

The general iterator for Simpson's rule is:

$$I(a, b, N) = \sum_i^N \frac{\Delta x}{3} [f(x_i) + 4f(x_i + \Delta x) + f(x_i + 2\Delta x)]$$

where, after each step, we increment by $2\Delta x$. This is because if we incremented by only Δx , we would be over-sampling our function, or in other words, our "rectangles" would overlap each other.

To make this look like Newman's form, we fix the integration between $x \in [a, b]$ and, for illustrative purposes, choose $N = 4$ steps, b is then fixed to be $b = a + 8\Delta x$ since we are incrementing by $2\Delta x$.

Explicitly writing out the summation yields:

$$I(a, b, 4) = I_1(a, b, 4) + I_2(a, b, 4) + I_3(a, b, 4) + I_4(a, b, 4)$$

where each I_i is the i th term in the summation. Writing out each I_i yields:

$$I_1 = \frac{\Delta x}{3} [f(a) + 4f(a + \Delta x) + f(a + 2\Delta x)]$$

$$I_2 = \frac{\Delta x}{3} [f(a + 2\Delta x) + 4f(a + 3\Delta x) + f(a + 4\Delta x)]$$

$$I_3 = \frac{\Delta x}{3} [f(a + 4\Delta x) + 4f(a + 5\Delta x) + f(a + 6\Delta x)]$$

$$I_4 = \frac{\Delta x}{3} [f(a + 6\Delta x) + 4f(a + 7\Delta x) + f(b)]$$

where in the last summation, $a + 8\Delta x = b$ by definition. The middle term in each I_i contains an odd prefactor attached to Δx which stops at $7 = N - 1$. The first term in the I_4 summation has $6 = N - 2$ attached to Δx . The values of these prefactors will ultimately fix where the summation ends.

Adding all the I_i terms together, we can rewrite the summation as:

$$I(a, b, N) \sim \frac{\Delta x}{3} \left[f(a) + f(b) + \sum_n 4f[a + (2n - 1)\Delta x] + 2f[a + 2n\Delta x] \right]$$

where the prefactor $2n + 1$ is always odd, and $2n$ is always even. However, the stopping point of the summation over n depends whether n is even or odd. We can make this dependence explicit and write the summation in the form of Newman's equation:

$$I(a, b, N) = \frac{\Delta x}{3} \left[f(a) + f(b) + \sum_{n, \text{ odd}}^{N-1} 4f[a + n\Delta x] + \sum_{n, \text{ even}}^{N-2} 2f[a + n\Delta x] \right]$$