Geometric Brownian

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Outline

Modeling Stock Prices

- In finance, we are always interested in the return on stocks.
- The Normal distribution is a typical distribution model for return on the stock; indeed equivalent to modeling the value of the stock as a lognormal distribution.
- Assume that the return on the stock is normally distributed distribution of stock price with annual mean μ and annual standard deviation σ.
- Denote by S_t the value of the asset at time t and $S_{t+\Delta t}$ claims denoting the the value Δt periods later. Thus, the percentage change (or return) of the value of the stock between times t and $t + \Delta t$ is approximated by

$$-\log(S_t)+\log(S_{t+\Delta t}) = \log(\frac{S_{t+\Delta t} - S_t + S_t}{S_t}) = \log(1+r_{\Delta t}) \approx r_{\Delta t},$$

where, $r_{\Delta t} = \frac{S_{t+\Delta t}-S_t}{S_t}$

Geometric Brownian Motion:

- Another way to write the stock price at time $t + \Delta t$ is

$$S_{t+\Delta t} = S_t \exp(\mu \Delta t + \sigma Z \Delta t), \quad Z \sim N(0,1)$$

- This is the discrete analogue of the geometric diffusion:

$$\frac{dS}{S} = \mu dt + \sigma dB$$

where dB is a "Brownian Motion" (or Weiner) process with

$$dB = Z\sqrt{dt}$$

- In the geometric Brownian motion, the change in the log S between time 0 and T has a Normal distribution with

$$\log(S_T) - \log(S_0) \sim N((\mu - \frac{1}{2}\sigma^2)T, \sigma^2 T)$$

where S_0 is the initial price of the stock while S_T is the stock price T periods later.

- This is equivalent S_T having a log-normal distribution:

$$\log(S_T) \sim N(\log(S_0) + (\mu - \frac{\sigma^2}{2})T, \sigma^2 T)$$

- Therefore, the recursive procedure for simulating values of S at $0 = t_0 < t_1 < \cdots < t_n$:

$$S(t_{i+1}) = S(t_i) exp([\mu - \frac{1}{2}\sigma^2](t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} Z_{i+1})$$

- It is straightforward to show that the mean is given by :

$$E(S_T) = S_0 e^{\mu T}$$

- Variance is

$$Var(S_T) = S_0^2 e^{2\mu T}(e^{\sigma^2 T} - 1).$$