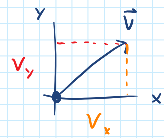
7.19.22

Tuesday, July 19, 2022 9:34 AM

4 - Vectors

Review vectors: Mag & direction. Draw rays.



Add vectors

↳ Coordinates, Components

Notation: (V_x, V_y) or $\vec{V} = V_x \hat{i} + V_y \hat{j}$

New: $\vec{V} = \begin{bmatrix} V_x \\ V_y \end{bmatrix}$, 2 rows, 1 col

"2-by-1" vector

Matrix multiplication ↓

Matrices: objects that "act" on vectors

$$\begin{pmatrix} \# & \# \\ \# & \# \end{pmatrix} \begin{matrix} \leftarrow \text{row 1} \\ \leftarrow \text{row 2} \end{matrix}$$

↑ Col 1 ↑ Col 2 "2 by 2"

"Rows & Columns rule" ↓

Rule: # of matrix cols = # of vector rows

$$\begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \text{ 2 rows}$$

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \quad 2 \times \boxed{2 \quad 2} \times 1 \leftarrow$$

Match

How to compute matrix multiplication: Matrix goes on the left, vector goes on the right.

$$\vec{A} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \quad M = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}$$

$$\vec{B} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} M_1 A_1 + M_2 A_2 \\ M_3 A_3 + M_4 A_2 \end{pmatrix}$$

$M\vec{A} \neq \vec{A}M$

$2 \times \cancel{2}\ \cancel{2} \times 1$ ↓ 2×1

$2 \times \boxed{1 \quad 2} \times 2$ Don't match

Exercises: Multiply the following

① $\vec{A} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$, $M = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}$, $\vec{A} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$, $M = \begin{pmatrix} M_1 & 0 \\ 0 & M_1 \end{pmatrix}$ What does M do to $\vec{A}$?

"Main diagonal"

Matrix Multiplicative identity.

rescale by M_1

$$\begin{pmatrix} A_1 M_1 \\ A_2 M_1 \end{pmatrix} = M_1 \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

② $\vec{A} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$, $M = \begin{pmatrix} M_1 & M_2 & M_3 \\ M_4 & M_5 & M_6 \\ M_7 & M_8 & M_9 \end{pmatrix}$, $\vec{A} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$, $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ What does M do to $\vec{A}$?

"off diagonal"

③ $M_1 = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}$ $M_2 = \begin{pmatrix} M_A & M_B \\ M_C & M_D \end{pmatrix}$

Hint: think about the rows & columns rule.