Probability Space (Ω, F, P)

Ω : Sample space
- Set of all outcomes
- Ex. 1) Roll of dice, $\Omega = \{1, 2, \cdots 3\}$
  2) can be infinite, $\Omega = \mathbb{R}$

F : Event space
- Set of Subsets ~~to which pro~~
- Ex. Roll was even $\{2, 4, 6\}$
  Temp was ≤ 70

P : Probability measure
- Axioms of Probability
  (i) $P(A) \geq 0$
  (ii) $P(\Omega) = 1$
  (iii) If $A_1, A_2, \ldots A_n$ are disjoint events,

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

Properties :

(i) $P(A^c) = 1 - P(A)$

(ii) Union Bound, for any events A, B $\quad P(A \cup B) \leq P(A) + P(B)$

$$\boxed{\text{Posterior} = \frac{\text{liklihood} \times \text{Prior}}{\text{evidence}}}$$

Conditional Probability :

Let B be s.t. $P(B) > 0$

then

$$P(A|B) \triangleq \frac{P(A \cap B)}{P(B)}$$

Bayes Rule

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$= \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|A^c)\,P(A^c)}$$

Example : (From wikipedia)

A test for a disease produces 1% False positive & false negative. 0.5% of people have that disease. If a randomly selected person tests positive on that test, what is the probability ~~he~~ ~~is~~ that he has disease ?

$$P(+|ND) = 0.01$$

$$P(-|D) = 0.01$$

$$P(D) = 0.005$$

$$P(D|+) = ?$$

$$P(D|+) = \frac{P(+|D)\,P(D)}{P(+|D)\,P(D) + P(+|ND)\,P(ND)}$$

$$= \frac{0.99 \times 0.005}{0.99 \times 0.005 + 0.01 \times (0.995)}$$

$$\approx 0.332$$

Barc. RM

Entropy of a distribution :

Let P be distribution on $\mathcal{X}$

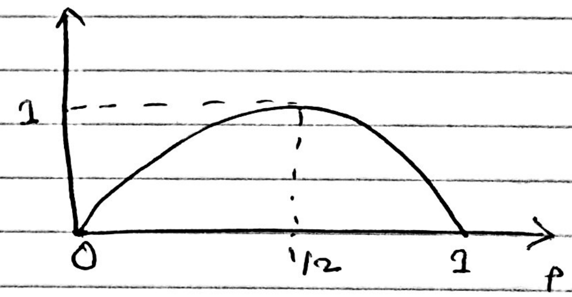
i.e. $\sum_x P_x = 1$ $\quad P_x \geq 0$

Then $H(P) \triangleq \sum_x - P_x \log P_x$

Ex. 1 Let

$X = 1$ w.p. p
$\quad = 0$ w.p. $1-p$

$H(P) = - p\log p - (1-p)\log(1-p)$



Ex. 2
Let

$$X = \begin{cases} a & \text{w.p. } 1/2 \\ b & \text{w.p. } 1/4 \\ c & \text{w.p. } 1/8 \\ d & \text{w.p. } 1/8 \end{cases}$$

$H(X) = \frac{1}{2}\log 2 + \frac{1}{4}\log 4 + \frac{1}{8}\log 8 + \frac{1}{8}\log 8$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + 3/8$$

$$= 1 + 3/4 = 7/4,$$

Suppose we wish to determine value X in the minimum number of binary question.

		# of Q	Prob
Is	$X = a$	1	1/2
	$X = b$	2	1/4
	$X = c$	3	1/8
	$X = d$	4	1/8

$$E[Q] = 7/4$$

For any discrete RV, minimum expected number of questions lie between

$H(X)$ & $H(X)+1$