область определения(sqrt(-eps), acos(1 + eps))

(WA) Проверить, что ответ влезает в int

#### Содержание Strategy.txt Проверить руками сэмплы Подумать как дебагать после написания Выписать сложные формулы и все +-1 flows/hungary.cpp Проверить имена файлов 3 Прогнать сэмплы geometry/basic3d.cpp Переполнения int, переполнения long long Выход за границу массива: \_GLIBCXX\_DEBUG Переполнения по модулю: в псевдо-онлайн-генераторе, в функциях-обертках geometry/halfplanes.cpp Проверить мультитест на разных тестах Прогнать минимальный по каждому параметру тест Прогнать псевдо-максимальный тест(немного чисел, geometry/polygon tangents.cpp но очень большие или очень маленькие) Представить что не зайдет и заранее написать geometry/primitives.cpp assert'ы, прогнать слегка модифицированные тесты 11 graphs/edmonds matching.cpp ..... cout.precision: в том числе в интерактивных задачах Удалить debug-output, отсечения для тестов, вернуть оригинальный тахп, удалить 13 graphs/kuhn.cpp \_GLIBCXX\_DEBUG 14 graphs/min automaton.cpp Вердикт может врать Если много тестов (>3), дописать в конец каждого теста ответ, чтобы не забыть 16 math/golden search quad eq.cpp 10 (WA) Потестить не только ответ, но и содержимое значимых массивов, переменных (WA) Изменить тест так, чтобы ответ не менялся: поменять координаты местами, сжать/растянуть координаты, поменять ROOT дерева (WA) Подвигать размер блока в корневой или битсете (WA) Поставить assert'ы, возможно написать чекер c assert'om (WA) Проверить, что программа не печатает что-либо неожиданное, что должно попадать под PE: inf - 2, не лекс. мин. решение, одинаковые числа вместо разных, неправильное количество чисел, пустой ответ, перечитать output format (TL) cin -> scanf -> getchar (TL) Упихать в кэш большие массивы, поменять местами for'ы или измерения массива (RE) Проверить формулы на деление на 0, выход за

### 2 flows/dinic.cpp

```
1namespace Dinic {
2 const int maxn = 100100;
 3struct Edge {
       int to;
       11 c, f;
 6
       Edge(int to, 11 c): to(to), c(c), f(0) {}
 7}:
 9 vector<Edge> es;
10 vector<int> g[maxn];
11 int q[maxn], d[maxn], pos[maxn];
12 int N, S, T;
14 void addEdge(int u, int v, ll c) {
       g[u].push_back(sz(es));
        es.emplace_back(v, c);
16
       g[v].push_back(sz(es));
17
18
        es.emplace_back(u, 0);
19}
20
21bool bfs() {
       fill(d, d + N, maxn);
22
       d[S] = 0, q[0] = S;
int rq = 1;
24
25
       forn (lq, rq) {
            int u = q[lq];
for (int id: g[u]) {
    if (es[id].c == es[id].f)
26
27
                       continue;
                  int v = es[id].to;
                  if (d[v] == maxn) {
    d[v] = d[u] + 1;
31
                       q[rq++] = \bar{v};
33
34
35
            }
36
37
       return d[T] != maxn;
38 }
39
4011 dfs(int u, ll curf) {
41    if (u == T)
            return curf;
42
43
       11 ret = 0:
       for (int &i = pos[u]; i < sz(g[u]); ++i) {
  int id = g[u][i];
  int v = es[id].to;</pre>
44
45
46
47
             11 delta = min(curf, es[id].c - es[id].f);
            if (delta == 0 || d[v] != d[u] + 1)
48
49
                  continue;
            delta = dfs(v, delta);
curf -= delta;
50
51
            ret += delta;
52
             es[id] f += delta;
53
            es[id ^ 1].f -= delta;
if (curf == 0)
54
55
                  return ret;
56
57
58
       return ret;
59 }
6111 dinic(int S, int T) {
       Dinic::S = S, Dinic::T = T;
62
63
       11 res = 0;
       while (bfs()) {
            fill(pos, pos + N, 0);
while (ll cur = dfs(S, infl))
65
66
67
                  res += cur;
68
69
       return res;
70}
71
72} // namespace Dinic
73
74 void test() {
       Dinic::N = 4;
75
       Dinic::addEdge(0, 1, 1);
76
77
       Dinic::addEdge(0, 2, 2);
       Dinic::addEdge(2, 1, 1);
Dinic::addEdge(1, 3, 2);
78
79
       Dinic::addEdge(2, 3, 1);
        cout << Dinic::dinic(0, 3) << endl; // 3</pre>
84 LR - поток находит не максимальный поток.
85 Добавим новый сток S' и исток T'. Заменим ребро (u, v, l, r) 86 LR-сети на ребра (u, T', l), (S', v, l), (u, v, r-l). 87 Добавим ребро (T, S, k). Ставим значение k=inf, пускаем поток.
88 Проверяем, что все ребра из S' насыщены (иначе ответ не
89 существует). Бинпоиском находим наименьшее к, что величина
90 потока не изменится. Это k - величина МИНИМАЛЬНОГО потока,
91 удовлетворяющего ограничениям. */
```

# 3 flows/hungary.cpp

```
1// left half is the smaller one
 2namespace Hungary {
 3 const int maxn = 505;
 4 int a [maxn] [maxn];
 5int p[2][maxn];
 6 int match[maxn]
7bool used[maxn];
8int from[maxn];
9 int mind[maxn];
10 int n, m;
11
12 int hungary(int v) {
13  used[v] = true;
       int u = match[v];
       int best = -1;
15
       forn (i, m + 1) {
17
           if (used[i])
                continue
            int nw = a[u][i] - p[0][u] - p[1][i];
if (nw <= mind[i]) {</pre>
20
                 mind[i] = nw;
                 from[i] = v;
            if (best == -1 || mind[best] > mind[i])
24
25
26
27
       v = best;
       int delta = mind[best];
28
29
       forn (i, m + 1) \{
            if (used[i]) {
30
                p[1][i] -= delta;
31
                 p[0][match[i]] += delta;
32
            } else
33
                mind[i] -= delta;
34
35
       if (match[v] == -1)
36
37
            return v:
38
       return hungary(v);
39 }
40
41 void check() {
      int edges = 0, res = 0;
42
       forn (i, m)
43
            if (match[i] != -1) {
44
45
                 ++edges;
                 assert(p[0][match[i]] + p[1][i] == a[match[i]][i]);
46
47
                 res += a[match[i]][i];
            } else
48
      assert(p[1][i] == 0);
assert(res == -p[1][m]);
forn (i, n) forn (j, m)
assert(p[0][i] + p[1][j] <= a[i][j]);
49
50
51
52
53}
54
55 int run() {
56 forn (i, n)
57
          p[0][i] = 0;
       forn (i, m + 1) {
   p[1][i] = 0;
58
59
            match[i] = -1;
60
62
       forn (i, n) {
            match[m] = i;
63
            fill(used, used + m + 1, false);
64
            fill(mind, mind + m + 1, inf);
fill(from, from + m + 1, -1);
65
66
            int v = hungary(m);
67
            while (v != m) {
    int w = from[v];
68
                 match[v] = match[w];
70
71
            }
72
73
       check();
       return -p[1][m];
77} // namespace Hungary
```

92

# 4 flows/mincost.cpp

pot[i] += dist[i];

```
return true;
                                                                                94}
1namespace MinCost {
2const ll infc = 1e12;
                                                                                95
                                                                                96bool push() {
                                                                                       //2 variants
                                                                                97
 4struct Edge {
                                                                                       //if (!fordBellman())
       int to;
                                                                                       if (!dikstra())
 6
       ll c, f, cost;
                                                                                           return false;
                                                                                       ++totalFlow;
8
       Edge(int to, ll c, ll cost): to(to), c(c), f(0), cost(cost)
                                                                                       int u = T;
9
                                                                                       while (u != S) {
10};
                                                                                           int e = fromEdge[u];
                                                                               104
11
                                                                               105
                                                                                            totalCost += edge[e].cost;
12 int N, S, T;
                                                                                           edge[e].f++;
edge[e ^ 1].f-
                                                                               106
13 int totalFlow;
                                                                               107
14ll totalCost;
                                                                                           u = edge[e ^ 1].to;
                                                                               108
15 const int maxn = 505;
                                                                               109
                                                                                       }
16 vector < Edge > edge;
                                                                               110
                                                                                       return true;
17 vector < int > g [maxn];
                                                                               111}
18
                                                                              112
19 void addEdge(int u, int v, ll c, ll cost) {
                                                                               113 //min-cost-circulation
       g[u].push_back(edge.size());
                                                                               11411 d[maxn][maxn];
       edge.emplace_back(v, c, cost);
                                                                               115 int dfrom[maxn][maxn]:
22
       g[v].push_back(edge.size());
                                                                               116 int level [maxn];
       edge.emplace_back(u, 0, -cost);
                                                                               117 void circulation() {
24}
                                                                                      while (true) {
                                                                               118
                                                                                           int q = 0;
fill(d[0], d[0] + N, 0);
25
                                                                               119
2611 dist[maxn];
                                                                               120
27int fromEdge[maxn];
                                                                                            forn (iter, N) {
    fill(d[iter + 1], d[iter + 1] + N, infc);
                                                                               121
                                                                               122
29bool inQueue[maxn];
                                                                               123
                                                                                                 forn (u, N)
30bool fordBellman() {
                                                                                                     for (int e: g[u]) {
   if (edge[e].c == edge[e].f)
                                                                               124
       forn (i, N)
                                                                               125
            dist[i] = infc;
32
                                                                               126
                                                                                                              continue;
       dist[S] = 0;
inQueue[S] = true;
33
                                                                                                          int v = edge[e].to;
ll ndist = d[iter][u] + edge[e].cost;
if (ndist >= d[iter + 1][v])
                                                                               127
34
                                                                               128
       vector<int> q;
35
                                                                               129
36
       q.push_back(S);
                                                                               130
                                                                                                               continue;
       for (int ii = 0; ii < int(q.size()); ++ii) {</pre>
37
                                                                                                          d[iter + 1][v] = ndist;
                                                                               131
           int u = q[ii];
inQueue[u] = false;
38
                                                                               132
                                                                                                          dfrom[iter + 1][v] = e;
39
                                                                                                     }
                                                                               133
           for (int e: g[u]) {
    if (edge[e].f == edge[e].c)
40
                                                                                                q ^= 1;
                                                                               134
41
                                                                               135
                                                                                           }
42
                     continue:
                                                                               136
                                                                                            int w = -1;
                int v = edge[e].to;
43
                                                                               137
                                                                                            ld mindmax = 1e18;
                lint v = edge[e].to;
ll nw = edge[e].cost + dist[u];
if (nw >= dist[v])
44
                                                                                            forn (u, N) {
                                                                               138
45
                                                                               139
                                                                                                ld dmax = -1e18;
                     continue:
46
                                                                               140
                                                                                                 forn (iter, N)
                dist[v] = nw;
47
                                                                               141
                                                                                                     dmax = max(dmax,
                fromEdge[v] = e;
48
                                                                                                         (d[N][u] - d[iter][u]) / ld(N - iter));
                                                                               142
                if (!inQueue[v]) {
49
                                                                                                if (mindmax > dmax)
    mindmax = dmax, w = u;
                                                                               143
                     inQueue[v] = true;
50
                                                                               144
                     q.push_back(v);
51
                                                                               145
                }
52
                                                                               146
                                                                                            if (mindmax >= 0)
           }
53
                                                                               147
                                                                                                break;
54
                                                                               148
                                                                                            fill(level, level + N, -1);
       return dist[T] != infc;
55
                                                                                            int k = N;
                                                                               149
56 }
                                                                               150
                                                                                            while (level[w] == -1) {
57
                                                                                                level[w] = k;
                                                                               151
5811 pot[maxn];
                                                                                                 w = edge[dfrom[k--][w] ^ 1].to;
                                                                               152
59bool dikstra() {
                                                                               153
       typedef pair<11, int> Pair;
                                                                               154
                                                                                            int k2 = level[w];
61
       priority_queue<Pair, vector<Pair>, greater<Pair>> q;
                                                                                            ll delta = infc;
                                                                               155
62
       forn (i, N)
                                                                                            while (k2 > k) {
                                                                               156
63
           dist[i] = infc;
                                                                                                int e = dfrom[k2--][w];
                                                                               157
64
       dist[S] = 0;
                                                                                                 delta = min(delta, edge[e].c - edge[e].f);
                                                                               158
       q.emplace(dist[S], S);
while (!q.empty()) {
65
                                                                               159
                                                                                                 w = edge[e ^ 1].to;
66
                                                                               160
67
            int u = q.top().second;
                                                                                            k2 = level[w];
                                                                               161
68
           11 cdist = q.top().first;
                                                                                            while (k2 > k) {
                                                                               162
           q.pop();
69
                                                                                                int e = dfrom[k2--][w];
                                                                               163
70
            if (cdist != dist[u])
                                                                                                 totalCost += edge[e].cost * delta;
                                                                               164
71
                 continue;
                                                                                                edge[e].f += delta;
edge[e ^ 1].f -= delta;
                                                                               165
           for (int e: g[u]) {
   int v = edge[e].to;
72
                                                                               166
73
                                                                                                w = edge[e ^ 1].to;
                                                                               167
74
                if (edge[e].c == edge[e].f)
                                                                                            }
                                                                               168
75
                     continue;
                                                                               169
76
                11 w = edge[e].cost + pot[u] - pot[v];
                                                                               170}
77
                assert(w >= 0);
                                                                               171} // namespace MinCost
                11 ndist = w + dist[u];
78
                                                                               172
                if (ndist >= dist[v])
79
                                                                               173 int main() {
                     continue;
                                                                                      main() {
MinCost::N = 3, MinCost::S = 1, MinCost::T = 2;
MinCost::addEdge(1, 0, 3, 5);
MinCost::addEdge(0, 2, 4, 6);
Thilo (MinCost::nuch()):
                                                                               174
                dist[v] = ndist;
81
                                                                               175
                fromEdge[v] = e;
                                                                               176
                q.emplace(dist[v], v);
83
                                                                                       while (MinCost::push());
                                                                               177
           }
                                                                                       cout << MinCost: totalFlow << ' '
                                                                               178
85
                                                                                            << MinCost::totalCost << '\n'; //3 33
                                                                               179
       if (dist[T] == infc)
86
                                                                               180 }
87
           return false;
       forn (i, N) {
           if (dist[i] == infc)
                continue;
```

#### geometry/basic3d.cpp

```
1struct Plane {
       pt v;
 3
       ld c;
       Plane(pt a, pt b, pt c) {
    v = ((b - a) % (c - a)).norm();
            this->c = a * v;
10
       ld dist(pt p) {
11
           return p * v - c;
13};
15pt projection(pt p, pt a, pt b) {
       pt v = b - a;
17
       if (ze(v abs2())) {
            //stub: bad line
            return a;
20
21
       return a + v * (((p - a) * v) / (v * v));
24pair<pt, pt> planesIntersection(Plane a, Plane b) {
25
       pt dir = a.v % b.v;
       if (ze(dir.abs2())) {
26
            //stub: parallel planes
return {pt{1e18, 1e18, 1e18}, pt{1e18, 1e18, 1e18}};
27
28
29
30
       ld s = a.v * b.v;
       pt v3 = b.v - a.v * s;
pt h = a.v * a.c + v3 * ((b.c - a.c * s) / (v3 * v3));
return {h, h + dir};
31
32
33
34 }
35
36pair<pt, pt> commonPerpendicular(pt a, pt b, pt c, pt d) {
       pt v = (b - a) % (d - c);
ld S = v.abs();
37
38
       if (ze(S)) {
39
            //stub: parallel lines
40
            return {pt{1e18, 1e18, 1e18}, pt{1e18, 1e18, 1e18}};
41
42
       v = v.norm();
43
       pt sh = v * (v * c - v * a);
pt a2 = a + sh;
44
45
       ld s1 = ((c - a2) % (d - a2)) * v;
pt p = a + (b - a) * (s1 / S);
46
47
       return {p, p + sh};
48
49 }
50
51/*
52\, \textit{Absolute error test}
53 testProjection: 1e1 -> -16.3
54 testProjection: 1e3 -> -14.1
55 testProjection: 1e4 -> -13.1
56 testProjection: 1e5 -> -12.3
57 testProjection: 1e6 -> -11.2
58 \ testPlanesIntersection: 1e1 \rightarrow -11.5
59\ testPlanesIntersection:\ 1e3\ ->\ -8.6
60 \ testPlanesIntersection: 1e4 \rightarrow -8.3
61 testPlanesIntersection: 1e5 -> -7.4
62 \ testPlanesIntersection: \ 1e6 \ -> \ -6.5
63\ test Common Perpendicular:\ 1e1\ ->\ -13.5
64 testCommonPerpendicular: 1e3 -> -11.4
65 testCommonPerpendicular: 1e4 -> -10.5
66 testCommonPerpendicular: 1e5 -> -8.7
67 testCommonPerpendicular: 1e6 -> -8.6
68 */
```

# geometry/chan.cpp

```
1mt19937 rr(111);
2ld rndEps() {
       return (ld(rr()) / rr.max() - 0.5) * 1e-7;
 4 }
 6typedef tuple<int, int, int> Face;
 7 const ld infc = 1e100;
9 int n;
10pt p[maxn];
11
12 namespace Chan {
13pt _p[maxn];
14
15ld turny(int p1, int p2, int p3) {
16    return (p[p2].x - p[p1].x) * (p[p3].y - p[p1].y) -
17    (p[p3].x - p[p1].x) * (p[p2].y - p[p1].y);
18}
19
20//replace\ y\ with\ z
211d turnz(int p1, int p2, int p3) {
22  return (p[p2].x - p[p1].x) * (p[p3].z - p[p1].z) -
23  (p[p3].x - p[p1].x) * (p[p2].z - p[p1].z);
24 }
25
26ld gett(int p1, int p2, int p3) {
       if (p1 == -1 || p2 == -1 || p3 == -1)
             return infc;
        ld ty = turny(p1, p2, p3);
if (ty >= 0)
31
            return infc;
32
33
             return turnz(p1, p2, p3) / ty;
34 }
35
36 void act(int i) {
       if (p[i] onHull) {
37
             p[p[i].nx].pr = p[i].pr;
p[p[i].pr].nx = p[i].nx;
38
39
40
        } else {
41
             p[p[i].nx].pr = p[p[i].pr].nx = i;
42
       p[i].onHull ^= 1;
43
44}
45
461d updt(vector<int> &V) {
       if (V.empty())
47
48
            return infc:
        int id = V.back();
49
        if (p[id].onHull)
50
             return gett(p[id].pr, p[id].nx, id);
51
52
             return gett(p[id].pr, id, p[id].nx);
53
54}
55
56//builds lower hull
57vector<int> buildHull(int 1, int r) {
       if (1 + 1 >= r) {
    p[1] .pr = p[1] .nx = -1;
58
59
             p[1].onHull = true;
60
61
             return {};
       }
62
        int mid = (1 + r) / 2;
63
       auto L = buildHull(1, mid);
auto R = buildHull(mid, r);
65
        reverse(all(L));
66
67
        reverse(all(R));
       int u = mid - 1, v = mid;
while (true) {
68
69
70
            if (p[u].pr != -1 &&
71
                       (turny(p[u].pr, u, v) <= 0))
             u = p[u].pr;
else if (p[v].nx != -1 &&
72
73
                       (turny(u, v, p[v].nx) <= 0))
75
                  v = p[v].nx;
76
             else
77
                  break;
78
79
        ld t[6];
        t[0] = updt(L);
t[1] = updt(R);
        vector<int> A;
        while (true) {
             t[2] = gett(p[u].pr, v, u);
            t[3] = gett(u, p[u].nx, v);
t[4] = gett(u, p[v].nx, v);
t[5] = gett(u, p[v].nx, v);
ld nt = infc;
89
             int type = -1;
             forn (i, 6)
```

```
if (t[i] < nt)
                        nt = t[i], type = i;
              if (nt >= infc)
 95
                   break;
 97
              if (type == 0) {
                   act(L.back());
if (L.back() < u)
 98
 99
                        A.push_back(L.back());
101
                    L.pop_back();
              t[0] = updt(L);
} else if (type == 1) {
103
                   act(R.back());
104
105
                    if (R.back() > v)
                        A.push_back(R.back());
                   R.pop_back();
t[1] = updt(R);
107
108
109
              } else if (type == 2) {
                   A.push_back(u);
110
              u = p[u].pr;
} else if (type == 3) {
111
112
                 A.push_back(u = p[u].nx);
else if (type == 4) {
113
114
              A.push_back(v = p[v].pr);
} else if (type == 5) {
115
116
                   A.push_back(v);
117
                    v = p[v].nx;
118
119
120
         assert(L.empty() && R.empty());
121
122
         p[u].nx = v, p[v].pr = u;
for (int i = u + 1; i < v; ++i)
    p[i].onHull = false;</pre>
123
124
125
         for (int i = sz(A) - 1; i >= 0; --i) {
126
              int id = A[i];
if (id <= u || id >= v) {
   if (u == id)
127
128
129
                   u = p[u].pr;
if (v == id)
130
131
                         v = p[v].nx;
132
                   act(id);
133
134
              } else {
135
                   p[id].pr = u, p[id].nx = v;
136
                    act(id);
137
                   if (id >= mid)
138
                         v = id;
139
                   else
140
                         u = id;
141
              }
         }
142
143
         return A:
144}
145
146//faces are oriented ccw if look from the outside
147 vector < Face > getFaces() {
148 forn (i, n) {
148
              _p[i] = p[i];

p[i].x += rndEps();

p[i].y += rndEps();

p[i].z += rndEps();
149
150
151
152
              p[i].id = i;
153
154
155
         sort(p, p + n, [](const pt &a, const pt &b) {
156
                        return a.x < b.x;
157
         vector<Face> faces;
158
         forn (q, 2) {
159
              auto movie = buildHull(0, n);
160
161
              for (int x: movie) {
                   int id = p[x].id;
int pid = p[p[x].pr].id;
int nid = p[p[x].nx].id;
if (!p[x].onHull)
162
163
164
165
166
                         faces.emplace_back(pid, id, nid);
167
                    else
                         faces.emplace_back(pid, nid, id);
168
169
                   act(x):
170
              forn (i, n) {
171
                   p[i] y *= -1;
p[i] z *= -1;
172
173
174
175
         forn (i, n)
p[i] = _p[i];
176
177
178
         return faces;
179 }
180
181} //namespace Chan
```

# 7 geometry/halfplanes.cpp

```
11d det3x3(line a, line b, line c) {
2    return a.c * (b.v % c.v)
            + b.c * (c.v % a.v)
            + c.c * (a.v % b.v);
 5 }
 7//check: bounding box is included
 8vector<pt> halfplanesIntersection(vector<line> 1) {
       sort(all(1), cmpLine); //the strongest constraint is first
       1.erase(unique(all(1), eqLine), 1.end());
       int n = sz(1);
11
       vi st;
13
       forn (iter, 2)
           forn (i, n) {
                while (sz(st) > 1) {
15
                    int j = st.back(), k = *next(st.rbegin());
17
                     if (1[k].v % 1[i].v <= eps |
                              det3x3(1[k], 1[j], 1[i]) \le eps)
                         break;
19
                     st.pop_back();
20
21
22
                st.push_back(i);
           }
24
25
      vi pos(n, -1);
bool ok = false;
26
      forn (i, sz(st)) {
    int id = st[i];
27
28
29
           if (pos[id] != -1) {
                st = vi(st.begin() + pos[id], st.begin() + i);
30
                ok = true;
31
                break:
32
           } else
33
               pos[id] = i;
34
35
      if (!ok)
36
           return {};
37
38
39
       vector<pt> res;
      pt M{0, 0};
int k = sz(st);
40
41
      forn (i, k) {
    line l1 = l[st[i]], l2 = l[st[(i + 1) % k]];
42
43
44
           res.push_back(linesIntersection(11, 12));
           M = M + res.back();
45
46
      M = M * (1. / k);
for (int id: st)
47
48
           if (l[id].signedDist(M) < -eps)</pre>
49
               return {};
50
51
       return res;
52 }
```

### 8 geometry/polygon.cpp

```
1bool pointInsidePolygon(pt a, pt *p, int n) {
       double sumAng = 0;
3
       forn (i, n) {
 4
            pt A = p[i], B = p[(i + 1) \% n];
 5
             if (pointInsideSegment(a, A, B))
                  return true;
7
             sumAng += atan2((A - a) \% (B - a), (A - a) * (B - a));
8
9
       return fabs(sumAng) > 1;
10}
12//check: p is oriented ccw
13bool segmentInsidePolygon(pt a, pt b, pt *p, int n) {
       if (!pointInsidePolygon((a + b) * .5, p, n))
             return false;
       if (ze((a - b) abs()))
17
            return true;
       forn (i, n) {
            pt c = p[i];
19
             if (ze((a - c) % (b - c)) && (a - c) * (b - c) < -eps) {
20
21
                  //point inside interval
22
                  pt pr = p[(i + n - 1) % n];
                  pt pr = p[(1 + n - 1) % n];
pt nx = p[(1 + 1) % n];
if ((c - pr) % (nx - c) > eps)
    return false;
ld s1 = (pr - a) % (b - a);
ld s2 = (nx - a) % (b - a);
24
25
26
27
28
                  if ((s1 > eps || s2 > eps) &&
29
                            (s1 < -eps || s2 < -eps))
30
                       return false;
31
32
33
             //interval intersection
            pt d = p[(i + 1) % n];
ld s1 = (a - c) % (d - c);
ld s2 = (b - c) % (d - c);
34
35
36
             if (s1 \ge -eps \&\& s2 \ge -eps)
37
                  continue;
38
             if (s1 <= eps && s2 <= eps)
39
                  continue:
40
41
             s1 = (c - a) \% (b - a);

s2 = (d - a) \% (b - a);

if (s1 >= -eps \&\& s2 >= -eps)
42
43
44
                  continue;
45
             if (s1 <= eps && s2 <= eps)
46
47
                  continue;
48
49
             return false;
50
51
       return true;
52 }
```

# 9 geometry/polygon tangents.cpp

```
1struct Cmp {
       pt M, v0;
       bool operator()(const pt &a, const pt &b) {
   pt va{v0 * (a - M), v0 % (a - M)};
   pt vb{v0 * (b - M), v0 % (b - M)};
            return cmpAngle(va, vb);
 9};
11struct Hull {
12
       vector<pt> h;
13
       int n;
14
       void build() {
15
            sort(all(h));
16
            h.erase(unique(all(h)), h.end());
17
            vector<pt> top, bot;
for (auto p: h) {
    while (sz(bot) > 1 && (p - bot.back()) %
18
19
20
                           (p - *next(bot.rbegin())) >= -eps)
21
22
                      bot.pop_back();
                 bot.push_back(p);
while (sz(top) > 1 && (p - top.back()) %
23
24
                           (p - *next(top.rbegin())) <= eps)</pre>
25
                      top.pop_back();
26
27
                 top.push_back(p);
            }
28
            if (sz(top))
29
            top.pop_back();
reverse(all(top));
30
31
            if (sz(top))
32
33
                 top.pop_back();
            h = bot;
34
            h.insert(h.end(), all(top));
35
36
            n = sz(h);
       }
37
38
       bool visSide(pt a, int i) {
    return (h[(i + 1) % n] - a) % (h[i % n] - a) > eps;
39
40
41
42
43
       bool vis(pt a, int i) {
            return visSide(a, i) || visSide(a, i + n - 1);
44
45
46
47
       bool isTangent(pt a, int i) {
48
            return visSide(a, i) != visSide(a, i + n - 1);
49
50
       int binSearch(int 1, int r, pt a) {
                                           r not necessarily
52
             //tricky binsearch; l <
            while (abs(l - r) > 1) {
53
54
                 int c = (1 + r) / 2;
55
                  if (vis(a, c))
                      1 = c;
56
57
                      r = c;
            }
            assert(isTangent(a, 1));
60
            return 1 % n;
61
62
63
       //check: n >= 3
       pair<int, int> tangents(pt a) {
65
            assert(n >= 3);
66
            pt M = (h[0] + h[1] + h[2]) * (1. / 3);
67
            if (a == M)
68
            return {-1, -1};
Cmp cmp{M, h[0] - M};
69
70
            //assert(is_sorted(all(h), cmp));
71
            int pos = upper_bound(all(h), a, cmp) - h.begin();
pt L = h[(pos + n - 1) % n], R = h[pos % n];
if ((R - L) % (a - L) >= -eps)
72
73
74
                 return {-1, -1}; //point inside hull
75
            int pos2 = upper_bound(all(h), M*2-a, cmp) - h.begin();
76
            assert(pos % n != pos2 % n);
77
            if (pos > pos2)
pos2 += n;
78
79
            return {binSearch(pos, pos2, a),
binSearch(pos + n - 1, pos2 - 1, a)};
80
81
       }
82
83 }:
```

# 10 geometry/primitives.cpp

```
1struct line {
        \frac{1}{1}d c; // v * p = c
        //check: p1 != p2
        line(pt p1, pt p2) {
    v = (p2 - p1).rot();
    v = v * (1. / v.abs());
 8
             c = v * p1;
 9
10
11
12
        // Convert from ax + by + c = 0
13
14
         //check: a^2+b^2 > 0
15
        line(ld a, ld b, ld _c): v(pt{a, b}), c(-_c) {
             ld d = v.abs();
16
             v = v * (1. / d);
17
18
             c /= d;
19
20
         //check: v.abs() == 1
22
        ld signedDist(pt p) {
            return v * p - c;
24
25};
26
27 //check: a != b
28pt lineProjection(pt p, pt a, pt b) {
29    pt v = (b - a).rot();
30    ld s = (p - a) % (b - a);
31    return p + v * (s / v.abs2());
32 }
33
341d pointSegmentDist(pt p, pt a, pt b) {
35    if ((p - a) * (b - a) <= 0 | | ze((b - a).abs()))
        return (p - a) abs();
if ((p - b) * (a - b) <= 0)
37
        return (p - b).abs();
return fabsl((p - a) % (p - b)) / (b - a).abs();
38
39
40 }
41
42pt linesIntersection(line 11, line 12) {
43    ld d = l1.v.x * 12.v.y - l1.v.y * 12.v.x;
        if (ze(d)) {
44
45
             if (eq(11.c, 12.c)) {
                  //stub: equal lines
46
             } else {
47
48
                  //stub: empty intersection
49
             return pt{1e18, 1e18};
50
51
        ld dx = 11.c * 12.v.y - 11.v.y * 12.c;
ld dy = 11.v.x * 12.c - 11.c * 12.v.x;
return pt{dx / d, dy / d};
52
53
54
55 }
56
57pt linesIntersection(pt a, pt b, pt c, pt d) {
        ld s = (b - a) \% (d - c);
if (ze(s)) {
58
59
              //stub: parallel or equal lines
60
61
             return pt{1e18, 1e18};
62
        ld s1 = (c - a) \% (d - a);
63
64
        return a + (b - a) * (s1 / s);
65 }
66
67bool pointInsideSegment(pt p, pt a, pt b) {
68     if (!ze((p - a) % (p - b)))
69
             return false;
        ld prod = (a - p) * (b - p);
return ze(prod) || prod < 0;</pre>
71
        if (ze(prod)) {
72
73
              //stub: coincides with segment end
74
             return true;
75
76
        return prod < 0;
77}
78
79bool checkSegmentIntersection(pt a, pt b, pt c, pt d) {
        if (ze((a - b) % (c - d))) {
             if (pointInsideSegment(a, c, d) |
                  pointInsideSegment(b, c, d) || pointInsideSegment(c, a, b) ||
83
                  pointInsideSegment(d, a, b)) {
                   //stub: intersection of parallel segments
                   return true;
87
             return false;
        forn (iter, 2) {
```

```
s1 = (c - a) \% (b - a);

s2 = (d - a) \% (b - a);
             if (s1 > eps && s2 > eps)
                  return false;
 95
             if (s1 < -eps && s2 < -eps)
                  return false;
 97
             swap(a, c), swap(b, d);
 99
100
        return true;
101}
103 vector <pt> lineCircleIntersection(line 1, pt a, ld r) {
        ld d = l.signedDist(a);
104
105
        pth = a - 1.v * d;
        if (eq(fabsl(d), r))
107
            return {h};
        else if (fabsl(d) > r)
108
109
            return {};
        pt w = 1.v.rot() * Sqrt(sqr(r) - sqr(d));
110
        return {h + w, h - w};
111
112 }
113
114 vector <pt> circlesIntersction(pt a, ld r1, pt b, ld r2) {
        ld d = (a - b).abs();
115
        if (ze(d) && eq(r1, r2)) {
116
             //stub: equal circles
117
             return {};
118
119
        // intersection is non-empty iff
120
         // triangle with sides r1, r2, d exists
121
        ld per = r1 + r2 + d;
122
        ld mx = max(max(r1, r2), d);
123
        int num = 2;
if (eq(mx * 2, per)) {
    num = 1;
124
125
126
127
        } else if (mx * 2 > per)
128
            return {};
        ld part = (sqr(r1) + sqr(d) - sqr(r2)) / ld(2 * d);
pt h = a + (b - a) * (part / d);
129
130
        if (num == 1)
131
132
             return {h};
        ld dh = Sqrt(sqr(r1) - sqr(part));
pt w = ((b - a) * (dh / d)).rot();
return {h + w, h - w};
133
134
135
136 }
137
138 vector p circle Tangents p p, pt a, ld r) { 139    ld d = p a.abs();
140
        if (eq(r, d))
            return {p}
141
142
        else if (r >
                       d)
143
             return {};
144
        ld len = Sqrt(sqr(d) - sqr(r));
        vector<pt> res;
pt vec = (a - p) * (len / sqr(d));
for (int sgn: {-1, 1})
145
146
147
             res.push_back(p + vec.rotCw(pt{len, r * sgn}));
148
149
150}
151
152 vector < line > circles Bitangents (pt a, ld r1, pt b, ld r2) {
        ld d = (a - b).abs();
153
154
        if (ze(d) && eq(r1, r2)) {
             //stub: equal circles
155
             return {};
156
157
158
        vector<line> res;
        for (int s1: {-1, 1})
for (int s2: {-1, 1}) {
161
                 // inner tangent iff s1 != s2
// treat radii as signed
162
163
                  1d r = s2 * r2 - s1 * r1;
164
                  if (eq(fabsl(r), d)) {
165
166
                           incident tangents; need only one copy
                       if (s1 == 1)
167
                           continue;
168
                  } else if (fabsl(r) > d)
169
170
                      continue;
                  ld len = Sqrt(sqr(d) - sqr(r));
171
                  line l(a, a + (b - a).rotCw(pt{len, r}));
l.c -= s1 * r1;
172
173
174
                  res.push_back(1);
             }
175
176
        return res:
177 }
```

### 11 graphs/edmonds matching.cpp

```
2 vi e[maxn];
 3 int mt[maxn], p[maxn], base[maxn], b[maxn], blos[maxn];
 4 int q[maxn];
 5int blca[maxn]; // used for lca
 7 int lca(int u, int v) {
8    forn(i, n) blca[i] = 0;
        while (true) {
            u = base[u];
11
            blca[u] = 1;
            if (mt[u] == -1) break;
12
13
            u = p[mt[u]];
14
15
       while (!blca[base[v]]) {
16
             v = p[mt[base[v]]];
17
18
       return base[v];
19 }
20
21 void mark_path(int v, int b, int ch) {
22 while (base[v] != b) {
            blos[base[v]] = blos[base[mt[v]]] = 1;
23
            p[v] = ch;
24
25
             ch = mt[v];
             v = p[mt[v]];
26
27
28 }
29
30 int find_path(int root) {
       forn(i, n) {
   base[i] = i;
   p[i] = -1;
   b[i] = 0;
31
32
33
34
35
36
       b[root] = 1:
37
38
       q[0] = root;
       int lq = 0, rq = 1;
while (lq != rq) {
39
40
             int v = q[lq++];
41
             for (int to: e[v]) {
42
                  if (base[v] == base[to] || mt[v] == to) continue;
if (to==root || (mt[to] != -1 && p[mt[to]] != -1)) {
43
44
                       int curbase = lca(v, to);
forn(i, n) blos[i] = 0;
45
46
47
                       mark_path(v, curbase, to);
48
                       mark_path(to, curbase, v);
                       forn(i, n) if (blos[base[i]]) {
   base[i] = curbase;
49
50
                             if (!b[i]) b[i] = 1, q[rq++] = i;
52
                 } else if (p[to] == -1) {
   p[to] = v;
54
                       if (mt[to] == -1) {
55
56
                            return to;
57
                       to = mt[to];
                       b[to] = 1;
q[rq++] = to;
60
61
62
            }
63
64
65
       return -1;
66}
67
68int matching() {
       forn(i, n) mt[i] = -1;
69
        int res = 0;
70
        forn(i, n) if (mt[i] == -1) {
71
             int v = find_path(i);
if (v != -1) {
72
73
74
                  ++res;
75
                  while (v != -1) {
                       int pv = p[v], ppv = mt[p[v]];
mt[v] = pv, mt[pv] = v;
76
77
78
                       v = ppv;
79
80
            }
81
82
       return res:
```

```
graphs/euler cycle.cpp
  12
 1struct Edge {
      int to, id;
 3};
 5bool usedEdge[maxm];
 6 vector < Edge > g[maxn];
 7int ptr[maxn];
 9vector<int> cycle;
10 void eulerCycle(int u) {
      while (ptr[u] < sz(g[u]) && usedEdge[g[u][ptr[u]].id])
11
      ++ptr[u];
if (ptr[u] == sz(g[u]))
13
          return;
14
       const Edge &e = g[u][ptr[u]];
15
      usedEdge[e.id] = true;
16
       eulerCycle(e.to);
17
       cycle.push_back(e.id);
18
19
      eulerCycle(u);
20}
21
22int edges = 0;
23void addEdge(int u, int v) {
24  g[u].push_back(Edge{v, edges});
25
       g[v].push_back(Edge{u, edges++});
26 }
```

#### 13 graphs/kuhn.cpp

```
1bool dfs(int v) {
 2
       if (vis[v]) return false;
       vis[v] = true;
 3
       for (int i = 0; i < (int)e[v].size(); i++) {
   if (mt[e[v][i]] == -1) {
      mt[e[v][i]] = v;</pre>
 5
 6
 7
                  return true;
 8
 9
       }
       for (int i = 0; i < (int)e[v].size(); i++) {
    if (dfs(mt[e[v][i]])) {</pre>
10
11
                 mt[e[v][i]] = v;
13
                  return true;
15
       return false;
16
17}
19 . . .
20
21fill(pair, -1);
22 for (int run = 1; run; ) {
23 run = 0, fill(used, 0);
   forn(i, n)
25
       if (pair[i] == -1 && dfs(i))
          run = 1;
26
```

# 14 graphs/min automaton.cpp

```
1vi inc[maxn][A];
 2int lst[maxn], pos[maxn], part[maxn];
3 int lp[maxn], rp[maxn], nrp[maxn];
4 int upd[maxn], used[maxn], inq[maxn];
 5 vector<int> q;
6int dtime;
7int np; // number of classes
8 vector<int> toRefine[A];
10 void doSwap(int x, int y) {
11  swap(lst[pos[x]], lst[pos[y]]);
       swap(pos[x], pos[y]);
12
13}
14
15 void refine(const vi& a) {
16
       ++dtime;
17
       vector<int> updated;
18
       for (int x: a) {
            if (used[x] == dtime) continue;
19
20
            used[x] = dtime;
22
             int p = part[x];
            if (upd[p] != dtime) {
    upd[p] = dtime;
    nrp[p] = rp[p];
24
25
26
                  updated.pb(p);
            }
28
            doSwap(x, lst[nrp[p]-1]);
30
             --nrp[p];
31
32
       for (int p: updated) {
   if (lp[p] == nrp[p]) continue;
33
34
            lp[np] = nrp[p];
rp[np] = rp[p];
35
36
            rp[p] = nrp[p];
for (int i = lp[np]; i < rp[np]; ++i) {
    part[lst[i]] = np;</pre>
37
38
39
40
41
            if (inq[p] || rp[np] - lp[np] < rp[p] - lp[p]) {
42
                 inq[np] = 1;
43
                  q.push_back(np);
44
45
            } else {
                  inq[p] = 1;
46
47
                  q.push_back(p);
            }
48
49
50
             ++np;
       }
51
52 }
53
54 void solve() {
       forn(i, n) lst[i] = i;
55
       sort(lst, lst+n, [](int i, int j) {
    return col[i] < col[j];</pre>
56
57
58
       }):
59
       forn(i, n) {
   if (i && col[lst[i]] != col[lst[i-1]]) {
60
61
                 rp[np] = i;
lp[++np] = i;
62
63
64
            }
            part[lst[i]] = np;
65
            pos[lst[i]] = i;
66
67
68
       rp[np++] = n;
69
       forn(i, np) {
70
            inq[i] = 1;
71
72
            q.push_back(i);
73
75
       forn(i, q.size()) {
           int p = q[i];
inq[p] = false;
76
77
             forn(c, A) {
78
79
                 toRefine[c].clear();
                  for (int id = lp[p]; id < rp[p]; ++id) {
                      toRefine[c].insert(
81
                            toRefine[c].end(), all(inc[lst[id]][c]));
85
            forn(c, A) if (!toRefine[c].empty()) {
                 refine(toRefine[c]);
86
87
89
       forn(i, n) printf("%d\n", part[i] + 1);
```

#### 15 math/factor.cpp

```
1//WARNING: only mod <= 1e18
211 mul(11 a, 11 b, 11 mod) {
       ll res = a * b - (ll(ld(a) * ld(b) / ld(mod)) * mod);
       while (res < 0)
           res += mod;
       while (res >= mod)
           res -= mod;
       return res;
9}
10
11bool millerRabinTest(ll n, ll a) {
      if (gcd(n, a) > 1)
13
           return false;
       11 x = n - 1;
       int 1 = 0;
while (x % 2 == 0) {
15
17
           x /= 2;
      11 c = binpow(a, x, n);
for (int i = 0; i < 1; ++i) {
    11 nx = mul(c, c, n);</pre>
20
21
22
           if (nx == 1) {
                if (c != 1 && c != n - 1)
24
25
                     return false;
26
27
                     return true;
           }
28
29
            c = nx;
30
31
       return c == 1;
32 }
33
34bool isPrime(ll n) {
       if (n == 1)
35
           return false;
36
       if (n % 2 == 0)
37
           return n == 2;
38
      return n -- 2,

// < 2^32: 2, 7, 61

// < 3e18: 2, 3, 5, 7, 11, 13, 17, 19, 23

// < 2^64: 2, 325, 9375, 28178, 450775, 9780504, 1795265022

for (11 a = 2; a < min<11>(8, n); ++a)
39
40
41
42
           if (!millerRabinTest(n, a))
43
44
                return false;
       return true;
45
46 }
47
48 //WARNING: p is not sorted
49 void factorize(ll x, vector<ll> &p) {
      if (x == 1)
50
51
           return;
       if (isPrime(x)) {
52
53
           p.push_back(x);
54
           return;
55
       for (11 d: {2, 3, 5})
56
            if (x \% d == 0) {
57
58
                p.push_back(d);
59
                factorize(x / d, p);
60
                return;
61
62
       while (true) {
           11 x1 = rr() \% (x - 1) + 1;
63
           11 x2 = (mul(x1, x1, x) + 1) % x;
int i1 = 1, i2 = 2;
64
65
            while (true) {
66
67
                11 c = (x1 + x - x2) \% x;
                if (c == 0)
                     break;
70
                11 g = gcd(c, x);
                if (g > 1) {
71
                      factorize(g, p);
                      factorize(\tilde{x} / g, p);
73
                      return;
75
                if (i1 * 2 == i2) {
76
                      i1 *= 2;
                      x1 = x2;
79
                 ++i2;
81
                x2 = (mul(x2, x2, x) + 1) \% x;
```

# 16 math/golden search quad eq.cpp

```
11d f(1d x) {
       return 5 * x * x + 100 * x + 1; //-10 is minimum
 3}
 51d goldenSearch(ld l, ld r) {
       ld phi = (1 + sqrtl(5)) / 2;
ld resphi = 2 - phi;
       ld x1 = 1 + resphi * (r - 1);
ld x2 = r - resphi * (r - 1);
       ld f1 = f(x1);
10
       1d f2 = f(x2);
11
       forn (iter, 60) {
    if (f1 < f2) {
12
13
                 r = x2;
14
                  x2 = x1;
15
                  f2 = f1;
16
                  x1 = 1 + resphi * (r - 1);
17
                  f1 = f(x1);
18
            } else {
19
                  1 = x1;
20
                  x1 = x2;
21
                 f1 = f2;
x2 = r - resphi * (r - 1);
22
23
                  f2 = f(x2);
24
            }
25
       }
26
       return (x1 + x2) / 2;
27
28 }
29
30 int main() {
31
       std::cout << goldenSearch(-100, 100) << '\n';
32 }
33
34 vector<1d> sqrRoots(1d a, 1d b, 1d c) {
35     ld d = b * b - 4 * a * c;
36     if (ze(d))
            return {-b / (2 * a)};
37
        if (d < 0)
38
            return {};
39
       d = sqrtl(d);
40
       if (ze(b)) {
   ld x1 = -d / (2 * a);
41
42
            1d x2 = d / (2 * a);
43
44
            if (x1 > x2)
45
                 swap(x1, x2);
46
            return {x1, x2};
47
48
       ld sgn = b > 0 ? 1 : -1;
       ld x1 = (-b - sgn * d) / (2 * a);
ld x2 = c / (a * x1);
49
50
       if (x1 > x2)
52
            swap(x1, x2);
       return {x1, x2};
54 }
```

### 17 math/numbers.tex

```
• Simpson and Gauss numerical integration:
```

```
\int_{a}^{b} f(x) dx = (b - a)/6 \cdot (f(a) + 4(f(a + b)/2) + f(b))\int_{-1}^{1}, x_{1,3} = \pm \sqrt{0.6}, x_2 = 0; a_{1,3} = 5/9, a_2 = 8/9
```

- Large primes:  $10^{18} + 3, +31, +3111, 10^9 + 21, +33$
- FFT modules:

```
\begin{array}{lll} 1\,107\,296\,257 & 2^{25}\cdot 3\cdot 11 + 1 & 10 \\ 1\,161\,822\,209 & 2^{22}\cdot 277 + 1 & 3 \\ 1\,261\,007\,895\,663\,738\,881 & 2^{55}\cdot 5\cdot 7 + 1 & 6 \text{ (check)} \end{array}
```

• Fibonacci numbers:

```
1,2: 1

45: 1134 903 170

46: 1836 311 903 (max int)

47: 2971 215 073 (max unsigned)

91: 4660 046 610 375 530 309

92: 7540 113 804 746 346 429 (max i64)

93: 12 200 160 415 121 876 738 (max unsigned i64)
```

• Powers of two

```
\begin{array}{l} 2^{31} = 2\,147\,483\,648 = 2.1\cdot 10^9 \\ 2^{32} = 4\,294\,967\,296 = 4.2\cdot 10^9 \\ 2^{63} = 9\,223\,372\,036\,854\,775\,808 = 9.2\cdot 10^{18} \\ 2^{64} = 18\,446\,744\,073\,709\,551\,616 = 1.8\cdot 10^{19} \end{array}
```

• Highly composite numbers

```
\begin{array}{lll} - \leq 1000 \colon d(840) = 32, \leq 10^4 \colon d(9\,240) = 64 & & & & & & & & & & & \\ - \leq 10^5 \colon d(83\,160) = 128, \leq 10^6 \colon d(720\,720) = 240 & & & & & \\ - \leq 10^7 \colon d(8\,648\,640) = 448, \leq 10^8 \colon d(91\,891\,800) = 768 \frac{41}{42} \\ - \leq 10^9 \colon d(931\,170\,240) = 1344 & & & & & \\ - \leq 10^{11} \colon d(97\,772\,875\,200) = 4032 & & & & \\ - \leq 10^{12} \colon d(963\,761\,198\,400) = 6720 & & & \\ - \leq 10^{15} \colon d(866\,421\,317\,361\,600) = 26880 & & & \\ - \leq 10^{18} \colon d(897\,612\,484\,786\,617\,600) = 103680 & & \\ 51 & & & & \\ \end{array}
```

#### • Misc

- Расстояние между точками по сфере:  $L=R\cdot 55$  arccos( $\cos\theta_1\cdot\cos\theta_2+\sin\theta_1\cdot\sin\theta_2\cdot\cos(\varphi_1-\varphi_2)$ ), 56 где  $\theta$ —широты (от  $-\frac{\pi}{2}$  до  $\frac{\pi}{2}$ ),  $\varphi$ —долготы (от  $-\pi_{58}$  до  $\pi$ ).
- Объём шарового сегмента:  $V=\pi h^2(R-\frac{1}{3}h)$ , где 61 h высота от вершины сектора до секущей плос- 62 кости 64
- Площадь поверхности шарового сегмента:  $S = 2\pi Rh$ , где h высота.
- Интеграл дуги:  $y(x) = \sqrt{r^2 x^2}$ ,  $\int y(x) dx = \frac{1}{2} (xy + r^2 \arctan \frac{x}{y}) + C$
- Bell numbers: 0:1, 1:1, 4:15, 5:52,722:2, 3:5,10:115975, 746:203,8:4140, 9:21147, 7:877, $11:678570, \quad 12:4213597, \quad 13:27644437,$ 14:190899322,75}  $17.82864869804, \frac{7611}{77}$  pi(11 x) { if (x > L) 16:10480142147, 18:682076806159, 19:5832742205057, 20:51724158235372,  $\frac{1}{78}$ 21:474869816156751, 22:4506715738447323,79} 23:44152005855084346
- $\begin{array}{l} \bullet \quad \text{Catalan numbers: } 0:1, \, 1:1, \, 2:2, \, 3:5, \, 4:14, \, 5:42, \, 6:132, \, 7:429, \, 8:3 \\ 8:1430, \, 9:4862, \, 10:16796, \, 11:58786, \, 12:208012, \, 13:742900, \, 84 \\ 14:2674440, \quad 15:9694845, \quad 16:35357670, \quad 17:129644790, \, 86 \\ 18:477638700, \quad 19:1767263190, \quad 20:6564120420, \, 87 \\ 21:24466267020, \quad 22:91482563640, \quad 23:343059613650, \, 89 \\ 24:1289904147324, \, 25:4861946401452 \end{array}$

# 18 math/stuff.cpp

```
1const int M = 1e6;
 2int phi[M];
 3void calcPhi() {
       for (int i = 1; i < M; ++i)
       for (int i = 1; i < m; ++1)
    phi[i] = i;
for (int j = 1; j < M; ++j)
    for (int i = 2 * j; i < M; i += j)
        phi[i] -= phi[j];</pre>
 8
9}
10 int inv[M];
11 void calcInv() {
12
        inv[1] = 1;
        for (int i = 2; i < M; ++i) {
   inv[i] = mul(sub(0, mod / i), inv[mod % i]);</pre>
14
15
             assert(mul(i, inv[i]) == 1);
16
17}
18 int gcd(int a, int b, int &x, int &y) {
        if (a == 0) {
    x = 0, y = 1;
19
20
             return b:
22
        int x1, y1;
        int g = gcd(b % a, a, x1, y1);
x = y1 - x1 * (b / a);
        y = x1;
        assert(a * x + b * y == g);
28
        return g;
30 int crt(int mod1, int mod2, int rem1, int rem2) {
31    int r = (rem2 - (rem1 % mod2) + mod2) % mod2;
       int x, y;
int g = gcd(mod1, mod2, x, y);
assert(r % g == 0);
32
33
35
        x \%= mod2;
36
        if (x < 0)
37
             x += mod2;
38
40
        int ans = (x * (r / g)) % mod2;
        ans = ans * mod1 + rem1;
        assert(ans % mod1 == rem1):
        assert(ans % mod2 == rem2);
44
45
        return ans:
46 }
47
48 // primes to N
49 const 11 n = 1000000000000LL:
50 const 11 L = 1000000;
51int small[L+1];
5211 large[L+1];
53 void calc_pi() {
        for (int i = 1; i <= L; ++i) {
    small[i] = i-1;
    large[i] = n / i - 1;</pre>
        for (11 p = 2; p <= L; ++p) {
    if (small[p] == small[p-1]) continue;</pre>
             int cntp = small[p-1];
             11 p2 = p*p;
11 np = n / p;
for (int i = 1; i <= min(L, n / p2); ++i) {</pre>
                   11 x = np / i;
if (x <= L) {</pre>
                        large[i] -= small[x] - cntp;
                   } else {
                        large[i] -= large[p*i] - cntp;
             for (int i = L; i >= p2; --i) {
                   small[i] -= small[i/p] - cntp;
             }
        if (x > L) return small[n/x];
        else return large[x];
81int main() {
        assert(phi[30] == 1 * 2 * 4);
        calcInv();
        int x, y;
        gcd(3, 5, x, y);
        gcd(15, 10, x, y);
crt(15, 13, 2, 5);
        crt(17, 3, 15, 2);
        return 0;
```

### 19 strings/automaton.cpp

```
1int t[maxn][26], lnk[maxn], len[maxn];
 2 int sz;
 3 int last;
 5void init() {
       sz = 3;
       last = 1;
 7
       forn(i, 26) t[2][i] = 1;
len[2] = -1;
 9
       lnk[1] = 2;
10
11}
13 void addchar(int c) {
      int nlast = sz++;
len[nlast] = len[last] + 1;
15
       int p = last;
for (; !t[p][c]; p = lnk[p]) {
    t[p][c] = nlast;
17
18
19
       int q = t[p][c];
if (len[p] + 1 == len[q]) {
20
21
           lnk[nlast] = q;
22
23
           24
25
26
27
28
29
                t[p][c] = clone;
30
31
32
       last = nlast;
33
```

# 20 strings/eertree.cpp

```
1char buf[maxn];
 2 char *s = buf + 1;
 3 int to[maxn][2];
4 int suff[maxn];
 5int len[maxn];
6 int sz;
7int last;
9 const int odd = 1;
10 const int even = 2;
11 const int blank = 3;
13 inline void go(int &u, int pos) {
      while (u != blank \&\& s[pos - len[u] - 1] != s[pos])
15
           u = suff[u];
16}
17
18 void add_char(int pos) {
      go(last, pos);
19
       int u = suff[last];
20
      go(u, pos);
int c = s[pos] - 'a';
21
22
       if (!to[last][c]) {
23
           to[last][c] = sz++;
len[sz - 1] = len[last] + 2;
assert(to[u][c]);
24
25
26
27
            suff[sz - 1] = to[u][c];
28
29
       last = to[last][c];
30}
31
32 void init() {
33
       sz = 4;
      sz = 4;
to[blank][0] = to[blank][1] = even;
len[blank] = suff[blank] = inf;
len[even] = 0, suff[even] = odd;
34
35
      36
37
38
39 }
40
41 void build() {
      init();
scanf("%s", s);
for (int i = 0; s[i]; ++i)
42
43
44
           add_char(i);
45
46 }
```

#### strings/hashes.cpp 21

```
4typedef long long 11;
 5typedef unsigned long long ull;
   static const int MA = 1e9 + 7, MB = 1e9 + 9;
11
   num() { }
13
    num( int x ) : a(x), b(x) { }
    num( int a, int b ) : a(a), b(b) { }
    num operator + ( const num &x ) const { return num((a + x.a) % 16
    → MA, (b + x.b) % MB); }

num operator - (const num &x) const { return num((a + MA - x.a) % MA, (b + MB - x.b) % MB); }
    num operator * ( int x ) const { return num(((11)a * x) % MA,
          ((11)b * x) % MB); }
    num operator * ( const num &x ) const { return num(((11)a * \rightarrow x.a) % MA, ((11)b * x.b) % MB); }
    bool operator == ( const num &x ) const { return a == x.a && b 24
     \rightarrow == x.b; }
    explicit operator 11 () const { return (11)a * MB + b + 1; }
23};
24
25template <class hash_t>
26 struct StrComparator {
27
   static const int P:
   static vector<hash_t> deg;
30
31
    const char *s;
   hash_t *h;
32
33
    34
35
      h = new hash_t[n + 1];
      h \Gamma 0 1 = 0:
36
      forn(i, n)
h[i + 1] = h[i] * P + s[i];
37
38
39
      deg.reserve(n);
      while (sz(deg) <= n)
40
41
         deg.push_back(*deg.rbegin() * P);
    }
42
43
    hash_t substr( int i, int len ) const { return h[i + len] -
44
     → h[i] * deg[len]; }
45
    int lcp( int i, int j ) {
  int L = 0, R = n - max(i, j);
  while (L < R) {
  int M = (L + R + 1) / 2;
}</pre>
46
47
48
49
50
         if (substr(i, M) == substr(j, M))
          L = M;
51
52
         else
53
          R = M - 1:
55
      return L;
56
57
    int cmp( int a, int b ) {
58
59
      int LEN = n - max(a, b), L = lcp(a, b);
      return L < LEN ? (int)s[a + L] - s[b + L] : b - a;
60
61
63 bool operator() ( int i, int j ) { return cmp(i, j) < 0; }
65template <class hash_t> vector <hash_t>
      StrComparator<hash_t>::deg(1, hash_t(1));
66template <class hash_t> const int StrComparator<hash_t>::P =
  \hookrightarrow max(239, rand());
       StrComparator<num> h(n, s);
70 /**
71 * Usage:
       StrComparator<num> h(length, s); // int length, char *s
h.substr(0, 3) == h.substr(1, 3); // сравнение на равенство
       подстрок за О(1)
       h.cmp(2, 3); // сравнение на больше-меньше суффиксов за
       O(logn)
       int p[n]; forn(i, n) p[i] = i;
76
       sort(p, p+n, h); // copmuposamь суффиксы, суф.нассив за O(n\log^2 2n)
78 */
```

#### strings/suffix array.cpp 22

```
1string s;
3 int sa[maxn], new_sa[maxn], cls[maxn], new_cls[maxn],
          cnt[maxn], lcp[maxn];
7void build() {
      n_cls = 256;
      forn(i, n) {
          sa[i] = i;
10
          cls[i] = s[i];
11
12
13
      for (int d = 0; d < n; d = d ? d*2 : 1) {
14
          forn(i, n) new_sa[i] = (sa[i] - d + n) % n;
15
          forn(i, n_cls) cnt[i] = 0;
          forn(i, n) ++cnt[cls[i]];
17
          forn(i, n_cls) cnt[i+1] += cnt[i];
18
          for (int i = n-1; i >= 0;
19
               sa[--cnt[cls[new_sa[i]]]] = new_sa[i];
21
          n cls = 0:
          forn(i, n) {
              if (i && (cls[sa[i]] != cls[sa[i-1]] ||
                       cls[(sa[i]+d)%n] != cls[(sa[i-1]+d)%n])) {
25
26
                   ++n cls:
              }
              new_cls[sa[i]] = n_cls;
28
          }
29
          ++n cls:
30
          forn(i, n) cls[i] = new_cls[i];
31
      }
32
33
      /\!/\ cls\ is\ also\ a\ inv\ perm\ of\ sa\ if\ a\ string\ is\ not\ cyclic
34
      // (i.e. a position of i-th lexicographical suffix) int val = 0;
35
36
      forn(i, n) {
    if (val) --val;
37
38
          if (cls[i] == n-1) continue;
int j = sa[cls[i] + 1];
39
40
          41
42
               ++val .
43
          lcp[cls[i]] = val;
44
      }
45 }
46
47 int main() {
48
      cin >> s;
      s += '$';
      n = s.length();
      build();
52
      forn(i, n) {
          cout << s.substr(sa[i]) << endl;</pre>
54
          cout << lcp[i] << endl;</pre>
      }
55
56}
```

95

97

100

101

103

105

107

108

109

110

111

112

113

114

115

116

119

120

121

122

123

124

125

#### 23strings/ukkonen.cpp

```
1string s;
2 const int alpha = 26;
4namespace SuffixTree {
      struct Node {
6
          Node *to[alpha];
          Node *lnk, *par;
8
          int 1, r;
9
10
          Node(int 1, int r): 1(1), r(r) {
               memset(to, 0, sizeof(to));
lnk = par = 0;
12
13
14
      };
15
      Node *root, *blank, *cur;
16
17
      int pos;
18
19
      void init() {
20
          root = new Node(0, 0);
          blank = new Node(0, 0);
22
          forn (i, alpha)
              blank->to[i] = root;
          root->lnk = root->par = blank->lnk = blank->par = blank; 118
24
25
          cur = root;
26
          pos = 0;
27
28
      int at(int id) {
30
          return s[id] - 'a';
31
32
33
      void goDown(int 1, int r) {
34
          if (1 >= r)
35
               return;
36
          if (pos == cur->r) {
               int c = at(1);
37
               assert(cur->to[c]);
cur = cur->to[c];
38
39
               pos = min(cur->r, cur->l + 1);
40
41
          } else {
42
               int delta = min(r - 1, cur->r - pos);
43
44
               1 += delta;
45
               pos += delta;
46
          goDown(1, r);
47
      }
48
49
      void goUp() {
   if (pos == cur->r && cur->lnk) {
50
51
               cur = cur->lnk;
52
               pos = cur->r;
53
54
               return;
55
56
          int 1 = cur->1, r = pos;
57
          cur = cur->par->lnk;
          pos = cur->r;
58
59
           goDown(1, r);
60
61
62
      void setParent(Node *a, Node *b) {
63
          assert(a);
64
           a->par = b;
65
          if (b)
66
               b \rightarrow to[at(a \rightarrow 1)] = a;
67
68
69
      void addLeaf(int id) {
70
           Node *x = new Node(id, inf);
71
           setParent(x, cur);
72
73
74
      void splitNode() {
75
           assert(pos != cur->r);
           Node *mid = new Node(cur->1, pos);
76
77
           setParent(mid, cur->par);
78
           cur->1 = pos;
79
           setParent(cur, mid);
          cur = mid;
81
82
83
      bool canGo(int c) {
          if (pos == cur->r)
85
               return cur->to[c];
           return at(pos) == c;
86
87
89
      void fixLink(Node *&bad, Node *newBad) {
          if (bad)
               bad->lnk = cur;
```

```
bad = newBad;
       void addCharOnPos(int id) {
           Node *bad = 0;
           while (!canGo(at(id))) {
               if (cur->r != pos) {
    splitNode();
                    fixLink(bad, cur);
                   bad = cur;
               } else {
                   fixLink(bad, 0);
               addLeaf(id);
               goUp();
           fixLink(bad, 0);
           goDown(id, id + 1);
       int cnt(Node *u, int ml) {
           if (!u)
               return 0;
           int res = min(ml, u->r) - u->1;
           forn (i, alpha)
              res += cnt(u->to[i], ml);
           return res;
       void build(int 1) {
           init();
           forn (i, 1)
               addCharOnPos(i);
       }
126 }:
```

#### 24 structures/fenwick.cpp

```
1//BEGIN ALGO
2struct Fenwick {
3
      int *t;
      int n;
5
      Fenwick(int *a, int len): n(len) {
           t = new int[n];
           memset(t, 0, sizeof(int) * n);
for (int i = 0; i < n; ++i) {</pre>
9
10
               inc(i, a[i]);
11
      }
12
13
      ~Fenwick() {
14
15
           delete[] t;
16
17
      void inc(int 1, int delta) {
           for (int i = 1; i < n; i = (i | (i + 1))) {
19
               t[i] += delta;
20
21
22
23
24
      int sum(int r) {
25
           int result = 0;
26
           for (int i = r; i \ge 0; i = (i & (i + 1)) - 1) {
27
               result += t[i];
28
29
           return result;
      }
30
31
      int sum(int 1, int r) {
32
33
           return sum(r) - sum(l - 1);
34
35};
36
37 //END ALGO
```

# 25 structures/heavy light.cpp

```
2 vi e[maxn];
 4namespace HLD {
 5int p[maxn], s[maxn], h[maxn], root[maxn];
6Rmq rmq[maxn];
8void dfs1(int v, int anc) {
      s[v] = 1;
if (anc != -1) e[v].erase(find(all(e[v]), anc));
10
      for (int to: e[v]) {
11
          p[to] = v;
h[to] = h[v] + 1;
12
13
          dfs1(to, v);
s[v] += s[to];
14
15
16
17}
18
19 void dfs2(int v, int rt) {
      root[v] = rt;
20
      if (e[v] empty()) {
21
          rmq[rt] = Rmq(h[v] - h[rt] + 1);
22
23
          return:
24
25
      int mxv = e[v][0];
      for (int to: e[v]) {
    if (s[to] > s[mxv]) mxv = to;
26
27
28
      for (int to: e[v]) {
    dfs2(to, to == mxv ? rt : to);
29
30
31
32}
33
34 int get(int u, int v) {
35
      int res = 0;
      int t;
36
      37
38
39
          40
41
42
43
44
          }
45
          res = max(res, t);
      }
46
47
      int r = root[u];
48
      if (h[u] > h[v]) {
49
          t = rmq[r] get(h[v] - h[r], h[u] - h[r] + 1);
      } else {
50
         t = rmq[r].get(h[u] - h[r], h[v] - h[r] + 1);
52
53
      return max(res, t);
54}
55
56 void put(int v, int x) {
57
      rmq[root[v]].put(h[v] - h[root[v]], x);
58}
59
60 void init() {
     const int ROOT = 0;
61
      h[0] = 0;
      dfs1(ROOT, -1);
dfs2(ROOT, ROOT);
66} // namespace HLD
```

### 26 structures/ordered set.cpp

```
1 #include <ext/pb_ds/assoc_container.hpp>
 2 #include <ext/pb_ds/tree_policy.hpp>
 4typedef __gnu_pbds::tree<int, __gnu_pbds::null_type,
           std::less<int>,
           __gnu_pbds::rb_tree_tag,
           __gnu_pbds::tree_order_statistics_node_update> oset;
 9 #include <iostream>
11int main() {
12
      oset X;
13
      X insert(1);
      X.insert(2);
14
15
      X.insert(4);
16
      X.insert(8);
      X.insert(16);
17
18
      std::cout << *X.find_by_order(1) << std::endl; // 2
19
      std::cout << *X.find_by_order(2) << std::endl; // 4
20
      std::cout << *X.find_by_order(4) << std::endl; // 16
std::cout << std::boolalpha <</pre>
21
22
           (end(X)==X.find_by_order(6)) << std::endl; // true</pre>
23
24
25
      std::cout << X.order_of_key(-5) << std::endl; // 0</pre>
                                                           // 0
      std::cout << X.order_of_key(1) << std::endl;
26
      std::cout << X.order_of_key(3) << std::endl;
std::cout << X.order_of_key(4) << std::endl;
27
28
       std::cout << X.order_of_key(400) << std::endl; // 5
29
30 }
```

### 27 structures/treap.cpp

```
1struct node {
      int x, y;
2
3
      node *1, *r;
      node(int x) : x(x), y(rand()), 1(r=NULL) {}
5};
7 void split(node *t, node *&l, node *&r, int x) {
      if (!t) return (void)(l=r=NULL);
9
      if (x \le t -> x) {
10
          split(t->1, 1, t->1, x), r = t;
11
      } else {
12
          split(t->r, t->r, r, x), l = t;
13
14 }
15
16 node *merge(node *1, node *r) {
17
     if (!1) return r;
      if (!r) return 1;
      if (1->y > r->y) {
19
          l->r = merge(l->r, r);
20
          return 1;
      } else {
          r - > 1 = merge(1, r - > 1);
24
          return r;
25
26}
27
28node *insert(node *t, node *n) {
29
     node *1, *r;
      split(t, 1, r, n->x);
30
      return merge(1, merge(n, r));
31
32 }
33
34node *insert(node *t, int x) {
35
     return insert(t, new node(x));
36}
37
38node *fast_insert(node *t, node *n) {
      if (!t) return n;
39
      node *root = t:
40
      while (true) {
41
          if (n->x < t->x) {
42
              if (!t->1 | | t->1->y < n->y) {
43
                   split(t->1, n->1, n->r, n->x), t->1 = n;
44
45
                  break;
              } else {
46
47
                   t = t -> 1;
              }
48
          } else {
49
              if (!t->r | | t->r->y < n->y) {
50
51
                   split(t->r, n->l, n->r, n->x), t->r = n;
52
                   break;
53
              } else {
54
                   t = t - > r;
55
          }
56
57
      }
58
      return root;
59 }
60
61node *fast_insert(node *t, int x) {
62
      return fast_insert(t, new node(x));
63}
64
65 int main() {
      node *t = NULL;
66
      forn(i, 1000000) {
          int x = rand();
68
69
          t = fast_insert(t, x);
70
```





