

$$\vec{b}(t) = \vec{r}_1 + (\vec{r}_2 - \vec{r}_1) t$$

$$\text{dist} = \sqrt{b_x^2 + b_y^2 + b_z^2}$$

\uparrow \uparrow \uparrow
 lav t

$$\text{lav} = f(t, t^2)$$

↓ solve

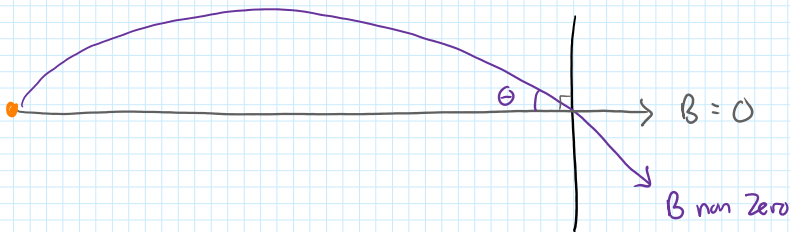
$$t = \text{---}$$

$$F = q \vec{v} \times \vec{B}$$

$$F = q \vec{c} \times \vec{B}$$

$$\vec{a} = \frac{q}{E} \vec{c} \times \vec{B}$$

Top Down

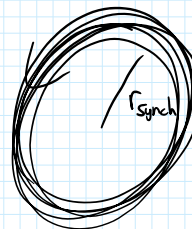


Compare B nonzero to B zero

$$\phi_{B \neq 0} - \phi_{B=0}$$

$$E = \frac{r_{\text{earth}}}{r_{\text{synch}}} \leftarrow E, B$$

valid only if $E \ll 1$



$$B \sim \frac{1}{r}$$

valid only if $\epsilon \ll 1$

$$B \sim \frac{1}{r}$$

Getting r_{synch}

$m \neq 0$, classical, e^- in $\vec{B} = (0,0,1)$

$$F = ma$$

↓

$$q \vec{v} \times \vec{B} = m \boxed{a_{\text{centripetal}}} \quad \text{why is } a_{\text{cent}} = \frac{v^2}{r}$$

$$= m \frac{v^2}{r}$$

$$\dot{v} = \frac{v^2}{r}$$

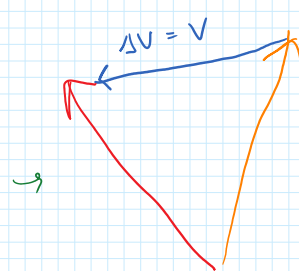
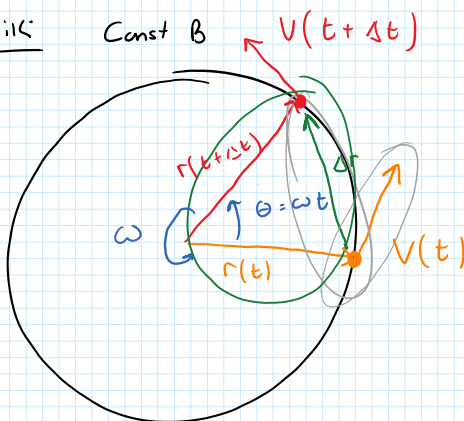
$$v^{-2} dv = \frac{1}{r} dt, \quad \text{const } r, \quad \text{const } B \star$$

$$-\frac{1}{v} = \frac{1}{r} t + C.$$

$$\underline{r = vt + C}$$

Synchrotron

With Const B



$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t}$$

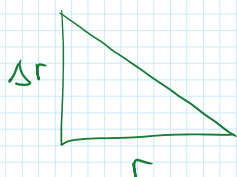
$$\frac{\Delta r}{\Delta t} = \Delta v$$

\vec{v} are always \perp to \vec{r}

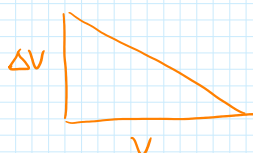
So the position and velocity triangles are mathematically similar (same scaling)



Position



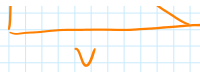
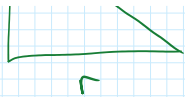
Velocity



$$\frac{\Delta r}{r} = \frac{\Delta v}{v}$$

$$\Delta v = v \frac{\Delta r}{r}$$

$$\Delta v = v \frac{\Delta r}{r}$$

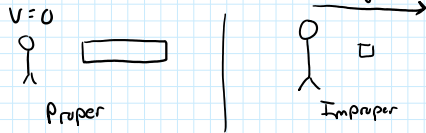


$$a = \frac{\Delta v}{\Delta t} = \frac{v \frac{\Delta r}{r}}{\Delta t} = \frac{v}{r} \left(\frac{\Delta r}{\Delta t} \right) = \frac{v^2}{r}$$

Potential problems

① length contraction

only in direction of motion



$$\begin{aligned} 6 \times 1 &= 2 \times 3 \\ l &= \gamma \quad l' \\ \text{Proper} &= \gamma \text{ (Improper)} \end{aligned}$$

• Always moving \perp to \vec{r} (location)

• Change Δr ? Yes

$$\Delta r \rightarrow \gamma \Delta r$$

$$\frac{\gamma \Delta r}{r} = \frac{\Delta v}{v} \rightarrow \frac{v}{r} \gamma \frac{\Delta r}{\Delta t} = \boxed{\gamma \frac{v^2}{r} = a_{\text{cent}}}$$

• $\gamma = \gamma(v)$, what v is this?

v in γ is the same v as the v in v^2/r

• $r_{\text{synch}}?$

$$F = qV \times B$$

$$\boxed{m \gamma} \frac{v^2}{r} = qV \times B$$

Total Energy E

$$E \frac{v^2}{r} = qV \times B$$

Punchline

non relativistic

$$F_{\text{cent}} = (m) a_{\text{cent}}$$

relativistic

$$F_{\text{cent}} = (E) a_{\text{cent}}$$

$$r_{\text{synch}} = \frac{E v^2}{qVB} = \boxed{\frac{E v}{qB}} \quad \text{relativistic}$$

$$v^2 = \frac{qB}{E} \quad \text{Curvature} \quad K \quad vB$$

location from origin

$$\hookrightarrow V^2 = \frac{q}{E} K V B$$

$$V^2(\quad) = \frac{q}{E} \underbrace{K(r)}_{\text{location from origin}} V(\quad) \underbrace{B(r)}$$

$$K(\vec{r}) = \frac{E V}{q B(\vec{r})}$$

$$K = \frac{E V}{q \#/r} \quad \text{For us } B(r) \sim \#/r$$

$$\underline{K} = \#_2 \underline{r} \quad \text{Matches intuition}$$