8.31.22

Wednesday, August 31, 2022 5:00 PM

Transverse Mass

Goal : We want to measure / deduce the mass of the W boson

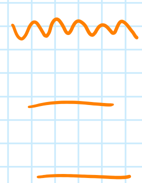
Weak force

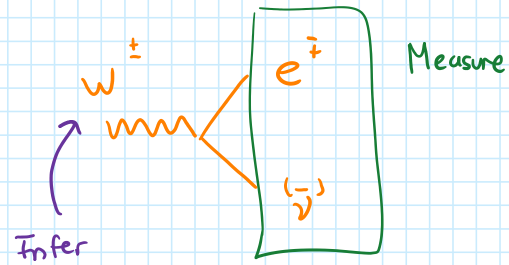
Theory $\longleftarrow$ $W^+ e^- \bar{\nu} + W^- e^+ \nu$

Pictures : W :

e :

ν :





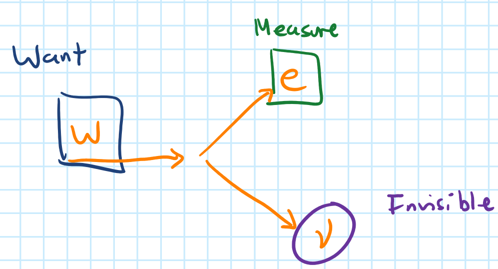
Given this interaction, the W decays into electron and neutrino.

Assign 4 momentum to everything :

$$P_W^{\mu} = (E_W, \vec{P}_W) \leftarrow E^2 = p^2 + m^2, \quad (P_W)^{\mu}(P_W)_{\mu} = m_W^2$$

$$P_e^{\mu} = (E_e, \vec{P}_e)$$

$$P_\nu^{\mu} = (E_\nu, \vec{P}_\nu)$$



4 - Momentum is conserved

$P_W^{\mu} = P_e^{\mu} + P_\nu^{\mu}$, now take a copy of each side and multiply them according to the metric,

$$(P_W)^{\mu}(P_W)_{\mu} = (P_e + P_\nu)^{\mu}(P_e + P_\nu)_{\mu}, \quad (A+B)^2 = A^2 + B^2 + 2AB$$

$$\shortparallel$$

$$m_W^2 = P_e^{\mu} P_{e\mu} + P_\nu^{\mu} P_{\nu\mu} + 2 P_e^{\mu} (P_\nu)_{\mu}$$

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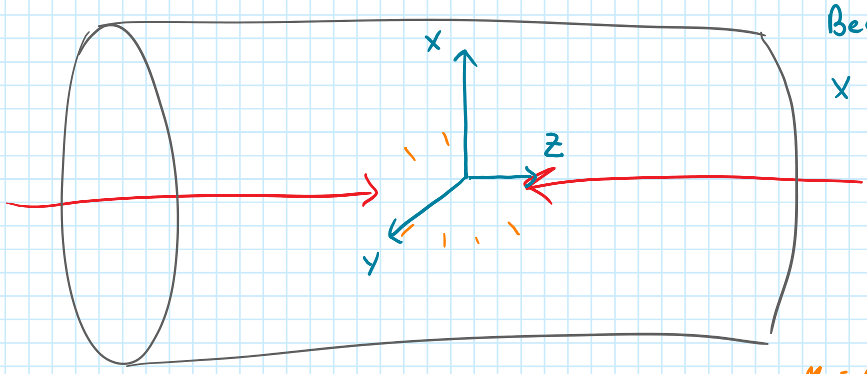
$$\star \quad m_W^2 = m_e^2 + m_\nu^2 + 2\left(E_e E_\nu - \vec{p}_e \cdot \vec{p}_\nu\right)$$

BUT, we don't know anything about the ν. Specifically E_ν.

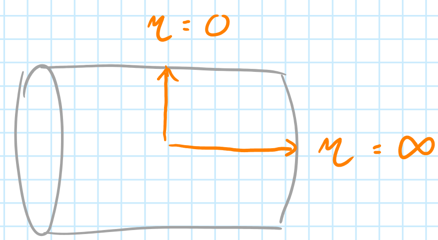
To get around this, we define "Transverse" variables



Beams travel in z

x & y are transverse to the beam direction.

Rapidity : $\eta = \frac{1}{2} \ln\left(\frac{E + p_z}{E - p_z}\right) \rightarrow$



- This is an angle that tells you the trajectory of the outgoing particles.

- This is invariant under boosts in the z direction.

Check: $E \rightarrow \gamma E - \gamma \beta p_z$

$p_z \rightarrow -\gamma \beta E + \gamma p_z$

$$\frac{E + p_z}{E - p_z} \rightarrow \frac{\gamma E - \gamma\beta p_z + (-\gamma\beta E + \gamma p_z)}{\gamma E - \gamma\beta p_z - (-\gamma\beta E + \gamma p_z)}$$

$$= \frac{E + p_z - \beta(E + p_z)}{E - p_z + \beta(E - p_z)}$$

$$= \frac{(E + p_z)}{(E - p_z)}\boxed{\frac{[1 - \beta]}{[1 + \beta]}}$$

Really this is $\ln\left(\frac{(E + p_z)}{\ldots}\right) + \ln\left(\frac{1 - \beta}{\ldots}\right)$.

Really this is $\ln\left(\frac{(E+P_z)}{(E-P_z)}\right) + \underbrace{\ln\left(\frac{1-\beta}{1+\beta}\right)}_{\text{Ignore for reasons not important right now}}$.

Using rapidity, define two more quantities

$$\sinh \eta = P_z / E_T$$
$$\cosh \eta = E / E_T$$

$$E_T^2 \equiv \vec{P}_T^{\,2} + m^2$$

Hyperbolic Sin & Cosine

Transverse Energy

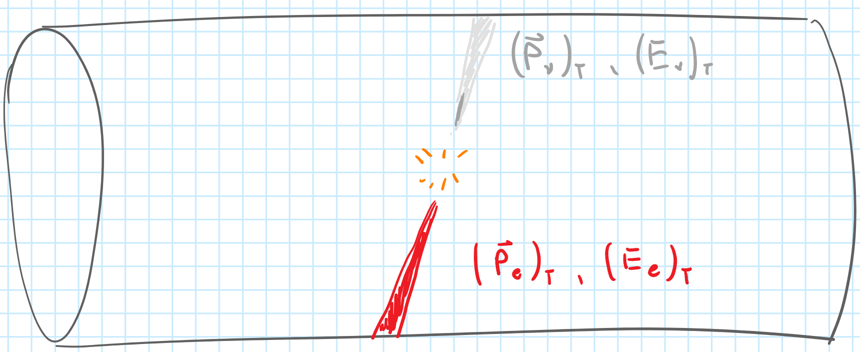
Transverse Momentum

$P_{ex} P_{\nu x} + P_{ey} P_{\nu y}$

$\boxed{\vec{P}_{Te} \cdot \vec{P}_{T\nu}} + P_{ze} P_{z\nu}$

$$\star \quad m_W^2 = m_e^2 + m_\nu^2 + 2\left(E_e E_\nu - \vec{P}_e \cdot \vec{P}_\nu \right)$$

$$= m_e^2 + m_\nu^2 + 2\Big((E_T)_e \cosh(\eta_e) \times (E_T)_\nu \cosh(\eta_\nu) - (\vec{P}_T)_e \cdot (\vec{P}_T)_\nu - P_{ez} P_{\nu z} \Big)$$



We don't actually know/care about the P_z for either.

Assume there is NO longitudinal momentum of the e & ν => Only transverse.

$$\star \quad M_W^2 \geq \underbrace{m_e^2 + m_\nu^2 + 2\left[(E_T)_e (E_T)_\nu \cosh^2(\Delta\eta) - (\vec{P}_T)_e \cdot (\vec{P}_T)_\nu \right]}_{\equiv \text{ Transverse Mass, } M_T^2\left((P_e)^\mu, (P_\nu)^\mu\right)}$$

RHS are all #'s that we know.

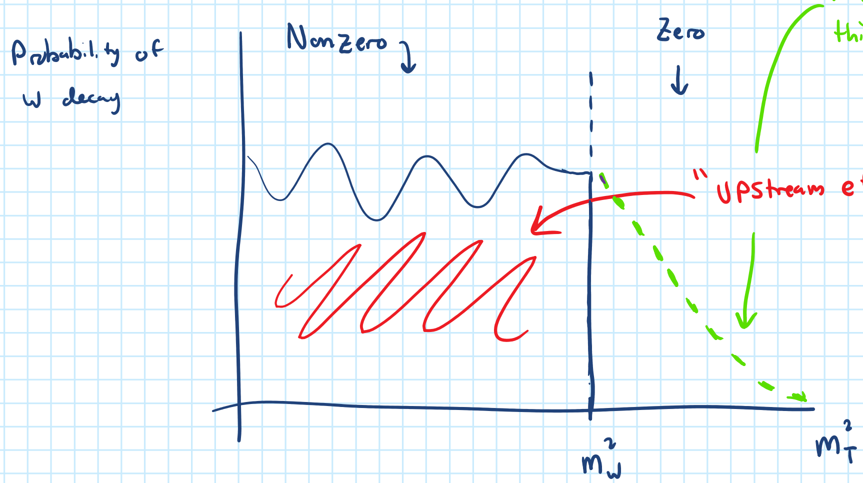
Each event will have a different RHS.

The largest the RHS will ever be is M_W^2

Nonzero ... Zero

Not sure where this comes from

The largest the RHS will ever be is M_W



Not sure where this comes from

"Upstream effects", LHC

$pp \to W + \text{Junk}$, $W \to e\nu$

There is nothing stopping the W from having P_T to begin with.

It must recoil against the "Junk"

	PP	W	Junk
P_T :	0	$(P_T)_W$	$(-P_T)_W$

$W \to e \; \nu$

$(P_T)_e \qquad (P_T)_\nu$

$(P_T)_W = (P_T)_e + (P_T)_\nu$

If $(P_T)_W = 0$

Then $(P_T)_e = (-P_T)_\nu$

In general, the W will have nonzero (P_T)

Next time: Recursive Jigsaw Method

See what you can find.

- what is it good for?
- when would you use it?
- what makes it useful?
- what makes it so costly?

M_{T2}

Why can't we do M_T twice?

https://arxiv.org/pdf/1004.2732.pdf

Section 3.2 talks about Transverse Mass

Section 4.1 talks about (Transvers Mass)_2