1. Motivated by the Fibonacci formula consider the recursion :

$$U_{i+1} = (U_i + U_{i-1}) \text{ mod } 1.$$

   Analyze and visualize the values generated in the unit cube (by plotting the triplets (U_i, U_{i+1}, U_{i+2})) on which all the points fall. What do you observe ? In you opinion is this a good generator for uniform distribution ?

2. Consider the *lagged Fibonacci generator* :

$$N_{i+1} = (N_{i-\nu} - N_{i-\mu}) \text{ mod } M \text{ , for suitable } \nu, \mu.$$

   Now consider the specific recursion :

$$U_{i+1} = (U_{i-17} - U_{i-5}).$$

   This recursion immediately produces floating point numbers U_i in the unit interval. In the event that $U_i < 0$, then set $U_i = U_i + 1$. Note that here you need 17 initial values before you can start the generator.

   (a) Use the linear congruence generator to generate the first 17 values of U_i.

   (b) Then generate the values of U_i (say for 1000, 10000 and 100000 values).

   (c) For each of the above set of values plot (U_i, U_{i+1}) and also draw a bar diagram. What are your observations ?

3. Consider the exponential distribution with mean θ :

$$F(x) = 1 - e^{-x/\theta} \text{ , } x \geq 0.$$

   Generate the values of X using the algorithm $X = -\theta \log(1 - U)$, where $U \sim \mathcal{U}[0, 1]$. Form a probability distribution (with frequencies of very small intervals). Plot the distribution function of these generated values (with midpoints of each interval against its frequency, and join by smooth curve), and the actual distribution function (using the above formula). You can do this plot for various values of the number of observations generated. Also, give the corresponding values of the sample mean and variance. Compare the values of mean and variance to see whether they converge to actual values.

4. Consider the Arcsin law with the distribution :

$$F(x) = \frac{2}{\pi} \text{ arc sin } \sqrt{x} \text{ , } 0 \leq x \leq 1.$$

   Generate the values of X making use of the inverse transformation :

$$X = \sin^2\left(\frac{U\pi}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos(U\pi), \text{ where } U \sim \mathcal{U}[0, 1].$$

   Form a probability distribution (with frequencies of very small intervals). Plot the distribution function of these generated values (with midpoints of each interval against its frequency, and join by smooth curve), and the actual distribution function (using the above formula). You can do this plot for various values of the number of observations generated. Also, give the corresponding values of the sample mean and variance.