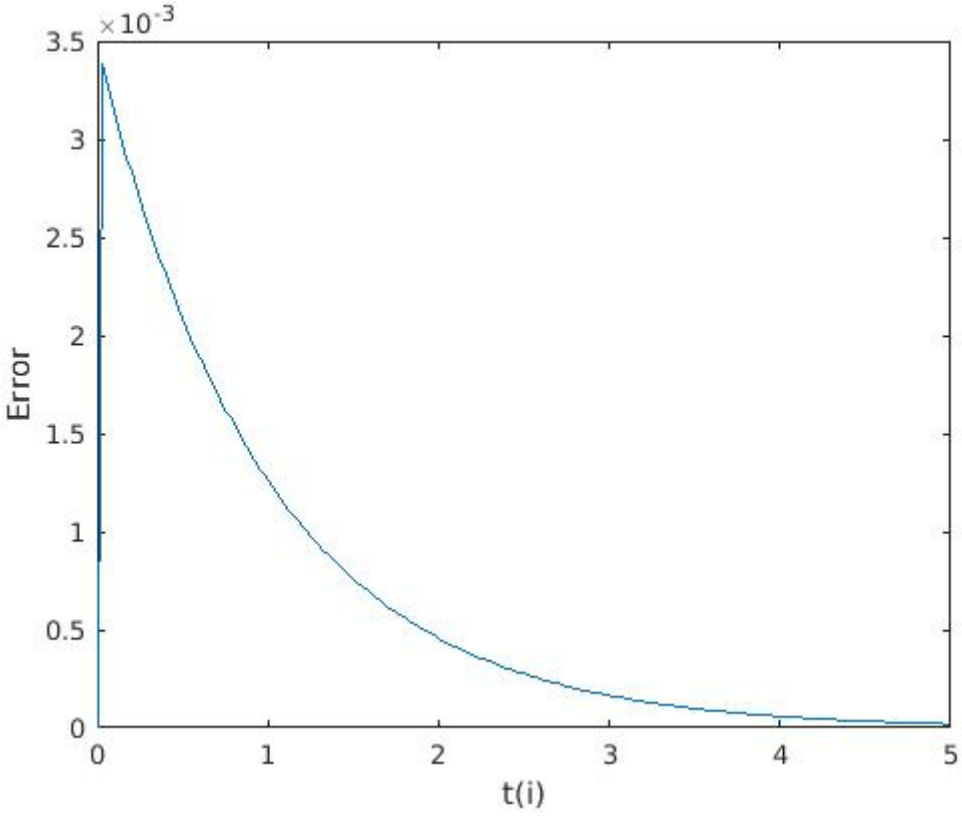
For h = 0.05:



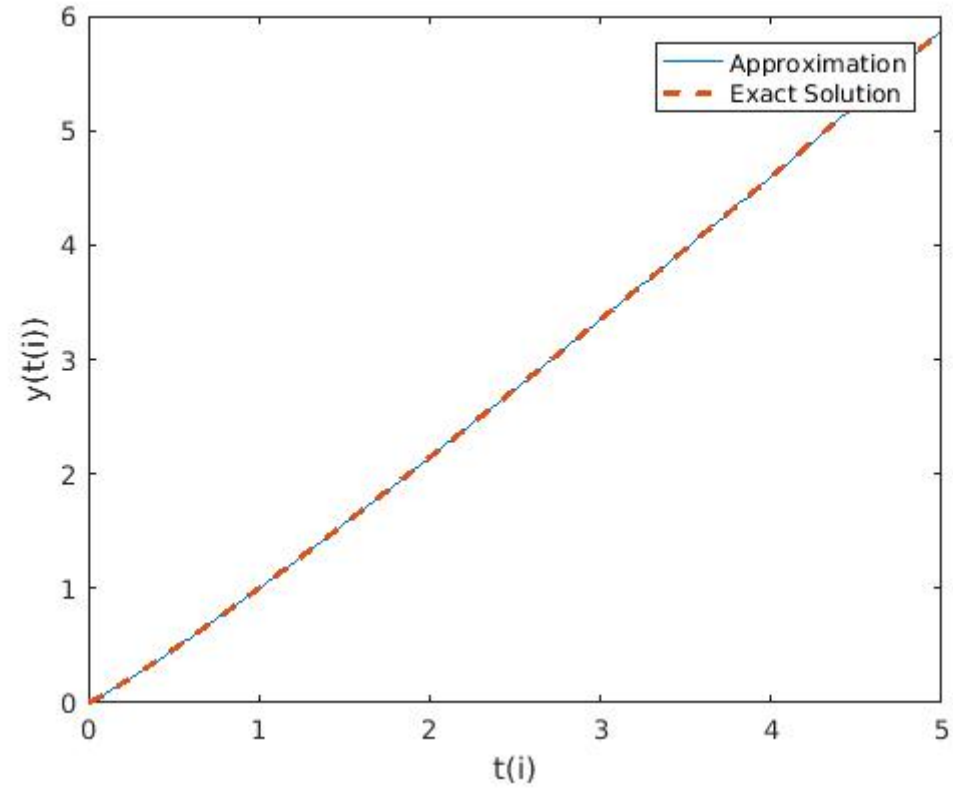


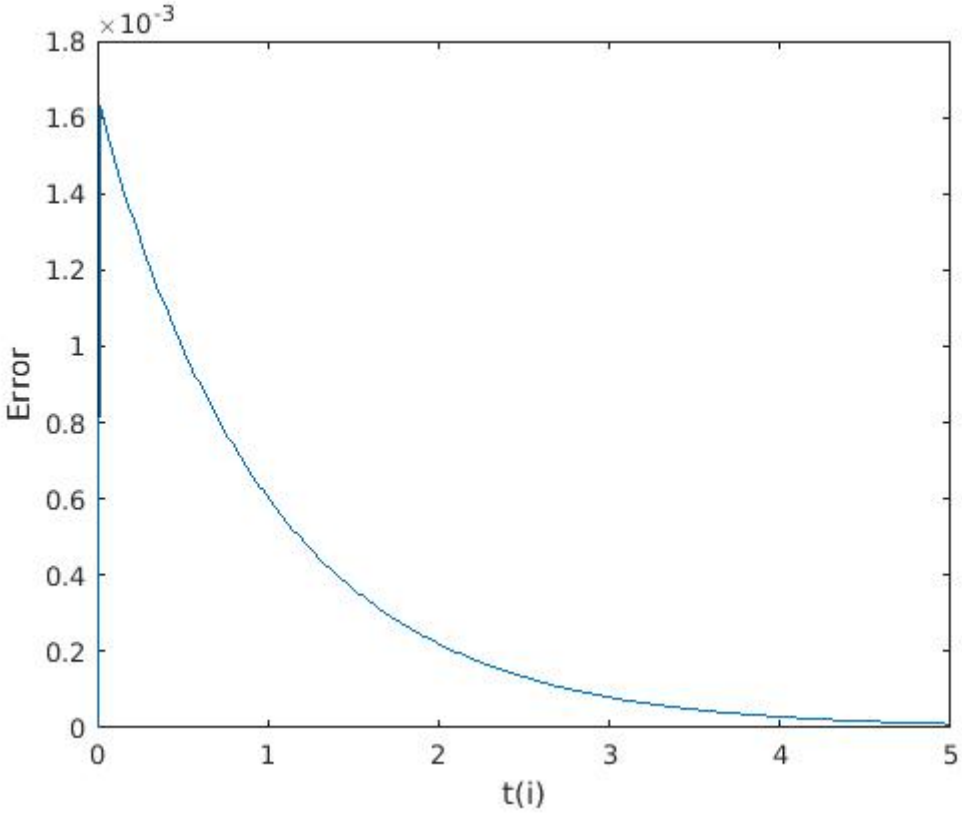
For h = 0.025:


Approximation
Exact Solution
y(t(i))
t(i)
0
1
2
3
4
5
6

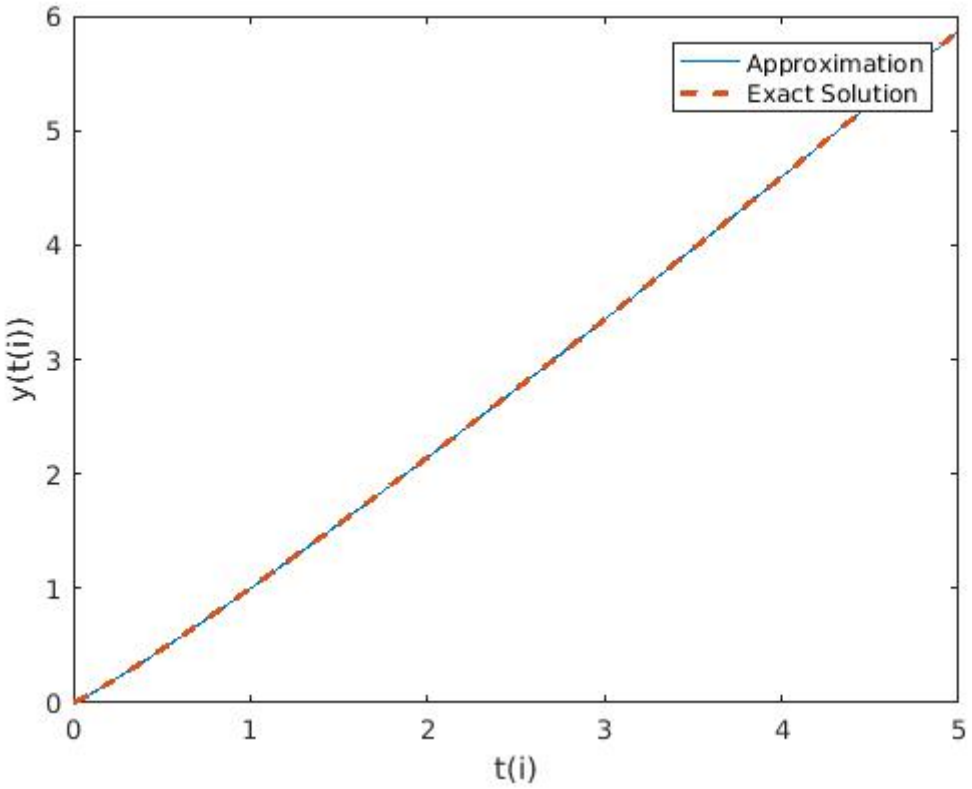

×10^-3
Error
t(i)
0
0.5
1
1.5
2
2.5
3
3.5
4
5

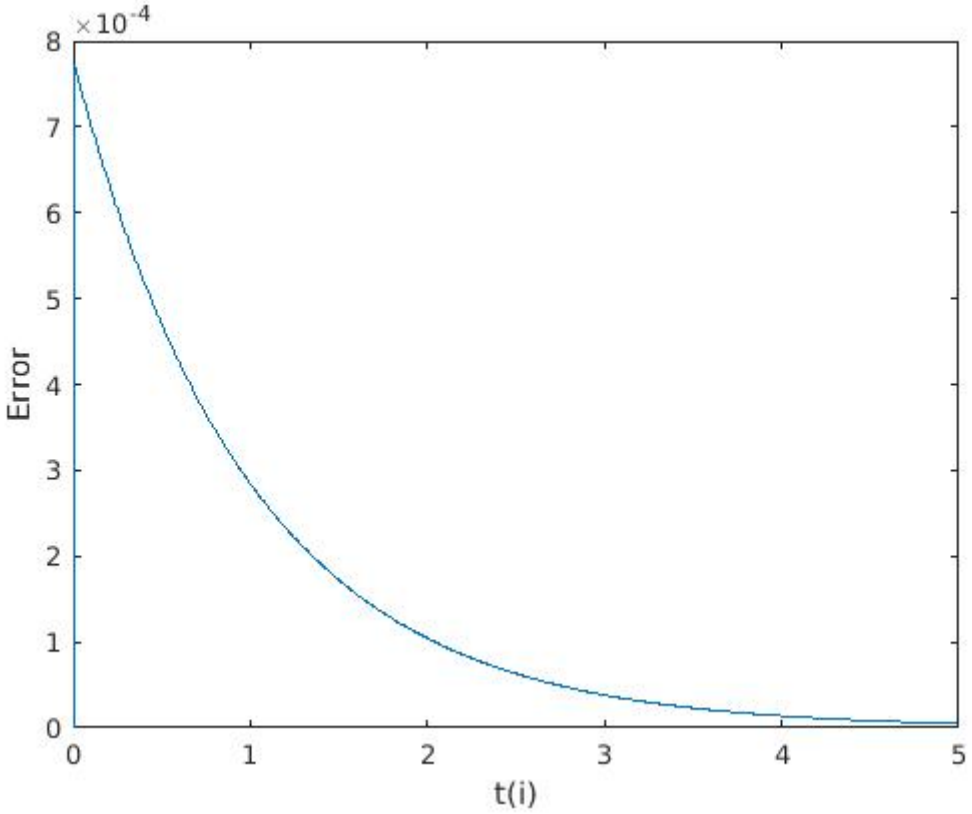
For h = 0.0125:


Approximation
Exact Solution
y(t(i))
t(i)
0
1
2
3
4
5
6


×10^-3
Error
t(i)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
1
2
3
4
5

For h = 0.00625:





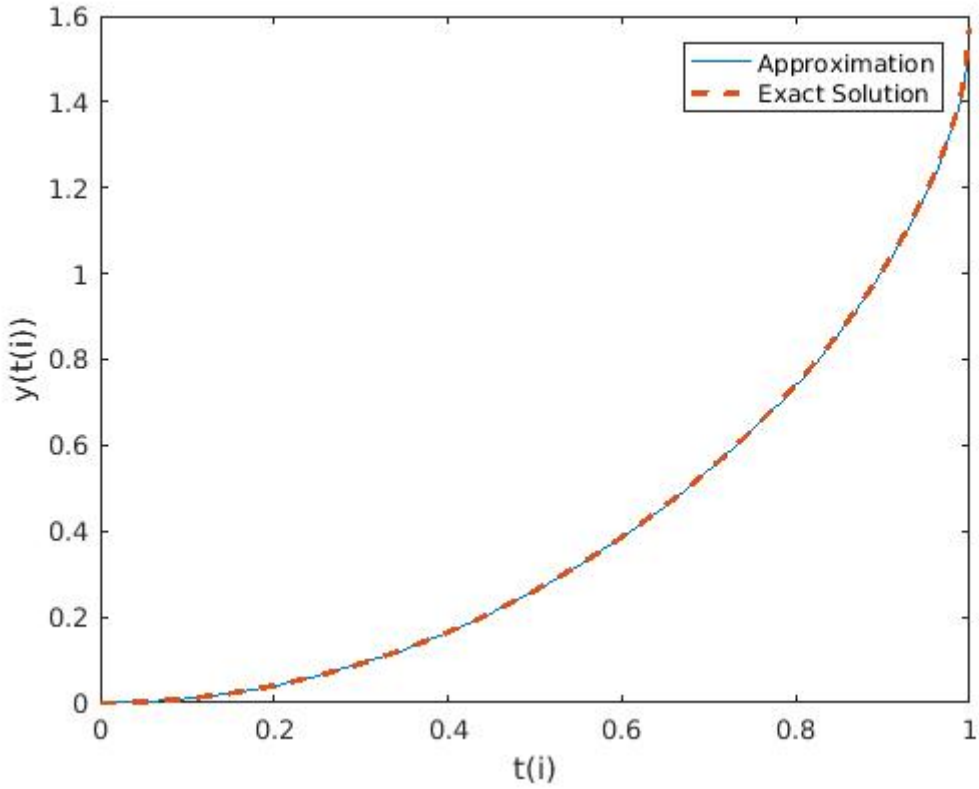
Problem 2: The table of values obtained is:

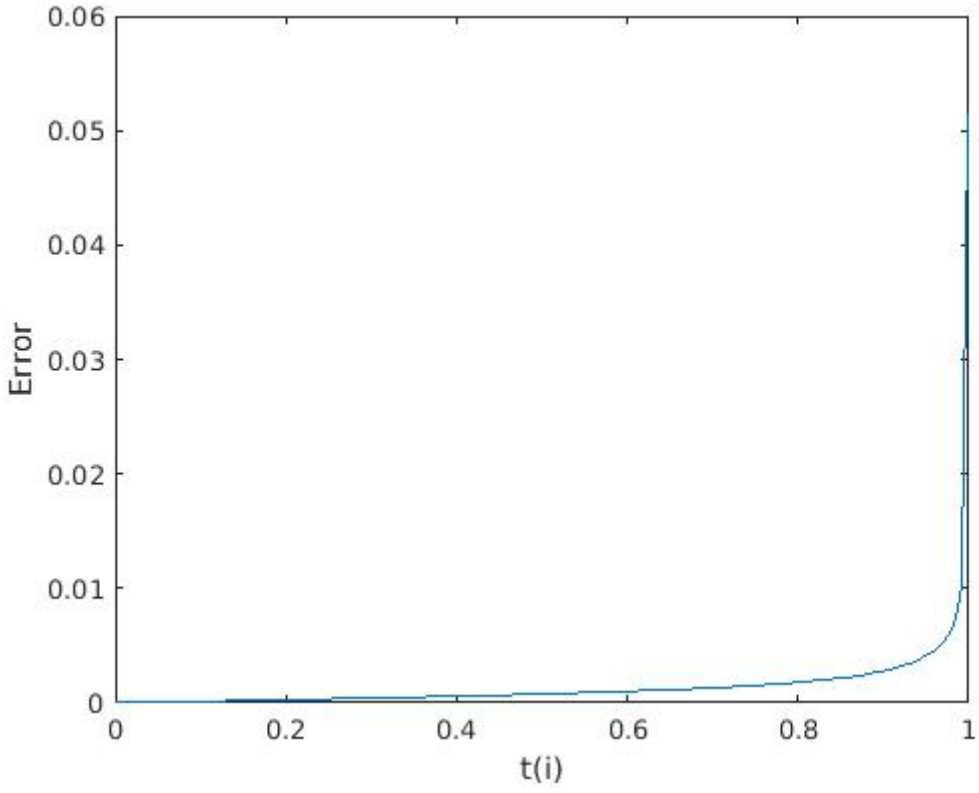
t	Approx	Exact	Error
0.007812	0.000051	0.000061	0.000010
0.015625	0.000223	0.000244	0.000021
0.023438	0.000518	0.000549	0.000032
0.031250	0.000935	0.000977	0.000042
0.039062	0.001474	0.001526	0.000053
0.046875	0.002135	0.002198	0.000063
0.054688	0.002918	0.002992	0.000074
0.062500	0.003824	0.003909	0.000084
0.070312	0.004853	0.004948	0.000095
0.078125	0.006004	0.006110	0.000106
0.085938	0.007278	0.007394	0.000116
0.093750	0.008675	0.008802	0.000127
0.101562	0.010195	0.010333	0.000138
0.109375	0.011839	0.011987	0.000148
0.117188	0.013606	0.013765	0.000159
0.125000	0.015496	0.015666	0.000170
0.132812	0.017511	0.017691	0.000180
0.140625	0.019650	0.019841	0.000191
0.148438	0.021913	0.022115	0.000202
0.156250	0.024301	0.024515	0.000213
0.164062	0.026815	0.027039	0.000224
0.171875	0.029453	0.029688	0.000235
0.179688	0.032218	0.032464	0.000246
0.187500	0.035108	0.035366	0.000257
0.195312	0.038126	0.038394	0.000268
0.203125	0.041269	0.041549	0.000279
0.210938	0.044541	0.044831	0.000291
0.218750	0.047940	0.048242	0.000302
0.226562	0.051467	0.051780	0.000313
0.234375	0.055123	0.055447	0.000325
0.242188	0.058908	0.059244	0.000336
0.250000	0.062822	0.063170	0.000348
0.257812	0.066867	0.067227	0.000359
0.265625	0.071043	0.071414	0.000371
0.273438	0.075350	0.075733	0.000383
0.281250	0.079789	0.080183	0.000395
0.289062	0.084360	0.084767	0.000407
0.296875	0.089065	0.089484	0.000419
0.304688	0.093904	0.094334	0.000431
0.312500	0.098877	0.099320	0.000443
0.320312	0.103986	0.104441	0.000455
0.328125	0.109230	0.109698	0.000468
0.335938	0.114612	0.115092	0.000480
0.343750	0.120131	0.120624	0.000493

0.351562	0.125789	0.126295	0.000506
0.359375	0.131587	0.132106	0.000519
0.367188	0.137525	0.138056	0.000532
0.375000	0.143604	0.144149	0.000545
0.382812	0.149826	0.150384	0.000558
0.390625	0.156190	0.156762	0.000571
0.398438	0.162700	0.163285	0.000585
0.406250	0.169355	0.169953	0.000599
0.414062	0.176156	0.176769	0.000613
0.421875	0.183105	0.183732	0.000627
0.429688	0.190203	0.190844	0.000641
0.437500	0.197452	0.198107	0.000655
0.445312	0.204852	0.205522	0.000670
0.453125	0.212406	0.213090	0.000684
0.460938	0.220113	0.220813	0.000699
0.468750	0.227977	0.228691	0.000715
0.476562	0.235998	0.236728	0.000730
0.484375	0.244178	0.244924	0.000746
0.492188	0.252519	0.253280	0.000761
0.500000	0.261022	0.261799	0.000777
0.507812	0.269689	0.270483	0.000794
0.515625	0.278523	0.279333	0.000810
0.523438	0.287524	0.288351	0.000827
0.531250	0.296696	0.297540	0.000844
0.539062	0.306039	0.306901	0.000862
0.546875	0.315557	0.316437	0.000879
0.554688	0.325252	0.326149	0.000898
0.562500	0.335125	0.336041	0.000916
0.570312	0.345180	0.346115	0.000935
0.578125	0.355419	0.356373	0.000954
0.585938	0.365845	0.366818	0.000973
0.593750	0.376460	0.377454	0.000993
0.601562	0.387268	0.388282	0.001014
0.609375	0.398272	0.399306	0.001035
0.617188	0.409474	0.410530	0.001056
0.625000	0.420879	0.421957	0.001078
0.632812	0.432490	0.433591	0.001100
0.640625	0.444311	0.445434	0.001123
0.648438	0.456346	0.457492	0.001147
0.656250	0.468598	0.469769	0.001171
0.664062	0.481073	0.482268	0.001195
0.671875	0.493774	0.494995	0.001221
0.679688	0.506708	0.507955	0.001247
0.687500	0.519879	0.521153	0.001274
0.695312	0.533292	0.534594	0.001302

0.703125	0.546954	0.548285	0.001331
0.710938	0.560871	0.562231	0.001360
0.718750	0.575048	0.576439	0.001391
0.726562	0.589494	0.590917	0.001423
0.734375	0.604216	0.605672	0.001456
0.742188	0.619222	0.620713	0.001490
0.750000	0.634521	0.636047	0.001526
0.757812	0.650121	0.651684	0.001563
0.765625	0.666033	0.667634	0.001601
0.773438	0.682268	0.683909	0.001642
0.781250	0.698836	0.700520	0.001684
0.789062	0.715752	0.717480	0.001728
0.796875	0.733028	0.734802	0.001774
0.804688	0.750680	0.752503	0.001823
0.812500	0.768723	0.770598	0.001874
0.820312	0.787177	0.789106	0.001929
0.828125	0.806060	0.808047	0.001987
0.835938	0.825395	0.827443	0.002048
0.843750	0.845207	0.847321	0.002113
0.851562	0.865523	0.867707	0.002184
0.859375	0.886374	0.888633	0.002259
0.867188	0.907796	0.910136	0.002340
0.875000	0.929828	0.932256	0.002429
0.882812	0.952516	0.955041	0.002525
0.890625	0.975915	0.978545	0.002631
0.898438	1.000087	1.002834	0.002748
0.906250	1.025106	1.027984	0.002878
0.914062	1.051063	1.054088	0.003025
0.921875	1.078068	1.081260	0.003192
0.929688	1.106257	1.109641	0.003384
0.937500	1.135804	1.139414	0.003610
0.945312	1.166937	1.170818	0.003881
0.953125	1.199965	1.204179	0.004214
0.960938	1.235327	1.239965	0.004638
0.968750	1.273680	1.278886	0.005206
0.976562	1.316106	1.322134	0.006027
0.984375	1.364636	1.372011	0.007375
0.992188	1.424139	1.434420	0.010281
1.000000	1.519125	1.570796	0.051671

The figures obtained are:


Approximation
Exact Solution
y(t(i))
t(i)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


Error
t(i)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.2
0.4
0.6
0.8
1

Problem 3: Using the Runge-Kutta method of order four to determine the units of potassium hydroxide formed after 0.2s , we get 2079.408617 units as the answer.

The table of values obtained is:

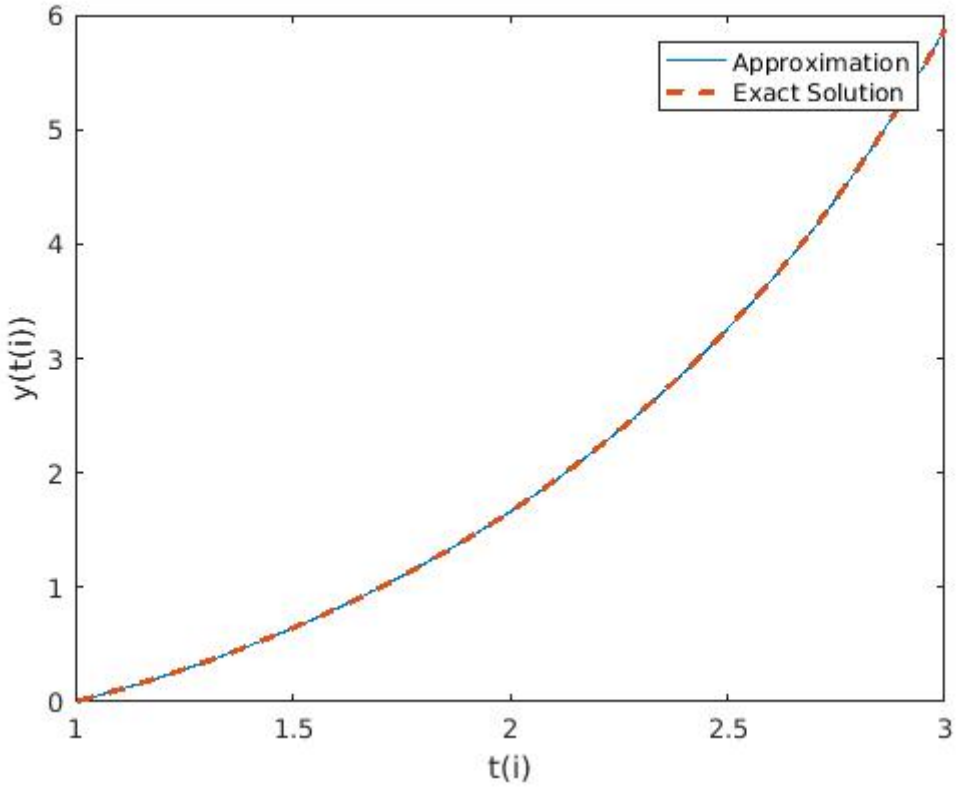
t	Approx
0.001000	219.534990
0.002000	375.342074
0.003000	495.011252
0.004000	591.552569
0.005000	672.097472
0.006000	740.960860
0.007000	800.943572
0.008000	853.962135
0.009000	901.383021
0.010000	944.213200
0.011000	983.215089
0.012000	1018.979157
0.013000	1051.971576
0.014000	1082.566508
0.015000	1111.068607
0.016000	1137.729068
0.017000	1162.757326
0.018000	1186.329738
0.019000	1208.596127
0.020000	1229.684802
.	
.	
.	
0.180000	2045.828213
0.181000	2047.607345
0.182000	2049.375201
0.183000	2051.131912
0.184000	2052.877608
0.185000	2054.612417
0.186000	2056.336463
0.187000	2058.049871
0.188000	2059.752762
0.189000	2061.445254
0.190000	2063.127466
0.191000	2064.799512
0.192000	2066.461506
0.193000	2068.113560
0.194000	2069.755783

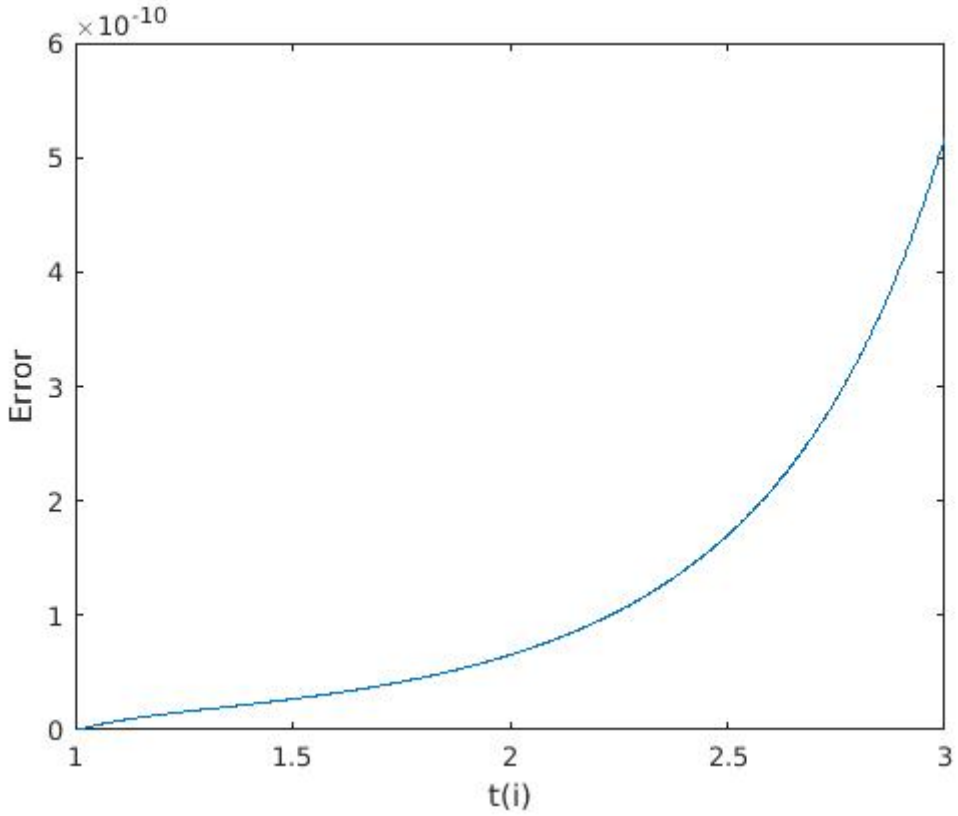
0.195000 2071.388284
0.196000 2073.011169
0.197000 2074.624543
0.198000 2076.228508
0.199000 2077.823166
0.200000 2079.408617

Problem 4:

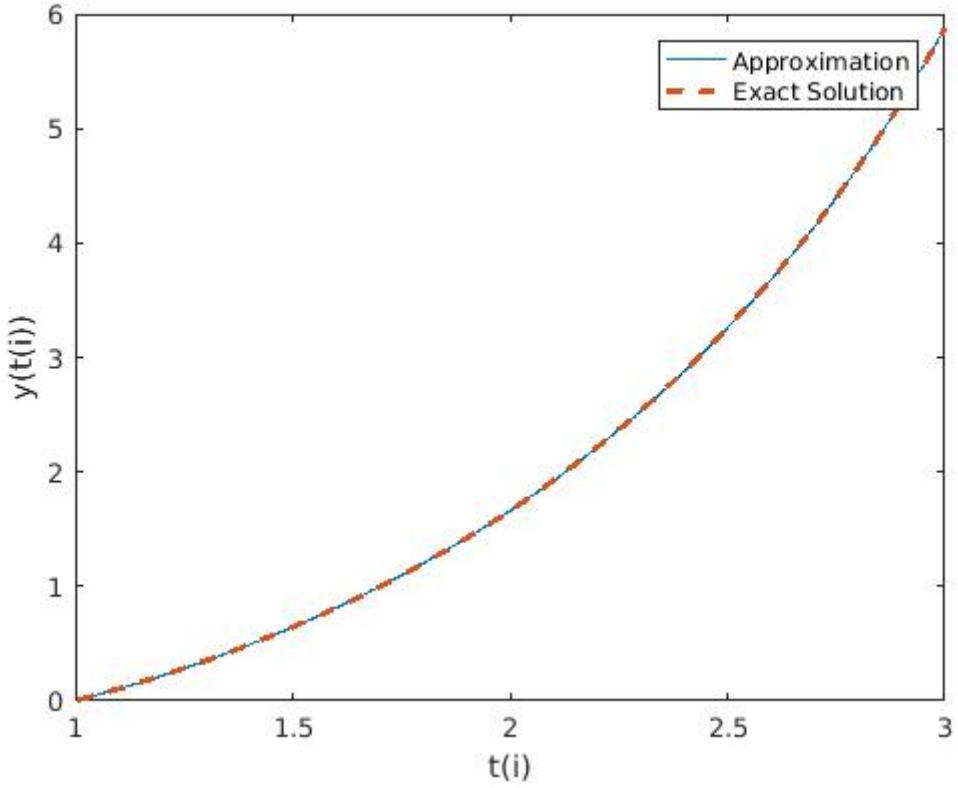
Part a) Using exact starting values:

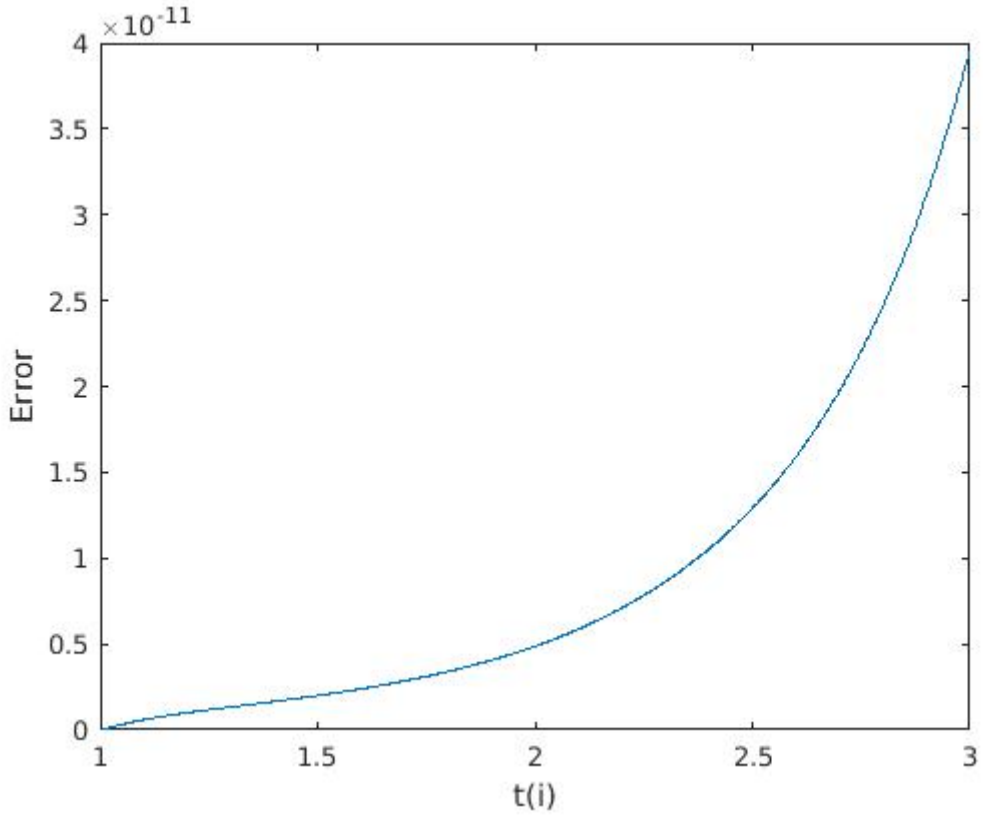
By Adams-Bashforth method:





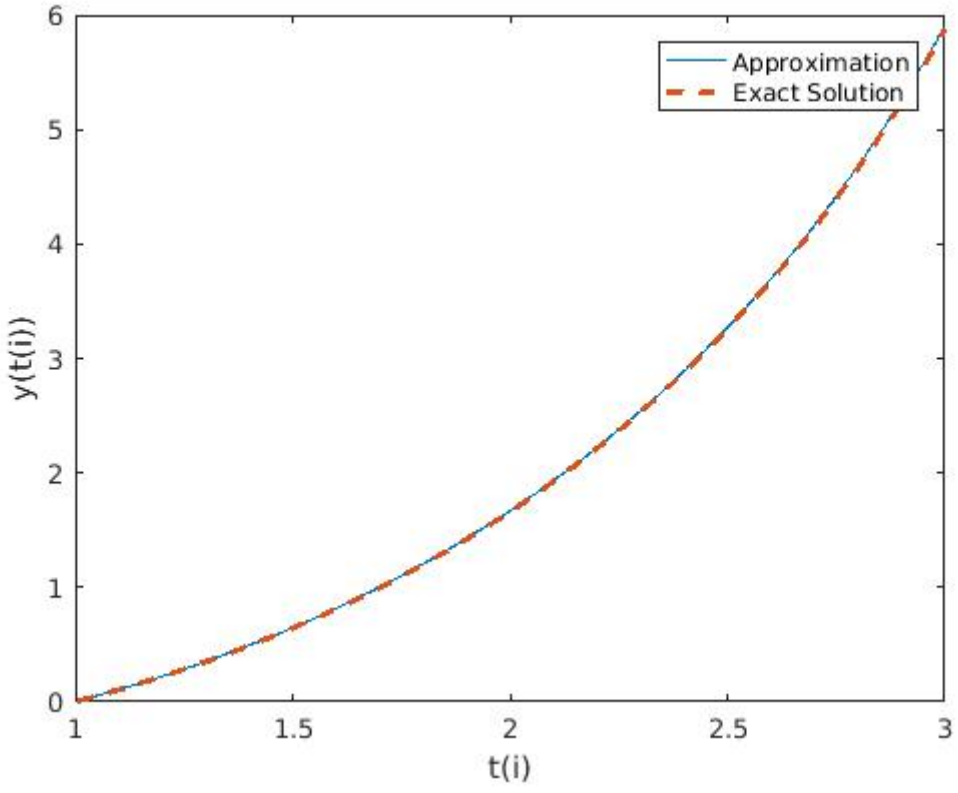
By Adams-Moulton method:

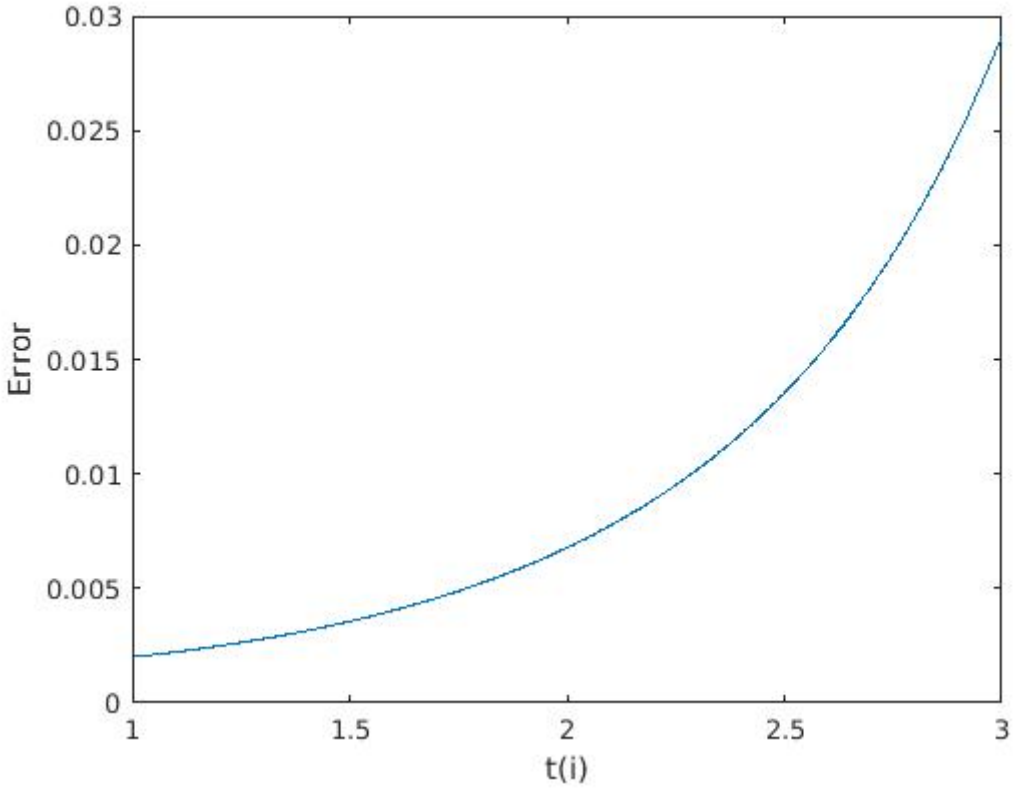




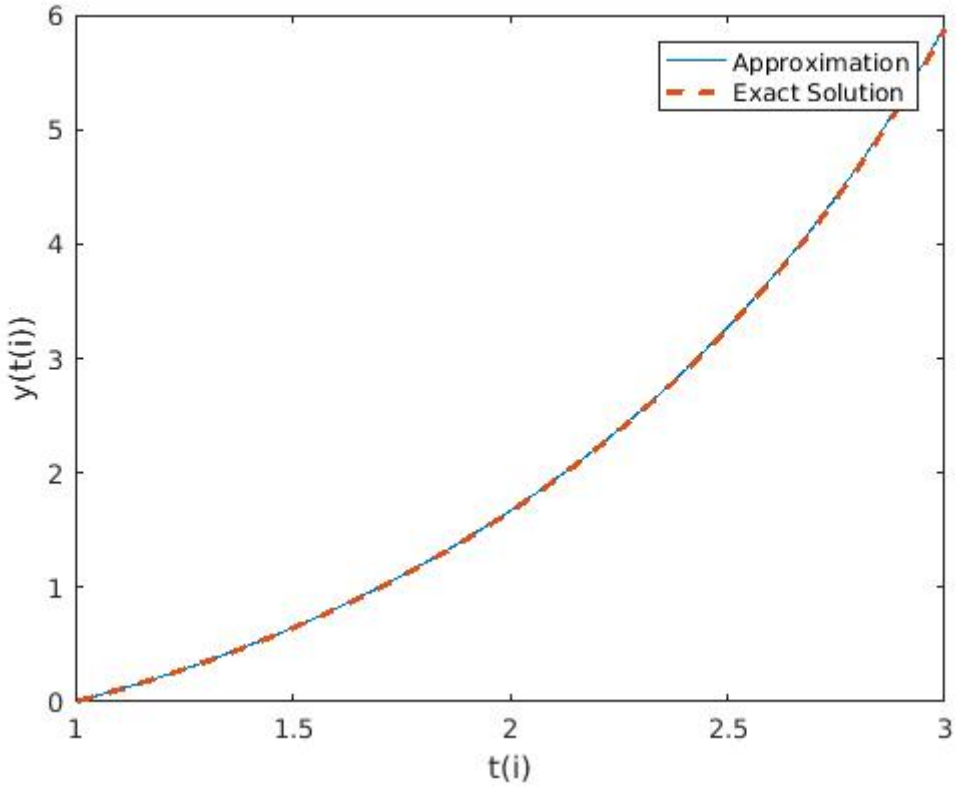
Using starting values obtained from RK method of order 4:

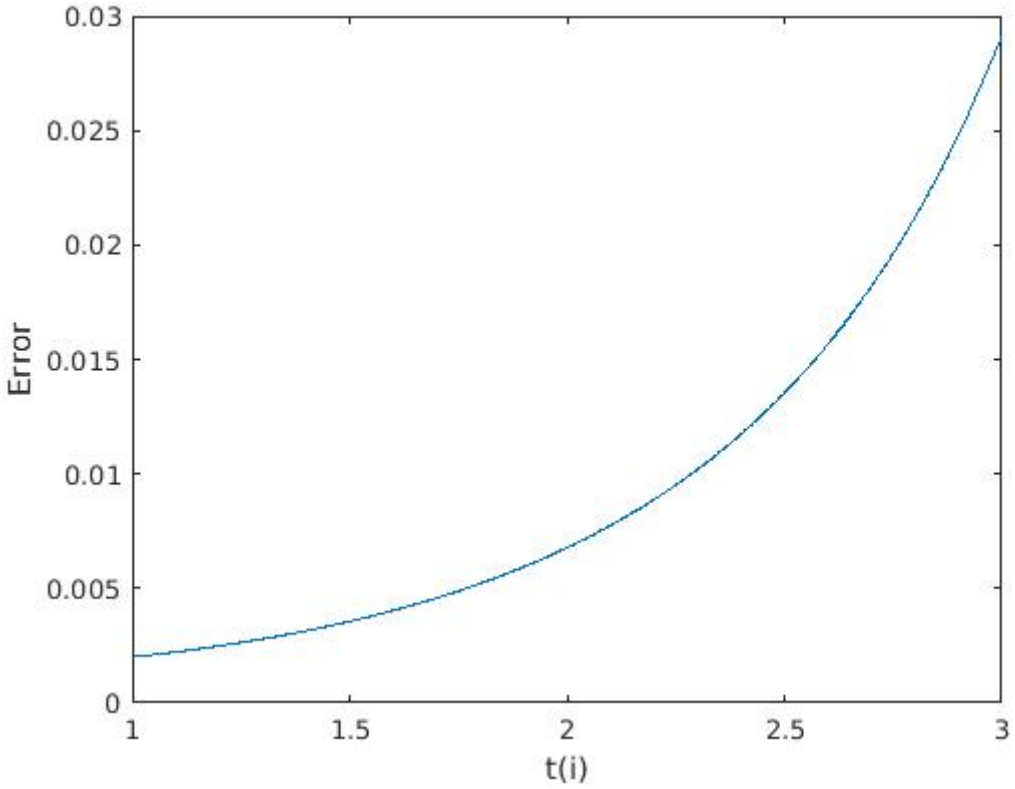
By Adams-Bashforth method:





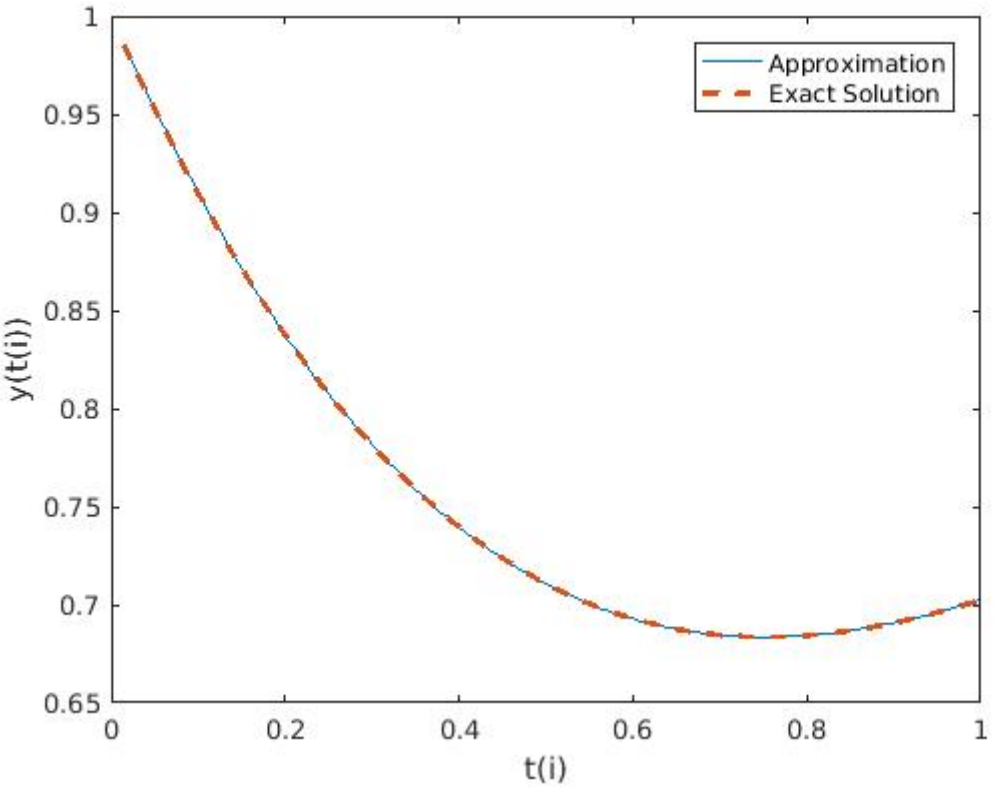
By Adams- Moulton method:

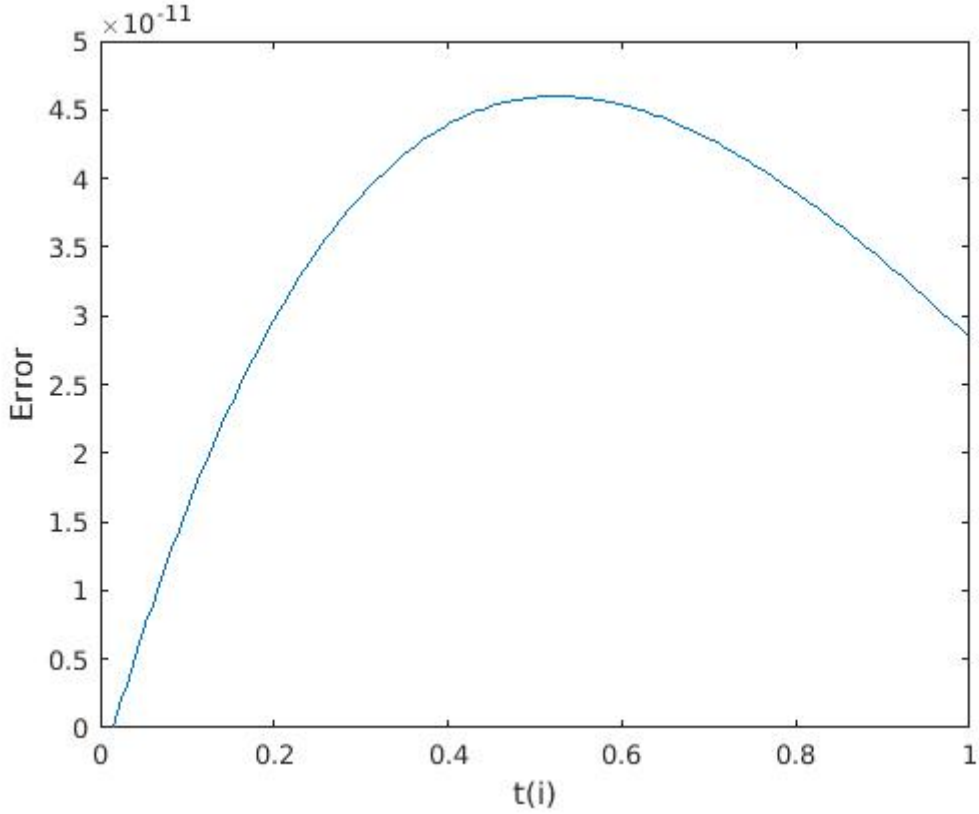




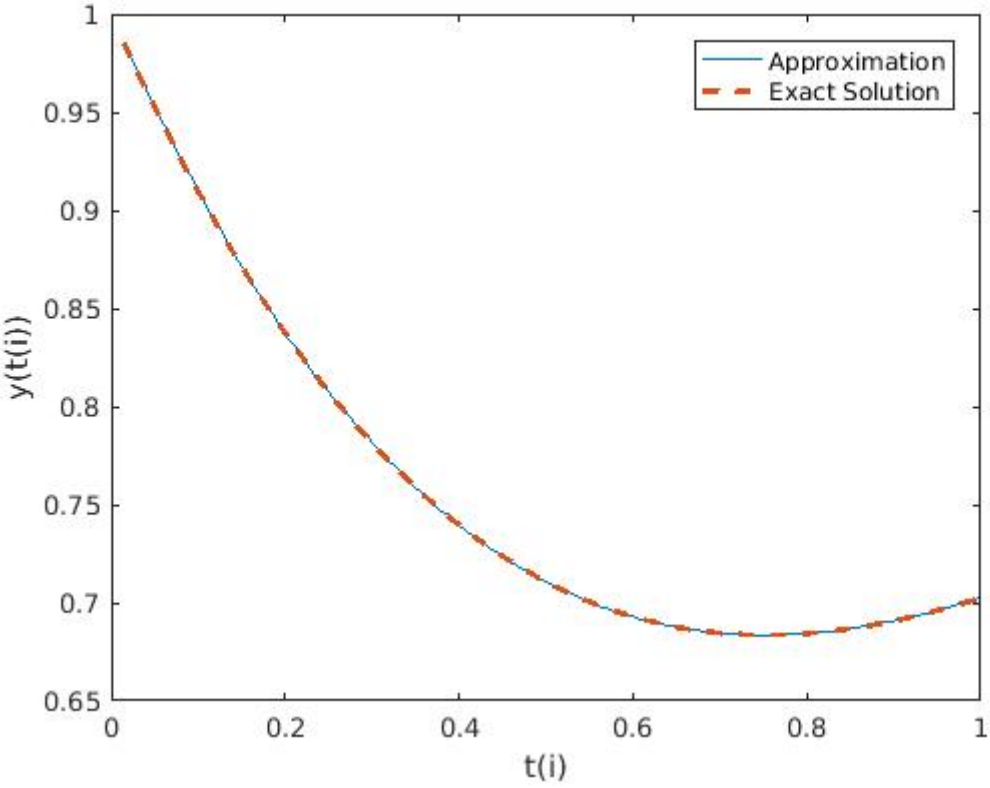
Part b) Using exact starting values:

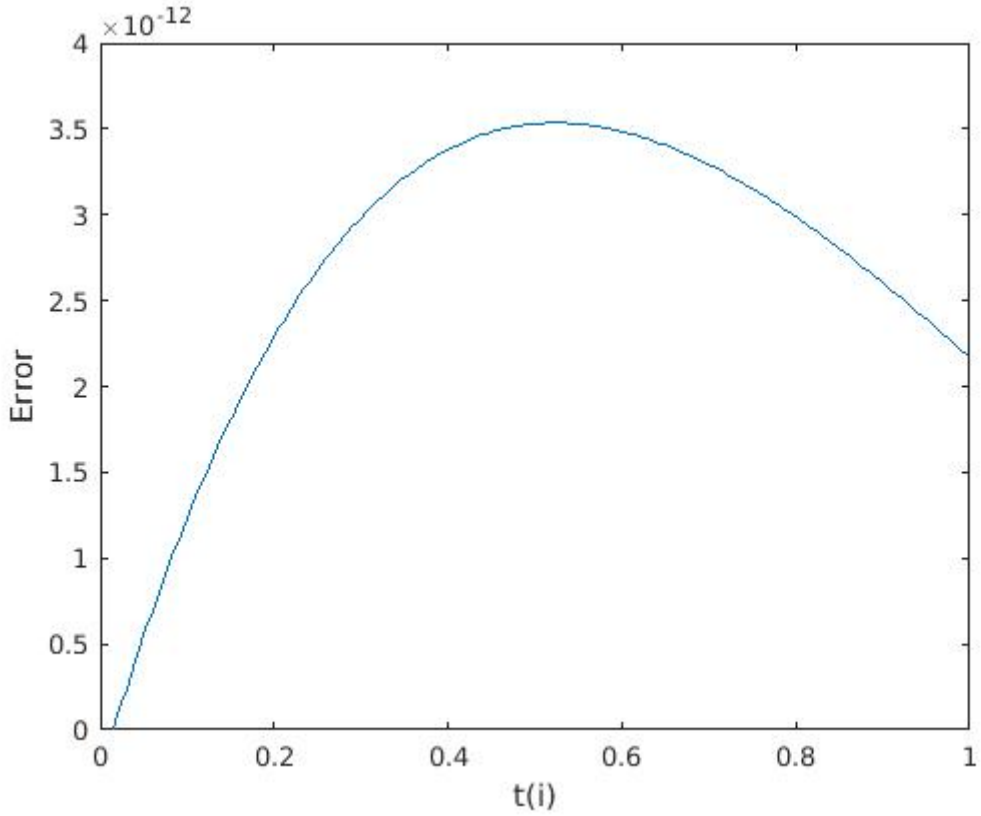
By Adams-Bashforth method:





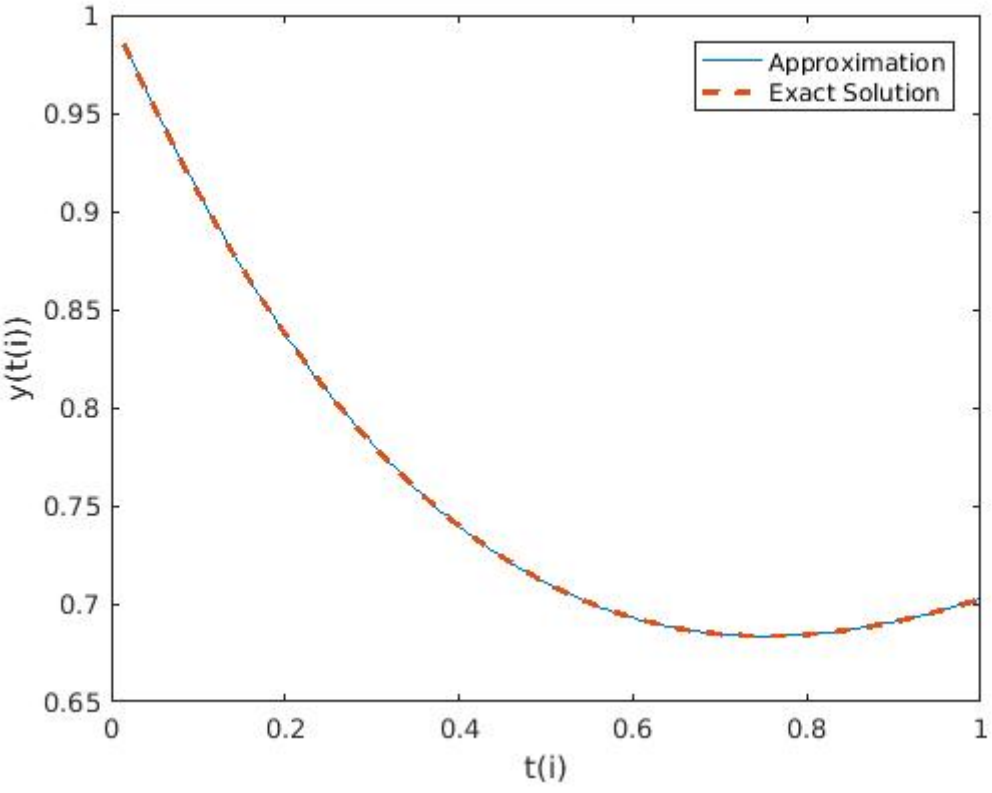
By Adams-Moulton method:

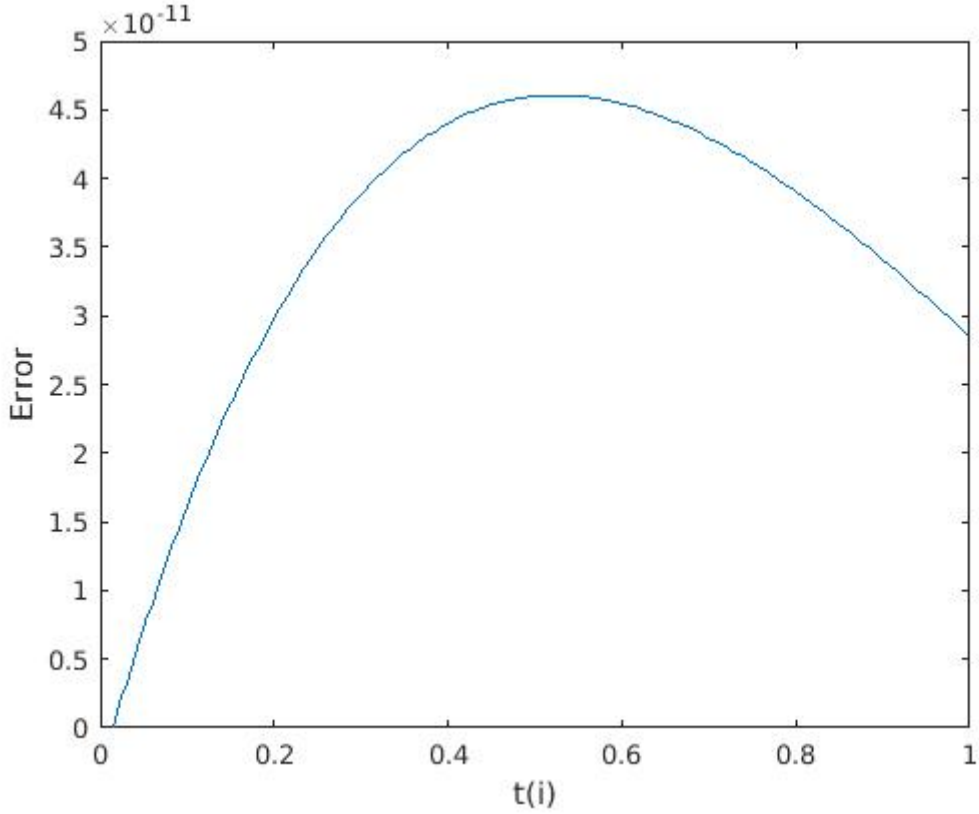




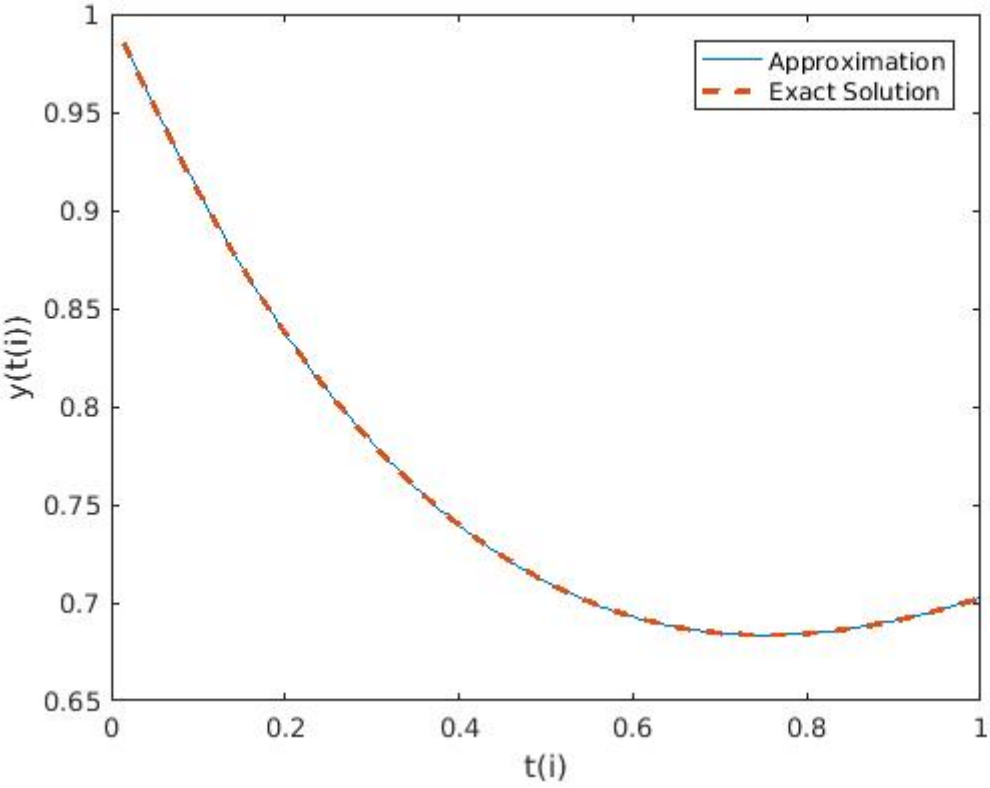
Using starting values obtained from RK method of order four:

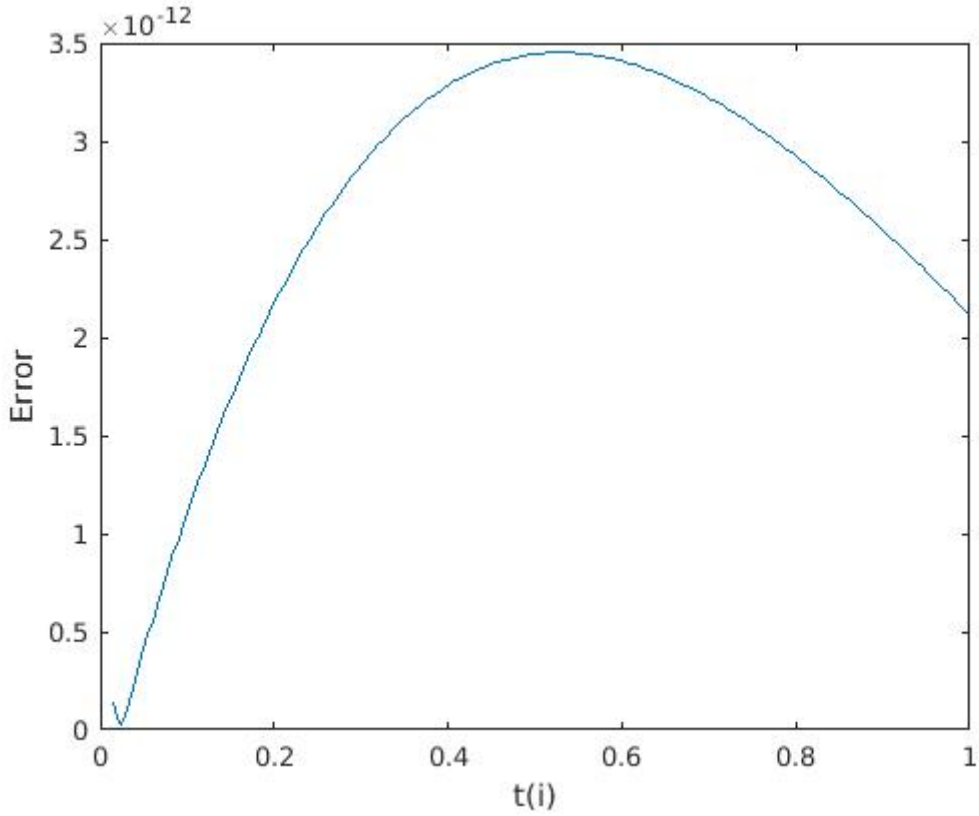
By Adams-Bashforth method:





By Adams-Moulton method:

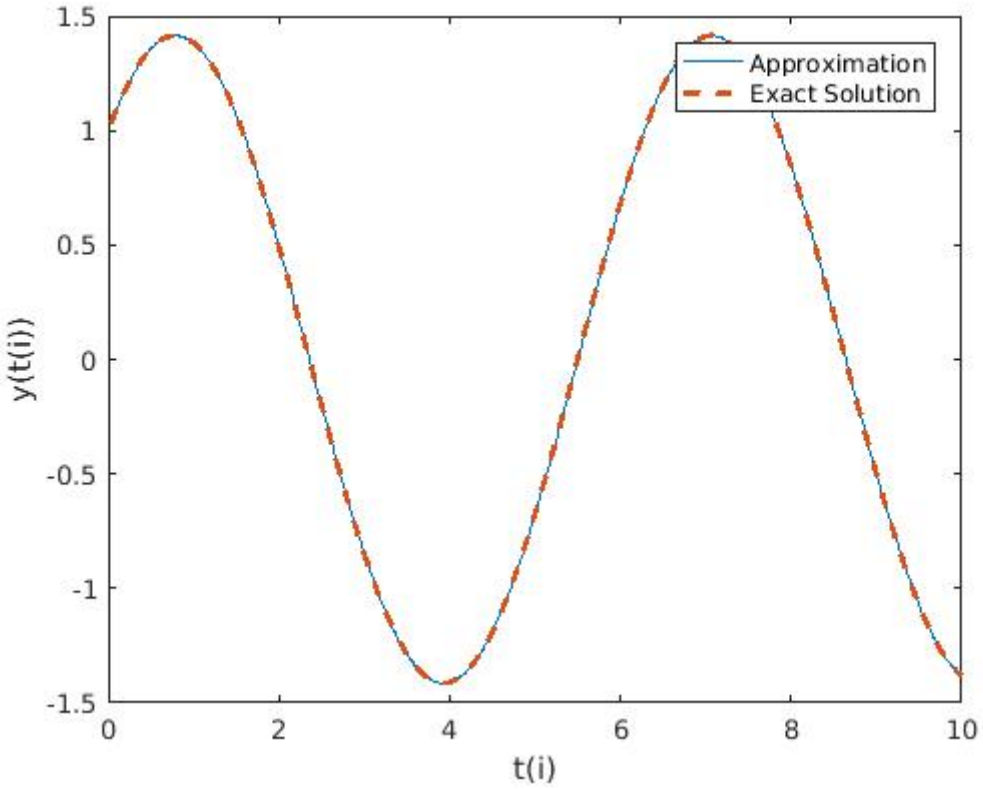


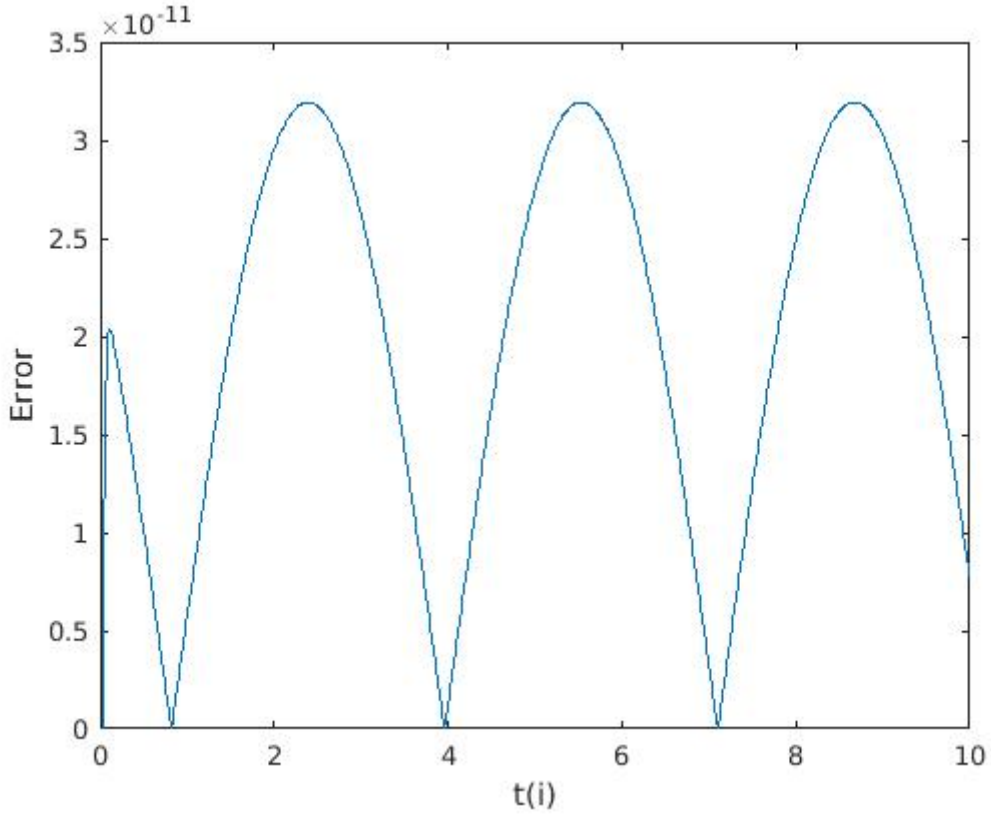


Problem 5: For h=0.01, we get:

Adams- Bashforth method doesn't converge.

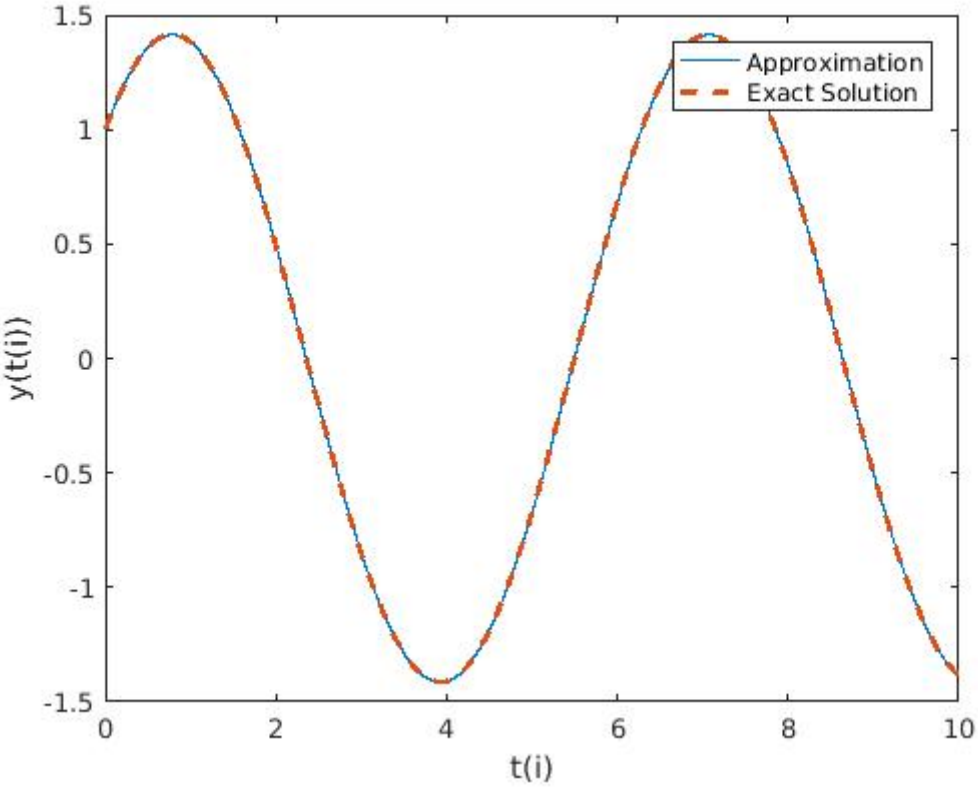
By Adams-Moulton, we get:

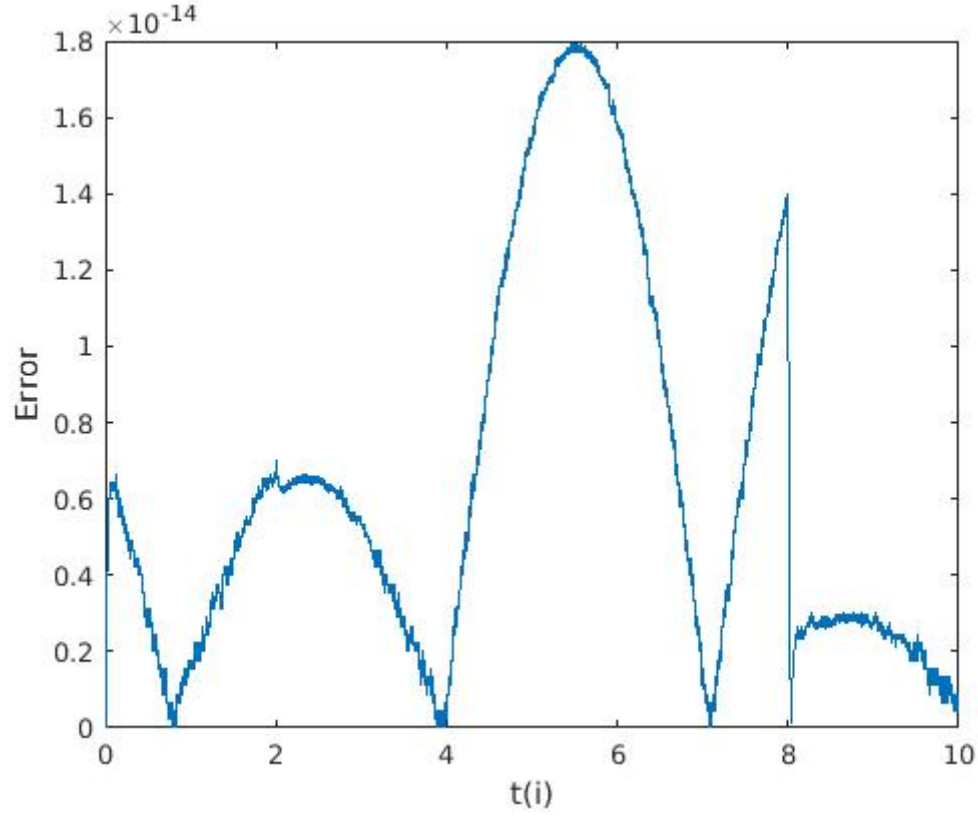

Approximation
Exact Solution
y(t(i))
t(i)
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10


×10^-11
Error
t(i)
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10

For h = 0.001,

By Adams – Bashforth, we get:





By Adams-Moulton, we get: