

# LAB 08

## ◆ Question 1

Following values were obtained in one of the sample run of the program to estimate I :

$$I = E(\exp(\sqrt{U}))$$

The 95% confidence interval for M = 100 is ( 1.99991 , 2.000316 )  
The estimated mean of I is 2.02655

The 95% confidence interval for M = 1000 is ( 1.99991 , 2.000316 )  
The estimated mean of I is 1.994623

The 95% confidence interval for M = 10000 is ( 1.99991 , 2.000316 )  
The estimated mean of I is 1.99753

The 95% confidence interval for M = 1e+05 is ( 1.99991 , 2.000316 )  
The estimated mean of I is 2.001461

## ◆ Question 2

Anthithetic Variable were used in the following way to increase the efficiency of estimator by decreasing variance :

$$Y = (\exp(\sqrt{U_i}) + \exp(\sqrt{1 - U_i}) ) / 2$$

Following values were obtained in one of the sample run of the program to estimate  $I = E(\exp(\sqrt{U}))$  :

95% confidence interval for M = 100 is ( 1.99991 , 2.000316 ).  
Var(Y1) = 0.1836068 Var(Y) = 0.001070836  
Variation reduction percentage from variance of (Y1)/2 is : 98.83356  
Variation reduction percentage from variance of Y1 is : 99.41678

95% confidence interval for M = 1000 is ( 1.99991 , 2.000316 ).  
Var(Y1) = 0.199652 Var(Y) = 0.001090033  
Variation reduction percentage from variance of (Y1)/2 is : 98.90807  
Variation reduction percentage from variance of Y1 is : 99.45403

95% confidence interval for M = 10000 is ( 1.99991 , 2.000316 ).  
Var(Y1) = 0.1947 Var(Y) = 0.001073072  
Variation reduction percentage from variance of (Y1)/2 is : 98.89772  
Variation reduction percentage from variance of Y1 is : 99.44886

95% confidence interval for M = 1e+05 is ( 1.99991 , 2.000316 ).  
Var(Y1) = 0.1953412 Var(Y) = 0.001088484  
Variation reduction percentage from variance of (Y1)/2 is : 98.88556  
Variation reduction percentage from variance of Y1 is : 99.44278

### ◆ Question 3

Sqrt(U) is used as a control variable and efficiency of estimator used to estimate  $I = E(\exp(\sqrt{U}))$  was improved by reducing the variance.

Following values were obtained in one of the sample run of the program to estimate  $I = E(\exp(\sqrt{U}))$  :

The 95% confidence interval for M = 100 is ( 2.000831 , 2.001474 )  
Original Variance = 0.193989 Reduced Variance = 0.002698865  
Variance Reduction Percentage = 98.60875

The 95% confidence interval for M = 1000 is ( 2.000831 , 2.001474 )  
Original Variance = 0.193989 Reduced Variance = 0.002698865  
Variance Reduction Percentage = 98.60875

The 95% confidence interval for M = 10000 is ( 2.000831 , 2.001474 )  
Original Variance = 0.193989 Reduced Variance = 0.002698865  
Variance Reduction Percentage = 98.60875

The 95% confidence interval for M = 1e+05 is ( 2.000831 , 2.001474 )  
Original Variance = 0.193989 Reduced Variance = 0.002698865  
Variance Reduction Percentage = 98.60875