Noise Fundamentals

1. Introduction

Electrical noise is often considered a nuisance of little consequence; however, it is an important characteristic based on fundamental concepts, and understanding this phenomenon is crucial when measuring small signals where noise can limit measurement precision or completely obscure the signal. Some level of noise is intrinsic in any dissipative electrical system. Noise is a consequence of basics principles such as the second law of thermodynamics and the quantitation of electrical charge, and its characteristics depend on the type of electrical system being measured.

The objective of this lab is to study the fundamentals of two different types of electrical noise: Johnson noise and shot noise. The former is caused by the random thermal fluctuations of electrons, and from measurements of Johnson noise in a resistor it is possible to determine a value for Boltzmann's constant. Shot noise is caused by the granularity of electrical charge, and measurement of this phenomenon enable one to determine the change of an electron. During this lab, you will learn how noise is modeled and on what parameters these models depend on, and how noise is quantified and measured.

A picture of the equipment that will be used for these measurements is shown in Figure 1, and subsequent sections will refer to these components during the set-up portion of the experiment. Whenever you turn-on the equipment to perform measurements, you will need to wait 20 to 30 minutes for the electronics to stabilize before starting measurements. If after 30 minutes the measurements are still unstable, contact the lab staff.

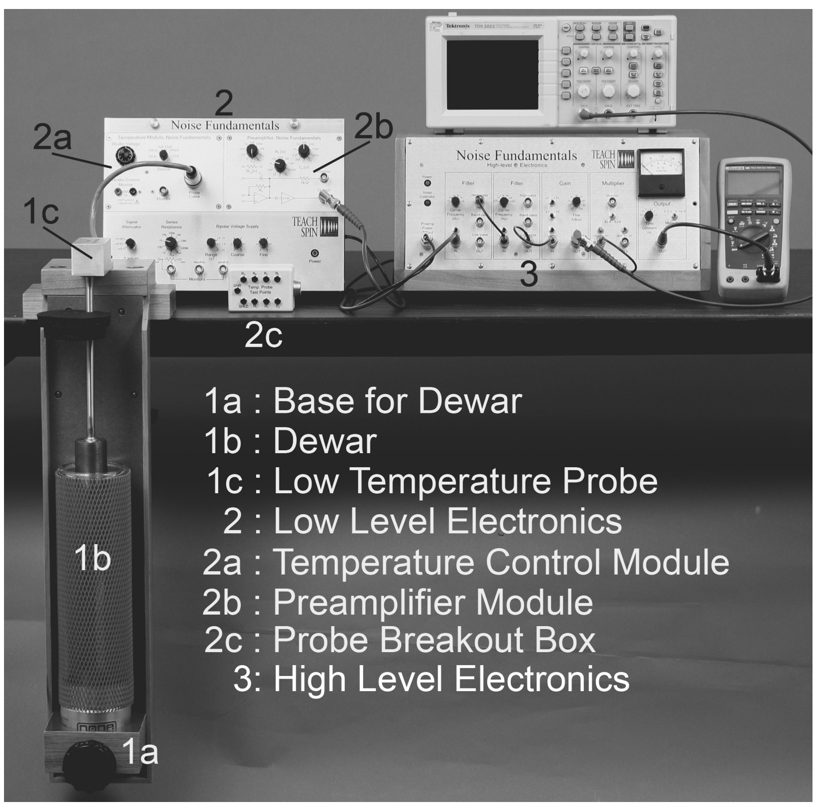

Figure 1. A picture of the equipment used in this experiment.

The low-level electronics box (LLE) contains a preamplifier module, a temperature module for monitoring and controlling the temperature of the low-temperature probe, and some additional electrons for calibrating and measuring shot noise in the lower panel. The shot noise experiment will require reconfiguring the internal wiring in the LLE box. When you get to this point, contact the instructor or lab staff to have this done.

2. Johnson Noise

2.1 Theoretical Background

In any electrical conductor at a finite temperature, the electrons are thermally agitated, and this movement produces a randomly fluctuating voltage across the conductor in which they reside. The average of this voltage is zero; however, the size of the fluctuating voltage can be quantified by time averaging the square of the voltage, $\langle V^2(t)\rangle$. John Johnson first measured this noise in 1926, and shortly after his observation, Harry Nyquist developed an explanation. See Reference 2. The fluctuating voltage contains many frequency components. In fact for an ideal resistor, the noise is "white", meaning the power spectral density is constant up to high frequencies comparable to the thermal energy.

There are several approaches to describing this phenomenon, and the one presented here follows Nyquist's original discussion based on black-body emissions from a resistor in thermal equilibrium with its surroundings. To understand this, consider two resistors connected to each other with a lossless transmission line of length L. The source resistor R_s will be modeled as an ideal resistor with an emf V_J for the Johnson noise. The resistors and characteristic impendence of the transmission line are all matched so that energy travelling down the line will be absorbed without reflection. This situation is depicted in Figure 2.

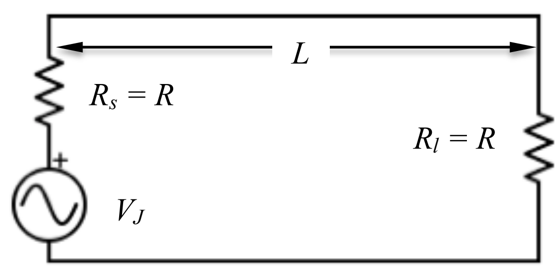

Figure 2. A long lossless transmission line of length L terminated at both ends with impedance-matched resistors.

For propagating electromagnetic modes, the boundary condition requires the voltages at the ends of the transmission line be equal. This is satisfied when $n\lambda = L$ where λ is the wavelength of the mode and n is an integer. If c is the velocity of the waves in the transmission line and f is the frequency, $c = f\lambda$, and the boundary condition becomes $f = nc/L$. From this expression, it is clear that the number of propagating modes per unit frequency is L/c. Now assume the whole system is in thermal equilibrium at a temperature T. The average energy for each mode is given by the Planck distribution

$$\epsilon(f) = \frac{hf}{e^{hf/k_BT} - 1} \tag{1}$$

where h is Planck's constant and k_B is Boltzmann's constant. The average energy per unit time with frequencies in the range from f to $f + \Delta f$ incident on the source resistor is given by

$$P_i = \left(\frac{c}{L}\right)\left(\frac{L}{c}\Delta f\right)\epsilon(f) = \Delta f \frac{hf}{e^{hf/k_BT} - 1} \tag{2}$$

In this expression, the first factor is rate at which energy is absorbed by the source resistor, the second factor is the number of modes in the frequency range and the last factor is the energy per mode.

By the principle of detailed balance, this power is equal to the power emitted by the source resistor. The thermal voltage V_J in the source resistor generates a current I_J in the transmission line. Hence the average power emitted by R_s is

$$P_e = R_s\langle I_J^2\rangle = R_s\left\langle\left(\frac{V_J}{R_s + R_l}\right)^2\right\rangle = \langle V_J^2\rangle\frac{R_s}{(R_s + R_l)^2} = \frac{\langle V_J^2\rangle}{4R} \quad . \tag{3}$$

The fact that $R_s = R_l = R$ was used in the last step. Equating $P_e = P_i$ yields

$$\langle V_J^2\rangle = 4R\Delta f \frac{hf}{e^{hf/k_BT} - 1} \quad . \tag{4}$$

If $hf \ll k_BT$,

$$\frac{hf}{e^{hf/k_BT} - 1} \quad \longrightarrow \quad k_BT \quad , \tag{5}$$

and with Eq. (5), Eq. (4) becomes

$$\langle V_J^2(t)\rangle = 4k_B RT\Delta f \quad . \tag{6}$$

This equation is known as Nyquist's theorem. Notice that $\langle V_J^2(t)\rangle$ is linear in R, T and Δf, and by measuring $\langle V_J^2(t)\rangle$ as a function of these quantities, it is possible to determine a value for Boltzmann's constant. In the remainder of Section 2, you will measure Johnson noise as a function of R and T to verify Eq. (6), and from these data determine k_B.

2.2 Measurement of Johnson Noise as a Function of Resistance

For this part of the experiment, the Johnson noise will be measured with different values of the resistance while holding the temperature and bandwidth constant. To do this, you will use the pre-amplifier module in the LLE box. The basic circuit is shown in Figure 3. You can see that the preamplifier amplifies the noise signal from the resistor R_{IN} with a gain of 6. The output from the pre-amplifier is further increased by a factor of 100 with a second op-amp. The actual wiring diagram for the pre-amplifier module is shown in Figure 4.

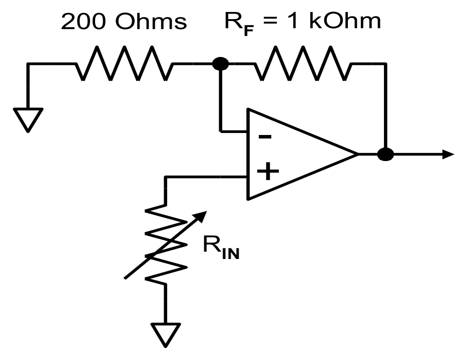

Figure 3. Schematic of pre-amplifier for Johnson noise measurement.

With R_f set to 1 kΩ for a gain of 6, select the source resistor to be $R_{in} = 1$ kΩ on the LLE box. The feedback capacitance is not connected in this configuration. Next connect the output from the LLE to an oscilloscope. A sensitive scale of perhaps 10 mV/division will be needed to see a signal. Set the horizontal axis to 5 μs/division and trigger close to zero volts. Notice that the noise is rather small and fluctuates a lot.

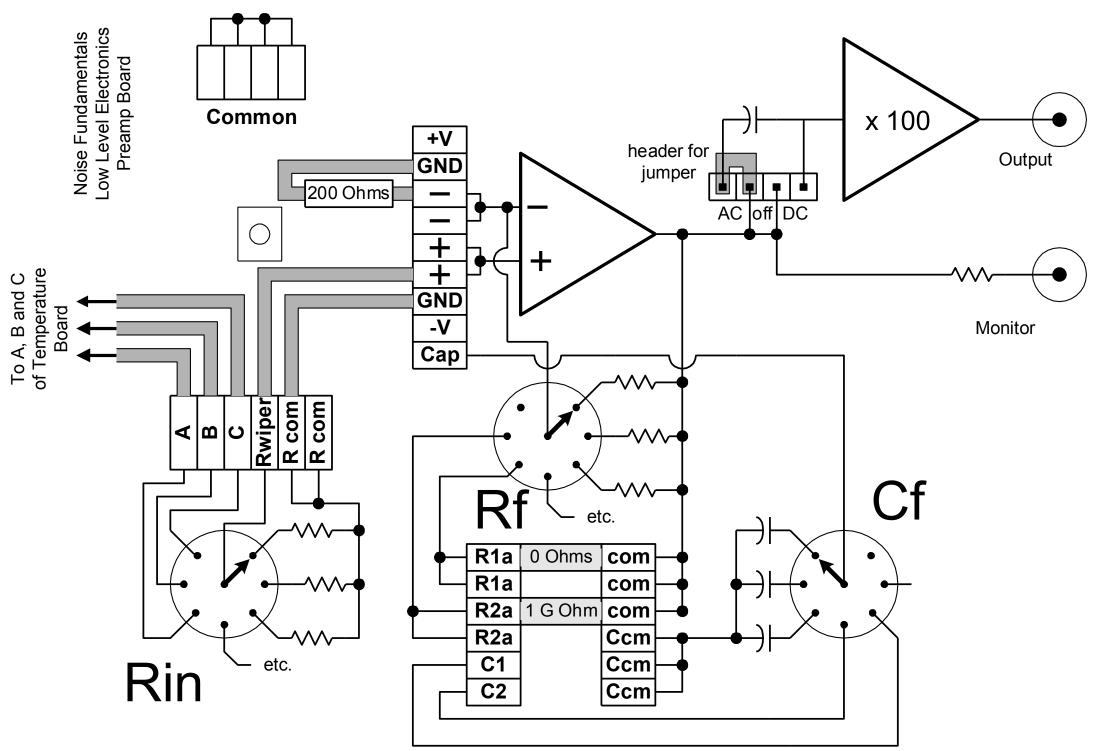

Figure 4. Wiring diagram for the pre-amplifier module in the LLE. The input resistors are selected with R_{in}. By changing R_f, it is possible to control the feedback on the op-amp and adjust the gain. After the first op-amp, the signal is further amplified by a factor of 100 at the output.

For the next step, additional processing of the signal will be performed with the high-level electronics (HLE) box. Notice that there are two filter modules on the HLE for limiting the frequency range of the signal. Each of these filters will provide a single cutoff, one for lower frequencies and the other for higher frequencies. Connect the signal from the output on the LLE to the input of the first filter, and then connect the high-pass output of the first filter to the input of the second filter. Finally connect the low-pass output of the second filter to the gain module before sending the signal to the oscilloscope input. The low-pass and high-pass outputs only allows frequencies above and below the corner frequency settings, respectively, and with these two modules it is possible to limit the frequencies to band determined by the two corner frequency settings. With the cut-off frequencies set to 0.1 kHz and 100 kHz, the HLE gain at 300 and all the modules AC coupled view the single on the oscilloscope. The signal is much larger now, and a lower sensitivity scale can be used on the oscilloscope. Notice how the signal changes as you change the source resistance. Knowing the amplification of the signal, is the size of signal consistent with the prediction of Nyquist's theorem? Do not proceed if there is a significant discrepancy.

To better quantify this signal for comparison to Nyquist's theorem, the signal must be squared and averaged. This is accomplished electronically with the last two modules in the HLE box: the multiplier and the final output modules. The multiplier multiplies two signals, ether the inputs on A and B (AxB) or the input on A times itself (AxA). This result appears at the *Monitor* connector as $V_{Mon}(t) = V_A^2(t)/10$ V and the *OUT* connecter after being averaged over the selected time interval as $\propto \langle V_A^2(t)\rangle$. If an oscilloscope is connected to the *Monitor* connector, you will notice the signal is always positive, unlike the output from the amplifier. To convince yourself that the output of the multiplier is really squared, switch the scope to the x-y mode and connect V_A to the x-channel and V_{Mon} to the y-channel. The trace should look parabolic. The averaged output from the multiplier shows up on the analog meter above the output module, and a digital multimeter (DMM) can also be used to measure this at the *OUT* connector. The ELE can handle voltages in the range of ± 10 V. Being a random signal, noise can occasionally have large fluctuations. To avoid clipping the signal with these uncommon events, adjust the HLE gain so that the final signal is ~1V. In the future, always check that the voltage at the output of the HLE amplifier is below ± 10 V and the V_{Out} does not exceed ~1 V to insure the signals are not distorted.

The voltage V_{Out} will still vary a little, but these fluctuations will decrease as the averaging time interval is increased. Notice that the signal takes considerably longer than the averaging time to settle down. Once somewhat stable, average several values recorded over a period of time. If $V(t)$ is the time-dependent noise signal in the source resistor over the frequency range determined by the filters, $V_{Out} = \langle V^2(t)\rangle (G_1 G_2)^2/10\ V$ is the signal at the *OUT* connector where G_1 and G_2 are the gains of the pre-amplifier module and HLE amplifier, respectively.

The discussion above assumes the electronics are ideal and do not add any noise to the signal from the source resistor. Unfortunately, this is not true. The instrumentation noise has the same characteristics as the Johnson noise from the resistor, so it is not possible to identify it in the signal. However, it is possible to correct for it. A real amplifier can be modeled as an ideal amplifier with a noise voltage V_n added to its source. When the input of the amplifier with a gain of G is connect to a source of Johnson noise, its output is given by $V_{amp} = G\left(V_J(t) + V_n(t)\right)$. The time average of this voltage squared is

$$\langle V_{amp}^2(t)\rangle = G^2 \left\langle \left(V_J(t) + V_n(t)\right)^2 \right\rangle = G^2 \left(\langle V_J^2(t)\rangle + 2\underbrace{\langle V_J(t) V_n(t)\rangle}_{=0} + \langle V_n^2(t)\rangle \right) = G^2\left(\langle V_J^2(t)\rangle + \langle V_n^2(t)\rangle\right) \quad .$$

The second term is zero because V_J and V_n are uncorrelated time dependence quantities, so the time average of their product is zero. Taking into account the added noise from the instrumentation, the signal at the *OUT* connecter is

$$V_{Out} = \left(\langle V_J^2\rangle + \langle V_n^2\rangle\right)(G_1 G_2)^2/10\ V \quad . \tag{7}$$

If the Johnson noise is reduced to an insignificant value, V_n can be directly measured. This is accomplished by setting $R_{\text{in}} = 1\ \Omega$. Assuming the instrumentation noise does not change with the source resistance, the measurement at 1 Ω can be used to correct the measurements with a larger source resistance where the Johnson noise is observable. If the bandwidth of the measurement is changed, V_n must be remeasured. For a more detailed discussion of instrumentation noise, see Reference 5.

For this part of the experiment, measure the noise for each source resistance R_{in} and correct for the contribution from the instrumentation. From these data, calculate the noise power spectral density given by $S = \langle V_f^2 \rangle / \Delta f$ where Δf is the effective noise bandwidth of the instrument, which is related to the corner frequency settings on the HLE filters. Using a model for the filter response, Δf can be calculated for difference values of the high-pass cutoff f_{HP} and low-pass cutoff f_{LP}. The results of this calculation are displayed in Table 1. Details on how this calculation is performed are given at the end of Section 2.3. Plot these data with error bars and comment on the relationship to the theoretical prediction. (Hint: examine the data on a log-log plot.) The internal source resistors have a tolerance of 0.1% up to 1 MΩ and 1% for higher resistances. Calculate Boltzmann's constant carefully take into account the uncertainty of this measurement.

Table 1: The effective noise bandwidth Δf in Hz for the ELE box filters. Numbers were calculated using a model for an ideal filter response, ignoring systematic effects. The uncertainties of these numbers is ~4%. Systematic effects may increase the values by $1\% \pm 1\%$ and $3\% \pm 1\%$ for the 33 kHz and 100 kHz columns, respectively.

		f_{LP}					
		0.33 kHz	1.0 kHz	3.3 kHz	10 kHz	33 kHz	100 kHz
f_{HP}	10 Hz	355	1,100	3,654	11,096	36,643	111,061
	30 Hz	333	1,077	3,632	11,074	36,620	111,039
	100 Hz	258	1,000	3,554	10,996	36,543	110,961
	300 Hz	105	784	3,332	10,774	36,321	110,739
	1000 Hz	9.0	278	2,576	9,997	35,543	109,961
	3000 Hz	0.4	28	1,051	7,839	33,324	107,740

2.3 Measurement of Johnson Noise as a Function of Temperature

In this section, the Johnson noise will be measured when the source resistor is at different temperatures between 77 K and 370 K. The same electronics will be used to measure and process the noise signal, but some addition equipment is needed to change and control the temperature. To reduce the temperature, liquid nitrogen (LN_2) cryogen, which has a boiling point of 77 K, is used. Being very cold, LN_2 must be thermally isolated from the room temperature environment. Otherwise, it will quickly dissipate. To do this, a Dewar, which is a glass container with an evacuated space between the inner and outer walls like a thermos bottle, is used to hold the cryogen. The source resistors, $R_{\text{a}} = 10\ \Omega, R_{\text{b}} = 10\ \text{k}\Omega$ and $R_{\text{c}} = 100\ k\Omega$, are mounted on the end of a long probe that is inserted into the cryogen from the top (see Figures 5 and 6). The support tube of the probe and the electrical leads attached to the source resistors have a very low thermal conductivity to reduce the heat load to the cryogen. Figure 7 shows a transdiode, which is a transistor with the base connected to the collector, mounted to the bottom of the probe to monitor the temperature. When a constant current of 10 μA is applied, the voltage drop across this device is ~400 mV, and this voltage increases by ~2 mV/K as the temperature is decreased. Once calibrated, the strong temperature dependence of the transdiode makes it an idea temperature transducer. Finally, there is an ~75 Ω electrical heater on the probe for controlling the temperature. A transdiode's calibration is posted on iLearn under this lab.

Unlike the measurements in Section 2.2, the source resistors are now connected to the pre-amplifier in the LLE box by much longer leads that run down the temperature probe. To protect the signals from external interference, a shielded coaxial cable is used; however, a significant amount of capacitance (~100 pF) exists between the signal lead and ground with this cable. This capacitance affects the frequency dependence of the noise signal so that it is no longer flat at the input of the pre-amplifier. A correction for this problem will be discussed later.

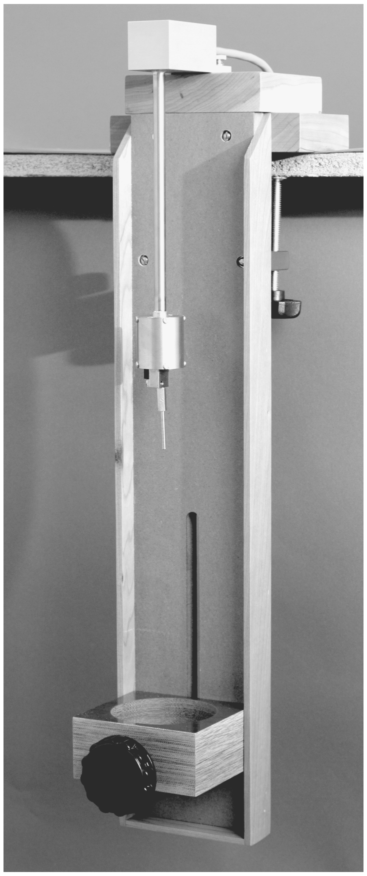

Figure 5. Temperature probe with Dewar support. Dewar is removed.

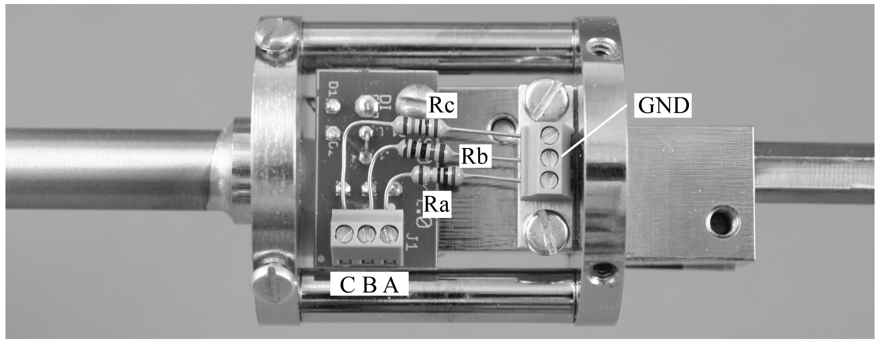

Figure 6. An interior view of the temperature probe end showing the three source resistors, R_a, R_b and R_c.

A breakout box, which can be connected directly to the probe cable, provides 8 test points for checking the components in the probe. All three source resistors are connected to a common ground. The heater is connected to H_1 and H_2. D_1 and D_2 are both connected to the emitter of the transdiode, and the collector/base are connected to ground. A multimeter in diode-testing mode should read ~0.5 V if the positive lead is connected to either D_1 or D_2.

To perform the temperature-dependent measurements, connect the temperature probe cable to the temperature module on the LLE box. The *Current Source* knob selects the current for the transdiode. The temperature calibration depends on the current level, and for the calibration you have, the current should be set to 10 μA. The voltage across the transdiode is measured at the *Monitor* connector on the temperature module. Finally, a control and connector are provided for adjusting the heater voltage and measuring the heater current. Each source resistor, R_a, R_b or R_c, in the temperature probe can be selected by setting R_{in} to A_{ext}, B_{ext} or C_{ext}, respectively, on the pre-amplifier module.

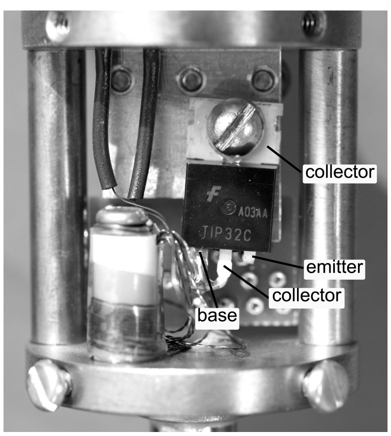

Figure 7. Interior view of the other side of the temperature probe end showing the transdiode for measuring the temperature.

Begin with a bandwidth of ~10 kHz by setting the high-pass cutoff to 1 kHz and the low-pass cutoff to 10 kHz. Following a procedure analogue to the one in Section 2.2, measure V_{out} for the source resistors in the probe at room temperature and the corresponding resistances in the pre-amplifier module. Make sure the signals are not distorted or saturated. Also, try a few different bandwidths. Most likely, the values measured for the resistors in the probe will be smaller than the values for the resistors in the pre-amplifier module, and the discrepancy will increase with larger source resistors and larger bandwidths. The reason for the discrepancy is the probe capacitance reduces the effective bandwidth. From this exercise, decide on a measurement strategy that can accommodate the added probe capacitance.

The next step is to cool the probe down with LN_2. With the Dewar in place and the support lowered to its lowest position, pour ~1 liter of LN_2 into the Dewar. Remember to follow the safety procedures for handling cryogens. Wait for the boiling to subside and place the black foam insulating cover over the mouth of the Dewar. Carefully raise the base of the Dewar until the probe makes contact with the LN_2, and position it so that the bottom plate of the probe is immersed in the liquid. Later for measurements above 77 K, the Dewar can be lowered slightly so that only the cold figure is in the LN_2 to reduce the cooling power of the cryogen, thereby reducing the amount of heat that must be added with the heater.

When the LN_2 has settled down and the temperature has stabilized, repeat the noise measurements on both the resistors in probe and pre-amplifier module. The gains may have to be readjusted to keep the electronics in their optimal range. Even though the resistors in the probe have high tolerances, they do not necessarily apply when the temperature of the resistor is changed a lot. Therefore, use the breakout box to measure the source resistances in the probe at each temperature. Repeat the measurements at a couple of temperatures. The temperature can be increased with the heater. As the temperature is increased, more heater power will be needed, and at some point, you may have to reduce the cooling power of the cryogen by lowering the Dewar. Once you are done collecting data, carefully remove the Dewar and dispose of the LN_2. Leave the probe suspended in the Dewar support as it warms up and never touch the end while it is cold.

After correcting for instrumentation noise, calculate the noise power spectral density given by $S = \langle V_J^2 \rangle / \Delta f$. Unfortunately, the effective bandwidths in Table 1 are inaccurate because of the extra capacitive coupling in the probe leads, and this effect must be included in the calculation of Δf. Before considering the probe capacitance, lets examine the effect of the two filters. The TeachSpin HLE box uses two-pole state-variable filters, and to a very good approximation, they can be modeled with a Butterworth filter response. (See Reference 5 for an explanation of these filters.) The filter-transmission functions for the low-pass filter and high-pass filter are given by

$$G_{LP} = \frac{1}{\sqrt{1 + (f/f_{LP})^4}} \quad \text{and} \quad G_{HP} = \frac{(f/f_{HP})^2}{\sqrt{1 + (f/f_{HP})^4}} \quad , \tag{8}$$

respectively. Each frequency component of the noise voltage coming from the pre-amplifier module is multiplied by these two functions as it goes through the two filters. From there the signal is amplified a second time before being squared by the multiplier. If f_1 and f_2 define the lower and upper cutoffs for the bandpass of the measurement instrument, the noise power spectral density S can be defined in relation to the output of the multiplier with the expression

$$\langle V_J^2 \rangle = \int_{f_1}^{f_2} S\, df = \int_0^{\infty} S (G_{LP} G_{HP})^2\, df = S \int_0^{\infty} (G_{LP} G_{HP})^2\, df \quad . \tag{9}$$

S is pulled out of the integral because it is frequency independent for Johnson noise. It is possible to calculate the noise power spectral density with $S = \langle V_J^2 \rangle / \Delta f$ using

$$\Delta f = \int_0^{\infty} (G_{LP} G_{HP})^2\, df \quad , \tag{10}$$

and in fact the values in Table 1 are numerically calculated with Eq. (10) using the filter-transmission functions in Eq. (8).

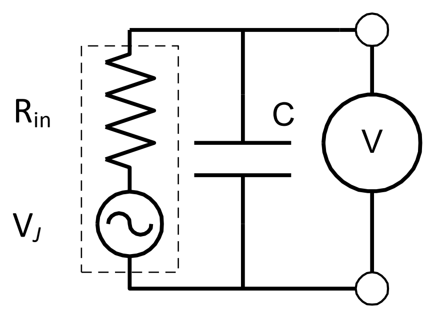

Figure 8. Diagram showing the effect of the capacitance in the leads on the measured noise voltage.

Now that we understand how to model the filter response, lets include the capacitive affects of the leads. Recall that there is a capacitive coupling to ground associated with the leads. The Johnson noise in the source resistor can be modeled as an ideal noiseless resistor in series with a Johnson-noise emf V_J. This combination is attached across the input of the pre-amplifier. The capacitive coupling of the leads is included by adding a capacitance C in parallel with the source resistor-emf combination. This is depicted in Figure 8. The circle inclosing the V represents the pre-amplifier. For the probe in this experiment, $C = 100$ pF. From the diagram, the combination of the lead capacitance and the source resistance forms a low-pass RC filter, which can attenuate the high-frequency components of the noise. The transmission function of the probe is

$$G_C = \frac{1}{\sqrt{1 + (f/f_C)^2}} \tag{11}$$

where the corner frequency is given by $f_C = 1/2\pi R_{\text{in}} C$. For the temperature probe, the effective noise bandwidth is found by numerically integrating

$$\Delta f = \int_{0}^{\infty} (G_C G_{LP} G_{HP})^2 \, df \quad . \tag{12}$$

For the report, plot the noise power spectral density as a function of temperature and demonstrate the measurement is consistent with Nyquist's theorem. Make sure to include error bars in your plot. Next determine Boltzmann's constant with an uncertainty and compare it with what was found in Section 2.2.

3. Shot Noise

3.1 Introduction

While Johnson noise results from the thermal fluctuations of electrons, shot noise is present because of charge quantization. This phenomenon is associated with the movement of discrete particles that obey Poisson statistics, as with electrons in a tunnel junction and photons in a photodiode. Walter Schottky first described it in 1918 while studying current fluctuations in vacuum tubes.

Lets consider a vacuum tube with a flow of electrons from the cathode to the anode. If the number of electrons n hitting the anode is measured during a succession of short time intervals Δt, this number will fluctuate about a mean value $\langle n \rangle$. This fluctuation Δn would result in a fluctuating current called shot noise, which is defined by

$$I_{\text{shot}}^2 \equiv \frac{1}{N} \sum_{i=1}^{N} (I_i - \langle I \rangle)^2 \tag{13}$$

where I_i is the current measured in each time interval, N is the number of measurements and $\langle I \rangle$ is the averaged current. By the definition of a current, $I_i = en_i/\Delta t$ and $\langle I \rangle = e\langle n \rangle/\Delta t$. So Eq. (13) becomes

$$I_{\text{shot}}^2 = \left(\frac{e}{\Delta t}\right)^2 \frac{1}{N} \sum_{i=1}^{N} (n_i - \langle n \rangle)^2 \quad . \tag{14}$$

Since the arrival of electrons is random and uncorrelated, this process is described by Poisson statistics, like radioactive decay. In this case, the RMS value of n is equal to $\sqrt{n}$, and the shot noise current is given by

$$I_{\text{shot}}^2 = \left(\frac{e}{\Delta t}\right)^2 n = \frac{e\langle I \rangle}{\Delta t} \quad . \tag{15}$$

According to the Nyquist-Shannon sampling theorem, the bandwidth of the instrument used to perform the measurement is related to the time interval of the measurement by the expression $\Delta f = 1/2\Delta t$. With this expression, Eq. (15) is

$$I_{\text{shot}}^2 = 2e\langle I \rangle \Delta f = 2eI_{dc}\Delta f \tag{16}$$

where in the last step, the average current is replaced with the DC current. Eq. (16) is known as Schottky's theorem.

3.2 Experimental Set-Up for Shot Noise Measurements

For this part of the experiment, a current of uncorrelated electrons is needed, and this is obtained from an illuminated photodiode. Before proceeding, the LLE box must be internally reconfigured with a photodiode and a small incandescent light bulb. Ask the lab staff to do this. For shot noise measurements, a transimpedance amplifier, which reverse biases the photodiode, has some advantages over the pre-amplifier arrangement used for the Johnson noise measurements. The problems with the other configuration include a breakdown in the linearity between photocurrent and illumination and additional capacitive coupling that limits the bandwidth.

The circuit that will be used is displayed in Figure 9. The photodiode is reverse biased at 12V. Since the non-inverting input is grounded, feedback through R_{f} will actively hold the inverting input at near-zero potential as well. This ensures the voltage drop across the photodiode is maintained at 12 V. Because of the very high input impedance of the op-amp, all the photodiode current I_{PD} passes through R_{f}. From Kirchhoff's voltage law,

$$0 - I_{\text{PD}}R_{\text{f}} = V_0 \quad \Rightarrow \quad V_0 = -R_{\text{f}}I_{\text{PD}} \quad . \tag{17}$$

So the output voltage V_0 is proportional to the photocurrent, which includes both a DC component I_{dc} and an AC component $I_{\text{shot}}(t)$. In effect, this is a current to voltage converter. Also shown in Figure 9 is the DC power supply for the light source. The wiring diagram for the pre-amplifier module and the bipolar power supply is shown in Figure 10.

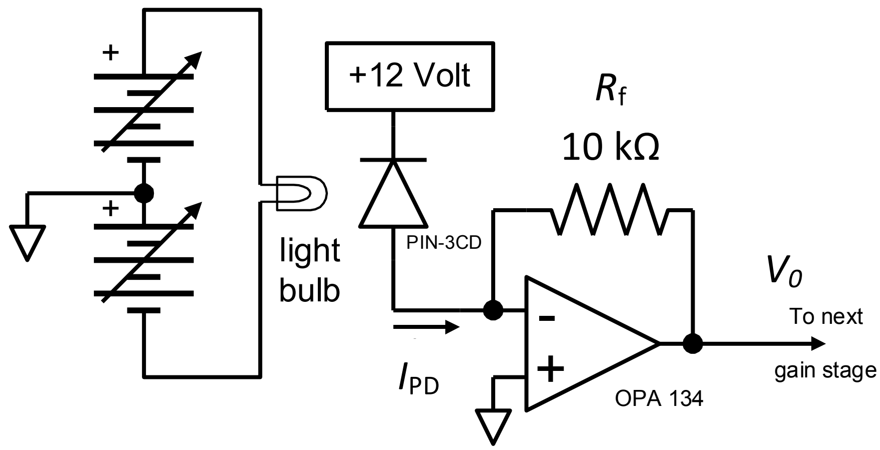

Figure 9. Schematic diagram for a transimpedance amplifier (i.e., current-to-voltage converter for reverse-biasing the photodiode and connecting it to the first stage of the preamplifier module in the LLE box

The signal from the photodetector also contains a noise contribution $V_n(t)$ from the instrumentation that includes the op-amp and the Johnson noise from R_{f}. The total signal from the first stage of the pre-amplifier is $V_s = V_0(t) + V_n(t) = -R_{\text{f}}\big(I_{dc} + I_{\text{shot}}(t)\big) + V_n(t)$. This signal is DC coupled to the *Monitor* output, and a multimeter in DC volts mode connected to this output will average the time-dependent signals to zero and only detect the DC component $-R_{\text{f}}I_{dc}$. The AC component of the signal, which includes all noise contributions with $f \geq 16$ Hz, goes to the next amplifier in the pre-amplifier module. The *Output* connector from the LLE box is $100\times(-R_{\text{f}}I_{\text{shot}} + V_n)$.

Configure the HLE unit in the same way that was used for the Johnson noise measurements, except without the high-pass filter. With all the components AC coupled, a high-pass cutoff of 16 Hz is imposed on the signal. The final output from the HLE unit is

$$V_{Out} = (R_{\text{f}}^2\langle I_{\text{shot}}^2\rangle + \langle V_n^2\rangle)(100\times G_2)^2/10\ V \quad . \tag{18}$$

It is possible that V_n may contain contributions from other components further down the chain from the first stage of the pre-amplifier. Notice that the AC signals, both the shot noise and the instrumentation noise, add in quadrature when squared because these signals are uncorrelated.

3.3 Experimental Procedure and Data Analysis for Shot Noise

Begin by setting $R_{\text{f}} = 10$ kΩ and $f_{LP} = 100$ kHz. Increase the light bulb's power supply until you see evidence of a photocurrent at *Monitor* connector of the LLE box with a multimeter set to DC volts. Verify that the magnitude of the *Monitor* voltage increases with the light intensity, and adjust the light intensity so that $I_{dc} = 10$ μA. ($V_{Mon} = -100$ mV.) Remember to adjust the gain on the HLE box, G_2, to maintain the multiplier output in the range 0.6 V to 1.2 V.

First investigate how the bandwidth of the instrument affects the shot noise by measuring the noise signal with different setting for f_{LP}. Remember, $f_{HP} = 16$ Hz because of the AC coupling in the instrumentation. As the magnitude of the signal changes it may be necessary to adjust the gain of the HLE box to keep the multiplier in the optimal range. Plot the noise power spectral density as a function of bandwidth with error bars. Is the noise power spectral density flat? This trend may eventually break down at large bandwidths because of intrinsic capacitances in the instrumentation. For future measurements, f_{LP} should be set below this breakdown point.

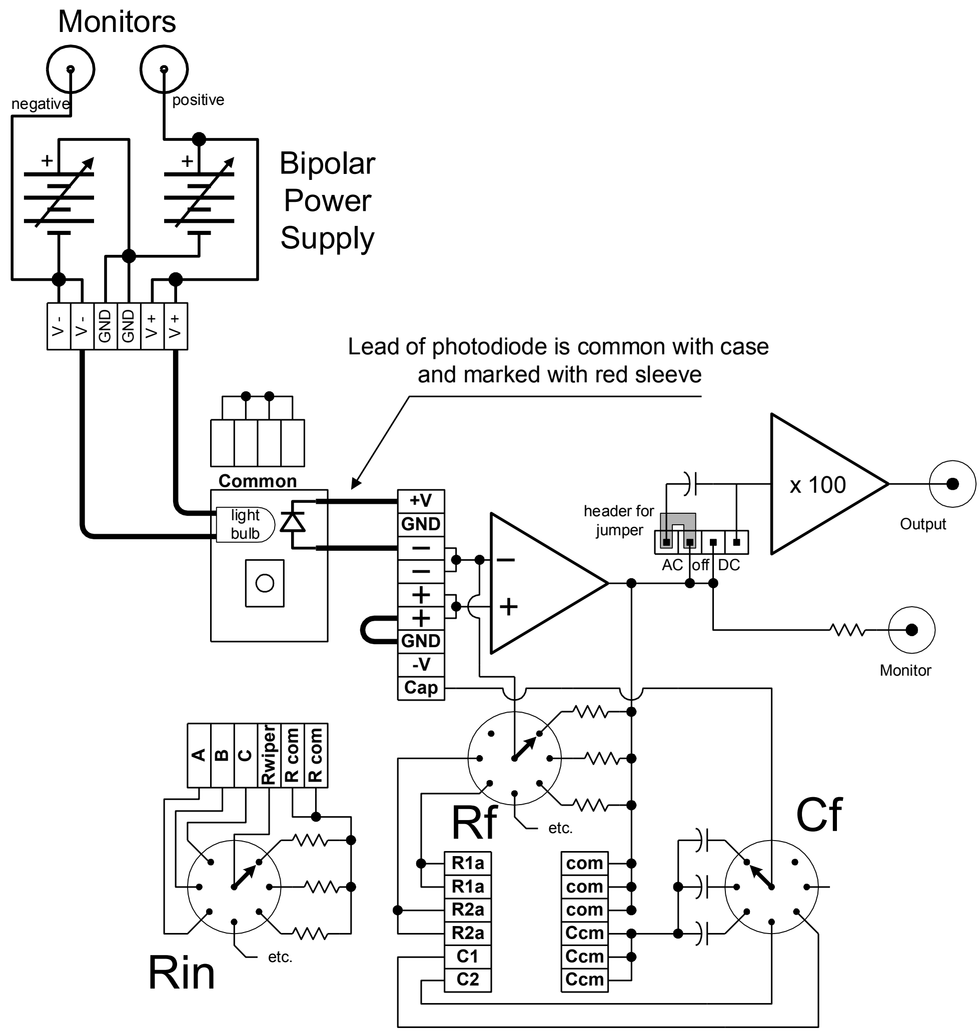

Figure 10. Wiring diagram for the LLE box when configured for shot-noise measurements.

Next, measure the shot noise as a function of the DC photocurrent up to ~100 μA. Your report should include a plot of the noise power spectral density as a function of the DC photocurrent with error bars. Is Schottky's theorem satisfied? Determine a value for the charge of an electron with an uncertainty.

There are a couple of points to keep in mind while performing the shot noise measurements. To begin, the first stage of the pre-amplifier has a small DC offset in the range of ± 2 mV. With the light source off, measure this DC offset and then subtract it from your measurements of I_{dc} to get a more accurate value of the DC photocurrent. Second, do not discount the possibility that the multimeter is interfering with the measurement. The meter can either generate its own noise or the test leads can pick up interference. If present, this additional noise may be injected into the *Monitor* connector of pre-amplifier module. To check for this, see if the output of the multiplier changes when the multimeter is disconnected from the *Monitor* output. Finally, recall that your measurement includes a noise contribution for the instrumentation. The OPA134 op-amp, which is the first stage of the pre-amplifier, has a noise voltage of $\sim 8\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$. The feedback resistor on this op-amp has a comparable amount of Johnson noise, and there may be contributions of other components as well. These uncorrelated noise sources add in quadrature, and must be being subtracted to determine $\langle I_{\mathrm{shot}}^2 \rangle$. Since these sources are independent of the photocurrent, the best way to correct for this is to measure the signal with the light source turned off and then subtract this value from the measurements when the light source is on.

4. References

1. Many books on statistical mechanics and thermal physics have a discussion on Johnson noise, and several different approaches can be found in the literature. See Charles Kittel and Herbert Kroemer, *Thermal Physics*, 2nd edition (Freeman, 1980). A more advanced treatment can be found in Frederick Reif, *Fundamentals of Statistical and Thermal Physics* (Waveland, 2008) or Charles Kittel, *Elementary Statistical Physics* (Wiley, 1958).

2. The two original papers on Johnson noise are J. B. Johnson, "Thermal Agitation of Electricity in Conductors," Phys Rev **32**, 97-109 (1928) and H. Nyquist, "Thermal Agitation of Electric Charge in Conductors," Phys Rev **32**, 110-113 (1928).

3. The original paper on shot noise is W. Schottky, "Über spontane Stromschwankungen in verschieden Elektrizitätsleitern," Annalen der Physik **57**, 541-567 (1918).

4. A more advanced treatment of shot noise can be found in A. van der Ziel, *Noise in Solid State Devices and Circuits* (Wiley, 1986).

5. For a discussion on electronics, electrical measurements and op-amps see Paul Horowitz and Winfield Hill, *The Art of Electronics*, 3rd edition (Cambridge University, 2015). This book also has a discussion on Johnson and shot noise.