Information Sheet: You may rip this page off of the exam, but leave the other pages intact.

Definitions

Throughout this exam, you may use the following definitions:

- An integer x is a perfect square if there exists an integer k such that $x = k^2$.
- An integer x divides an integer y if and only if $x \neq 0$ and there exists integer k such that $y = kx$.
- An integer x is even if there exists an integer k such that $x = 2k$.
- An integer x is odd if there exists an integer k such that $x = 2k + 1$.

Direct Proof Framework

Statement. *For all x in the domain, if $P(x)$, then $Q(x)$.*

Proof. Let x be an arbitrary member of the domain. Suppose $P(x)$. We want to show $Q(x)$.

...

Therefore, $Q(x)$, as desired. □

Proof by Contrapositive Framework

Statement. *For all x in the domain, if $P(x)$, then $Q(x)$.*

Proof. Let x be an arbitrary member of the domain. We will prove the contrapositive. Suppose $\neg Q(x)$. We want to show $\neg P(x)$.

...

Therefore, $\neg P(x)$, as desired. □

Proof by Contradiction Framework

Statement. *For all x in the domain, if $P(x)$, then $Q(x)$.*

Proof. We will proceed by contradiction. Suppose there exists an x in the domain such that $P(x)$ and $\neg Q(x)$.

...

This is a contradiction. Therefore, the statement if $P(x)$, then $Q(x)$ is true for all x in the domain. □

Proof by Cases Framework

Statement. *For all x in the domain, if $P(x)$ or $Q(x)$, then $R(x)$.*

Proof. Let x be an arbitrary member of the domain. We will proceed by cases: First, $P(x)$, and second, $Q(x)$. We want to show $R(x)$.

Case 1: Suppose $P(x)$.

...(This part of the proof can be a direct proof, proof by contrapositive, proof by contradiction, etc.)

Therefore, $R(x)$, as desired.

Case 2: Suppose $Q(x)$.

...(This part of the proof can be a direct proof, proof by contrapositive, proof by contradiction, etc.)

Therefore, $R(x)$, as desired. □