Derivadas - Integrales.

$f(x) = \text{sen}(x)$, $f'(x) = \cos(x)$ y $\int \text{sen}\,x\,dx = -\cos(x) + k$

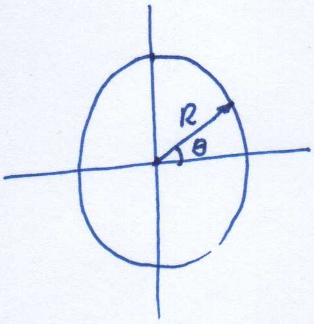
$f(x) = \cos(x)$, $f'(x) = -\text{sen}(x)$ y $\int \cos x\,dx = \text{sen}(x) + k$

$f(x) = a x^n$, $f'(x) = a n x^{n-1}$ y $\int a x^n dx = a \frac{x^{n+1}}{n+1} + k$

$f(x) = \ln(x)$, $f'(x) = \frac{1}{x}$ y $\int \frac{dx}{x} = \ln(x) + k$

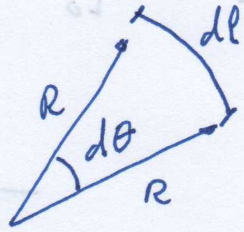
$f(x) = a e^{+bx}$, $f'(x) = +a b e^{bx}$ y $\int a e^{bx} dx = \frac{a}{b} e^{bx} + k$

——— o ———



Longitud circunferencia

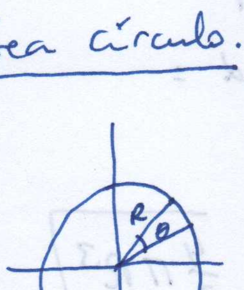
$$dl = R\,d\theta$$



$$L = \int_0^{2\pi} R\,d\theta = R\left[2\pi - 0\right] = 2\pi R.$$

——— o ———

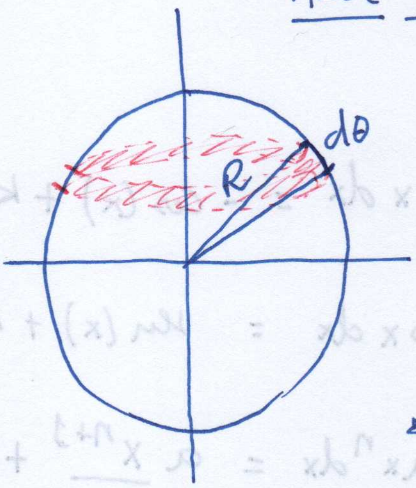
Área círculo.

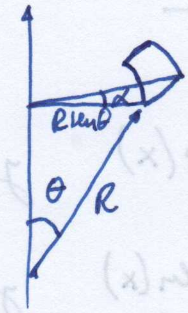


$$S = \int_{\theta=0}^{2\pi}\int_{r=0}^{R} r\,d\theta\,dr = \int_0^R r\,dr \cdot \int_0^{2\pi} d\theta =$$

$$\boxed{S = \frac{R^2}{2} \cdot 2\pi = \pi R^2}$$

Área Esfera.





$$dS = R\,\text{sen}\,\theta\, d\alpha \cdot R\, d\theta$$

$$dS = R^2\, \text{sen}\,\theta\, d\theta\, d\alpha$$

*) θ se recorre desde la vertical, de 0 a π.

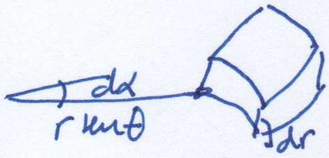
*) α da la vuelta entera, 2π. $\alpha \in [0, 2\pi]$ $\theta \in [0, \pi]$

$$S = \int dS = \int_{\alpha=0}^{2\pi}\int_{\theta=0}^{\pi} R^2\, \text{sen}\,\theta\, d\theta\, d\alpha = R^2 \cdot \int_0^{\pi} \text{sen}\,\theta\, d\theta \cdot \int_{\alpha=0}^{2\pi} d\alpha =$$

$$= 2\pi R^2 \cdot (-\cos\theta)\Big|_0^{\pi} = 2\pi R^2 (-\cos\pi + \cos 0) = 2\pi R^2 (-(-1) + 1) =$$

$$\boxed{S = 4\pi R^2}$$

Volumen Esfera.



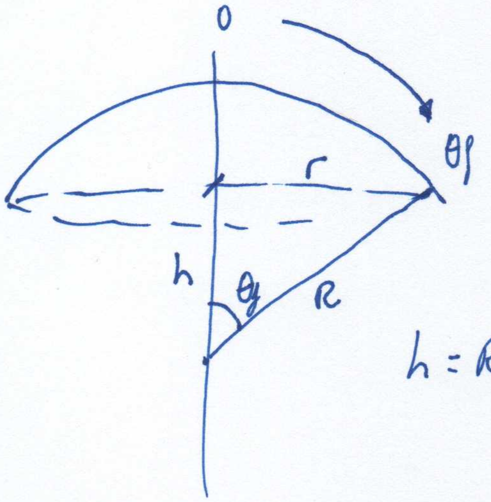
$$\cancel{dV = dr\, d\alpha \cdot r\, \text{sen}\,\theta\, d\alpha}$$

$$dV = dr \quad r\, \text{sen}\,\theta\, d\alpha \cdot r\, d\theta$$

$$V = \int_0^R \int_{\alpha=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 dr\, \text{sen}\,\theta\, d\theta\, d\alpha = \int_0^R r^2 dr \cdot \int_0^{\pi} \text{sen}\,\theta\, d\theta \cdot \int_0^{2\pi} d\alpha =$$

$$= \frac{R^3}{3} \cdot \underbrace{\left(-\cos(\theta)\Big|_0^{\pi}\right)}_{2} \cdot 2\pi = \frac{4\pi}{3} R^3 = \boxed{\frac{4}{3}\pi R^3}$$

Area casquete esférico.



$$dS = R \sin\theta \, d\theta \, d\alpha$$

integrando como en la esfera.

$$S = 2\pi R^2 (-\cos\theta) \Big|_0^{\theta_f}$$

$$h = R \cdot \cos\theta_f \Rightarrow \cos\theta_f = h/R = \frac{\sqrt{R^2 - r^2}}{R}$$

$$S = 2\pi R^2 \left[-\frac{\sqrt{R^2 - r^2}}{R} + 1 \right]$$

$R = D/2 \quad r = d/2$

$$S = 2\pi R \left[R - \sqrt{R^2 - r^2} \right]$$

$$\boxed{S = 2\pi \frac{D}{2} \left[\frac{D}{2} - \sqrt{\frac{D^2}{4} - \frac{d^2}{4}} \right] = \frac{\pi D}{2} \left(D - \sqrt{D^2 - d^2} \right)}$$

—o—

Dureza Brinell

$$HB = \frac{F}{S} = \frac{2F}{\pi D \left(D - \sqrt{D^2 - d^2} \right)}$$