

1. Consider the following algorithm (done in class) to sample from a distribution :

- (i) Generate a uniform $U \sim \mathcal{U}[0, 1]$.
 - (ii) Find $K \in \{1, 2, \dots, n\}$ such that $q_{K-1} < U \leq q_K$.
 - (iii) Set $X = c_K$.
- (a) Use the above algorithm to generate exponential variates.
- (b) Using the same algorithm to generate discrete uniform variates on $\{1, 3, 5, 7, 9\}$.

2. Use the acceptance-rejection method to generate samples from a distribution with density function:

$$f(x) = 20x(1-x)^3.$$

- (a) Take $\mathcal{U}[0, 1]$ as the known density function g . You have to first find out the smallest constant c that satisfies the required inequality ($f(x) \leq cg(x)$).
 - (b) Then, with the smallest such constant c , generate a random sample from f . Check if these values converge. Also, keep a count of number of iterations needed to find one variate.
 - (c) Finally, compute the average of all these values, and see how it compares with the value of c that you have chosen.
 - (d) Now, repeat the above experiment with two values of c higher than the smallest value that you have chosen. What are your observations ?
3. Consider the problem of generating a discrete random variable X that takes one of the values $1, 2, \dots, 10$ with corresponding probabilities $0.11, 0.12, 0.09, 0.08, 0.12, 0.10, 0.09, 0.09, 0.10, 0.10$. Using the discrete uniform distribution on $1, 2, \dots, 10$ as the base (i.e., in place of g), generate a random sample from X , again with two possible values of the constant c . What is your conclusion ?