7.26.22

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4 - Vectors for real.

3 - Vectors : $\vec{x} = (x, y, z)$

$\vec{v} = (v_x, v_y, v_z)$

Contain information ONLY about space

4 - Vector :

index

$$\underbrace{X^{\mu}}_{\text{4-Position}} = (ct, \overbrace{x, y, z}^{\vec{x}})$$

take $\frac{d}{dt}$

$$\underbrace{U^{\mu}}_{\text{4-Velocity}} = (c, v_x, v_y, v_z)$$

multiply by $\gamma m c$

$$\underbrace{P^{\mu}}_{\text{4-Momentum}} = (\gamma E, \gamma P_x c, \gamma P_y c, \gamma P_z c), \text{ "}\gamma\text{" means that we may not in the rest frame}$$

Measured in the rest frame of the object

Natural Units

$c = 1, \hbar = 1$

$\downarrow$

$E = mc^2$ & $E = f h$

If $c = 1$, then $\frac{m}{s} = 1$

so meters & seconds mean the same thing

Notation :

$X^{\mu} = (ct, \vec{x})$, μ indexes the components

$\mu = 0, 1, 2, 3$

$X^0 = ct$, "$\mu = 0$" as the "time" or "energy" component

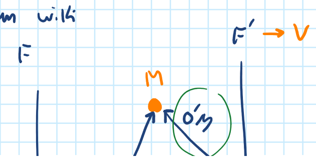
$X^3 = z$

$X^{\mu} = (ct, x^i)$

Spatial index, $i = 1, 2, 3$

How do 4 - Vectors Transform

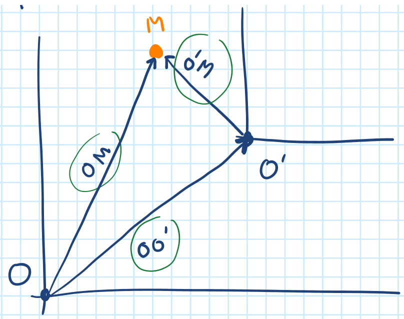
From wiki



Frame F' move at speed V relative to frame F.

$$\vec{OM} = \vec{OO'} + \vec{O'M}$$

For simplicity, choose M to



$\vec{OM} = \vec{OO'} + \vec{O'M}$

For simplicity, choose M to be along the x-axis ($y = z = 0$)

$$x = vt + x'/\gamma$$

Solve for x'

rest frame

$$\boxed{x' = \gamma(x - vt)}$$

Different Frame

How fast is moving

This is how spatial components transform under special relativity

"Lorentz Transformation"

What about time?

$x' = \gamma(x - vt)$, invert all primes

$x = \gamma(x' - vt')$, solve for t'

$$-\left(\frac{x}{\gamma} - x'\right)\frac{1}{v} = t' \qquad t' = \frac{\gamma x' - x}{v}$$

$$-\left(\frac{x}{\gamma} - \gamma(x - vt)\right)\frac{1}{v} = t'$$

$$-\frac{1}{\gamma v}\left(x - \gamma^2(x - vt)\right) = t'$$

$$-\frac{1}{\gamma v}\left(x - \gamma^2 x + \gamma^2 vt\right) = t'$$

$$-\frac{1}{\gamma v}\left(x(1 - \gamma^2) + \gamma^2 vt\right) = t', \qquad \gamma^2 = \frac{1}{1 - (v/c)^2}$$

$$\frac{-x(1 - \gamma^2)}{\gamma v} - \gamma t = t'$$

$$1 - \gamma^2 = \frac{1 - (v/c)^2 - 1}{1 - (v/c)^2} = \boxed{-\frac{(v/c)^2}{1 - (v/c)^2}} = -\frac{v^2}{c^2}\gamma^2$$

$$\frac{+x\left(-v^2/c^2\, \gamma^2\right)}{\gamma v} - \gamma t = t'$$

The result is correct but there is a sign error somewhere in the derivation.

$$\boxed{t' = \gamma t - \gamma\frac{xv}{c^2}}$$

This is how the time components transform.

The point

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \qquad x'^\mu = \begin{pmatrix} \gamma(t - xv/c^2) \\ \gamma(x - vt) \\ y \\ z \end{pmatrix}$$

Formulate this as a matrix.

Want matrix M that satisfies

$M x^\mu = x'^\mu$, ① M must be a 4x4

$$\underbrace{\begin{pmatrix} \gamma/c & -\gamma\frac{v}{c^2} & 0 & 0 \\ -\gamma\frac{v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{M\text{, "Lorentz Matrix"}} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma t - \gamma x v/c^2 \\ \gamma x - \gamma v t \\ y \\ z \end{pmatrix}$$

Lorentz matricies & Notation

$(ct, x, y, z) = x^\mu$

$$\begin{pmatrix} \dots \end{pmatrix} = M^\mu{}_\nu$$

row #

column #

What the entries are

$$\boxed{M^\mu{}_\nu X^\nu} = X'^\mu$$

Why would we ever do this?

$$\begin{pmatrix} - & - & - \end{pmatrix}\begin{pmatrix} - \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \\ - \\ - \end{pmatrix}$$

Notational shorthand.

$$M^\mu{}_\nu X^\nu = X^\mu$$

① $X^\mu = X^\nu = X^\sigma = X^\delta = \dots$

We are only choosing a greek letter to index X, doesn't matter which letter.

☆ ② Rule: repeated indicies are summed over

③ Free indicies must match.

ex: $M^i{}_j = \begin{pmatrix} M^1{}_1 & M^2{}_1 & M^3{}_1 \\ M^1{}_2 & M^2{}_2 & M^3{}_2 \\ M^1{}_3 & M^2{}_3 & M^3{}_3 \end{pmatrix}$ $\quad X^i = \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}$

Compute $\underline{M^i{}_j X^j}$ using rule ②.

$$X'^{i} = M^{i}{}_{j} X^{j}$$

☆ pick $i = 2$

$$X'^{2} = M^{2}{}_{j} X^{j}$$

$$= M^{2}{}_{1} X^{1} + M^{2}{}_{2} X^{2} + M^{2}{}_{3} X^{3} \longrightarrow \begin{pmatrix} \cdot \\ \\ \cdot \end{pmatrix}$$

<u>ex:</u>

$$A^{i}{}_{j} = \begin{pmatrix} A^{1}{}_{1} & A^{1}{}_{2} \\ A^{2}{}_{1} & A^{2}{}_{2} \end{pmatrix} \qquad B^{i}{}_{j} = \begin{pmatrix} B^{1}{}_{1} & B^{1}{}_{2} \\ B^{2}{}_{1} & B^{2}{}_{2} \end{pmatrix}$$

Compute $A^{i}{}_{j} B^{j}{}_{K} = C^{i}{}_{K}$